In Synchrony with the Heavens

Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization

Volume One • The Call of the Muezzin

DAVID A. KING

BRILL

IN SYNCHRONY WITH THE HEAVENS

VOLUME ONE

THE CALL OF THE MUEZZIN

ISLAMIC PHILOSOPHY THEOLOGY AND SCIENCE

Texts and Studies

EDITED BY

H. DAIBER and D. PINGREE

VOLUME LV

IN SYNCHRONY WITH THE HEAVENS

Studies in Astronomical Timekeeping and Instrumentation in Medieval Islamic Civilization

(STUDIES I-IX)

VOLUME ONE

THE CALL OF THE MUEZZIN

BY

DAVID A. KING

BRILL
LEIDEN · BOSTON
2004

Illustration on the cover: see fig. 2.1, page 861.

This book is printed on acid-free paper.

Library of Congress Cataloging-in-Publication Data

The Library of Congress Cataloging-in-Publication Data is available on
http://catalog.loc.gov

ISSN 0169-8729
ISBN 90 04 12233 8

PRINTED IN THE NETHERLANDS

To the memory of Ḥabash (Baghdad), Ibn Yūnus (Cairo),
Abu ʻl-ʻUqūl (Taiz), al-Khalīlī (Damascus), and all the others,
in gratitude for the privilege of an encounter with their minds.

Individual parts of this work are dedicated to the teachers and colleagues
who helped me appreciate the achievements of these scholars
in their cultural contexts.

This page intentionally left blank

TABLE OF CONTENTS

Note: Vol. 2, entitled *Instruments of Mass Calculation* and containing Studies X-XV, is to be published by Brill Academic Publishers in 2004. This contains:

This page intentionally left blank

PREFACE

The obligation of Muslims to pray at specific times in a specific direction gave rise to a substantial literature during the period between *ca*. 750 and 1900. The prescriptions were interpreted at two different levels, namely, by the astronomers, who proposed mathematical solutions, and by the scholars of the religious law, who proposed non-mathematical solutions. This dichotomy between mathematical science and folk science, now to some extent documented, is not known to have led to any strife.

This particular aspect of the activity of the Muslim astronomers belongs to what I have called "Astronomy in the Service of Islam".[1] But it might also be considered as Islam's service to the astronomers:[2] it brought them work and perhaps occasionally recognition. But today, even in the Islamic world, their activities and achievements have been completely forgotten. Muslim readers in particular should appreciate that virtually all of the materials presented in this book have never been researched before in modern times: therefore, they should not be surprised to find that some of them have called for a reinterpretation of hypotheses proposed by previous generations who did not have access to these materials. In fact, only those Muslim astronomers whose works were used in the medieval Europe have received much recognition in modern times. It is my firm opinion that the history of Islamic astronomy merits study for its own sake, as part of the history of Islamic civilisation generally.[3] Furthermore, that history cannot be written from texts alone. In this study I have recourse to not a few texts, but my main sources are astronomical tables, most without any accompanying text, and also astronomical instruments.

Virtually none of the materials I present in this book were known in medieval Europe. Nevertheless, as the reader will learn, European astronomers from the 16th to the 20th century produced tables of the same kind as the earlier Islamic ones, without any inkling that they were not the first to do so.

The literature of the Muslim astronomers relating to timekeeping in general and the regulation of the prayer-times in particular is treated here first (**Parts I-II**), because at the time I worked on it over 20 years ago I had no conception that the legal scholars addressed the same topics at a different level. However, I have since included at least the arithmetical schemes for timekeeping proposed by the legal scholars and some of their comments on the definitions of the times of prayer (**III-IV**). I also investigate the social background of the Muslim astronomers charged with the regulation of the times of prayer (**V**) and a significant technical aspect of astronomical timekeeping and instrumentation, namely, universalism (**VI**). Some of the material in the scientific sources – in this case, a table for orienting massive ventilators

---

[1] This is the title of one of the volumes of reprints of some of my early publications (*Studies*, C).

[2] This point is made in Ragep, "Freeing Astronomy from Philosophy", p. 51.

[3] See the surveys in King, "Islamic Astronomy", and Saliba, "Islamic Astronomy and Astrology", as well as *idem*, "The Astrologer in Islamic Society". None of our generation has produced anything to really replace Nallino, "Islamic Astronomy" (1921).

on the rooftops of Cairene architecture in an astronomically-significant direction – has proven to be of prime importance for our understanding of other aspects of Islamic civilization – in this case, the layout of cities and religious architecture (**VIIa-b**). New evidence for the *Islamic* origin of the highly-sophisticated grids on some recently-discovered Islamic world-maps for finding the direction and distance to Mecca is also presented (**VIIc**). Furthermore, it is not without interest to compare the activities of Muslims involved with timekeeping with those of Christian monks in the European Middle Ages, some of whom also cared about timekeeping (**VIII**). And, finally for this volume, I present some reflections occasioned by the horrendous events of Autumn, 2001, and their aftermath (**IX**). In the second volume, I present a survey of Islamic instrumentation (**X**), and detailed studies of two varieties of instruments of Islamic origin known only from European sources (**XIIa-b**). Certain Islamic and medieval European instruments are based on an approximate formula for timekeeping, and I have here documented the history of that formula during a millennium (**XI**). I have appended various studies on individual astrolabes of particular historical significance (**XIIIa-d**) and the explicit and implicit geographical data on a large group of instruments (**XIV**). The volume concludes with an ordered checklist of early medieval Islamic and European instruments (**XV**).

As the title of this book implies, it contains a series of 'Studies', and since these deal with related subjects, there is a substantial amount of repetition, which anyone reading the whole text will surely find irksome, but I have chosen to preserve the original integrity of the individual 'Studies' and have used cross-referencing in the notes to the place where a given topic is treated most fully.

For convenience I use the term "medieval" for the era under discussion, which extends from the 9th to the 19th century, since the astronomical timekeeping that was practiced during that time in the Islamic world was neither "classical" nor "modern". However, the term appears only in the title of this book and very occasionally thereafter. I trust it will be clear to readers that the term "medieval" has no negative connotations, not least because much of the highly sophisticated material that I describe had no counterpart in any other civilization, neither before nor after. In the same way, "Islamic" refers to a civilization rather than a religion. "Islamic civilization" was the milieu in which Muslims, Jews and Christians could live and work together, and in which there was discussion of differences of opinion; their *convivencia* was not always a delight for all concerned, but generally it functioned better centuries ago than it does now.

In a sense, this work is a supplement to a work that does not exist yet: an overview of Islamic mathematical astronomy in general. In 1956, E. S. Kennedy published "A Survey of Islamic Astronomical Tables", listing some 125 astronomical handbooks with tables of the genre known as "*zīj*".[4] Of these, at that time, only those of al-Khwārizmī, al-Battānī, and al-Bīrūnī had been published. The first, which survives only in a Latin translation of an Andalusī recension, has been published optimally given the circumstances: Latin text, English translation and commentary. The second survives more or less in tact: the Arabic text has been published with a translation and commentary unfortunately in Latin. The third has been published in Arabic, and there is a Russian translation. Now these are important works, but they are only three out

[4] Kennedy, "*Zīj* Survey".

of some 125 which Kennedy was able to identify. In the 1970s, mainly during our Cairo days, Kennedy and I upped the number of known *zīj*es to over to 200. Also I had started to unearth vast numbers of tables that were not at all of the kind standard in *zīj*es.[5] Yet today, half a century after Kennedy started work on *zīj*es, not one more *zīj* has been published in its entirety. Now each *zīj* contains a couple of hundred pages of tables and text, so there are good reasons why only the hardy would get involved with a *zīj*. We were all delighted when Benno van Dalen undertook to prepare a much more detailed survey of the some 225 Islamic *zīj*es now known, and his Frankfurt-based project has made very satisfactory progress, so that a publication is in sight. In the meantime, however, the *Encyclopaedia of Islam* was progressing to the letter "Z", and I undertook to write the article on *zīj*es, with Julio Samsó contributing the early Eastern Islamic and Andalusī sections, and Bernard Goldstein the section on Hebrew *zīj*es. Such works contain:

– Tables for calendar conversion;
– Tables of trigonometric functions;
– Tables of stellar coordinates;
– Tables of geographical longitudes and latitudes;
– Tables of functions relating to spherical astronomy, such as solar declination and right and oblique ascensions; tables for timekeeping (not common in *zīj*es);
– Tables of solar, lunar and planetary mean-motions and equations; lunar and planetary latitudes; planetary stations; lunar and planetary visibility; equation of time;
– Tables for computing conjuctions and eclipses; and, last but not least,
– Tables for mathematical astrology.

Someone could (and should) write a book on each of these categories. Inevitably, what we wrote for the *Encyclopaedia* was far longer than what the Editors wanted or could accept. A short version was prepared for the *Encyclopaedia*, and was published with several illustrations. The longer version was published in *Suhayl* (Barcelona) in 2001, without any illustrations.[6] It was subtitled "An Interim Report" because Benno van Dalen has a much more detailed survey in store for us, and for the time being I refer the interested reader to his website.[7] My point is that the subject of this book, Islamic astronomical timekeeping, is just one part of a much greater entity, astronomy in medieval Islamic civilization.[8]

In two parallel studies, I have investigated the ways in which Muslims handled the determination of the sacred direction towards Mecca. In the first, I show how the Muslim astronomers treated the problem as one of mathematical geography, using formulae, tables and

[5] King, "Islamic Astronomical Tables".

[6] King & Samsó (with Goldstein), "Islamic Astronomical Handbooks and Tables".

[7] www.rz.uni-frankfurt.de/~dalen/index.htm (2003).

[8] See the overviews mentioned in n. 3 above. The interested reader will find over a hundred extracts from Islamic scientific manuscripts in *Cairo ENL Survey*, pp. 195-331, but should bear in mind that these are mainly from manuscripts post-dating *ca*. 1300. More representative pictorial overviews could be prepared from the collections in Turkey, Iran or Europe. UNESCO is about to publish a volume on illustrated scientifio manuscripts in Iranian collections, edited by Ziva Vesel. This is a step in the right direction, but it should be remembered that some of the most important scientific manuscripts are not illustrated at all. Nobody has taken seriously my list of Islamic scientific works which should be published in facsimile editions using the best manuscripts (King, "Islamic Scientific Manuscripts", pp. 128-134).

maps.[9] In the second, not yet published, I describe the ways in which the Muslim legal scholars developed a sacred folk geography based on astronomical horizon phenomena for the same ultimate purpose.[10] (I have not included the materials presented in the valuable 2000 study by Mònica Rius dealing in detail with materials relating to the qibla in al-Andalus and the Maghrib: the author is currently engaged in preparing an English version of her original Spanish text.[11]) We have also gained a good measure of control over medieval Islamic instruments: my catalogue of them, still in preparation, provides a useful research tool but is a long way from publication.[12] A task for the future is to confront the vast corpus of Islamic manuscripts dealing with different aspects of instrumentation.[13]

Around 1990 my assistant Kurt Maier in Frankfurt valiantly typed the text of **I** and **II**, but for all the symbols and formulae, into a computer. The formulae I typed in myself, with imposing curly brackets around multi-tiered numerators and denominators. I did not look at the new text of **I** and **II** for close to 10 years. In 1990 I published **III** as a separate article, and in 1996 I published **V**. Taking advantage of a sabbatical in the autumn of 1999 I spent some weeks on the entire text, inserting all the symbols and formulae (for the second time, for the first disappeared when I upgraded the software),[14] and cross-references and updating the notes

---

[9] King, *Mecca-Centred World-Maps*, pp. 47-127, is devoted to a survey of solutions to the qibla problem (see also n. 17).

[10] The study listed as King, *The Sacred Geography of Islam*, is not yet published, but a summary has appeared in the article "Makka. iv. As centre of the world" in EI_2, repr. in King, *Studies*, C-X.

[11] Rius, *La Alquibla en al-Andalus y al-Magrib* (in Spanish, with original texts edited in Arabic). An English version is planned.

[12] See King, "Medieval Instrument Catalogue", A-C, and also **VIII**. A table of contents from 1991 is to be found at www.uni-frankfurt.de/fb13/ign/instrument-catalogue.html.

[13] See already Sédillot-*père* and *fils*, *al-Marrākushī*, for a much-neglected medieval encyclopaedic work on instrumentation, and now Charette, *Mamluk Instrumentation*, on a remarkable 14th-century treatise describing over 100 instrument-types, and *idem* & King, *The Universal Astrolabe of Ibn al-Sarrāj: Innovation in Medieval Islamic Science* (forthcoming), on the most spectacular Islamic astronomical instrument also from the 14th century and the associated texts.

[14] It was naïve of me to think that formulae of all things would survive successive versions of software over several years. But I could not predict in advance the other computer problems I would have. Since many readers with a sense of humour have enjoyed my previous horror stories about computers (*Mecca-Centred World-Maps*, p. xxi, and *Ciphers of the Monks*, p. 19), I am confident that the following tales will also amuse them.

Many of my early problems were caused mainly by the fact that MSWord cannot handle large texts. Ever cautious, I had the text of this book saved in three separate files. I wonder how many modern people know that with software available in the mid 1990s it was not possible to insert more than a few dozen automatic cross-references in a text of such size before the computer would freeze: the other few hundred cross-references had to be inserted manually and fixed up at the stage of printing. Or that the English and German versions of MSWord 6 are not compatible and switching computers with different versions during a project can lead to unexpected consequences. But my favorite story this time round is what happened to my iMac bought in Germany in early 2000 with an English system and German software: over the months, the thing invariably froze at some stage of my work. I learned to live with this, not least because the same thing happens with my iBook purchased in the US. But when the time before the freezing of the iMac became intolerably short, I sought assistance from Apple, only to learn to my horror that I had purchased a computer without a serial number, one of that kind which "fell off some truck". So it was not guaranteed and Apple was not interested to help me with an unregistered computer. But my "dealer" had forgotten to send me a bill for the computer anyway, and his bill arrived only in December, 2000, in the very same week that I found out that the computer was not for real. I ignored the bill, and he ignored my telephone call to tell him that there was a severe problem with the computer. If and when his lawyer contacts me I shall send him this footnote.

The Frankfurt-based company Mac Org, the new Apple dealers in Frankfurt and the best I have ever worked with, kindly reinserted a new system and a new English version of Word 98. But the computer crashed upon

and bibliography, trying by means of subtle cross-references to suppress duplicated references, as well as making an attempt to improve the deathless prose, as far as this is possible in such a text. The word "table" occurs some 4,000 times in this book, and many people will not find descriptions of one table after another very exciting.[15] Also, when I prepared this new version,

---

opening (fortunately before I took it home), and it was ascertained that this English version of Word was defective, so I had to be content with a German version. At least this did not cause the computer to crash upon opening, but it had the annoying habit of always beginning spelling checks whenever files were opened, and then freezing. When one is dealing with hundreds of pages of coded text (such as a$f@$@ to produce oblique ascensions as α_{ϕ}) one does not need to be told that there are too many misspelled words for Word to control. It took me two days to turn off the spelling check (there are two places where this can be done, and the more obvious of the two was jammed because defective). Also the version I acquired had no subprogram for inserting formulae at all. Since my formulae were by now all one-liners (see the text to n. I-1:38) I conceded myself a victory against the system. It was about the same time that my Epson 760 Color Printer developed the singularly annoying habit of printing only the top half of the page for about one-third of every assignment.

In January 2001, some two weeks after converting the text to Word 95, I discovered that the three files on which the book was preserved had self-destructed: the texts of all three files were punctuated by large chunks of gibberish. I had to go back to the last backups on MSWord 6 and try to remember what changes I had made ... .

In August 2001, when both my iBook and iMac were defunct (constantly crashing) and I was spending some weeks in France, I ordered an English iMac with English keyboard from Apple in Ireland to be delivered in France ("delivery within three days"). Apple kindly sent me a French iMac with French keyboard, which I could have purchased locally had I wanted. It took a total of *four weeks* and several phone-calls a day, each time to different people of divers nationalities, each talking a multiplicity of languages, and most of them in Cork, Ireland, to get rid of the French computer and, only thereafter, to receive what I had ordered: the real English Apple arrived (from the Czech Republic) a week before I left France to return to Frankfurt. But just before I left, a French friend who is an Apple wizard came over to organize for me all new installations: he promptly changed the system to a more recent French one, also installing a French version of MSWord 2000. This combination actually seems to work, although frankly I prefer MSWord 5 because it was adequate for all sensible purposes and it did what I wanted, not *vice versa*. My Epson printer by this time had developed the habit of registering almost-new ink-cartridges as empty, and thirsting for new cartridges before it would print anything. Nevertheless I was able to squeeze the entire text out of it before returning to Frankfurt for the Winter Semester.

The final version of the book submitted to E. J. Brill in the Summer of 2002 was somehow achieved under the above-mentioned circumstances. But their printers could not receive the text by email, nor could they open the "superdisk" I sent them containing the text in four chunks. Then we found to our joint surprise that the powers that be do not make "superdisk" readers anymore, no doubt because like Eurochecks, they were so useful. Only in April, 2003, could the printers access the text.

My Epson Stylus Color 760 printer died of exhaustion in October, 2003, just when I had finished proof-reading the text and when I wanted to print a collection of chunks of corrected text and the indexes for Brill. I spent half a day travelling from my home in rural France to the city of Aubenas, where I purchased a C42 Plus. This turned out to be missing the CD to install it on my computer. The installation package from the Epson website is unusable. I spent another half day going to Nîmes to buy another printer.

A few days later, my iBook expired, taking with it half of the files saved on it, for which the back-ups were in Frankfurt. The materials for Volume 2 are all somehow retrievable somewhere, and interested readers will find new stories about this in my next publication.

The various chunks of Arabic text were all prepared in Cairo on a typewriter in the 1970s, then retyped on a computer in New York in the 1980s using al-Kaatib software. They all had to be retyped in the 1990s and early 2000s in various parts of the world using hardware unknown to me and software whose use is beyond me. This was achieved by Michael Kreutz in Bochum (**IV**), Sally Ragep in Norman, Oklahoma (**V**), Mónica Herrera in Frankfurt (**VIIb**), and François Charette in Frankfurt (**XIIa**). To dwell on the problems we encountered formatting these texts and making final changes would fill another page: my favourite was when the Arabic in the footnotes came out backwards. The formidable task of preparing the final printouts from the extremely volatile computer files was achieved by Mohammed Abu Zayed and Mónica Herrera in Frankfurt.

There were days at the turn of the Millennium when I yearned for the 1970s: at that time I was privileged to have the impeccable combination of a highly competent Egyptian secretary with two splendid Italian typewriters for English and Arabic.

[15] In 1973 I gave a lecture entitled "Medieval Islamic Prayer-Tables" at the American University of Beirut.

I had a much more broader respect for manuscripts than I had in my youth, so I have tried to add as much as is feasible about the provenance and date of the manuscripts. If such information is not given, the reader may assume that I am no longer prepared to hazard a guess from memory. I note that most colleagues do not care much about the origin of the manuscripts (or the provenance of the instruments) they are studying, being happy enough to have a copy or two from which to work. Nevertheless, the original environments of the manuscripts (and instruments) are extremely important. This study was much facilitated by the availability of some first-rate catalogues of Arabic scientific manuscripts, of which those by Wilhelm Ahlwardt (1893) for Berlin, McGulkin de Slane (1883-95) for Paris, and Henri-Paul-Joseph Renaud (1941) for El Escorial, are exemplary.

Apart from various studies by Glen Van Brummelen and François Charette,[16] the only other contributions to the subject of astronomical timekeeping over the 20-or-so years during which most of the text of this book (I mean, **I** and **II**) lay dormant are a series of derivative publications of my own – mostly surveys and encyclopaedia articles – which all contain references to, and promises concerning a mysterious "**SATMI**". The fact that **I** and **II** were not published years ago, as they should have been, and that **III**, **V**, **VIa**, and **VIb**, were published without **I** and **II** being accessible, has resulted in the rather cumbersome form in which the present work is presented. And the reader should also be aware that François Charette's *Mathematical Instrumentation in Fourteenth-Century Egypt and Syria*, published by E.J. Brill in 2003, could be considered as an *instant supplement* to the present work, since it deals with tables as well as instruments. Indeed, it features numerous tables and instruments that are not represented here at all. In the last few years, numerous Internet-sites have appeared dealing with Islamic prayer-times on the one hand, and historical instruments on the other: as far as I am aware, none add anything to the historical materials contained in this book. They do, however, offer the means to generate prayer-times for any date and any locality, and to purchase copies of medieval instruments. To François Charette too go my heart-felt thanks for numerous corrections to the penultimate version of this text, and no less for preparing all of the computer graphics.

None of my research over the last 20 years would have been possible without the infrastructure of the Institute for the History of Science in Frankfurt. I have to thank our Institute's librarian, Ryszard Dyga, and my CEO, Wolf-Dieter Wagner, for countless favours over many years, not least with regard to the acquisition of research materials – letters requesting this or that, library searches for this or that, ordering photos of this, preparing photocopies of that, ... Inevitably, this often had to be done in a hurry so that I could disappear for weeks on end and work in isolation, ...

Most of the photographs used to illustrate the two volumes of this book were kindly provided by the Deutsche Staatsbibliothek in Berlin; the Egyptian National Library in Cairo; the Chester Beatty Library in Dublin; the Museo di Storia della Scienza in Florence; the National Maritime

---

A young lady walked out a couple of minutes after I started: I later found out that she was interested in medieval Islamic furniture.

[16] See Van Brummelen, "al-Khalīlī's Auxiliary Tables"; and Charette, "Najm al-Dīn's Monumental Table", and *idem*, *Mamluk Instrumentation*.

Museum in Greenwich; the Süleymaniye and Topkapı Libraries in Istanbul; the British Museum and the British Library in London; the Bodleian Library and the Museum of History of Science in Oxford; the Bibliothèque Nationale de France in Paris; and Princeton University Library. It is a pleasure to record my gratitude to these splendid institutions. I also wish to thank Trudy Kamperveen and Tanja Cowall of Brill Academic Publications for their patience and understanding, and especially Ciska Palm of Palm Produkties, who somehow produced this book from a few computer files and a pile of photographs.

Whilst the materials presented here will be new to most Western readers, they will be completely new to Muslim readers unfamiliar with Western writings on the history of Islamic science.[17] (Alas virtually nothing of our research over the past 50 years is known in the Islamic world outside Turkey, Iran and India; neither is it ever mentioned by those Muslim savants who write in the West on the "new" "Islamic science".) There is nothing on the subject of medieval Islamic astronomical timekeeping available for the modern Arabic reader other than a short article of mine in a general work on Islamic science (*Mawsūʿat taʾrīkh al-ʿulūm al-ʿarabiyya*) published in Beirut in 1997 and not widely available in the Muslim world anyway. In any case, no Muslim readers should be surprised that a study based on over 500 medieval manuscripts that no-one in modern times had ever looked at previously, should contain materials that are new and exciting, and which radically alter – I venture to say, improve – our picture of science in the medieval Islamic world.

[17] My book on the qibla (n. 9) was generously sponsored by the al-Furqan Islamic Heritage Foundation in London. The present work could not be sponsored by the Foundation because a colleague from C.N.R.S., Paris, reported to them that it contained not enough new material to merit their support. No other foundation or interested individual could be found to sponsor the publication, hence this book is somewhat more expensive than my book on the qibla, which was likewise mainly based on materials that had never been identified or studied previously.

PREFACE 2: SOME BRIEF REMARKS ON THE ISLAMIC SCIENTIFIC HERITAGE IN THE CONTEXT OF THE TRANSMISSION OF SCIENTIFIC IDEAS

Some out-dated notions wide-spread amongst the "informed public" and even amongst historians of science are that:

(1) The Muslims were fortunate enough to be the heirs to the sciences of Antiquity.

(2) They cultivated these sciences for a few centuries but never really achieved much that was original.

(3) They provided, mainly in Islamic Spain, a milieu in which eager Europeans emerging out of the Dark Ages could benefit from these Ancient Greek sciences once they had learned how to translate them from Arabic into Latin.

Islamic science, therefore, one might argue, is of no consequence *per se* for the development of global science and is important only insofar as it marks a rather obscure interlude between a more sophisticated Antiquity and a Europe that later became more civilized.

What happened in fact was something rather different. The Muslims did indeed inherit the sciences of Greek, Indian and Persian Antiquity. But within a few decades they had created out of this potpourri a new science, now written in Arabic and replete with new Muslim contributions, which flourished with innovations until the 15th century and continued thereafter without any further innovations of consequence until the 19th. Most of this activity took place in the Islamic East. Not much of this Islamic science was known in al-Andalus, and even less of it was available for any Europeans, however eager. In astronomy, the Europeans rescued from al-Andalus outdated and mutually inconsistent works such as the astronomical handbooks of al-Khwārīzmī and al-Battānī (respectively representing the first Arabic redactions of the Indo-Persian and Greek traditions) and produced therefrom the unhappy mixture known as the *Toledan Tables*, influential all over Europe for a few centuries, including England. Indeed when I was a graduate student some 30 years ago I read a 1952 paper by Otto Neugebauer and Olaf Schmidt entitled "Indian Astronomy at Newminster in 1428". The main tables discussed were some tables of the tangent of the solar declination multiplied by a very curious coefficient. They were very much out of place in medieval England (see now **I-7.1.1** and **10.1**).

In 1997 my colleague, the eminent British historian of medieval science, Charles Burnett, published a book entitled *The Introduction of Arabic Learning into England*. It is an excellent book, not least because it deals with a subject on which Burnett is more qualified to write than anyone else. I take issue only with the title, my problem being the expression "Arabic learning" in the context of the ultimate destination, "medieval England". Since, as it happens, only a modest fraction of the grand total of Muslim scholarship was available in al-Andalus, and only a very small fraction of that ever made it to England in the Middle Ages, Burnett's title might be construed as a little misleading: in fact, apparently no Englishman in the Middle Ages ever came into contact with the full force of "Muslim scholarship" or "Arabic learning", for which he would have had to go to Baghdad or Cairo or Rayy, rather than, say, Toledo. When English scholars looked at Arabic scientific manuscripts in Oxford in the 17th century they were in for

a few surprises: see Gerald Toomer's *Eastern Wisedome and Learning: The Study of Arabic in Seventeenth-Century England* (1996) and Gül Russell's *'Arabick' Interest in 17th-Century England* (1994). But by that time, it was already a bit late for actually exploiting the scientific bounty; the materials discovered tended to be more of historical interest. The real surprises came during the 19th and 20th centuries when historians from a multiplicity of national backgrounds investigated Islamic scientific manuscripts in libraries all over Europe and then in the Near East. Their investigations revealed a intellectual tradition of proportions that no medieval or Renaissance European could ever have imagined: anyone who might doubt this should look at the monumental bio-bibliographical writings of Heinrich Suter, Carl Brockelmann and Fuat Sezgin,[1] and, in the case of astronomy, also the early studies of J.-J. and L.-A. Sédillot, C. A. Nallino, H. Suter or E. S. Kennedy. These investigations are by no means over, and there is much more material to be unearthed in manuscript libraries and museums around the world. When investigating manuscripts that nobody has looked at for centuries it highly likely that one will unearth materials that call for a complete re-evaluation of some aspects of Muslim activity in a particular branch of knowledge. The task of the historian is to organize and present these materials. The task of the reader is to be prepared for a few surprises.

The present study introduces some of the materials from a very special trend in the Muslim scientific heritage. These materials have become known only in the past 30 years. Indeed, it has been my priviledge to look at most of them for the very first time. Inevitably, they modify the overall picture we have of Islamic science. And it so happens that the particular intellectual activity that inspired these materials is related to the religious obligation to pray at specific times. The material presented here makes nonsense of the popular modern notion that religion inevitably impedes scientific progress, for in this case, the requirements of the former actually inspired the progress of the latter for centuries. But the same material also enables us to witness the eventual drying up of initiative in Islamic science.[2] The reason for this phenomenon is seldom ever mentioned by historians of science who write on the decline of the scientific tradition in the Islamic world: within the medieval context, all of the problems had been solved, some many times over. Nowhere is this clearer than in the kind of sources that I describe in this book.

---

[1] As I was reading the proofs of this book, a monumental new publication – Boris A. Rosenfeld and Ekmeleddin Ihsanoğlu, *Mathematicians, Astronomers and Other Scholars of Islamic Civilisation and their Works (7th-19th C.)*, Istanbul, 2003 – appeared. This is an English version of Galina P. Matvievskaya and Boris A. Rosenfeld, *Mathematicians and Astronomers of the Islamic Middle Ages and Their Works*, Moscow, 1983 (three volumes in Russian), supplemented with information from Ekmeleddin İhsanoğlu *et al.*, *Ottoman Astronomical / Mathematical / Geographical Literature*, Istanbul, 1997-2000 (altogether six volumes in Turkish). This new work is a mine of bio-bibliographical information on over 1,700 Muslim scientists and is of course an important contribution to our field. It is a useful addition to the standard bio-bibliographical works of Suter, Brockelmann and Sezgin, which should still be used alongside it. In the present volumes I have not included references to this new work.

[2] A useful essay by a scholar who is familiar with the scope and depth of Islamic science is Al-Hassan, "Decline of Islamic Science" (1994). It is significant that this essay deals with the decline that took place in and after the 16th century, not least because the available sources – available now to us, but not known to our predecessors or not previously studied – show that scientific initiative had not dried up before that time.

STATEMENT ON PREVIOUS PUBLICATION OF PARTS OF THIS WORK

Parts of this work have been published previously. They are included here because they are all essential to the overall picture provided by the ensemble of studies. They have been revised and brought into line with the other studies; in particular the references and bibliography have been updated.

Volume 1:
General remarks:
- ♦ **Parts I**, **II**, **IV**, **VIIc** and **IX** are published here for the first time.
- ♦ **Part VIII** is here published in English for the first time.
- ♦ The reader should please keep in mind that **Parts I-II** were written almost 30 years ago, **Parts IV** and **VIIb** some 20 years ago, **Parts III** and **VI** some 10 years ago. **Parts VIIc** and **IX** were written just a few months before this volume went to press.

Remarks to specific sections of this volume:
- ♦ **Parts I-II** have not been published previously.
- ♦ An earlier version of **Part III** was published in *Oriens* 32 (1990), pp. 191-249.
- ♦ **Part IV** has not been published previously.
- ♦ An earlier version of **Part V** was published in *Tradition, Transmission, Transformation: Proceedings of Two Conferences on Premodern Science Held at the University of Oklahoma,* F. Jamil Ragep & Sally P. Ragep, with Steven J. Livesey, eds., Leiden, New York & Cologne: E. J. Brill, 1996, pp. 285-346.
- ♦ Earlier versions of **Parts VIa** and **VIb** were published in *From Ancient Omens to Statistical Mechanics*: *Essays on the Exact Sciences Presented to Asger Aaboe*, J. Lennart Berggren & Bernard R. Goldstein, eds., *Acta Historica Scientiarum Naturalium et Medicinalium* (Copenhagen) 39 (1987), pp. 121-132; and *A Way Prepared*: *Essays on Islamic Culture in Honor of Richard Bayly Winder*, Farhad Kazemi & Robert B. McChesney, eds., New York: New York University Press, 1988, pp. 153-184, respectively.
- ♦ Earlier versions of **Parts VIIa** and **VIIb** were published in *Journal for the History of Astronomy* 26 (1995), pp. 253-274, and *Journal of the American Oriental Society* 104 (1984), pp. 97-133, respectively.
- ♦ **Part VIIc**, previously unpublished, supplements my recent book *World-Maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science*, Leiden: E. J. Brill, and London: Al-Furqan Islamic Heritage Foundation, 1999.
- ♦ A German translation of an earlier version of **Part VIII** was published as "Aspekte angewandter Wissenschaften in Moscheen und Klöstern", *Berichte zur Wissenschafts-geschichte (Organ der Gesellschaft für Wissenschaftsgeschichte)* 18 (1995), pp. 85-95 and 137-149.
- ♦ **Part IX** has not been published previously.

Volume 2:
General remarks:
Parts XI, **XIIa-b**, **XIIIa** and **XV** have not been published previously.
Parts X and **XIIIc** are here published in English for the first time.

Remarks to specific sections of the second volume:

- An Italian translation of an earlier version of **Part X** was published as "Strumentazione astronomica nel mondo medievale islamico", in *Gli strumenti*, Gerard L'E. Turner, ed., Turin: Giulio Einaudi, 1991, pp. 154-189 and 581-585. The text has been considerably expanded.
- **Part XI** has not been published previously.
- The section of **Part XIIa** dealing with the Baghdad treatise has appeared separately as "A *Vetustissimus* Arabic Treatise on the *Quadrans Vetus*", in *Journal for the History of Astronomy* 33 (2002), pp. 237-255.
- A summary of **Part XIIb** entitled "14th-Century England or 9th-Century Baghdad? New Insights on the Origins of the Elusive Astronomical Instrument Called *Navicula de Venetiis*" was published in *Astronomy and Astrology from the Babylonians to Kepler – Essays Presented to Bernard R. Goldstein on the Occasion of his 65th Birthday*, Peter Barker, Alan C. Bowen, José Chabás, Gad Freudenthal and Tzvi Langermann, eds., a special issue of *Centaurus* 44 (2003), pp. ••-••. **•FIX IN PROOF•**
- **Part XIIIa** has not been published before.
- **Part XIIIb** is modified from my article "Early Islamic Astronomical Instruments in Kuwaiti Collections", in Arlene Fullerton & Géza Fehérvári, eds., *Kuwait: Art and Architecture – Collection of Essays*, Kuwait (no publisher stated), 1995, pp. 76-96.
- A French version of **Part XIIIc** was part of my contribution "L'astronomie en Syrie à l'époque islamique", to the exhibition catalogue *Syrie – mémoire et civilisation*, Sophie Cluzan, Éric Delpont and Jeanne Mouliérac, eds., Paris: Flammarion (Institut du Monde Arabe), 1993, pp. 386-395 and 432-443.
- A modified version of **Part XIIId** was published as "A Remarkable Italian Astrolabe from *ca*. 1300 – Witness to an Ingenious Tradition of Non-Standard Astrolabes", in *MUSA MUSAEI: Studies on Scientific Instruments and Collections in Honour of Mara Miniati*, Marco Beretta, Paolo Galluzzi and Carlo Triarico, eds., Florence: Leo S. Olschki, 2003, pp. 29-52.
- **Part XIIIe** was first published as "An Astrolabe from 14th-Century Christian Spain with Inscriptions in Latin, Hebrew and Arabic – A Unique Testimonial to an Intercultural Encounter" in *Suhayl – Journal for the History of the Exact and Natural Sciences in Islamic Civilisation* (Barcelona) 3 (2002/03), pp. 9-156.
- **Part XIV** was first published as "Bringing Astronomical Instruments back to Earth – The Geographical Data on Medieval Astrolabes to *ca*. 1100", in *Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North*, Leiden: E. J. Brill, 1999, pp. 3-53. [This new version omits a significant error in the original, identified in the preamble.]
- **Part XV** has not been published previously.

MAIN BIBLIOGRAPHY AND BIBLIOGRAPHICAL ABBREVIATIONS

Notes: The reader will find here the main modern sources for the history of Islamic astronomy and astronomical instrumentation, including conference proceedings and exhibition catalogues. Some additional sources on instruments will be included in Vol. 2. Many studies have been reprinted in various collected volumes, notably those published by Variorum; those individual studies related to astronomical timekeeping and related aspects of mathematical astronomy which are particularly relevant to this study are also listed here. The abbreviations introduced below are used throughout this book. Further bibliographical lists are given in **VIII** (comparison of aspects of Islamic and medieval European science) and **X** (in-strumentation), and the works cited there are listed below without abbreviated titles. Earlier publications reprinted in this volume are indicated by diamonds. Detailed references to other works cited which are not directly related to our main subject are presented *ad loc*.

Aaboe Festschrift: *From Ancient Omens to Statistical Mechanics*: *Essays on the Exact Sciences Presented to Asger Aaboe*, J. Lennart Berggren and Bernard R. Goldstein, eds., *Acta Historica Scientiarum Naturalium et Medicinalium* (Copenhagen), 39 (1987).

Ackermann, "Instruments of Humphrey Cole": Silke Ackermann, "Scientific Instruments", in *eadem*, ed., *Humphrey Cole*, pp. 29-95.

— , "London BM *Quadrans vetus*": see EPACT.

— , "Sloane Astrolabe": see EPACT.

— , "Umarbeitung eines Astrolabs": *eadem*, "Mutabor: Die Umarbeitung eines mittelalterlichen Astrolabs im 17. Jahrhundert", in *Frankfurt IGN Festband*, pp. 193-209.

— , "Zinner Archive": *eadem*, "Dormant Treasures – The Zinner-Archive at Frankfurt University", *Nuncius – Annali di Storia della Scienza* (Florence) 16:2 (2001), pp. 711-722.

— , ed., *Humphrey Cole*: *eadem*, ed., *Humphrey Cole: Mint, Measurement and Maps in Elizabethan England*, (British Museum, Occasional Paper, no. 126), London: The British Museum, 1998.

– & Cherry, "Three Medieval English Quadrants": *eadem* and John Cherry, "Richard II, John Holland and Three Medieval Quadrants", *AS* 56 (1999), pp. 3-23.

Adıvar, Adnan, *La science chez les Turcs Ottomans*, Paris: G. P. Maisonneuve, 1939.

AHES: *Archive for History of Exact Sciences*.

Āghā Buzurg, *al-Dharīʿa*: Āghā Buzurg Muḥammad Muḥsin al-Tihrānī, *al-Dharīʿa ilā taṣānīf al-Shīʿa*, 25 vols., Najaf, later Tehran, 1965-1978.

Ahlwardt: see *Berlin Catalogue*.

AIHS: *Archives Internationales d'Histoire des Sciences*.

AIOS: *Arabische Instrumente in orientalistischen Studien*, Fuat Sezgin *et al*, eds., 6 vols., Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1990-91, repr. as *Islamic Astronomy and Mathematics*, vols. 85-90 (1998), with 6 further volumes *ibid*., vols. 91-96 (1998). [Reprints of studies from the 19th and early 20th century: a monumental total of over 5,000 pages.]

Aleppo 1976 Symposium Proceedings: *Proceedings of the First International Symposium for the History of Arabic Science*, Ahmad Y. al-Hassan *et al*., eds., 2 vols., Aleppo: Institute for the History of Arabic Science, 1978. Contains:

- D. A. King "Astronomical Timekeeping in Fourteenth-Century Syria", in II, pp. 75-84, repr. in *idem*, *Studies*, A-X.
- Fritz W. Zimmermann, "The Dustūr al-Munajjimīn of MS Paris, BN ar. no. 5968", in II, pp. 184-192.

Algiers BN Catalogue: E. Fagnan, *Catalogue général des manuscrits de la Bibliothèque Nationale d'Algérie* ... , first published Paris, 1893, 2nd edn., Algiers: Bibliothèque Nationale d'Algérie, 1995.

Ali: see al-Bīrūnī, *Geography*.

al-ʿĀmilī, *Aʿyān al-Shīʿa*: Muḥsin al-Amīn al-Ḥusaynī al-ʿĀmilī, *Aʿyān al-Shīʿa*, 56 vols., Damascus: Mabaʿat Ibn Zaydūn, 1935-1962.

Amsterdam NK 1990 Exhibition Catalogue: Anthony J. Turner, ed., *time*, Amsterdam: Tijd voor Tijd Foundation, 1990. [Catalogue of an exhibition at the Nieuwe Kirk, Amsterdam, in 1990.]

Anagnostakis, "Astrolabe Ecliptic Divisions": Christopher Anagnostakis, "How to Divide the Ecliptic on an Astrolabe", in *Aaboe Festschrift*, pp. 133-144.

Analema – Boletín de la Asociación de Amigos de los Relojes de Sol (Madrid).

Ankara (AÜDTCF) Dergisi and *Yayınlari*: *Ankara Üniversitesi Dil ve Tarih-Coğrafya Fakültesi Dergisi* and *Yayınları*.

Ansari & Ghori, "Two Treatises on Astronomical Instruments": S. M. Raza Ansari and S. A. Khan Ghori, "Two Treatises on Astronomical Instruments by ʿAbd al-Munʿim al-ʿĀmilī and Qāsim ʿAlī al-Qāyinī", *Delhi 1985 Colloquium Proceedings*, pp. 215-225.

Anthiaume & Sottas, "*Quadrans novus*": A. Anthiaume and J. Sottas, *L'astrolabe-quadrant du Musée des antiquités de Rouen – recherches sur les connaissances mathématiques, astronomiques et nautiques au moyen âge*, Paris: Librairie G. Thomas, 1910, repr. Paris: Librairie Alain Brieux, 1988.

Apian: see Röttel.

Araştırma: *Araştırma,* published by Ankara Üniversitesi Dil ve Tarih-Coğrafya Fakültesi, Felsefe Araştırmaları Enstitüsü.

Arcari & Costamagna, "Acquafredda Sundial", A-B: Barbara Arcari and Andrea Costamagna, "L'orologio a ore canoniche dell'Acquafredda", *L'Astronomia* (Rome) 211 (iuglio 2000), pp. 44-50 (A); and *eidem*, "Il quadrante solare dell'abbazia dell'Acquafredda", in *eidem*, *L'abbazia dell'Acquafredda* , Lenno (Como), (Comune di Lenno, Biblioteca Comunale Vittorio Antonini, Quaderno n. 3), 2002, pp. 115-121, and fig. 18 (B).

Archinard, "Cadrans solaires rectilignes": Margarida Archinard, "Les cadrans solaires rectilignes", *Nuncius* (Florence) 3:2 (1988), pp. 149-181.

— , "Geneva *Navicula*": *eadem*, "*Navicula de Venetiis* : une acquisition prestigieuse du Musée d'histoire des sciences [Genève]", *Genava*, N.S., 43 (1995), pp. 87-94, published separately with the same pagination, Geneva: Musée d'histoire des sciences, n.d. [1995].

— , "Unequal Hour Diagram": *eadem*, "The Diagram of Unequal Hours", *AS* 47 (1990), pp. 173-190.

Arnaldi & Schaldach, "Roman Cylinder Dial": Mario Arnaldi and Karl-Heinz Schaldach, "A Roman Cylinder Dial: Witness to a Forgotten Tradition", *JHA* 28 (1997), pp. 107-117.

Arvanitaki, "L'heure arabe": G. Arvanitaki, "Théorie de l'heure arabe", *Memoires de l'Institut Égyptien* 8 (1915), pp. 21-55.

AS: *Annals of Science*.

al-Aṣbaḥī, *al-Yawāqīt*: References in **IV** are to MS Cairo Dār al-Kutub *mīqāt* 948,1, fols. 1v-53r, of al-Aṣbaḥī's *Kitāb al-Yawāqīt fī ʿilm al-mawāqīt*.

ASP: *Arabic Science and Philosophy* (Cambridge University Press).

Assemani, *Globus arabicus*: Simon Assemani, *Globus caelestis cufico-arabicus Veliterni Musei Borgiani*, Padua, 1790, repr. in *IMA*, vol. 91 (1998), pp. 63-305, plus fold-outs.

Astrolabica: *Astrolabica* (Paris: Société Internationale de l'Astrolabe). [See especially no. 5: *Astrolabica N° 5: Etudes 1987-1989*, Anthony J. Turner, ed., Paris: Institut du Monde Arabe & Société Internationale de l'Astrolabe, 1989.]

Atkinson, "Points of the Compass": T. D. Atkinson, "Points of the Compass", in *ERE*, X, pp. 73-88.

Aʿyān al-Shīʿa: see al-ʿĀmilī, *Aʿyān al-Shīʿa*.

Azzawi, *History of Astronomy in Iraq*: ʿAbbās al-ʿAzzāwī, *Taʾrīkh ʿilm al-falak fī ʾl-ʿIrāq wa-ʿalāqātihi bi-ʾl-aqṭār al-islāmiyya wa-ʾl-ʿarabiyya fī ʾl-ʿuhūd al-tāliya li-ayyām al-ʿAbbāsiyyīn min sanat 656 H = 1258 M ilā sanat 1335 H = 1917 M* [= *A History of Astronomy in Iraq and its relations with Islamic and Arab regions in the period following the Abbasids, from 1258 A.D. to 1917 A.D.*], Baghdad: Iraqi Scientific Academy (al-Majmaʿ al-ʿilmī al-ʿIrāqī, 1958. [This Arabic work is independent of any earlier European bio-bibliographical works on the subject.]

Bagheri, "al-Kāshī on the Visible Horizon": Mohammad Bagheri, "A New Treatise by al-Kāshī on the Depression of the Visible Horizon", in *Bellagio 2000 Conference Proceedings*, pp. 357-368.

Bagvi, M. B. A. *Determination of the Direction of Qibla and the Islamic Timings*, Lahore: Sh. Muhammed Ashraf, 1972.

Baloch, "Measurement of Space and Time in Lower Sind": N. A. Baloch, "Measurement of Space and Time in the Lower Indus Valley of Sind", in *Islamabad 1979 Conference Proceedings*, pp. 168-196.

Barceló: see Labarta & Barceló.

Barmore, "Turkish Mosque Orientation": Frank E. Barmore, "Turkish Mosque Orientation and the Secular Variation of the Magnetic Declination", *Journal of Near Eastern Studies* 44 (1985), pp. 81-98.

Barthold, "Orientierung der ersten Moscheen": Wilhelm Barthold, "Die Orientierung der ersten muhammedanischen Moscheen", *Der Islam* 18 (1929), pp. 245-250.

Bashear, "Qibla Musharriqa": Suliman Bashear, "Qibla Musharriqa and Early Muslim Prayer in Churches", *The Muslim World* 81 (1991), pp. 267-282.

BEA: *Biographical Encyclopedia of Astronomers*, Thomas Hockey, ed., to be published by Kluwer Academic Publishers, Dordrecht.

Bennett, "Oxford *Navicula*": see EPACT.

Beer, "Astronomical Dating of Works of Art": Arthur Beer, "Astronomical Dating of Works of Art", *Vistas in Astronomy* 9 (1967), pp. 177-223.

Bellagio 2000 Conference Proceedings: *From China to Paris: 2000 Years Transmission of Mathematical Ideas*, (Boethius – Texte und Abhandlungen zur Geschichte der Mathematik und der Naturwissenschaften, Band 46), Yvonne Dold-Samplonius, Joseph W. Dauben, Menso Folkerts and Benno van Dalen, eds., Stuttgart: Franz Steiner, 2002. [Proceedings of a conference held at the Rockefeller Foundations's Research and Conference Center, Bellagio, during May, 2000.]

Benoit, "Cadran solaire monastique": J. L. Benoit, "Un cadran solaire monastique du XIIe siècle", *Le Cosmos* 59/1238 (1908), pp. 435-437.

Berggren, "Archimedes among the Ottomans": J. Lennart Berggren, "Archimedes among the Ottomans", in *Aaboe Festschrift*, pp. 101-109.

— , "Al-Bīrūnī on Plane Maps of the Sphere": *idem*, "Al-Bīrūnī on Plane Maps of the Sphere", *JHAS* 6 (1982), pp. 47-96.

— , "Four Analemmas for the Qibla": *idem*, "A Comparison of Four Analemmas for Determining the Azimuth of the Qibla", *JHAS* 4 (1980), pp. 69-80.

— , "Ḥabash's Analemma for Azimuth Curves": *idem*, "Ḥabash's Analemma for Representing Azimuth Circles on the Astrolabe", *ZGAIW* 7 (1991/92), pp. 23-30.

— , *Islamic Mathematics*: *idem*, *Episodes in the Mathematics of Medieval Islam*, New York, *etc*.: Springer, 1986.

— , "al-Kūhī on Astrolabe Construction": *idem*, "Abū Sahl al-Kūhī's Treatise on the Construction of the Astrolabe with Proof: Text, Translation and Commentary", *Physis* 31 (1994), pp. 141-252.

— , "Origins of the 'Method of the *Zīj*es'": *idem*, "The Origins of al-Bīrūnī's "Method of the *Zīj*es" in the Theory of Sundials", *Centaurus* 28 (1985), pp. 1-16.

— , "Spherical Trigonometry in the *Jāmiʿ Zīj*": *idem*, "Spherical Trigonometry in Kūshyār ibn Labbān's *Jāmiʿ Zīj*", in *Kennedy Festschrift*, pp. 15-33.

Berggren: see also *Aaboe Festschrift*.

Bergmann, *Innovationen im Quadrivium*: Werner Bergmann, *Innovationen im Quadrivium des 10. und 11. Jahrhunderts: Studien zur Einführung von Astrolab und Abakus im lateinischen Mittelalter*, Stuttgart: Franz Steiner, 1985.

Berlin Catalogue: Wilhelm Ahlwardt, *Die Handschriften-Verzeichnisse der Königlichen Bibliothek zu Berlin*, 5. Band: *Verzeichnis der arabischen Handschriften*, Berlin: A. Asher, 1893.

Biarne, "Le temps du moine": J. Biarne, "Le temps du moine d'après les premières règles monastiques d'Occident (IVe-VIe siècles)", in *Le temps chrétien de la fin de l'Antiquité au Moyen-Age, IIIe-XIIIe siècles*, (Colloques Internationaux du Centre National de la Recherche Scientifique, no. 604), Paris, 1984, pp. 99-128. [Not consulted.]

Bielefeld Catalogue: *Wissenschaftliche Instrumente und Sonnenuhren – Kunstgewerbesammlung der Stadt Bielefeld / Stiftung Huelsmann*, Dirk Syndram, ed., Munich: Callwey, 1989.

Bilfinger, *Die mittelalterlichen Horen*: Gustav Bilfinger, *Die mittelalterlichen Horen und die modernen Stunden – Ein Beitrag zur Kulturgeschichte*, Stuttgart: W. Kohlhammer, 1892.

al-Bīrūnī, *Astrolabe Construction*: al-Bīrūnī, *Istīʿāb al-wujūh al-mumkina fī ṣanʿat al-asṭurlāb*, al-Sayyid Muḥammad Akbar Jawādī al-Ḥusaynī, ed., with introductory notes by Ayatallāh Ḥasan Ḥasan Zāde al-Āmulī, Mashhad: Islamic Research Foundation, Astan Quds Razavi, 1380 HS [= 2001/02].

— , *Astrology*: R. Ramsay Wright, transl., *The Book of Instruction in the Elements of the Art of Astrology by ... al-Bīrūnī*, London: Luzac & Co., 1934, repr. Baghdad: al-Muthannā, n.d.

— , *Chronology*: C. Edward Sachau, *Chronologie orientalischer Völker von Alberuni*, Leipzig: F. A. Brockhaus & Otto Harrassowitz, 1923, repr. Baghdad: Maktabat al-Muthannā, n.d., translated as C. E. Sachau, *The Chronology of Ancient Nations – An English Version of the Arabic Text of the Athár-ul-bákiya of Albírúní, or 'Vestiges of the Past,' collected and reduced to writing by the author in A.H. 390-1, A.D. 1000*, London, 1879, repr. Frankfurt am Main: Minerva, 1984.

— , *India*: E. C. Sachau, *Alberuni's India*, Delhi, *etc*.: S. Chand & Co., 1964 reprint.

— , *Maqālīd*: Marie-Thérèse Debarnot, ed. and trans., *al-Bīrūnī: Kitāb Maqālīd ʿilm al-hayʾa. La trigonométrie sphérique chez les Arabes de l'est à la fin du Xe siècle,* Damascus: Institut Français de Damas (Publication no. 114), 1985.

— , *Qānūn*: al-Bīrūnī, *al-Qānūn al-Masʿūdī,* [Max Krause, ed.], 3 vols., Hyderabad, Deccan: Osmania Oriental Publications, 1954-1956.

— , *Shadows*: al-Bīrūnī, *Ifrād al-maqāl fī amr al-ẓilāl*, no. 2 in *Rasāʾil al-Bīrūnī*, Hyderabad: Osmania Oriental Publications, 1948; with translation and commentary in Edward Stewart Kennedy, *The Exhaustive Treatise on Shadows by Abū al-Rayḥān Muḥammad b. Aḥmad al-Bīrūnī*, 2 vols., Aleppo: Institute for the History of Arabic Science, 1976.

— , *Taḥdīd*: al-Bīrūnī, *Kitāb Taḥdīd nihāyāt al-amākin*, ed. P. Bulgakov, *Majallat Maʿhad al-makhṭūṭāt al-ʿarabiyya* (Cairo) 8 (1962). [I have not seen the Ankara 1962 edn. by Muḥammad ibn Tāwīt al-Ṭanjī.] Translation in Jamil Ali, *The Determination of the Coordinates of Cities ... by al-Bīrūnī*, Beirut: American University of Beirut Press, 1967. Commentary in Edward Stewart Kennedy, *A Commentary upon Bīrūnī's Kitāb Taḥdīd [nihāyāt] al-amākin, An 11th Century Treatise on Mathematical Geography*, Beirut: American University of Beirut Press, 1973.

Blachère *et al.*, *Dictionnaire Arabe-Français-Anglais*: Régis Blachère *et al.*, *Dictionnaire Arabe-Français-Anglais*, 3 vols. to date, Paris: G.-P. Maisonneuve et Larose, 1967 to present.

Bonine, "The Morphogenesis of Iranian Cities": Michael E. Bonine, "The Morphogenesis of Iranian Cities", *Annals of the Association of American Geographers* 69 (1979), pp. 208-224.

— , "Sacred Direction and City Structure in Morocco": *idem*, "The Sacred Direction and City Structure: A Preliminary Analysis of the Islamic Cities of Morocco", *Muqarnas: An Annual on Islamic Art and Architecture* 7 (1990), pp. 50-72.

Borst, *Astrolab und Klosterreform*: Arno Borst, *Astrolab und Klosterreform an der Jahrtausendwende*, Heidelberg: Carl Winter, 1989. [Problematic.]

Bosworth, *Islamic Dynasties*: C. Edmund Bosworth, *The New Islamic Dynasties – A chronological and genealogical manual*, New York: Columbia University Press, 1996.

Bremner, "Shadow Table in Mul.Apin": Robert W. Bremner, "The Shadow Length Table in Mul.Apin", in *Graz 1991 Conference Proceedings*, pp. 367-382.

Brentjes, "The Republic of Letters in the Middle East": Sonja Brentjes, "The Interests of the Republic of Letters in the Middle East, 1550-1700", *Science in Context* 12:3 (1999), pp. 435-468.

Brentjes: see also *Frankfurt 2001 Conference Proceedings*.

Brice, *Atlas*: Brice, William C., ed., *An Historical Atlas of Islam*, Leiden: E. J. Brill, 1981.

Brice & Imber & Lorch, "Equatorial Semicircle": William Brice, Colin Imber and Richard P. Lorch, "The Dāʾire-yi Muʿaddel of Seydī ʿAlī Reʾīs", *Seminar on Early Islamic Science* (University of Manchester) 1 (1976). [Reviewed in King, *Studies*, B-XIII.]

Brieux, "Authenticité des astrolabes": Alain Brieux, "Les astrolabes: tests d'authenticité", *Art et curiosité*, sept. 1974 (unpaginated).

Brieux & Maddison, *Répertoire*: Alain Brieux and Francis R. Maddison, *Répertoire des facteurs d'astrolabes et de leurs œuvres*, in press (Paris: C.N.R.S.).

Brockelmann, *GAL*: Carl Brockelmann, *Geschichte der arabischen Litteratur*, 2 vols., 2nd edn., Leiden: E. J. Brill, 1943-49, and Supplementbände, 3 vols., Leiden: E. J. Brill, 1937-42.

Bruin, *al-Bīrūnī Newsletter*: A series of 38 mimeographed articles prepared between 1966 and 1971 by the late Professor Frans Bruin, then of the American University of Beirut. [For a complete list see the introduction to **XIIa**. Copies are available at the Department of History of Science and Medicine at Yale University; the University Library at the American University of Beirut; and the Institutes for History of Science in Frankfurt and Hamburg.] In this book the following are cited:

"The Astronomical Observatory of Ulugh Beg in Samarqand", *al-Bīrūnī Newsletter* 9 (1967).
"The Observational Instruments of al-ʿUrḍī ... ", *ibid.* 10 (1968).
"The Astronomical Observatory of Naṣīr al-Dīn al-Ṭūsī", *ibid.* 13 (1968).
"The Construction of Instruments ... by ... al-ʿĀmilī ... ", *ibid.* 15 (1968).
"The Fakhrī Sextant in Rayy", *ibid.* 19 (1969).
"The Instruments of Taqī al-Dīn ... ", *ibid.* 29 (1970).
"A Treatise on Small Instruments by ... al-Khāzinī ... ", *ibid.* 30 (1970).
"The Planispheric Astrolabe: Its History, Theory, and Use", *ibid.* 33, 34 and 35 (1970).

Brusa, "Navicelle": Giuseppe Brusa, "Le navicelle orarie di Venezia", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* 5 (1980), pp. 51-59.

Brussels 1984 Exhibition Catalogue: *La mesure du temps dans les collections Belges*, Brussels: Société Générale de Banque, 1984. [Catalogue of an exhibition at the Société Générale de Banque during 26.1.-7.4.1984.]

Brussels BR 2001 Exhibition Catalogue: *Des agrimensores romains aux arpenteurs du XVIe siècle*, Jan De Graeve and Jean Mosselmans, eds., Brussels: UGEB, 2001. [Catalogue of an exhibition organized by the Union des Géomètres-Experts immobiliers de Bruxelles held at the Bibliothèque Royale de Belgique during 22.11-21.12.2001.]

BSIS: *Bulletin of the Scientific Instrument Society*.

Buchner, "Antike Reiseuhren": Edmund Buchner, "Antike Reiseuhren", *Chiron* 1 (1971), pp. 457-482.

— , *Die Sonnenuhr des Augustus*: *idem*, *Die Sonnenuhr des Augustus*, Mainz: Philipp von Zabern, 1982.

Burnett, "ʿAbd al-Masīḥ of Winchester": Charles Burnett, "ʿAbd al-Masīḥ of Winchester", in *North Festschrift*, pp. 159-169.

— , "Antioch as a Link between Arabic and Latin Culture": *idem*, "Antioch as a Link between Arabic and Latin Culture in the Twelfth and Thirteenth Centuries", in *Occident et Proche-Orient : Contacts scientifiques au temps des Croisades, Actes du colloqie de Louvain-la-Neuve, 24 et 25 mars 1997*, Isabelle Draelants, Anne Tihon, Baudouin van den Abeele, eds., Turnhout (B): Brepols, 2000, pp. 1-78.

— , *The Introduction of Arabic Learning into England*: *idem*, *The Introduction of Arabic Learning into England*, (The Panizzi Lectures, 1996), London: The British Library, 1997.

— , "The Translation of Arabic Science into Latin": *idem*, "The Translation of Arabic Science into Latin: A Case of Alienation of Intellectual Property?", *Bulletin of the Royal Institute for Inter-Faith Studies* (Amman, Jordan) 4:1 (2002), pp. 145-157.

Burnett, ed, *Adelard of Bath*: *Adelard of Bath – An English Scientist of the Early Twelfth Century*, Charles Burnett, ed., (Warburg Institute Surveys and Texts, XIV), London: The Warburg Institute, University of London, 1987.

Cairo ENL Catalogue: David A. King, *A Catalogue of the Scientific Manuscripts in the Egyptian National Library* (in Arabic), 2 vols., Cairo: General Egyptian Book Organization, 1981 and 1986.

Cairo ENL Survey: *idem*, *A Survey of the Scientific Manuscripts in the Egyptian National Library* (in English), (American Research Center in Egypt, Catalogs, vol. 5), Winona Lake, Ind.: Eisenbrauns, 1987.

Caldo, "Astrolabi di Palermo": L. Caldo, "Astrolabi del Museo Nazionale di Palermo", *Pubblicazione dell' Osservatorio astronomico di Palermo*, *Memorie*, no. 65, 1936, repr. in *Coelum* 8 (June 1938), pp. 101-108, and 8 figs. [Not reprinted in *AIOS* or *IMA*.]

Calvo, *Ibn Bāṣo and his Universal Plate*: Emilia Calvo Labarta, *Abū ʿAlī al-Ḥusayn ibn Bāṣo (m. 716/1316): Risālat al-Ṣafīḥa al-ŷāmiʿa li-ŷamīʿ al-ʿurūḍ (Tratado sobre la lámina general para todas las latitudas)*, Madrid: Consejo Superior de Investigaciones Científicas and Instituto de Cooperación con el Mundo Árabe, 1993.

— , "Ibn Bāṣo's Universal Plate": *eadem*, "Ibn Bāṣo's Astrolabe [Plate] in the Maghrib and East", in *Vernet Festschrift*, II, pp. 755-767.

— , "Ibn Bāṣo's *Ṣafīḥa Mujayyaba*": *eadem*, "Transformation of Coordinates in Ibn Bāṣo's *al-Risāla fī 'l-ṣafīḥa al-mujayyaba dhāt al-awtār*", *JHAS* 12 (2001), pp. 3-21.

— , "La resolution graphique des questions astrologiques": *eadem*, "La resolution graphique des questions astrologiques à al-Andalus", *Actes du 3me Colloque Maghrébin sur l'Histoire des Mathématiques Arabes – Tipaza (Alger, Algérie) – 1-3 Décembre 1990*, Algiers: Association Algérienne d'Histoire des Mathématiques, 1998, pp. 31-44.

— , "The Use of Ibn Bāṣo's Universal Plate": *eadem*, "A Study of the Use of Ibn Bāṣo's Universal Astrolabe Plate", *AIHS* 50 (2000), pp. 264-295.

Cambridge DI 1998 Conference Proceedings: selected papers given at a conference "New perspectives on science in medieval Islam" held at the Dibner Institute, Cambridge, Mass., during Nov. 6-8, 1998, to appear.

Cambridge History of Iran, IV-VI: *The Cambridge History of Iran*, vol. IV: The Period from the Arab Invasion to the Saljuqs, (Richard N. Frye, ed.), vol. V: *The Saljuq and Mongol Periods* (John A. Boyle, ed.), and vol. VI: *The Timurid and Safavid Periods* (Peter Jackson and Laurence Lockhart, eds.), Cambridge: Cambridge University Press, 1975, 1968 and 1986. [See Kennedy, "Exact Sciences in Iran".]

Cambridge WMHS Catalogue: David J. Bryden, *The Whipple Museum of the History of Science, Catalogue 6: Sundials and Related Instruments*, Cambridge: The Whipple Museum of the History of Science, 1988.

Canobbio, "Fragment of a Spherical Astrolabe": Ernesto Canobbio, "An Important Fragment of a West Islamic Spherical Astrolabe", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* 1 (1976), pp. 37-41.

Cantor, *Geschichte der Mathematik*: Moritz Cantor, *Vorlesungen über die Geschichte der Mathematik* 2nd edn., 4 vols., repr. Leipzig: B. G. Teubner, 1913.

Carandell, "Analemma for the Qibla": Joan Carandell, "An Analemma for the Determination of the Azimuth of the Qibla in the *Risāla fī ʿilm al-ẓilāl* of Ibn al-Raqqām", *ZGAIW* 1 (1984), pp. 61-72.

— , *Ibn al-Raqqām sobre los cuadrantes solares*: *idem*, Risāla fī ʿilm al-ẓilāl *de Muḥammad Ibn al-Raqqām al-Andalusī: edicion, traducción y comentario*, Barcelona: Universidad de Barcelona, Instituto "Millás Vallicrosa" de Historia de la Ciencia Árabe, 1988.

Carboni, *Kitāb al-bulhān*: Stefano Carboni, *Il Kitāb al-bulhān di Oxford*, (*Eurasiatica* - Quaderni del Dipartimento di Studi Eurasiatici, Università degli Studi di Venezia, 6), Turin: Tirrenia, 1988.

Carra de Vaux, "L'astrolabe linéaire": Bernard Carra de Vaux, "L'astrolabe linéaire ou bâton d'al-Tousi", *Journal Asiatique*, 11. sér., 5 (1895), pp. 464-516, repr. in *AIOS*, III, pp. 181-233.

Casanova, "Cadran solaire syrien": Paul Casanova, "La montre du Sultan Nour ad Din (554 de l'Hégire = 1159-1160)", *Syria* 4 (1923), pp. 282-299, repr. in *AIOS*, IV, pp. 242-262.

Casulleras, "Andalusī Sundial": Josep Casulleras, "El último capítulo del *Kitāb al-Asrār fī natāʾiŷ al-afkār*", in *Vernet Festschrift*, II, pp. 613-653.

Cátedra & Samsó, *Astrología de Enrique de Villena*: Pedro M. Cátedra (ed. and transl.) with Julio Samsó (comm.), *Tratado de astrología atribuido a Enrique de Villena*, Madrid: Río Tinto Minera, 1980.

Caussin, "Table Hakémite": A. P. Caussin de Perceval, "Le livre de la grande table Hakémite", in *Notices et extraits des manuscrits de la Bibliothèque nationale*, 7 (An XII [= 1804]), pp. 16-240, with separate pagination in the separatum.

Chabás, "Astronomy in 15th-Century Salamanca": José Chabás, "Astronomy in Salamanca in the Mid-Fifteenth Century: The Tabulae resolutae", *JHA* 29 (1998), pp. 167-175.

— , "Cahier d'un croisier": *idem*, "Le cahier d'astronomie d'un croisier du XV[e] siècle", *Nuncius* (Florence) 12:1 (1997), pp. 3-16.

— & Goldstein, *Zacut*: *idem* and Bernard R. Goldstein, *Astronomy in the Iberian Peninsula: Abrahan Zacut and the Transition from Manuscript to Print*, (*Transactions of the American Philosophical Society* 90:2), Philadelphia, Pa.: 2000.

Chapoutot-Remadi: see Souissi & Chapoutot-Remadi.

Charette, "Der geflügelte Quadrant": François Charette, "Der geflügelte Quadrant: Ein ungewöhnliches Sinusinstrument aus dem 14. Jh.", in *Saltzer Festschrift*, pp. 25-36.

— , "al-Khwārizmī on the Astrolabe": *idem* with Petra Schmidl, "Scientific Initiative in 9th-Century Baghdad: al-Khwārizmī on the Astrolabe and Related Treatises", to appear.

— , "The Library of Muṣṭafā Ṣidqī": *idem*, "A View of Scientific Life in Mid-Eighteenth Century Istanbul: The Library of Muṣṭafā Ṣidqī" [revised version of a paper presented at the DAVO Annual Congress (German Middle East Studies Association for Contemporary Research and Documentation), Hamburg, December 1999], forthcoming.

— , *Mamluk Instrumentation*: *idem*, *Mathematical Instrumentation in Fourteenth-Century Egypt and Syria – The Illustrated Treatise of Najm al-Dīn al-Miṣrī*, Leiden: E. J. Brill, 2003.

— , "Najm al-Dīn's Monumental Table": *idem*, "A Monumental Medieval Table for Solving the Problems of Spherical Astronomy for all Latitudes", *AIHS* 48 (1998), pp. 11-64.

— , *Orientalisme et histoire des sciences: l'historiographie européenne des sciences islamiques et hindoues, 1784-1900*, Master's thesis, Départment d'histoire, Université de Montréal, 1995.

— & King, "Survey of Islamic Tables for Astrolabe Construction": *idem* and David A. King, "A Survey of Medieval Islamic Tables for Astrolabe Construction", to appear.

— & King, *The Universal Astrolabe of Ibn al-Sarrāj*: *idem* and David A. King, *The Universal Astrolabe of Ibn al-Sarrāj*, to appear.

— & Schmidl, "Habash's Universal Plate": *idem* and Petra G. Schmidl, "A Universal Plate for Timekeeping with the Stars by Ḥabash al-Ḥāsib: Text, Translation and Preliminary Commentary", *Suhayl* 2 (2001), pp. 107-159.

Charette: see also *Frankfurt 2001 Conference Proceedings* and King & Charette.

Cheikho, "al-Mukḥula": Louis Cheikho (= Shaykhū), "al-Mukḥula" (in Arabic), *al-Machriq* [= *al-Mashriq*] (Beirut) 10 (1907) [repr. Munich: Kraus-Thomson Organization G.m.b.H., 1980], pp. 75-80, repr. in *IMA*, vol. 91 (1998), pp. 370-375.

Chidyaq, "Indicateur de qibla": Hana Chidyaq, "Un indicateur de qibla signé Muḥammad ʿIzz al-Ṣabbāgh", *Astrolabica* 5 (1989), pp. 27-36.

Christie's London 08.04.1998 Catalogue: *Christie's South Kensington – Exceptional Scientific and Engineering Works of Art, Instruments and Models – 8 April 1998*, London: Christie's, 1998.

Colin & Renaud, "Abū Miqraʿ": Georges Colin and Henri-Paul-Joseph Renaud, "Note sur le "*muwaqqit*" marocain Abū Muqriʿ – ou mieux Abū Miqraʿ – al-Baṭṭīwī (XIII[e] s. J.C.)", *Hespéris* 25 (1933), pp. 94-96.

Comes, *Ecuatorios andalusíes* and "Ecuatorios andalusíes": Mercè Comes, *Ecuatorios andalusíes – Ibn al-Samḥ, al-Zarqālluh y Abū-l-Ṣalt*, Barcelona: Universidad de Barcelona, Facultad de Filología, *etc*., 1991; and a summary in the chapter "Los ecuatorios andalusíes" in *Madrid MAN 1992 Exhibition Catalogue*, pp. 75-87.

The Compendium: *The Compendium – Journal of the North American Sundial Society*.

Cook, "Muslim Material on Comets and Meteors": David Cook, "A Survey of Muslim Material on Comets and Meteors", *JHA* 30 (1999), pp. 131-160.

Cotter, "Nautical Tables": Charles H. Cotter, "The Development of Nautical Astronomical Inspection Tables in the Period from 1770 to 1919", *Vistas in Astronomy* 20 (1976), pp. 245-247.

Crombie, "Griechisch-arabische Naturwissenschaften und Abendländisches Denken": Alistair C., Crombie, "Griechisch-arabische Naturwissenschaften und Abendländisches Denken", in Gereon Sievernich and Hendrik Budde, eds., *Europa und der Orient 800-1900*, Gütersloh and Munich: Bertelsmann Lexikon Verlag GmbH, 1989, pp. 102-131.

Curtze, "Urkunden zur Geschichte der Trigonometrie": Maximilian Curtze, "Urkunden zur Geschichte der Trigonometrie im christlichen Mittelalter", *Bibliotheca Mathematica*, 3. Folge, 1 (1900), pp. 321-416.

Al-Daffa & Stroyls, *Studies:* Ali A. Al-Daffa and John S. Stroyls, *Studies in the Exact Sciences in Medieval Islam*, New York: John Wiley & Sons, 1984. [To be used with caution.]
Dallal, "Al-Bīrūnī on Climates": Ahmad Dallal, "Al-Bīrūnī on Climates", *AIHS* 34(1984), pp. 3-18.
— , "Ibn al-Haytham's Universal Solution for the Qibla": *idem*, "Ibn al-Haytham's Universal Solution for Finding the Direction of the *Qibla* by Calculation", *ASP* 5 (1995), pp. 145-193.
— , "Islamic Science, Medicine, and Technology": *idem*, "Science, Medicine, and Technology – The Making of a Scientific Culture", in Esposito, ed., *The Oxford History of Islam*, pp. 154-213.
Damascus Catalogue: Ibrahim Khoury, *Fihris makhṭūṭāt Dār al-Kutub al-Ẓāhiriyya*, vol. VI: *'Ilm al-hay'a wa-mulḥaqātuhu*, Damascus: Majma' al-lugha al-'Arabiyya, 1389 H [= 1969].
Daniel, "Greenwich *Navicula*": Christopher Daniel, "The Fifth Navicula", *Clocks* (Hemel Hempstead, U.K.) 14 (1992), p. 37. [On the *navicula* now in Greenwich.]
Davidian, "al-Bīrūnī on the Time of Day": Davidian, Marie-Louise, "al-Bīrūnī on the Time of Day from Shadow Lengths", *JAOS* 80 (1960), pp. 330-335, repr. in Kennedy *et al.*, *Studies*, pp. 274-279.
Davidian: see also Kennedy & Davidian.
Debarnot, *al-Bīrūnī's* Maqālīd: see al-Bīrūnī, *Maqālīd*.
— , "Islamic Trigonometry": Marie-Thérèse Debarnot, "Trigonometry", in *EHAS*, II, pp. 495-538.
— , "*Zīj* of Ḥabash": *eadem*, "The *Zīj* of Ḥabash al-Ḥāsib: A Survey of MS Istanbul Yeni Cami 784/2", in *Kennedy Festschrift*, pp. 35-69.
Dekker, "Astrolabes and Dates and Dead Ends": Elly Dekker, "Of Astrolabes and Dates and Dead Ends", *AS* 49 (1992), pp. 175-184. [A critical study of Torode, "Dating Astrolabes".]
— , "Cartographic Grids from Iran": *eadem*, "Cartographic Grids from Iran: An Early Version of the Retro-Azimuthal Orthographic Projection", *The Cartographical Journal* (The British Cartographic Society, London) 37:2 (2000), pp. 109-116. [A discussion of the grids on the maps described in King, *Mecca-Centred World-Maps* – see now **VIIc**.]
— , "*Quadrans novus*": *eadem*, "An Unrecorded Medieval Astrolabe Quadrant from *c*. 1300", *Annals of Science* 52 (1995), pp. 1-47.
Delambre, *HAMA*: Jean-Baptiste Delambre, *Histoire de l'astronomie du moyen âge*, Paris, 1819, repr. New York: Johnson Reprints, 1965.
Delehar, "Illustrations": Peter Delehar, "Illustrations of Scientific Instruments in the *Gentleman's Magazine* 1746-1796", in *Turner Festschrift*, pp. 383-394.
Delphin, "L'astronomie au Maroc": Gäetan Delphin, "L'astronomie au Maroc", *Journal Asiatique*, 8e ser., 17 (1891), pp. 177-201.
De Smet: see Poulle & De Smet.
Destombes, "Astrolabe carolingien": Marcel Destombes, "Un astrolabe carolingien et l'origine de nos chiffres arabes", *AIHS* 58-59 (1962), pp. 1-45, repr. in *idem*, *Contributions*, pp. 153-200.
— , "Chiffres coufiques": *idem*, "Les chiffres coufiques des instruments astronomiques arabes", *Physis* 2:3 (1960), pp. 197-210, repr. in *idem*, *Contributions*, pp. 127-140.
— , *Contributions*: G. Schilder *et al*., *M. Destombes (1905-1983): Contributions sélectionnées à l'Histoire de la Cartographie et des Instruments scientifiques,* Utrecht: Hes & Paris: A. G. Nizet, 1987.
— & Kennedy, "al-Ṣūfī on the Astrolabe": *idem* and Edward Stewart Kennedy, "Introduction to *Kitāb al-ʿAmal bil Asṭurlāb* [of al-Ṣūfī]", repr. in Kennedy *et al*., *Studies*, pp. 405-447.
de Vries *et al*., "Hafîr and Halazûn": Fer J. de Vries, William S. Maddux and Mac Oglesby, "Hafîr and Halazûn", *The Compendium – Journal of the North American Sundial Society* 6:2 (1999), pp. 23-26.
de Vries *et al*., "Regiomontanus Sundials": Fer J. de Vries, Mac Oglesby, William S. Maddux and Warren Thom, "Regiomontanus, Apian and Capuchin Sundials", *The Compendium* 5:4 (1998), pp. 13-19.
al-Dharīʿa: see Āghā Buzurg, *al-Dharīʿa*.
D'Haenens, "La clepsydre de Villiers": Albert D'Haenens, "La clepsydre de Villiers (1267) – Comment on mesurait et vivait le temps dans une abbaye cistercienne au XIIIe siècle", in Harry Kühnel, ed., *Klösterliche Sachkultur des Spätmittelalters*, Vienna: Österreichische Akademie der Wissenschaften, 1980.
d'Hollander, *L'Astrolabe*: Raymond d'Hollander, *L'Astrolabe – Histoire, théorie et pratique*, Paris: Institut océanographique, 1999.
Dicks, "Ancient Astronomical Instruments": D. R. Dicks, "Ancient Astronomical Instruments", *Journal of the British Astronomical Association* 64 (1954), pp. 77-85.
Dizer, "Equatorial Semicircle": Muammer Dizer, "The *dāʾirat al-muʿaddal* in Kandilli Observatory ... ", *JHAS* 1 (1977), pp. 257-260.
Dizer: see also *Kandilli Instrument Handbook*.
Djebbar, *Une histoire de la science arabe*: Ahmed Djebbar, *Une histoire de la science arabe – Introduction à la connaissance du patrioine scientifique des pays d'Islam*, Paris: Seuil, 2001.
DMA: *Dictionary of the Middle Ages*, 13 vols., New York: Charles Scribner's Sons, 1982-89.
Dobrzycki: see Kremer & Dobrzycki.

Dold-Samplonius, "Measuring the Muqarnas": Yvonne Dold-Samplonius, "Practical Arabic Mathematics: Measuring the Muqarnas by al-Kāshī", *Centaurus* 35 (1992), pp. 193-242.

Dorn, "Drei arabische Instrumente": Bernhard Dorn, "Drei in der Kaiserlichen Öffentlichen Bibliothek zu St. Petersburg befindliche astronomische Instrumente mit arabischen Inschriften", *Mémoires de l'Académie Impériale des Sciences de St.-Pétersbourg,* VII[e] série, 9:1 (1865), pp. 1-150 and 2 figs., repr. in *AIOS*, I, pp. 345-498.

Dozy, *Supplément aux dictionnaires arabes*: Reinhart Dozy, *Supplément aux dictionnaires arabes*, 2[nd] edn., 2 vols., Leiden: E. J. Brill & Paris: Maisonneuve Frères, 1927, repr. Beirut: Librarie du Liban, 1968.

Drecker, "Hermannus Contractus über das Astrolab": Joseph Drecker, "Hermannus Contractus über das Astrolab", *Isis* 16 (1931), pp. 200-219.

— , Theorie der Sonnenuhren: *idem*, *Die Theorie der Sonnenuhren*, vol. 1E of *Die Geschichte der Zeitmessung und der Uhren*, Ernst von Bassermann-Jordan, ed., Berlin & Leipzig: Vereinigung Wissenschaftlicher Verleger Walter de Gruyter & Co., 1925.

DSB: *Dictionary of Scientific Biography*, 14 vols. and 2 supp. vols., New York: Charles Scribner's Sons, 1970-80.

Dublin CB Arabic Catalogue: A. J. Arberry, *The Chester Beatty Library. A Handlist of the Arabic Manuscripts*, 7 vols., Dublin: Emery Walker (Ireland) Ltd. (vols. 1-2) and Hodges, Figgis & Co., Ltd. (vols. 3-7), 1955-64.

Dublin CB Persian Catalogue: A. J. Arberry, M. Minovi, and E. Blochet, *The Chester Beatty Library: A Catalogue of the Persian Manuscripts and Miniatures*, vol. I: MSS. 101-150, Dublin: Hodges Figgis & Co. Ltd., 1959.

Ducène, "*Kitāb Dalāʾil al-qibla* d'Ibn al-Qāṣṣ": Jean-Charles Ducène, "Le *Kitāb Dalāʾil al-qibla* d'Ibn al-Qāṣṣ: analyse des trois manuscrits et des emprunts d'Abū Ḥāmid al-Garnaṭī", *ZGAIW* 14 (2001), pp. 169-187.

EHAS: *Encyclopedia of the History of Arabic Science*, Roshdi Rashed, with Régis Morelon, eds., 3 vols., London: Routledge, 1996. [Also available in French as *Histoire de la science arabe*, 3 vols., Paris: Seuil, 1997, and in Arabic as *Mawsūʿat taʾrīkh al-ʿulūm al-ʿarabiyya*, 3 vols., Beirut: Markaz dirāsāt al-wiḥda al-ʿarabiyya & Muʿassasat ʿAbd al-Ḥamīd Shūmān, 1997.]

EI$_1$: *Encyclopaedia of Islam*, 1[st] edn., 4 vols., Leiden: E. J. Brill, 1913-1934, repr. in 9 vols., Leiden, *etc.*: E. J. Brill, 1987, and again in paper-back, 1993. [Also available in French and German.]

EI$_2$: *The Encyclopaedia of Islam*, new edn., 11 vols. and supplements, Leiden: E. J. Brill, 1960 to present, especially articles "Anwāʿ" (aspects of folk astronomy), "Asṭurlāb" (astrolabe), "ʿIlm al-hayʾa" (here: astronomy generally), "ʿIlm al-ḥisāb" (arithmetic), "Ḳibla" (religious and astronomical aspects of the sacred direction), "Ḳuṭb" (celestial pole), "Layl and Nahār" (aspects of folk astronomy), "Makka. iv: as centre of the world" (sacred geography), "Maṭāliʿ" (right and oblique ascensions), "Maṭlaʿ" (astronomical risings and settings), "Mayl" (declination and obliquity), "Mīḳāt. ii. astronomical aspects" (astronomical timekeeping), "Minṭaḳa" (ecliptic, zodiac and obliquity), "Mizwala" (sundial), "Nu<u>dj</u>ūm" (stars); "Nu<u>dj</u>ūm, ʿIlm aḥkām al-" (astrology), "Rīḥ" (winds in the folk-astronomical tradition), "Rubʿ" (quadrant), "Samt" (direction), "<u>Sh</u>akkāziyya" (universal astrolabic projections), "Taʾrī<u>kh</u>. 2. Era chronology in astronomical handbooks" (calendrics in *zīj*es), Ṭāsa" (magnetic compass), "Zī<u>dj</u>" (tables and astronomical handbooks) [a fuller version is in King & Samsó, "Islamic Astronomical Handbooks and Tables"]; and in the supplement: "Bādgīr" (ventilators). [Also available in French.]

EIr: *Encyclopaedia Iranica*, 10 vols. to date, London & New York: Routledge & Kegan Paul, 1983 to present.

Eisner, *Kalendarium of Nicholas of Lynn*: S. Eisner, ed., & *idem* and G. MacEoin, transl., *The Kalendarium of Nicholas of Lynn*, Athens, Ga.: University of Georgia Press (under the auspices of The Chaucer Library), 1980.

EJ: *Encylopaedia of Judaism*, 12 vols., New York & London: Funk & Wagnalls, 1901-1906.

Elwell-Sutton, L. P., *The Horoscope of Asadullāh Mīrzā – A Specimen of Nineteenth-Century Astrology*, Leiden: E. J. Brill, 1977.

Endreß, "Arabische Wissenschaftliche Literatur": Gerhard Endreß, "Die wissenschaftliche Literatur", Kapitel 8 in *Grundriß der Arabischen Philologie*, Band II: *Litteraturwissenschaft*, Helmut Gätje, ed., Wiesbaden: Dr. Ludwig Reichert, 1987, pp. 400-506.

EPACT: Website *Epact: Scientific Instruments of Medieval and Renaissance Europe*, located at www.mhs.ox.ac.uk/epact, including descriptions of individual instruments which can be accessed using the key-words ion the following artificial titles. See in particular Ackermann, "London BM *Quadrans Vetus*", and "Sloane Astrolabe"; Bennett, "Oxford *Navicula*"; Meliconi, "Brechte Plate", and "Oxford *Quadrans vetus*"; and A. J. Turner, "Florence *Navicula*", and "Florence *Quadrans vetus*".

ERE: *Encyclopaedia of Religion and Ethics*, James Hastings, ed., 12 vols., Edinburgh: T. & T. Clark, 1921.

Erlangen PMS SB: *Sitzungsberichte der Physikalisch-Medizinischen Sozietät zu Erlangen*.

Escorial Catalogue: Henri-Paul-Joseph Renaud, *Les manuscrits arabes de l'Escurial*, tome II, fascicule 3: *Sciences exactes et sciences occultes*, Paris: Paul Geuthner, 1941.

Esposito, ed., *The Oxford History of Islam*: *The Oxford History of Islam*, John L. Esposito, ed., Oxford & New York: Oxford University Press, 1999.

Evans, *Ancient Astronomy*: James Evans, *The History and Practice of Ancient Astronomy*, New York & Oxford: Oxford University Press, 1998.

Fantoni, *Orologi solari*: Girolamo Fantoni, *Orologi solari – Trattato completo di gnomonica,* Rome: Technimedia, 1988.

Fehérvári, "Quadrant of al-Mizzī": Géza Fehérvári, "An Eighth/Fourteenth-Century Quadrant of the Astrolabist al-Mizzī", *Bulletin of the School of Oriental and African Studies* (London) 36 (1973), pp. 115-117 and 2 pls.

Ferrari & Severino, *Meridiane islamiche*: Gianni Ferrari and Nicola Severino, *Appunti per uno studio delle meridiane islamiche*, privately distributed, 1997.

Field & Wright, "Gears from the Byzantines": Judith V. Field and Michael T. Wright, "Gears from the Byzantines: A Portable Sundial with Calendrical Gearing", *Annals of Science* 42 (1985), pp. 87-138, repr. in *eidem* and Hill, *Byzantine and Arabic Gearing*.

— & Hill & Wright, *Byzantine and Arabic Gearing*: *eidem* and Donald R. Hill, *Byzantine and Arabic Mathematical Gearing*, London: The Science Museum, 1985

Fierro & Samsó, eds., *The Formation of al-Andalus*, II: Maribel Fierro and Julio Samsó, eds., *The Formation of al-Andalus*, Part 2: Language, Religion, Culture and the Sciences, Aldershot, Brookfield, Vt.: Ashgate – Variorum, 1998.

Fletcher *et al*., *Index of Mathematical Tables*: A. Fletcher, J. C. P. Miller, L. Rosenhead, and L. J. Comrie, *An Index of Mathematical Tables*, 2 vols., Oxford: Blackwell, 1962.

Forcada, "*Anwāʾ* Books in al-Andalus": Miquel Forcada, "Books of *Anw*āʾ in al-Andalus", in Fierro & Samsó, eds., *The Formation of al-Andalus*, II, pp. 305-327.

— , "Folk Astronomy of Aḥmad b. Fāris": *idem*, "Astrology and Folk Astronomy: the *Mukhtaṣar min al-Anwāʾ* of Aḥmad b. Fāris", *Suhayl* 1 (2000), pp. 107-205.

— , "*Mīqāt* en los calendarios andalusíes", *idem*, "*Mīqāt* en los calendarios andalusíes", *al-Qanara* 11 (1990), pp. 59-69.

– , "The *Mukhtaṣar min al-Anwāʿ* of Aḥmad ibn Fāris": "A New Andalusian Astronomical Source from the Fourth/Tenth Century: The *Mukhtaṣar min al-Anwāʿ* of Aḥmad ibn Fāris", *in Vernet Festschrift*, II, 769-780.

— , "Les sources andalouses du Calendrier d'Ibn al-Bannā'", *idem*, "Les sources andalouses du Calendrier d'Ibn al-Bannā'", in *Actas del II Coloquio hispano-maroquí de ciencias históricas "Historia, ciencia y sociedad", Granada, 6-10 nov. 1989*, pp. 183-196.

Frank, *Zur Geschichte des Astrolabs*: Joseph Frank, *Zur Geschichte des Astrolabs*, Erlangen: Universitäts-Buchdruckerei, 1920, repr. in *AIOS*, IV, pp. 31-62.

— , "al-Khwārizmī über das Astrolab": *idem*, "Die Verwendung des Astrolabs nach al-Chwârizmî", *Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin* 3 (1922), repr. in *AIOS*, IV, pp. 125-159, with a review by Carl Schoy repr. *ibid*, pp. 160-161.

— , "Zwei arabische Instrumente": *idem*, "Über zwei astronomische arabische Instrumente", *Zeitschrift für Instrumentenkunde* 41 (1921), pp. 193-200, repr. in *AIOS*, IV, pp. 63-70.

— & Meyerhof, "Mogulisches Astrolab": *idem* and Max Meyerhof, "Ein Astrolab aus dem indischen Mogulreiche", *Heidelberger Akten der von-Portheim-Stiftung* 13 (Heidelberg: Carl Winter), 1925, reprinted in *AIOS*, IV, pp. 307-356.

Frank: see also Wiedemann & Frank.

Frankfurt IGN Festband: *Ad radices – Festband zum fünfzigjährigen Bestehen des Instituts für Geschichte der Naturwissenschaften Frankfurt am Main*, Anton von Gotstedter, ed., Stuttgart: Franz Steiner, 1994.

Frankfurt 2001 Conference Proceedings: Sonja Brentjes, Benno van Dalen and François Charette, eds., *Certainty, Doubt, Error: Aspects of the Practice of Pre- and Early Modern Science. ...* , in *Early Science and Medicine* (E. J. Brill, Leiden) 7:3 (2002), pp. 173-310. [Proceedings of a conference held in Frankfurt am Main during 17-18.11.2001.]

Fück, *Arabische Studien*: Johann Fück, *Die arabischen Studien in Europa bis in den Anfang des 20. Jahrhunderts*, Leipzig: Otto Harrassowitz, 1955.

Fuller, "Universal Rectilinear Dials": A. W. Fuller, "Universal Rectilinear Dials", *The Mathematical Gazette* (London) 41 (1957), pp. 9-24.

Garbers, "Thābit ibn Qurra über Sonnenuhren": Karl Garbers, "Ein Werk Ṯābit b. Qurra's über ebene Sonnenuhren", *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. A, Bd. 4 (1936).

García Avilés, *El tiempo y los astros*: Alejandro García Avilés, *El tiempo y los astros – Arte, ciencia y religion en la Alta Edad Media*, Murcia: Unversidad de Murcia, 2001.

García Franco, *Astrolabios en España*: Salvador García Franco, *Catálogo crítico de los astrolabios existentes en España*, Madrid, Instituto Histórico de Marina, 1945.

Gentleman's Magazine: see Delehar.
Ghori: see Ansari & Ghori.
Gibbs, *Greek and Roman Sundials*: Sharon L. Gibbs, *Greek and Roman Sundials*, New Haven & London: Yale University Press, 1976.
Gibbs: see also *Washington NMAH Catalogue*.
Gingerich, "Applications of Computers to the History of Astronomy": Owen Gingerich, "Applications of High-Speed Computers to the History of Astronomy", *Vistas in Astronomy* 9 (1967), pp. 229-236.
— , "Fake Astrolabes": *idem*, "Astronomical Scrapbook – Fake Astrolabes", *Sky & Telescope*, May, 1982, pp. 465-468.
— , "Paper Instruments with Moving Parts": *idem*, "Astronomical Instruments with Moving Parts", in *G. Turner Festschrift*, pp. 63-74.
— , "Sacrobosco Illustrated": *idem*, "Sacrobosco Illustrated", in *North Festschrift*, pp. 211-224.
— , "Zoomorphic Astrolabes": *idem*, "Zoomorphic Astrolabes and the Introduction of Arabic Star Names into Europe", in *Kennedy Festschrift*, pp. 89-104.
— & King & Saliba, "The ʿAbd al-Aʾimma Astrolabe Forgeries": *idem*, David A. King and George Saliba, "The ʿAbd al-Aʾimma Astrolabe Forgeries", *JHA* 3 (1972), pp. 188-198, repr. in King, *Studies*, B-VI.
Ginzel, *Handbuch der Chronologie*: F. K. Ginzel, *Handbuch der mathematischen und technischen Chronologie*, 3 vols., Leipzig: Hinrichs, 1906-14.
Girke, "Die frühesten islamischen Sternkataloge": Dorothea Girke, "Drei Beiträge zu den frühesten islamischen Sternkatalogen mit besonderer Rücksicht auf Hilfsfunktionen für die Zeitrechnung bei Nacht", Johann Wolfgang Goethe-Univer-sität, Frankfurt am Main, Institut für Geschichte der Naturwissenschaften, Preprint Series no. 8 (1988).
Glowatzki & Göttsche: *Die Tafeln des Regiomontanus*: E. Glowatzki and H. Göttsche: *Die Tafeln des Regiomontanus. Ein Jahrhundertwerk*, (Algorithmus. Studien zur Geschichte der Mathematik und der Naturwissenschaften), Munich: Institut für Geschichte der Naturwissenschaften, 1990.
Gockerell, "Telling Time without a Clock": Nina Gockerell, "Telling Time without a Clock", in *The Clockwork Universe: German Clocks and Automata 1550-1650*, Klaus Maurice and Otto Mayr, eds., New York: Neale Watson, 1980, pp. 131-143.
Goldstein, "Astronomical Instruments in Hebrew": Bernard R. Goldstein, "Description of Astronomical Instruments in Hebrew", in *Kennedy Festschrift*, pp. 105-141.
— , *The Astronomy of Levi ben Gerson*: *idem*, *The Astronomy of Levi ben Gerson (1288-1344)*, New York, *etc*.: Springer, 1985.
— , *Ibn al-Muthannâ on al-Khwārizmī*: *idem*, *Ibn al-Muthannâ's Commentary on the Astronomical Tables of al-Khwârizmî*, New Haven, Conn.: Yale University Press, 1967.
— , "A Medieval Table for Reckoning Time from Solar Altitude", *Scripta Mathematica* 27 (1963), pp. 61-66, repr. in Kennedy *et al*., *Studies*, pp. 293-298.
— , *Studies*: *idem*, *Theory and Observation in Ancient and Medieval Astronomy*, London: Variorum, 1985.
Goldstein Festschrift: *Astronomy and Astrology from the Babylonians to Kepler – Essays Presented to Bernard R. Goldstein on the Occasion of his 65th Birthday*, Peter Barker, Alan C. Bowen, José Chabás, Gad Freudenthal and Tzvi Langermann, eds., a special issue of *Centaurus* 44 (2003).
Goldstein: see also *Aaboe Festschrift*; Chabás & Goldstein; and King & Samsó, "Astronomical Handbooks and Tables".
Goldziher, "Nachmittagzeit im Islam": Ignaz Goldziher, "Die Bedeutung der Nachmittagzeit im Islam", *Archiv für Religionswissenschaft* 9 (1906), pp. 293-302.
Gouk, *Ivory Sundials of Nuremberg*: Penelope Gouk, *The Ivory Sundials of Nuremberg 1500-1700*, Cambridge: Whipple Museum of the History of Science, 1988.
Gotha Catalogue: Wilhelm Pertsch, *Die arabischen Handschriften der Herzoglichen Bibliothek zu Gotha*, 5 vols., Gotha: Fried. Andr. Perthes, 1878-1892, reprinted Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1987.
Graz 1991 Conference Proceedings: *Die Rolle der Astronomie in den Kulturen Mesopotamiens, Beiträge zum 3. Grazer Morgenländischen Symposion (23.-27. September 1991)*, Hannes D. Galter, ed., (Grazer Morgenländische Studien, Band 3), Graz, 1993.
Greenwich Astrolabe Booklet: *The Planispheric Astrolabe*, Greenwich: National Maritime Museum, 1976.
Greenwich Astrolabe Catalogue: Koenraad Van Cleempoel, ed., A catalogue of the astrolabes at the National Maritime Museum, Greenwich, to be published by Oxford University Press.
Greenwich Sundial Catalogue: Hester Higton, ed., *Sundials at Greenwich – A Catalogue of the Sundials, Nocturnals, and Horary Quadrants in the National Maritime Museum*, Oxford: Oxford University Press, 2002.
Gunella, "Abaco para transformar horas": Alessandro Gunella, "Un abaco para transformar horas iguales en disiguales y viceversa", *Analema* no. 32 (Mayo-Agosto 2001), pp. 12-13.

Gunella *et al.*, "Antecedentes del reloj de Rodrigo Zamorano": Alessandro Gunella, "Antecedentes del reloj de Rodrigo Zamorano", Giovanni Paltrinieri, "Sobre un instrumento de Oroncio Finé", and Manuel María Valdés, "Antecedentes del reloj universal de Rodrigo Zamorano (1581)", *Analema* no. 24 (Septiembre-Diciembre 1998), pp. 3-4, 5-7, and 8-9.

Gunther, *Astrolabes*: Robert T. Gunther, *The Astrolabes of the World*, 2 vols., Oxford: Oxford University Press, 1932, repr. in 1 vol., London: The Holland Press, 1976.

— , *Early Science in Oxford*, II and V: *idem*, *Early Science in Oxford*, vol. II: *Astronomy*, Oxford: Clarendon Press, 1923, and vol. V: *Chaucer and Messahalla on the Astrolabe*, Oxford: University Press, 1929.

Haddad: see Kennedy & Pingree & Haddad.

Härtel, "Ludolfus Borchdorp de Brunswik": Helmar Härtel, "Ludolfus Borchdorp de Brunswik", *Braunschweigisches Jahrbuch* 68 (1987), pp. 113-120. [Establishes the identity of the maker of an astrolabe from Einbeck, *ca*. 1330.]

Hahn, *Medieval Mensuration*: Nan L. Britt Hahn, *Medieval Mensuration:* Quadrans vetus *and* Geometrie due sunt partes principales, *Transactions of the American Philosophical Society* 72:8 (1984).

Hakim Mohammed Said: see *Islamabad 1979 Conference Proceedings*.

Hamadanizadeh, "Medieval Islamic Interpolation Schemes": Javad Hamadanizadeh, "A Survey of Medieval Islamic Interpolation Schemes", in *Kennedy Festschrift*, pp. 143-152.

Hartner, "Astrolabe", A-B: Willy Hartner, "The Principle and Use of the Astrolabe", a chapter in *A Survey of Persian Art from prehistoric times to the present*, Arthur Upham Pope, ed., 9 vols., London & New York: Oxford University Press, 1938-39, III, pp. 2530-2554, reprinted with the original plates in *Astrolabica*, no. 1, Feb. 1978, Paris: Société Internationale de l'Astrolabe, and article "Asṭurlāb" in EI_2, both reprinted in *idem*, *Oriens-Occidens*, Hildesheim: Georg Olms, 1968, pp. 287-311 and pp. 312-318.

— , *Oriens-Occidens*, I-II: *idem*, *Oriens-Occidens – Ausgewählte Schriften zur Wissenschafts- und Kulturgeschichte – Festschrift zum 60. Geburtstag*, Hildesheim: Georg Olms, 1968 (I), and *Oriens-Occidens – Ausgewählte Schriften zur Wissenschafts- und Kulturgeschichte, Band II*, Yasukatsu Maeyama, ed., Hildesheim, *etc*.: Georg Olms, 1984 (II).

— & Ruska, "Katalog": Willy Hartner and Julius Ruska, "Katalog der orientalischen und lateinischen Originalhandschriften, Abschriften und Photokopien des Instituts für Geschichte der Medizin und der Naturwissenschaften in Berlin", *Quellen und Studien zur Geschichte der Naturwissenschaften* 7:2/3 (1939), pp. 1-149.

Hartner Festschrift: *ΠΡΙΣΜΑΤΑ – Naturwissenschaftsgeschichtliche Studien – Festschrift für Willy Hartner*, Yasukatsu Maeyama and Walter G. Saltzer, eds., Wiesbaden: Franz Steiner, 1977.

Al-Hassan, "Decline of Islamic Science": Ahmad Y. al-Hassan, "Factors behind the Decline of Islamic Science after the Sixteenth Century", in Sharifah Shifa al-Attas, ed., *Islām and the Challenge of Modernity: Historical and Contemporary Contexts, Proceedings of the Inaugural Symposium on Islām and the Challenge of Modernity ... at ... the International Institute of Islamic Thought and Civilization in Kuala Lumpur*, August 1-5, 1994, Kuala Lumpur: International Institute of Islamic Thought and Civilization, pp. 351-389.

Hastings: see *ERE*.

Hawkins & King, "Orientation of the Kaʿba": see King, *Studies*, C-XII.

Heidarzadeh, "Astronomy in Iran": Tofigh Heidarzadeh, "From the Maragha School to the Darolfonun - A Historical Review of Astronomy in Iran from the 13th to the 19th Century", paper presented at the Symposium on Science and Technology in the Turkish and Islamic World, Istanbul, 3-5 June, 1994. [Not printed in the *Proceedings*.]

Heinen, *Islamic Cosmology*: Anton Heinen, *Islamic Cosmology: A Study of as-Suyūṭī's* al-Hayʾa al-sanīya fi-l-hayʾa al-sunnīya, Beiruter Texte und Studien (Orient-Institut der Deutschen Morgenländischen Gesellschaft), Band 27, Beirut (for Franz Steiner Verlag, Wiesbaden), 1982.

Helmecke, "Berliner Astrolab des Muḥammad Zamān": Gisela Helmecke, "Das Berliner Astrolab des Muḥammad Zamān al-Mašhadī", *Forschungen und Berichte der Staatlichen Museen zu Berlin, Hauptstadt der DDR* (Berlin: Akademie-Verlag) 25 (1985), pp. 129-142 and pls. 31-32.

Higgins, "Classification of Sundials": Kathleen Higgins, "The Classification of Sundials", *Annals of Science* 9 (1953), pp. 342-358.

Higton, *Portable Sundials*: Hester K. Higton, *Sundials – An Illustrated History of Portable Dials*, London: Philip Wilson Publishers, Ltd., 2001.

Higton: see also *Greenwich Sundial Catalogue*.

Hill, "Al-Bīrūnī's Mechanical Calendar": Donald R. Hill, "Al-Bīrūnī's Mechanical Calendar", *AS* 42 (1985), pp. 139-163, repr. in *idem*, *Studies*, XIV, and also in Field & Hill & Wright, *Byzantine and Arabic Gearing*.

— , *Islamic Science and Engineering*: *idem*, *Islamic Science and Engineering*, Edinburgh: Edinburgh University Press, 1994.

— , *Studies*: *idem*, *Studies in Medieval Islamic Technology: From Philo to al-Jazarī – from Alexandria to Diyār Bakr*, David A. King, ed., Aldershot, Brookfield USA, Singapore and Sydney: Ashgate-Variorum, 1998.
Hill: see also Field & Wright & Hill.
G. F. Hill, *Arabic Numerals in Europe*: G. F. Hill, *The Development of Arabic Numerals in Europe Exhibited in Sixty-Four Tables*, Oxford: Clarendon Press, 1915.
Hinks, "Retro-Azimuthal Equidistant Projection": A. Hinks, "A Retro-Azimuthal Equidistant Projection of the Whole Sphere", *Geographical Journal* 73:2 (1929), pp. 245-247.
Hinz, *Islamische Masse und Gewichte*: Walter Hinz, *Islamische Masse und Gewichte*, Leiden: E. J. Brill, 1955.
HM: *Historia Mathematica*.
History of Cartography: *The History of Cartography*, J. B. Harley and David Woodward, eds., vol. I: *Cartography in Prehistoric, Ancient, and Medieval Europe and the Mediterranean*; vol. II, book 1: *Cartography in the Traditional Islamic and South Asian Societies*, Chicago, Ill., & London: The University of Chicago Press, 1987 and 1992. [See also the review of II:1 by Karen C. Pinto in *Harvard Middle Eastern and Islamic Review* 1:1 (1994), pp. 109-115.]
History of Oriental Astronomy: see *New Delhi 1985 Colloquium Proceedings*.
Hogendijk, "Al-Khwārizmī's Sine of the Hours": Jan P. Hogendijk, " Al-Khwārizmī's Tables of the "Sine of the Hours" and the Underlying Sine Table", *Historia scientiarum* 42 (1991), pp. 1-12.
— , "Mathematical Astrology": *idem*, "Mathematical Astrology in the Islamic Tradition" (dealing with houses, rays and progressions), in *Cambridge DI 1998 Conference Proceedings*, to appear.
— , "al-Muʿtaman ibn Hūd": *idem*, "al-Muʾtaman ibn Hūd. 11th Century King of Saragossa and Brilliant Mathematician", *HM* 22 (1995), pp. 1-18.
— , "al-Nayrīzī on the Qibla at Baghdad": *idem*, "Al-Nayrīzī's Mysterious Determination of the Azimuth of the Qibla at Baghdad", *Sciamvs* 1 (2000), pp. 49-70.
— , "Seasonal Hour Lines on Astrolabes and Sundials": *idem*, "The Contributions by Abū Naṣr ibn ʿIrāq and al-Ṣāghānī to the Theory of Seasonal Hour Lines on Astrolabes and Sundials", *ZGAIW* 14 (2001), pp. 1-30.
— , "Tables for Casting the Rays": *idem*, "The Mathematical Structure of Two Islamic Astrological Tables for Casting the Rays", *Centaurus* 32 (1989), pp. 171-202.
— , "Three Islamic Lunar Crescent Visibility Tables": *idem*, "Three Islamic Lunar Crescent Visibility Tables", *JHA* 19 (1988), pp. 29-44.
Hogendijk: see also Luckey, *Thesis*.
Hoyland, *Seeing Islam as Others Saw it*: Robert G. Hoyland, *Seeing Islam as Others Saw it: A Survey and Evaluation of Christian, Jewish and Zoroastrian Writings on Early Islam*, Princeton, NJ: The Darwin Press, 1997.
HM: *Historia Mathematica*.
HS: *History of Science*.
IAM: see *Islamic Astronomy and Mathematics*.
Ibish, ed., *Editing Islamic Manuscripts on Science*: *Editing Islamic Manuscripts on Science*, Yusuf Ibish, ed., London: Al-Furqan Islamic Heritage Foundation, 1999.
Ibn al-Nadīm, *Fihrist*: Ibn al-Nadīm, *Kitāb al-Fihrist*, ed. Gustav Flügel (1871), repr. Beirut: Khayats, 1964, transl. by Bayard Dodge as *The Fihrist of al-Nadīm*, 2 vols., New York and London: Columbia University Press, 1970.
Ibn Nujaym, *Rafʿ al-ghishāʿ*: References in **IV** are to MS Cairo Ṭalʾat *majāmīʿ* 200,5 (fols. 27r-30r, copied 1733/34) of his treatise *Rafʿ al-ghishāʾ ʿan waqtayi ʾl-ʿaṣr wa-ʾl-ʿishāʾ*.
Ibn Raḥīq, *al-Mawāqīt*: References in **IV** are to MS Berlin Ahlwardt 5664 of his treatise on folk astronomy.
Ibn al-Raqqām: see Carandell.
Id, "Analemma for Ascensions": Yusuf Id, "An Analemma Construction for Right and Oblique Ascensions", *The Mathematics Teacher* 62 (1969), pp. 669-672, repr. in Kennedy *et al*., *Studies*, pp. 495-498.
İhsanoğlu, "Western Science in the Ottoman World": Ekmeleddin İhsanoğlu, "Introduction of Western Science to the Ottoman World: a Case Study of Modern Astronomy (1660-1860)", in *Istanbul 1987 Symposium Proceedings*, pp. 67-120.
— *et al*., *Ottoman Astronomical Literature*: *idem et al*., *Osmanlı astronomi literatürü tarihi – History of Astronomy Literature during the Ottoman Period* (in Turkish), 2 vols., (Studies and Sources on the History of Science, Series No. 7), Istanbul: Research Centre for Islamic History, Art and Culture (IRCICA), 1997.
— , *Ottoman Geographical Literature*: *eidem*, *Osmanlı coğrafya literatürü tarihi – History of Geographical Literature during the Ottoman Period* (in Turkish), 2 vols., (Studies and Sources on the History of Science, Series No. 9), Istanbul: Research Centre for Islamic History, Art and Culture (IRCICA), 2000.
— , *Ottoman Mathematical Literature*: *eidem*, *Osmanlı matematik literatürü tarihi – History of Astronomy Literature during the Ottoman Period* (in Turkish), 2 vols., (Studies and Sources on the History of Science, Series No. 8), Istanbul: Research Centre for Islamic History, Art and Culture (IRCICA), 1999.

İhsanoğlu: see also *Istanbul 1987 Symposium Proceedings*, and *Istanbul 1991 and 1994 Symposia Proceedings*.
Ilyas, *Astronomy of Islamic Times*: Mohammad Ilyas, *Astronomy of Islamic Times for the Twenty-first Century*, London & New York: Mansell, 1988.
— , *Islamic Calendar, Times & Qibla*: *idem*, *A Modern Guide to the Astronomical Calculations of Islamic Calendar, Times & Qibla*, Kuala Lumpur: Berita, 1984.
Imad-ad-Dean Ahmad, *Signs in the Heavens: A Muslim Astronomer's Perspective on Religion and Science*, Beltsville, Md.: Writers' Inc.-International, 1992.
Index Islamicus: *Index Islamicus,* numerous vols. to date, Cambridge: W. Heffer & Sons, starting in 1958, continued up to the present as *The Quarterly Index Islamicus*, London: Bowker-Saur, 4th edn. (including 1906-1999) available on CD-ROM from E. J. Brill, Leiden.
Ingolstadt Apian Exhibition: see Röttel.
Irani, "Arabic Numeral Forms": Rida A. K. Irani, "Arabic Numeral Forms", *Centaurus* 4 (1955), pp. 1-12, repr. in Kennedy *et al*., *Studies*, pp. 710-721.
— , *The Jadwal al-Taqwīm of Ḥabash al-Ḥāsib*, unpublished Master's dissertation, American University of Beirut, 1956.
Islamabad 1979 Conference Proceedings: *History and Philosophy of Science – Proceedings of the International Congress of the History and Phlosophy of Science, Islamabad, 8-13 December, 1979*, Hakim Mohammed Said, ed., Nazimabad, Pakistan: Hamdard Foundation Press, n.d. (*ca*. 1980).
Islamic Geography: *Islamic Geography*, Fuat Sezgin, with Mazen Amawi, Carl Ehrig-Eggert and Eckhard Neubauer *et al*., eds., 278 vols. to date, Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1992 to present. [Facsimile reprints of early writings, mainly 19th and early-20th century.]
Islamic Mathematics and Astronomy: *Islamic Mathematics and Astronomy*, *eidem*, eds., 112 vols. to date, Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1997 to present. [Reprints of texts and studies mainly from the 19th and early 20th centuries. Materials relating to al-Khwārizmī feature in vols. 1-7; al-Farghānī 9-10, 68; al-Battānī 11-13; Thābit ibn Qurra 21-22; Ibn Yūnus 25; al-Ṣūfī 26; Ibrāhīm ibn Sinān 27, 101; Abū Naṣr 28, 37; al-Bīrūnī 29-36, 74; Ibn al-Zarqālluh 39-40; al-Marrākushī 41-42; Ibn al-Bannāʿ 43-44; ʿUmar al-Khayyām 45-46; al-Ṭūsī *et al*. 47-51; Ulugh Beg 52-55; al-Kāshī 56, 84; Ibn al-Haytham 57-58, 75; Abu ʿl-Wafāʾ 60-61; al-Sijzī 66; Maimonides 67; Alfonso X 98-99; calendrics 64-65; astronomical instruments 85-96; Sayılı's *Observatory in Islam* 97; Nallino's *Taʾrīkh ʿilm al-falak* 100.]
Istanbul 1977 Symposium Proceedings: *Proceedings of the International Symposium on the Observatories in Islam (Istanbul, 1977)*, Muammer Dizer, ed., Istanbul: Milli Eğitim Basımevi, 1980.
Istanbul 1987 Symposium Proceedings: *Transfer of Modern Science & Technology to the Muslim World*, Ekmeleddin İhsanoğlu, ed., Istanbul: Reseach Centre for Islamic History, Art and Culture (IRCICA), 1982.
Istanbul 1991 and 1994 Symposia Proceedings: *Science in Islamic Civilisation*, Ekmeleddin İhsanoğlu and Feza Günergun, eds., Istanbul (IRCICA) 2000.
Istanbul: see also *Kandilli Instrument Handbook*.
Jachimowicz, "Islamic Cosmology": E. Jachimowicz, "Islamic Cosmology", in C. Blacker and M. Loewe, eds., *Ancient Cosmologies*, London: George Allen & Unwin Ltd., 1985, pp. 143-171.
Jackson, "Retro-Azimuthal Projections": J. E. Jackson, "Retro-Azimuthal Projections", *Survey Review* 19 (1968), pp. 319-328.
Janin, "Cadran solaire de Damas": Louis A. Janin, "Le cadran solaire de la Mosquée Umayyade à Damas", *Centaurus* 16 (1972), pp. 285-298, repr. in Kennedy & Ghanem, eds., *Ibn al-Shāṭir*, pp. 107-121.
— , "Un cadran solaire grec d'Afghanistan": *idem*, "Un cadran solaire grec à Aï Khanoum, Afghanistan", *L'Astronomie* (Paris) 92 (1978), pp. 357-362.
— , "Gnomonique tunisienne": *idem*, "Quelques aspects récents de la gnomonique tunisienne", *Revue de l'Occident musulman et de la Méditerranée* 23 (1977), pp. 202-221.
— , "Projection horizontale": *idem*, "Astrolabe et cadran solaire en projection horizontale", *Centaurus* 22 (1979), pp. 298-314.
— , "Ar-Rūdānī sur l'astrolabe sphérique": *idem*, "Un texte d'ar-Rūdānī sur l'astrolabe sphérique", *Annali dell'Istituto e Museo di Storia della Scienza di Firenze* 3 (1978), pp. 71-75. [See also Pellat, "L'astrolabe sphérique d'al-Rūdānī".]
Janin & King, "Cadran solaire d'Ibn Ṭūlūn": see King, *Studies*, B-XVI.
— & — , "Ibn al-Shāṭir's Compendium": see King, *Studies*, B-XII.
Janin & Rohr, "Deux astrolabes-quadrants turcs", *idem* and R. R. J. Rohr, "Deux astrolabes-quadrants turcs", *Centaurus* 19 (1975), pp. 108-124.
JAOS: *Journal of the American Oriental Society*.
Jaubert, "Capteurs de vents d'Égypte": Olivier Jaubert, "Capteurs de vents d'Égypte – Essai de typologie", *Annales islamologiques* (Institut Français d'Archéologie Orientale du Caire) 25 (1995), pp. 169-231.

Jensen, "Abū Naṣr's *Table of Minutes*": Claus Jensen, "Abū Naṣr's Approach to Spherical Trigonometry as Developed in His Treatise *The Table of Minutes*", *Centaurus* 16 (1971), pp. 1-19.

— , "The Lunar Theories of al-Baghdādī": *idem*, "The Lunar Theories of al-Baghdādī", *AHES* 8 (1971-72), pp. 321-328.

JHA: *Journal for the History of Astronomy*.

JHAS: *Journal for the History of Arabic Science* (Institute for the History of Arabic Science, Aleppo).

Jiminez, "La qibla extraviada": Alfonso Jiminez, "La qibla extraviada", in *Cuadernos de Madīnat al-Zahrāʿ*, vol. III, Cordova, 1991, pp. 189-209.

JNES: *Journal of Near Eastern Studies*.

Jones, *Byzantine Astronomical Manual*: Alexander Jones, *An Eleventh-Century Manual of Arabo-Byzantine Astronomy*, (*Corpus des astronomes byzantins* III), Amsterdam: J. C. Gieben, 1987.

JRAS: *Journal of the Royal Asiatic Society*.

Juschkewitsch, *Mathematik im MA*: Adolf P. Juschkewitsch, *Geschichte der Mathematik im Mittelalter*, (first published in Russian in 1961), Leipzig: B. G. Teubner, 1964. [See also Youschkevitsch, *Mathématiques arabes*.]

Kamal al-Din, *Tables*: Mathematical tables and graphs by Husayn Kamal al-Din (then Professor in the Civil Engineering Dept., Riyadh University) in 3 volumes bearing the titles: 1 *Kebla* [*sic*] *Directions for the Whole World*, 2 *Praying Time Tables for the Whole World*, 3 *Curves of Praying Times for the Whole World*, Cairo: Dār al-Fikr al-ʿArabī, for The Conference of Islamic Jurisprudence at the Islamic University of Imām Muḥammad ibn Saʿūd, Saudi Arabia, n.d. [*ca*. 1970 ?]. [Copy inspected at Aligarh Muslim University in 1978, by which time the book was no longer available at the publishers in Cairo.]

Kandilli Instrument Handbook: Muammer Dizer, *Astronomi hazineleri*, (B.Ü. Yayınları no. 404), Istanbul: Boğaziçi Üniversitesi and Kandilli Rasathanesi, 1986.

Kaye, *The Observatories of Jai Singh*, A-B: George R. Kaye, *The Astronomical Observatories of Jai Singh,* (*Archaeological Survey of India*, New Imperial Series, vol. XL), and *A Guide to the Observatories at Delhi, Jaipur*, *etc*., Calcutta, 1918 and 1920, both repr. in *IAM*, vol. 93.

Kennedy, "al-Bīrūnī on Prayer-Times": Edward Stewart Kennedy, "al-Bīrūnī on the Muslim Times of Prayer", in *The Scholar and the Saint: Studies in Commemoration of Abū'l-Rayḥān al-Bīrūnī and Jalāl al-Dīn al-Rūmī*, Peter J. Chelkowski, ed., New York: New York University Press, 1975, pp. 83-94, repr. in *idem et al*., *Studies*, pp. 299-310.

— , *al-Bīrūnī's* Shadows: see al-Bīrūnī, *Shadows*.

— , "Comets in Islamic Astronomy and Astrology": *idem*, "Comets in Islamic Astronomy and Astrology", *JNES* 16 (1957), pp. 44-51, repr. in *idem et al*., *Studies*, pp. 311-318.

— , "The Digital Computer and the History of the Exact Sciences": *idem*, "The Digital Computer and the History of the Exact Sciences", *Centaurus* 12 (1967), pp. 107-113, repr. in *idem et al*., *Studies*, pp. 385-393.

— , "The Equatorium of Abū al-Ṣalt": *idem*, "The Equatorium of Abū al-Ṣalt", *Physis* 12 (1970), pp. 73-81, repr. in *idem et al*., *Studies*, pp. 481-491.

— ,"Exact Sciences in Iran", A-C: *idem*, "The Exact Sciences during the Abbasid Period", *Cambridge History of Iran*, IV, pp. 378-395 (A); "The Exact Sciences in Iran under the Seljuqs and Mongols", *ibid*., V, pp. 659-679 (B); "The Exact Sciences in Iran under the Timurids", *ibid*., VI, pp. 568-580.

— , "History of Trigonometry": *idem*, "Overview of the History of Trigonometry" (first published 1969), repr. in *idem et al*., *Studies*, pp. 3-29.

— , "Ibn al-Aʿlam's Tables": "The Astronomical Tables of Ibn al-Aʿlam", *JHAS* 1 (1977), pp. 13-23.

— , "Ibn al-Raqqām's Tables": *idem*, "The Astronomical Tables of Ibn al-Raqqām, a Scientist of Granada", *ZGAIW* 11 (1997), pp. 35-72.

— , "Islamic Mathematical Geography": *idem*, "Mathematical Geography", in *EHAS*, I, pp. 185-201.

— , "al-Kāshī on Instruments": *idem*, "al-Kāshī's Treatise on Astronomical Observational Instruments", *JNES* 20 (1961), pp. 98-108, repr. in *idem et al*., *Studies*, pp. 394-404.

— , *al-Kāshī's Equatorium*: *idem*, *The Planetary Equatorium of Jamshīd Ghiyāth al-Dīn al-Kāshī (d. 1429) ...* , Princeton N.J.,: Princeton University Press, 1960.

— , "Spherical Astronomy in al-Kāshī's *Khāqānī Zīj*": see *idem*, *Studies*, VII.

— , *Studies*: *idem*, *Astronomy and Astrology in the Medieval Islamic World*, (Variorum Collected Studies Series: CS600), Aldershot & Brookfield, Vt.: Ashgate-Variorum, 1998. Contains:

I "Ḥabash al-Ḥāsib on the Melon Astrolabe" (with Richard P. Lorch), first publication.

VII "Spherical Astronomy in Kāshī's *Khāqānī Zīj*", *ZGAIW* 2 (1990), pp. 1-46.

— , "Al-Ṣūfī on the Celestial Globe": *idem*, "Al-Ṣūfī on the Celestial Globe", *ZGAIW* 5 (1989), pp. 48-93.

— , "*Zīj* Survey": *idem*, "A Survey of Islamic Astronomical Tables", *Transactions of the American Philosophical Society*, N. S., 46:2 (1956), pp. 123-177, repr. with separate pagination, n.d. [*ca*. 1990].

— & Davidian, "Al-Qāyinī on Twilight": *idem* and Marie-Louise Davidian, "Al-Qāyinī on the Duration of Dawn and Twilight", *JNES* 20 (1961), pp. 145-153, repr. in *idem et al*., *Studies*, pp. 284-292.

— & Debarnot, "Two Mappings of al-Bīrūnī": *idem* and Marie-Thérèse Debarnot, "Two Mappings Proposed by al-Bīrūnī", *Zeitschrift für Geschichte der arabisch-islamischen Wissenschaften* (Frankfurt am Main) 1 (1984), pp. 144-147, repr. in *idem*, *Studies*, V.

— & Faris, "Yaḥyā b. Abī Manṣūr on Solar Eclipses": *idem* and Nazim Faris, "The Solar Eclipse Technique of Yaḥyā b. Abī Manṣūr", *JHA* 1 (1970), pp. 20-38, repr. in *idem et al.*, *Studies*, pp. 185-203.

— & Ghanem, *eds., Ibn al-Shāṭir*: *idem* and Imad Ghanem, eds., *The Life and Work of Ibn al-Shāṭir: an Arab Astronomer of the Fourteenth Century*, Aleppo: Institute for the History of Arabic Science, 1976.

— & Id, "Ḥabash's Analemma for the Qibla": *idem* & Yusuf Id, "A Letter of al-Bīrūnī: Ḥabash al-Ḥāsib's Analemma for the Qibla", *Historia Mathematica* 1 (1973), pp. 3-11, repr. in Kennedy *et al.*, *Studies*, pp. 621-629.

— & Kennedy, *Islamic Geographical Coordinates*: *idem* and Mary Helen Kennedy, *Geographical Coordinates of Localities from Islamic Sources*, Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1987.

— & King, "Ibn al-Majdī's Tables for Calculating Ephemerides": *idem* and David A. King, "Ibn al-Majdī's Tables for Calculating Ephemerides", *JHAS* 4 (1980), pp. 48-68, repr. in King, *Studies*, A-VI.

— — , "Indian Astronomy in Fourteenth-Century Fez": *eidem*, "Indian Astronomy in Fourteenth-Century Fez: The Versified *Zīj* of al-Qusunṭīnī [*sic*: read al-Qusunṭīnī]", *JHAS* 6 (1982), pp. 3-45, repr. in King, *Studies*, A-VIII.

— & Krikorian-Preisler, "Projecting the Rays": *idem* and H. Krikorian-Preisler, "The Astrological Doctrine of Projecting the Rays", *Al-Abhath* 25 (1972), pp. 3-15, repr. in Kennedy *et al.*, *Studies*, pp. 372-384.

— & Kunitzsch & Lorch, *Melon Astrolabe*: *idem*, Paul Kunitzsch and Richard P. Lorch, *The Melon-Shaped Astrolabe in Arabic Astronomy*, (*Boethius: Texte und Abhandlungen zur Geschichte der Mathematik und der Naturwissenschaften*, Band 43), Stuttgart: Franz Steiner, 1999.

— & Pingree, *Astrological History of Māshāʾallāh*: *idem* and David Pingree, *The Astrological History of Māshāʾallāh*, Cambridge, Ma.: Harvard University Press, 1971.

— & Pingree & Haddad, *al-Hāshimī on Astronomical Tables*: *idem*, David Pingree and Fuad I. Haddad, *The Book of Reasons behind Astronomical Tables* (Kitāb ʿilal al-zījāt) *by ʿAlī ibn Sulaymān al-Hāshimī*, Delmar, N.Y.: Scholars' Facsimiles & Reprints, 1981.

Kennedy & Ukashah, "The Chandelier Clock of Ibn Yūnis": *idem* and Walid Ukashah, "The Chandelier Clock of Ibn Yūnis", *Isis* 60 (1969), pp. 543-545, repr. in Kennedy *et al.*, *Studies*, pp. 499-501.

Kennedy *et al.*, *Studies*: E. S. Kennedy, Colleagues and Former Students, *Studies in the Islamic Exact Sciences*, David A. King and Mary Helen Kennedy, eds., Beirut: American University of Beirut, 1983. Contains:

- "Overview of the History of Trigonometry", *31st Yearbook of the National Council of Teachers of Mathematics*, Washington, D.C., 1969, pp. 333-359, repr. here pp. 3-29.
- "The Solar Eclipse Technique of Yaḥyā b. Abī Manṣūr" (with Nazim Faris), *JHA* 1 (1970), pp. 20-38, repr. here pp. 185-203.
- Mark Lesley, Mark, "Bīrūnī on Rising Times and Daylight Lengths", *Centaurus* 5 (1957), pp. 121-141, repr. here pp. 253-273.
- Marie-Louise Davidian, "al-Bīrūnī on the Time of Day from Shadow Lengths", *JAOS* 80 (1960), pp. 330-335, repr. here pp. 274-279.
- Nadi Nadir, "Abū al-Wafāʿ on the Solar Altitude", *The Mathematics Teacher* 53 (1960), pp. 460-463, repr. here pp. 280-283.
- "Al-Qāyinī on the Duration of Dawn and Twilight" (with Marie-Louise Davidian), *JNES* 20 (1961), pp. 145-153, repr. here, pp. 284-292.
- Bernard R. Goldstein, "A Medieval Table for Reckoning Time from Solar Altitude", *Scripta Mathematica* 27 (1963), pp. 61-66, repr. here pp. 293-298.
- "al-Bīrūnī on the Muslim Times of Prayer", in *The Scholar and the Saint ...* , P. J. Chelkowski, ed., New York, 1975, pp. 83-94, repr. here pp. 299-310.
- "Comets in Islamic Astronomy and Astrology", *JNES* 16 (1957), pp. 44-51, repr. here, pp. 311-318.
- "The Astrological Doctrine of Projecting the Rays" (with H. Krikorian-Preisler), *Al-Abhath* 25 (1972), pp. 3-15, repr. here pp. 372-384.
- "The Digital Computer and the History of the Exact Sciences", *Centaurus* 12 (1967), pp. 107-113, repr. here pp. 385-393.
- "al-Kāshī's Treatise on Astronomical Observational Instruments", *JNES* 20 (1961), pp. 98-108, repr. here pp. 394-404.
- "The Equatorium of Abū al-Ṣalt", *Physis* 12 (1970), pp. 73-81, repr. here pp. 481-491.
- "The Chandelier Clock of Ibn Yūnis" (with Walid Ukashah), *Isis* 60 (1969), pp. 543-545, repr. here, pp. 499-501.
- Yusuf Id, "An Analemma Construction for Right and Oblique Ascensions", *The Mathematics Teacher* 62 (1969), pp. 669-672, repr. here pp. 495-498.
- "A Letter of al-Bīrūnī: Ḥabash al-Ḥāsib's Analemma for the Qibla" (with Yusuf Id), *Historia Mathematica* 1 (1973), pp. 3-11, repr. here pp. 621-629.
- Rida A. K. Irani, "Arabic Numeral Forms", *Centaurus* 4 (1955), pp. 1-12, repr. here pp. 710-721.

Kennedy Festschrift: *From Deferent to Equant*: *Studies in the History of Science in the Ancient and Medieval Near East in Honor of E. S. Kennedy*, David A. King and George Saliba, eds., *Annals of the New York Academy of Sciences* 500 (1987). Contains:

- Marie-Thérèse Debarnot, "The *Zīj* of Ḥabash al-Ḥāsib: A Survey of MS Istanbul Yeni Cami 784/2", pp. 35-69.
- Javad Hamadanizadeh, "A Survey of Medieval Islamic Interpolation Schemes", pp. 143-152.
- D. A. King, "Some Early Islamic Tables for Lunar Crescent Visibility", pp. 185-225.

• G. Rosinska, "Tables of Decimal Trigonometric Functions from *ca.* 1450 to *ca.* 1550", pp. 419-426.

Kennedy: see also al-Bīrūnī, *Geography*, and *idem*, *Shadows*; Destombes & Kennedy; and King, *Studies*, A-VI and A-VIII (joint papers).

Kessler: "Mecca-Oriented Architecture in Cairo", A-C: Christel Kessler, "Mecca-Oriented Architecture and Urban Growth of Cairo", *Atti del Terzo Congresso di Studi Arabi e Islamici* (Ravello, 1966), Naples, 1967, p. 425 (A); "Funerary Architecture within the City", *Colloque International sur l'Histoire du Caire*, Cairo, 1969, pp. 257-267 (B); and "Mecca-Oriented Urban Architecture in Mamluk Cairo: The Madrasa-Mausoleum of Sultan Shaʿbān II, in *In Quest of an Islamic Humanism: Arab an Islamic Studies in Memory of Mohamed Nowaihi*, Cairo: American University of Cairo Press, 1984, pp. 97-108 (C).

Khalili Collection: see *London Khalili Collection ... ...* .

al-Khwārizmī: see Suter, *al-Khwārizmī*; Neugebauer, *al-Khwārizmī*; Goldstein, *Ibn al-Muthannā on al-Khwārizmī*; and also King, "al-Khwārizmī", and Rosenfeld *et al*., eds., *Al-Khorezmi*.

ʿAbdallāh al-Khwārizmī, *Mafātīḥ al-ʿulūm*: Gerlof van Vloten, ed., *Mafâtîh al-olûm explicans vocabula technica scientiarum tam arabum quam peregrinorum auctore Abû Abdallah Mohammed ibn Ahmed ibn Jûsuf al-Kâtib al-Khowarezmi*, Leiden, 1895, also pirated edition: Cairo: Maṭbaʿat al-Sharq, n. d. [*ca*. 1960?], and Beirut, 1984 edn. [based on the 1895 edn.?].

Kimerling, "Adding the Qibla to Geographic Education": A. Kimerling, "Adding the Qibla to Geographic Education", *Journal of Geography* 101 (2002), pp. 20-26.

♦ King, "Architecture and Astronomy": David A. King, "Architecture and Astronomy: The Ventilators of Medieval Cairo and their Secrets", *JAOS* 104:1 (1984), pp. 97-133. [A new version is included here as **VIIb**.]

— , "Andalusī Astronomical Instruments": *idem*, "[Andalusī Astronomical Instruments]", in *Madrid & New York 1992 Exhibition Catalogue*, pp. 376-383.

— , "Andalusī Sundials": see *idem*, *Studies*, B-XV, also *idem*, "Cuadrantes solares andalusíes".

♦ — , "Aspekte": *idem*, "Aspekte angewandter Wissenschaften in Moscheen und Klöstern", *Berichte zur Wissenschaftsgeschichte* 18 (1995), pp. 85-95 and 137-149. [A translation of "Aspects of Science in Mosques and Monasteries", unpublished paper presented at the conference "Science and Religion in Islam, Judaism and Christendom" organised by Prof. David C. Lindberg, Madison, Wisc., April 15-17, 1993. A new version is included here as **IX**.]

— , "Astronomical Alignments in Medieval Islamic Religious Architecture": see *idem*, *Studies*, C-XIII.

— , "Astronomical Instruments between East and West": *idem*, "Astronomical Instruments between East and West", in *Kommunikation zwischen Orient und Okzident – Alltag und Sachkultur*, Harry Kühnel, ed., (*Sitzungsberichte der Philosophisch-Historischen Klasse der Österreichischen Akademie der Wissenschaften*, Band 619, *Veröffentlichungen des Instituts für Realienkunde des Mittelalters und der Frühen Neuzeit*, Nr. 16), Vienna: Österreichische Akademie der Wissenschaften, 1994, pp. 143-198.

— , "Astronomical Timekeeping in Medieval Cairo": see *idem*, *Studies*, A-IX.

— , "Astronomical Timekeeping in Ottoman Turkey": see *idem*, *Studies*, A-XII.

— , "Astronomical Timekeeping in Medieval Syria": see *idem*, *Studies*, A-X.

— , "Astronomy in Fatimid Egypt": *idem*, "Aspects of Fatimid Astronomy: From Hard-Core Mathematical Astronomy to Architectural Orientations in Cairo", in Marianne Barrucand, ed., *L'Égypte Fatimide: son art et son histoire – Actes du colloqie organisé à Paris les 28, 29 et 30 mai 1998*, Paris: Presses de l'Université de Paris-Sorbonne, 1999, pp. 497-517.

— , "Astronomy of the Mamluks": see *idem*, *Studies*, A-III.

— , "Astronomy in the Maghrib": *idem*, "On the History of Astronomy in the Medieval Maghrib", in *Études Philosophiques et Sociologiques Dédiées à Jamal ed-Dine Alaoui*, Université Sidi Mohamed Ben Abdallah, Publications de la Faculté des Lettres et des Sciences Humaines Dhar El Mahraz – Fès, N° Spécial 14 (Département de Philosophie, Sociologie et Psychologie), Fez, 1998 [published 1999], pp. 27-61.

— , "Astronomie en Syrie": *idem*, "L'astronomie en Syrie à l'époque islamique", in *Paris IMA 1993-94 Exhibition Catalogue*, pp. 386-395 and 432-443.

— , *Astronomy in Yemen*: *idem*, *Mathematical Astronomy in Medieval Yemen – A Bio-Bibliographical Survey*, (Publications of the American Research Center in Egypt), Malibu, Ca.: Undena Publications, 1983.

— ,"al-Bazdawī on the Qibla in Tranoxania": *idem*, "al-Bazdawī on the Qibla in Early Islamic Tranoxania", *JHAS* 7 (1983/1986), pp. 3-38.

— , "Byzantine Astronomy": *idem*, an essay review of Pingree, *Astronomical Works of Gregory Chioniades*, and Jones, "Byzantine Astronomical Manual", in *Isis* 82 (1991), pp. 116-118.

— , "Catalogue of Medieval Astronomical Instruments", A-B: *idem*, "Medieval Astronomical Instruments: A Catalogue in Preparation", *Bulletin of the Scientific Instrument Society* no. 31 (Dec., 1991), pp. 3-7, and "1992 – A Good Year for Medieval Astronomical Instruments", *ibid*. no. 36 (March, 1993), pp. 17-18. [For a table of contents of the Frankfurt catalogue see www.uni-frankfurt.de/fb13/ign/instrument-catalogue.html.]

– , "Cuadrantes solares andalusíes": *idem*, "Los cuadrantes solares andalusíes", in *Madrid MAN 1992 Exhibition Catalogue*, pp. 89-102.

— , *Ciphers of the Monks*: *idem*, *The Ciphers of the Monks – A Forgotten Number Notation of the Middle Ages*, (*Boethius: Texte und Abhandlungen zur Geschichte der Mathematik und der Naturwissenschaften*, Band 44), Stuttgart: Franz Steiner, 2001.
— , "Earliest Muslim Geodetic Measurements": *idem*, "Too Many Cooks ... – A Newly-Rediscovered Account of the First Muslim Geodetic Measurements", *Suhayl* 1 (2000), pp. 207-241.
— , "Earliest Qibla Methods": see *idem*, *Studies*, C-XIV.
— , "Geography of Astrolabes": *idem*, "Bringing Astronomical Instruments Back to Earth: The Geographical Data on Medieval Astrolabes (to *ca*. 1100)", in *North Festschrift*, pp. 3-53.
— , *Ibn Yūnus*: *idem*, *The Astronomical Works of Ibn Yūnus*, unpublished doctoral dissertation, Yale University, 1972. [Available from ProQuest.com (formerly University Microfilms, Ann Arbor, Mich.), no. 7229740]
— , "Ibn Yūnus and the Pendulum": *idem*, "Ibn Yūnus and the Pendulum: A History of Errors", *AIHS* 29 (1979), pp. 35-52.
— , "Islamic Astronomical Tables": see *idem*, *Studies*, A-I.
— , "Islamic Astronomy", A-B: *idem*, "Islamic Astronomy", in *Astronomy before the Telescope*, Christopher B. F. Walker, ed., London: British Museum Press, 1996, repr. 1999, pp. 143-174 (A); and "Mathematical Astronomy in Islamic Civilisation", in Selin, *Astronomy across Cultures*, pp. 585-613 (B).
— , "Islamic Folk Astronomy", A-B: *idem*, "Folk Astronomy in the Service of Religion: The Case of Islam", in *Astronomies and Cultures*, Clive L. N. Ruggles & Nicholas J. Saunders, eds., Niwot, Co.: University Press of Colorado, 1993 [published 1994], pp. 124-138 (A), with a summary in *Archaeoastronomy in the 1990s*, Clive L. N. Ruggles, ed., Loughborough (U.K.): Group D Publications, Ltd., 1993, p. 346; and *idem*, "Applications of Folk Astronomy and Mathematical Astronomy to Aspects of Muslim Ritual", *The Arabist (Budapest Studies in Arabic)*, 13-14 (1995) (*Proceedings of the XIVth UEAI Congress, Budapest, 1988*, Alexander Fodor, ed., Part I), pp. 251-274 (B).
— , "Islamic Scientific Manuscripts": *idem*, "Some Remarks on Islamic Scientific Manuscripts and Instrument and Past, Present and Future Research", in John Cooper, ed., *The Significance of Islamic Manuscripts*, London: Al-Furqan Islamic Heritage Foundation, 1992, pp. 115-144 (also available in Arabic).
— , "Islamic Trigonometric Tables": *idem*, "A Brief Survey of Medieval Islamic Trigonometric Tables", paper delivered at the Colloquium "Innovation et transmission dans les mathématiques arabes", Paris, Institut du Monde Arabe, 25-27 Oct., 1990. [As yet unpublished.]
— , "Italian Astrolabe": *idem*, "A Remarkable Italian Astrolabe from *ca*. 1300 – Witness to an Ingenious Islamic Tradition of Non-Standard Astrolabes", in *Miniati Festschrift*, pp. 29-52.
— , "Al-Khalīlī's Auxiliary Tables": see *idem*, *Studies*, A-XI.
— , "Al-Khalīlī's Qibla Table": see *idem*, *Studies*, A-XIII.
— , "al-Khwārizmī": *idem*, "al-Khwārizmī and New Trends in Mathematical Astronomy in the Ninth Century", *Occasional Papers on the Near East* (New York University, Hagop Kevorkian Center for Near Eastern Studies), 2 (l983), 43 pp.
— , "Kuwait Astrolabes": *idem*, "Early Islamic Astronomical Instruments in Kuwaiti Collections", in Arlene Fullerton & Géza Fehérvári, eds., *Kuwait: Art and Architecture – Collection of Essays*, Kuwait (no publisher stated), 1995, pp. 76-96.
— , "Lunar Equation Table": *idem*, "A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus", *Centaurus* 18 (1974), pp. 129-146, repr. in *idem*, *Studies*, A-V.
— , *Mecca-Centred World-Maps*: *idem*, *World-Maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science*, Leiden: E. J. Brill, and London: Al-Furqan Islamic Heritage Foundation, 1999.
— , "A Medieval Arabic Report on Algebra before al-Khwārizmī": *idem*, "A Medieval Arabic Report on Algebra before al-Khwārizmī", *Al-Masāq*: *Studia Arabo-Islamica Mediterranea* 1 (1988), pp. 25-32.
— , "Medieval Islamic Multiplication Tables", A-B: see *idem*, *Studies*, A-XIV and XV.
— , "Monumental Syrian Astrolabe": *idem*, "The Monumental Syrian Astrolabe in the Maritime Museum, Istanbul", in *Sayılı Memorial Volume*, II, pp. 729-735 and 10 pls.
♦ — , "Muwaqqits": *idem*, "On the Role of the Muezzin and the *Muwaqqit* in Medieval Islamic Society", in *Oklahoma 1992 & 1993 Conference Proceedings*, pp. 285-346. [A new version is presented here as **V**.]
— , "*Navicula*": see *idem*, "Universal Horary Dials".
— , "Neglected Astrolabe": *idem*, "The Neglected Astrolabe", in *Mathematische Probleme in Mittelalter – Der lateinische und arabische Sprachbereich*, Menso Folkerts, ed., Wiesbaden: Harrassowitz, 1996, pp. 45-55.
— , "Nuremberg Astrolabes": *idem*, "Die Astrolabiensammlung des Germanischen Nationalmuseums", in *Nuremberg GNM 1992-93 Exhibition Catalogue*, I, pp. 101-114, and II, pp. 568-602, 640-643.
♦ — , "Orientation of Islamic Architecture": *idem*, "The Orientation of Medieval Islamic Religious Architecture and Cities", *JHA* 26 (1995), pp. 253-274. [A new version is presented here as **VIIa**.]
— , "Origin of the Astrolabe": see *idem*, *Studies*, B-III.

— , "*Quadrans vetus*": *idem*, "A *Vetustissimus* Arabic Treatise on the *Quadrans Vetus*", *JHA* 33 (2002), pp. 237-255

— , "Review of Higton": *idem*, "Towards a History from Antiquity to the Renaissance of Sundials and Other Instruments for reckoning Times by the Sun and Stars", an essay review of *Greenwich Sundial Catalogue*, and Higton, *Portable Sundials*, to appear in *AS* 61 (2004), pp. 1-13.

— , "Review of Hill, *al-Jazarī*": *idem*, "Medieval Mechanical Devices", an essay review of Donald R. Hill's 1974 translation of al-Jazarī's *Book of Knowledge of Ingenious Mechanical Devices*, *HS* 13 (1975), pp. 284-289, repr. in *idem*, *Studies*, B-XX.

— , "Review of *London Khalili Collection Catalogue*": *idem*, "Cataloguing Islamic Astronomical Instruments", essay review in *Bibliotheca Orientalis* 57 (2000), cols. 247-258.

— , "Review of Sezgin, *GAS*, V": *idem*, "On the Sources for the Study of Early Islamic Mathematics", an essay review in *JAOS* 99 (1979), pp. 450-459.

— , "Review of Sezgin, *GAS*, VI": *idem*, "Early Islamic Astronomy", an essay review in *JHA* 12 (1981), pp. 55-59.

— , "Review of G. Turner, *Elizabethan Instruments*": review in *Journal of the History of Collections* (Oxford) 15:1 (2003), pp. 147-150.

— , "Review of Varisco & Smith, eds., *Al-Afḍal's Anthology*, in *Yemeni Update* 44 (2002) – see www.aiys.org/webdate/kngrev.html.

— , *The Sacred Geography of Islam*: *idem*, *The World about the Kaaba – The Sacred Folk Geography of Medieval Islam*, to be submitted to E. J. Brill.

♦ — , **SATMI**: This abbreviation has been used by the author since the mid 1970s to refer to the unpublished manuscript on which part of the present work (**I-II**) is based.

♦ — , "Shadow Schemes": *idem*, "A Survey of Medieval Islamic Shadow Schemes for Simple Timereckoning", *Oriens* 32 (1990), pp. 191-249. [A new version is presented here as **III**.]

— , "The *Shakkāzīya* Quadrant of al-Māridīnī": see *idem*, *Studies*, B-X.

— , "Star-Names on European Astrolabes": *idem*, "The Stars-Names on Three 14th-Century Astrolabes from Spain, France and Italy", in *Kunitzsch Festschrift*, pp. 307-333.

♦ — , "Strumentazione astronomica nel mondo medievale islamico", a chapter in *Gli strumenti*, Gerard L'E. Turner, ed., Turin: Giulio Einaudi, 1991, pp. 154-189 and pp. 581-585 (bibliography). [A new version of the English original is presented here as **X**.]

— , *Studies*, A: *idem*, *Islamic Mathematical Astronomy*, London: Variorum, 1986, 2nd rev. edn., Aldershot: Variorum, 1993. Includes reprints of:

- I "On the Astronomical Tables of the Islamic Middle Ages", *Studia Copernicana* 13 (1975), pp. 37-56.
- III "The Astronomy of the Mamluks", *Isis* 74 (1983), pp. 531-555.
- V "A Double-Argument Table for the Lunar Equation Attributed to Ibn Yūnus" *Centaurus* 18 (1974), pp. 129-146.
- VI "Ibn al-Majdī's Tables for Calculating Ephemerides" (with E. S. Kennedy), *JHAS* 4 (1980), pp. 48-68.
- VII "Some Astronomical Observations from Thirteenth-Century Egypt" (with Owen Gingerich), *JHA* 13 (1982), pp. 121-128.
- VIII "Indian Astronomy in Fourteenth-Century Fez: The Versified Zīj of al-Qusunṭīnī [*sic*: read al-Qusanṭīnī]" (with E. S. Kennedy), *JHAS* 6 (1982), pp. 3-45.
- IX "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun", *AHES* 10 (1973), pp. 342-394.
- X "Astronomical Timekeeping in Fourteenth-Century Syria", *Aleppo 1976 Symposium Proceedings*, II, pp. 75-84.
- XI "Al-Khalīlī's Auxiliary Tables for Solving Problems of Spherical Astronomy", *JHA* 4 (1973), pp. 99-110.
- XII "Astronomical Timekeeping in Ottoman Turkey", *Istanbul 1977 Symposium Proceedings*, pp. 245-269.
- XIII "Al-Khalīlī's Qibla Table", *JNES* 34 (1975), pp. 81-122.
- XIV "On Medieval Islamic Multiplication Tables", *HM* 1 (1974), pp. 317-323.
- XV "Supplementary Notes on Medieval Islamic Multiplication Tables", *HM* 6 (1979), pp. 405-417.
- XX "Medieval Mechanical Devices" (an essay review of Donald R. Hill's 1974 translation of al-Jazarī's *Book of Knowledge of Ingenious Mechanical Devices*), *HS* 13 (1975).

— , *Studies*, B: *idem*, *Islamic Astronomical Instruments*, London: Variorum, 1987, repr. Aldershot: Variorum, 1995. Includes reprints of:

- II "The Medieval Yemeni Astrolabe in the Metropolitan Museum of Art in New York", *ZGAIW* 2 (1985), pp. 99-122, and 4 (1987/88), pp. 268-269 (corrections).
- III "On the Origin of the Astrolabe According to the Medieval Arabic Sources", *JHAS* 5 (1981), pp. 43-83.
- VI "The ʿAbd al-Aʾimma Astrolabe Forgeries" (with Owen Gingerich and George Saliba), *JHA* 3 (1972), pp. 188-198.
- VII "On the Early History of the Universal Astrolabe in Islamic Astronomy and the Origin of the Term *Shakkāzīya* in Medieval Scientific Arabic", *JHAS* 3 (1979), pp. 244-257.
- VIII "The Astrolabe of ʿAlī al-Wadāʿī", previously unpublished.
- IX "The Astronomical Instruments of Ibn al-Sarrāj", previously-unpublished paper presented at the Second International Symposium for the History of Arabic Science, Aleppo, 1979.
- X "An Analog Computer for Solving Problems of Spherical Astronomy: The *Shakkāzīya* Quadrant of Jamāl al-Dīn al-Māridīnī", *AIHS* 24 (1974), pp. 219-242.
- XI "New Light on the *Zīj al-Ṣafāʾiḥ* of Abū Jaʿfar al-Khāzin", *Centaurus* 23 (1980), pp. 105-117.

XII "Ibn al-Shāṭir's *Ṣandūq al-Yawāqīt*: An Astronomical Compendium" (with Louis Janin), *JHAS* 1 (1977), pp. 187-256.
XIV "Osmanische astronomische Handschriften und Instrumente", first published 1985.
XV "Three Sundials from Islamic Andalusia", *JHAS* 2 (1978), pp. 358-392.
XVI "Le cadran solaire de la mosquée d'Ibn Ṭūlūn au Caire" (with Louis Janin), *JHAS* 2 (1978), pp. 331-357.
XVII "The Sundial on the West Wall of the Madrasa of Sultan Qaytbay in Jersualem" (with Archibald G. Walls), *art and architecture research papers* (London) 15 (1979), pp. 16-21.
XVIII "A Fourteenth-Century Tunisian Sundial for Regulating the Times of Muslim Prayer", in *Hartner Festschrift*, pp. 187-202.

— , *Studies*, C: *idem*, *Astronomy in the Service of Islam*, Aldershot: Variorum, 1993. Includes reprints of:
I "Science in the Service of Religion: The Case of Islam", *impact of science on society* (U.N.E.S.C.O.), no. 159 (1991), pp. 245-262.
II "Some Early Islamic Tables for Lunar Crescent Visibility", in *Kennedy Festschrift*, pp. 185-225.
III "Ibn Yūnus on Lunar Crescent Visibility", *JHA* 19 (1988), pp. 155-168.
IV "Lunar Crescent Visibility Predictions in Medieval Islamic Ephemerides", in *Quest for Understanding – Arabic and Islamic Studies in Memory of Malcolm H. Kerr*, S. Seikaly, R. Baalbaki, P. Dodd, eds., Beirut: American University of Beirut, 1991, pp. 233-251.
♦ VI "Universal Solutions in Islamic Astronomy", in *Aaboe Festschrift*, pp. 121-132. [A new version is included here as **VIa**.]
♦ VII "Universal Solutions to Problems of Spherical Astronomy from Mamluk Egypt and Syria", in *The Way Prepared*: *Essays on Islamic Culture in Honor of R. Bayly Winder*, Farhad Kazemi and Robert B. McChesney, eds., New York: New York University Press, 1987, pp. 153-184. [A new version is included here as **VIb**.]
XII "On the Orientation of the Kaʿba" (with Gerald S. Hawkins), *JHA* 13 (1982), pp. 102-109.
XIII "Astronomical Alignments in Medieval Islamic Religious Architecture", *Annals of the New York Academy of Sciences* 385 (1982), pp. 303-312.
XIV "The Earliest Mathematical Methods and Tables for Finding the Direction of Mecca", *ZGAIW* 3 (1986), pp. 82-149, with corrections listed *ibid*. 4 (1987/88), p. 270.

— , "Tables for Lunar Crescent Visibility": see *idem*, *Studies*, C-II.

— , "Three Sundials from Islamic Andalusia": see *idem*, *Studies*, B-XV.

— , "The Toledo Astrolabe": *idem*, "An Astrolabe from 14th-Century Christian Spain with Inscriptions in Latin, Hebrew and Arabic – A Unique Testimonial to an Intercultural Encounter", *Suhayl* 3 (2002/03), pp. 9-156.

— , "Tunisian Sundial": see *idem*, *Studies*, B-XVIII.

— , "Universal Astrolabe": see *idem*, *Studies*, B-VII.

♦ — , "Universal Horary Dials", *idem*, "On Universal Horary Dials for Determining Time by the Sun and Stars", in **XIIb**. [A summary is in King, "*Navicula*": "14th-Century England or 9th-Century Baghdad? New Insights on the Origins of the Elusive Astronomical Instrument Called the *Navicula de Venetiis*", in *Goldstein Festschrift*.]

♦ — , "Universal Horary Quadrant": *idem*, "The Universal Horary Quadrant for Timekeeping by the Sun", in **IXa**. [A shorter version is "A *Vetustissimus* Arabic Treatise on the *Quadrans Vetus*", *JHA* 33 (2002), pp. 237-255.]

— , "Universal Quadrant of al-Māridīnī": see *idem*, *Studies*, B-X.

♦ — , "Universal Solutions in Islamic Astronomy": see *idem*, *Studies*, C-VI.

♦ — , "Universal Solutions from Mamluk Egypt and Syria": see *idem*, *Studies*, C-VII.

— , "Urbino Astrolabe": *idem*, "The Astrolabe Depicted in the Intarsia of the *Studiolo* of Archduke Federico in Urbino", in *La scienza del Ducato di Urbino – The Science of the Dukedom of Urbino*, Flavio Vetrano, ed., Urbino: Accademia Raffaello, 2001, pp. 101-139.

— , "Yemeni Astrolabe": see *idem*, *Studies*, B-II.

— , "The *Zīj al-Ṣafāʾiḥ*" : see *idem*, *Studies*, B-XI.

— & Charette, *The Universal Astrolabe of Ibn al-Sarrāj*: *idem* and François Charette, *The Universal Astrolabe of Ibn al-Sarrāj: Innovation in Medieval Islamic Science*, to appear.

King & Gingerich, "Astronomical Observations from 13th-Century Egypt": *idem* and Owen Gingerich, "Some Astronomical Observations from Thirteenth-Century Egypt", *JHA* 13 (1982), pp. 121-128, repr. in King, *Studies*, A-VII.

♦ – with Dorothea Girke, "An Approximate Trigonometric Formula for Astronomical Timekeeping and Related Tables, Sundials and Quadrants from *ca.* 500 to *ca.* 1900", *Johann Wolfgang Goethe-Universität Frankfurt am Main, Institut für Geschichte der Naturwissenschaften*, *Preprint Series*, 1 (1988). [A new version is included here as **XI**.]

— & Kennedy, "Ibn al-Majdī's Tables": *idem* and E. S. Kennedy, "Ibn al-Majdī's Tables for Calculating Ephemerides", *JHAS* 4 (1980), pp. 48-68, repr. in King, *Studies*, A-VI.

— & Maier, "London Catalan Astrolabe": *idem* and Kurt Maier, "The Medieval Catalan Astrolabe of the Society of Antiquaries, London", in *Vernet Festschrift*, II, pp. 673-718.

— & Samsó, "Islamic Astronomical Handbooks and Tables": *idem* and Julio Samsó, with a contribution by Bernard R. Goldstein, "Astronomical Handbooks and Tables from the Islamic World (750-1900): An Interim Report", *Suhayl* 2 (2001), pp. 9-105. [A summary is in the article "Zīdj̲" in EI_2 (illustrated).]

— & Turner, "Regiomontanus' Astrolabe": *idem* and Gerard L'E. Turner, "The Astrolabe Presented by Regiomontanus to Cardinal Bessarion in 1462", *Nuncius – Annali di Storia della Scienza* (Florence) 9 (1994), pp. 165-206, repr. in Turner, *Studies*.

— & Walls, "Jerusalem Sundial": see *idem*, *Studies*, B-XVII.

King: see also *Cairo ENL Catalogue* and *Survey*, and *Kennedy Festschrift*.

King-*père*, *Geared to the Stars*: Henry C. King with John R. Milburn, *Geared to the Stars*, Toronto: University of Toronto Press, 1978.

Klemm, *Georg Hartmann*: Hans Gunther Klemm, *Georg Hartmann aus Eggolsheim (1489-1564) – Leben und Werk eines fränkischen Mathematikers und Ingenieurs*, Forschheim (D): Ehrenbürg-Gymnasium (*Wissenschaftliche und künstlerische Beiträge*, Heft 8), 1990, 104 pp.

Knobel, *Ulughbeg's Catalogue of Stars*: E. B. Knobel, *Ulughbeg's Catalogue of Stars*, Washington, D.C.: Carnegie Institution, 1917.

Knorr, "*Quadrans Vetus*": Wilbur R. Knorr, "The Latin Sources of *Quadrans Vetus*, and What They Imply for Its Authorship and Date", in Edith Sylla and Michael McVaugh, eds., *Texts and Contexts in Ancient and Medieval Science – Studies on the Occasion of John E. Murdoch's Seventieth Birthday*, Leiden, *etc*.: E. J. Brill, 1997, pp. 23-67.

— , "Sacrobosco's *Quadrans*": *idem*, "Sacrobosco's *Quadrans*: Date and Sources", *Journal for the History of Astronomy* 28 (1997), pp. 187-222.

Kragten, *Greenwich Navicula*: Jan Kragten, *The Venetian Ship of Sibton Abbey*, Eindhoven (NL), published privately, 1992.

— , *Navicula*: *idem*, *The Little Ship of Venice* [*sic*] *(Navicula de Venetiis)*, Eindhoven (NL), published privately, 1989.

Krause, "Stambuler Handschriften": Max Krause, "Stambuler Handschriften islamischer Mathematiker", *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, B 3:4 (1936), pp. 437-532.

Krause: see also al-Bīrūnī, *Qānūn*.

Kremer & Dobrzycki, "Peurbach and Marāgha Astronomy?": Richard L. Kremer and Jerzy Dobrzycki, "Peurbach and Marāgha Astronomy? The Ephemerides of Johannes Angelus and their Implications", *JHA* 27 (1996), pp. 187-237.

Kren, *Bibliography of Medieval Science and Technology*: Claudia Kren, *Medieval Science and Technology – A Selected, Annotated Bibliography*, New York and London: Garland Publishing, Inc., 1985.

— , "The Traveller's Dial": *eadem*, "The Traveller's Dial in the Late Middle Ages: The Chilinder", *Technology and Culture* 18 (1977), pp. 419-435;

Kunitzsch, "*Anwāʾ*-Tradition": Paul Kunitzsch, "Über eine *anwāʾ*-Tradition mit bisher unbekannten Sternnamen", *Sitzungsberichte der Bayerischen Akademie der Wissenschaften, Phil.-hist. Klasse*, Heft 5, (*Beiträge zur Lexikographie des Classischen Arabisch*, Nr. 4), Munich, 1983.

— , *Arabische Sternnamen in Europa*: *idem*, *Arabische Sternnamen in Europa*, Wiesbaden: Otto Harrassowitz, l959.

— , "Fachausdrücke der Astrolabliteratur": *idem*, "Glossar der arabischen Fachausdrücke in der mittelalterlichen europäischen Astrolabliteratur", *Nachrichten der Akademie der Wissenschaften in Göttingen*, I. Philologisch-Historische Klasse, Jahrgang 1982, Nr. 11, pp. 455-571. [The separatum is paginated 1-117.]

— , "Ibn Ḥabīb's *Book on the Stars*": *idem*, "ʿAbd al-Malik ibn Ḥabīb's *Book on the Stars*", *ZGAIW* 9 (1994), pp. 161-194.

– , "Al-Khwārizmī as a Source for the *Sententie astrolabii*": *idem*, "Al-Khwārizmī as a Source for the *Sententie astrolabii*", in *Kennedy Festschrift*, pp. 227-236, repr. in *idem*, *Studies*, IX.

— , "Messahalla on the Astrolabe: *idem*, "On the Authenticity of the Treatise on the Composition and Use of the Astrolabe Ascribed to Messahalla", *AIHS* 31 (1981), pp. 42-62.

— , *Sternkatalog des Almagest*: *idem*, *Claudius Ptolemäus – Der Sternkatalog des Almagest – Die arabisch-mittelalterliche Tradition*, 3 vols., Wiesbaden: Otto Harrassowitz, 1986-1991.

— , *Sternnomenklatur der Araber*: *idem, Untersuchungen zur Sternnomenklatur der Araber*, Wiesbaden: Otto Harrassowitz, 1961.

— , *Sternverzeichnisse*: *idem*, *Typen von Sternverzeichnissen in astronomischen Handschriften des zehnten bis vierzehnten Jahrhunderts*, Wiesbaden: Otto Harrassowitz, 1966.

— , *Studies*: *idem*, *The Arabs and the Stars*, Northampton: Variorum, 1989.

— , "Al-Ṣūfī and the Astrolabe Stars": *idem*, "Al-Ṣūfī and the Astrolabe Stars", *ZGAIW* 6 (1990), pp. 151-166.

— & Dekker, "The Stars on the Carolingian Astrolabe": *idem* and Elly Dekker, "The Stars on the Rete of the So-Called "Carolingian Astrolabe"", in *Vernet Festschrift*, II, pp. 655-672.

Kunitzsch & Lorch, "*Matāliʿ al-samt*": *idem* and Richard Lorch, "Abū Naṣr and Ḥabash on *matāliʿ al-samt*", *ZGAIW* 9 (1994), pp. 43-82.

Kunitzsch Festschrift: *Sic itur ad astra. Studien zur Geschichte der Mathematik und Naturwissenschaften. Festschrift für den Arabisten Paul Kunitzsch zum 70. Geburtstag*, Menso Folkerts and Richard P. Lorch, eds., Wiesbaden: Otto Harrassowitz, 2000.

Kunitzsch: see also Kennedy & Kunitzsch & Lorch.

Kurz, *European Clocks*: Otto Kurz, *European Clocks and Watches in the Near East*, London: The Warburg Institute, and Leiden: E. J. Brill, 1975.

Labarta & Barceló, "Un nuevo fragmento de reloj de sol andalusí": Ana Labarta and Carmen Barceló, "Un nuevo fragmento de reloj de sol andalusí", *al-Qanṭara* (Madrid) 16 (1995), pp. 147-150.

— , "Ocho relojes de sol hispanomusulmanes", *eaedem*, "Ocho relojes de sol hispanomusulmanes", al-*Qanṭara* (Madrid) 9 (1988), pp. 231-247.

Lacroix, *Science in the Middle Ages*: Paul Lacroix, *Science and Literature in the Middle Ages and Renaissance*, translated from the French, first published in 1878, repr. New York: Frederick Ungar, 1964.

de La Lande, *Bibliographie Astronomique*: Jérôme de La Lande, *Bibliographie Astronomique ; avec L'Histoire de l'Astronomie jusqu'à 1802*, Paris: Imprimerie de la République, An XI = 1803.

Landes, *Revolution in Time*: David S. Landes, *Revolution in Time – Clocks and the Making of the Modern World*, Cambridge, Mass, and London: The Belknap Press of Harvard University Press, 1983

Lane, *Lexicon*: Edward W. Lane, *An Arabic-English Lexicon*, 8 pts., London: Williams and Norgate, 1863, repr. Beirut: Librairie du Liban, 1968.

Langermann, *Ibn al-Haytham's Cosmology*: Y. Tzvi Langermann, *Ibn al-Haytham's* On the Configuration of the World, (Series: Harvard Dissertations in the History of Science), New York & London: Garland Publishing, Inc., 1990.

— , *Studies*: *idem*, *The Jews and the Sciences in the Middle Ages*, Aldershot: Variorum, 1999.

Lee, "Astronomical Tables of Al Farsi": Samuel Lee, "Notice of the Astronomical Tables of Mohammed Abibekr Al Farsi ... ... ", *Transactions of the Cambridge Philosophical Society* 1 (1822), pp. 249-265.

Lehr, *De Geschiedenis van het Astronomisch Kunstuurwerk*: André Lehr, *De Geschiedenis van het Astronomisch Kunstuurwerk – Zijn techniek en muziek*, The Hague: Martinus Nijhoff, 1981.

Leipzig Catalogue: K. Vollers, *Katalog der islamischen ... Handschriften der Universitätsbibliothek zu Leipzig* ... , Leipzig: Otto Harrassowitz, 1906, reprinted Osnabrück: Biblio Verlag, 1975.

Le Strange, *Lands of the Eastern Caliphate*: Guy Le Strange, *The Lands of the Eastern Caliphate*, *Mesopotamia, Persia and Central Asia from the Moslem Conquest to the Time of Timur*, Cambridge: Cambridge Universty Press, 1905, repr. London: Frank Cass & Co. Ltd., 1966, and again in *Islamic Geography*, vol. 85 (1993).

Lesley, Mark, "Bīrūnī on Rising Times and Daylight Lengths", *Centaurus* 5 (1957), pp. 121-141, repr. in Kennedy *et al*., *Studies*, pp. 253-273.

Levin, "Direction-Facing in Judaism and Islam": Daniel Z. Levin, "Which Way is Jerusalem? Which Way is Mecca? The Direction-Facing Problem in Religion and Geography", *Journal of Geography* (National Council for Geographic Education, Washington, D.C.) 101 (2002), pp. 27-37.

Lexikon des Mittelalters, 9 vols., Munich and Zurich: Artemis, 1980-99, especially articles "Astrologie", "Astronomie", "Instrumente", "Komputistik", "Mathematik".

Lindberg, David C., *The Beginnings of Western Science*, Chicago: University of Chicago Press, 1992.

Linton Collection Catalogue: *Scientific Instruments and Rare Books from the Collection of Leonard Linton, Point Lookout, N.Y.*, Paris: Alain Brieux, 1980.

Lippincott, "*Navicula*": Kristen Lippincott, "The Navicula Sundial", *Bulletin of the Scientific Instrument Society*, no. 35 (Dec., 1992), p. 22. [On the *navicula* now in Greenwich.]

Livingston, "Islamic Conical Sundial": John W. Livingston, "The *Mukḥula*, an Islamic Conical Sundial", *Centaurus* 16 (1972), pp. 299-308.

London BL Catalogue: (W. Cureton and C. Rieu), *Catalogus codicum manuscriptorum orientalium qui in Museo Britannico asservantur*, Part II: *Codices arabicos amplectens*, London: British Museum, 1871.

London BM Catalogue: F. A. B. Ward, *A Catalogue of European Scientific Instruments in the Department of Medieval and Later Antiquities of the British Museum*, London: British Museum Publications, 1981. [See also the essay review in Price, "Instruments in the British Museum".]

London Khalili Collection Catalogue: *The Nasser D. Khalili Collection of Islamic Art*, vol. XII: *Science, Tools and Magic*, by Francis Maddison and Emilie Savage-Smith, 2 pts., London: The Nour Foundation, in association with Azimuth Editions and Oxford University Press, 1997. [See King, "Cataloguing Islamic Instruments", for a review.]

London SM 1976 Exhibition Catalogue: Francis R. Maddison and Anthony J. Turner, *Catalogue of an Exhibition 'Science and Technology in Islam' ... , 1976*, [a catalogue of the exhibition The World of Islam, held at The Science Museum, London, in 1976], privately distributed [but unfortunately never published].

Lorch, "Astronomical Instruments of Jābir ibn Aflaḥ": see Richard P. Lorch, *Studies*, XVI.

— , "Al-Khāzinī's 'Sphere that Rotates by Itself'": see *idem*, *Studies*, XI.

— , "Mischastrolabien": Richard P. Lorch, "Mischastrolabien im arabisch-islamischen Kulturbereich", in *Frankfurt IGN Festschrift,* pp. 231-236.
— , "Rudolf of Bruges on the Astrolabe": *idem*, "The Treatise on the Astrolabe by Rudolf of Bruges", in *North Festschrift*, pp. 55-100.
— , "Al-Ṣaghānī on Projecting the Sphere": see *idem*, *Studies*, XVIII.
— , "Sine Quadrant": *idem*, "Some Early Applications of the Sine Quadrant", *Suhayl* 1 (2000), pp. 251-272.
— , "The *sphaera solida*": see *idem*, *Studies*, XII.
— , *Studies*: *idem*, *Arabic Mathematical Sciences – Instruments, Texts, Transmission*, Aldershot: Variorum, 1995. Contains *inter alia*:
XI "Al-Khāzinī's "Sphere that Rotates by Itself"", *JHAS* 4 (1980), pp. 287-329.
XII "The *sphaera solida* and Related Instruments", *Centaurus* 24 (1980), pp. 153-161.
XIII "Ḥabash al-Ḥāsib's Book on the Sphere and its Use" (with Paul Kunitzsch), *ZGAIW* 2 (1985), pp. 68-98.
XIV "The *Qibla*-Table Attributed to al-Khāzinī", *JHAS* 4 (1980), pp. 259-264.
XVI "The Astronomical Instruments of Jābir ibn Aflaḥ and the Torquetum", *Centaurus* 20 (1976), pp. 11-34.
XVII "A Note on the Horary Quadrant", *JHAS* 5 (1981), pp. 115-120.
XVIII "Al-Ṣaghānī's Treatise on Projecting the Sphere", in *Kennedy Festschrift*, pp. 237-252.
— , "Universal Horary Quadrant": see *idem*, *Studies*, XVII.
— & Kennedy, "Ḥabash al-Ḥāsib on the Melon Astrolabe": see Kennedy, *Studies*, I.
— & Kunitzsch, "Ḥabash on the Sphere": see *idem*, *Studies*, XIII.
Lorch: see also Kennedy & Kunitzsch & Lorch, and Kunitzsch & Lorch.
Luckey, "Das Analemma von Ptolemäus": Paul Luckey, "Das Analemma von Ptolemäus", *Astronomische Nachrichten*, Bd. 230, Nr. 5498 (1927), pp. 18-46.
— , "Beiträge", I/II: *idem*, "Beiträge zur Erforschung der islamischen Mathematik", *Orientalia*, N. S., 17 (1948), pp. 490-510, and 22 (1953), pp. 166-189, repr. in *IAM*, vols 96 and 56.
— , *Ibrāhīm ibn Sinān über die Sonnenuhren*: Jan Pieter Hogendijk, ed., *Die Schrift des Ibrāhīm b. Sinān b. Ṯābit über die Schatteninstrumente, übersetzt und erläutert von Paul Luckey (Phil. Diss. Tübingen, 1941)*, *Islamic Mathematics and Astronomy* vol. 101 (1999).
— , "Thābit ibn Qurra über die Sonnenuhren": *idem*, "Thābit b. Qurra's Buch über die ebenen Sonnenuhren", *ibid*., Abt. B, Bd. 4 (1937-38), pp. 95-148, repr. in *IMA*, vol. 22 (1997), pp. 95-148.
— , "Zur Entstehung der Kugeldreiecksrechnung", *Deutsche Mathematik* 5 (1940), pp. 405-446, repr. in *IMA*, vol. 77 (1998), pp. 137-178.
MacKay, "The Qibla of Ottoman Mosques in Greece": Pierre MacKay, "The Qibla of Ottoman Mosques in Greece", paper presented at the Annual Meeting of the American Oriental Society, San Francisco, April, 1980.
Maddison, "Early Astronomical Instruments": Francis R. Maddison, "Early Astronomical and Mathematical Instruments ... ", *HS* 2 (1963), pp. 17-50.
— , "15th-Century Spherical Astrolabe": *idem*, "A Fifteenth-Century Spherical Astrolabe", *Physis* 4 (1962), pp. 101-109.
— , "Instruments arabes": *idem*, "Observatoires portatifs: les instruments arabes à usage pratique", in *HSA*, I, pp. 139-172 (not in the English version, *EHAS*).
— , "Locks": *idem*, "al-Jazarī's Combination Lock: Two Contemporary Examples", in *The Art of Syria and the Jazīra 1100-1250* (*Oxford Studies in Islamic Art* 1), Julian Raby, ed., Oxford: Oxford University Press, 1985, pp. 141-157.
— , "Medieval Instruments": *idem*, "Medieval Scientific Instruments and the Development of Navigational Instruments in the XVth and XVIth Centuries", *Revista da Universidade de Coimbra* 24 (1969), printed separately, Agrupamento de Estudos de Cartografia Antiga, XXX, Secção de Coimbra, 1969.
— , "Rojas Astrolabe Projection": *idem*, "Hugo Helt and the Rojas Astrolabe Projection", *Revista da Faculdade de Ciências* (Coimbra) 39 (1966), pp. 195-251 and 2 fold-outs.
Maddison: see also *London Khalili Catalogue*, *London SM Exhibition Catalogue*, and *Oxford Billmeir Supplement*.
Madrid 1980 Conference Proceedings: *Historia de la ciencia árabe*, Madrid: Real Academía de Ciencias Exactas, Fîsicas y Naturales, 1981.
Madrid MAN 1992 Exhibition Catalogue: *El legado científico andalusí*, Juan Vernet and Julio Samsó, eds., Madrid: Ministerio de Cultura, 1992. [A catalogue of an exhibition at the Museo Arqueológico Nacional, Madrid, April-June, 1992.]
Madrid & New York 1992 Exhibition Catalogue: Jerrilynn D. Dodds, ed., *Al-Andalus – The Art of Islamic Spain*, New York: The Metropolitan Museum of Art, 1992. [Catalogue of an exhibition held at the Alhambra, Granada, and then at The Metropolitan Museum of Art, New York, in 1992.]
Maistrov, *Nauchnye pribory*: L. E. Maistrov, *Nauchnye pribory*, Moscow: Nauk. 1968. [A catalogue in Russian of scientific instruments in collections in the former Soviet Union.]

Manchester Catalogue: Alois Mingana, *Catalogue of the Arabic Manuscripts in the John Rylands Library*, Manchester: Manchester University Press, 1934.

Maranyi, "Zwischen ʿ*aṣr* and *mağrib* in Mekka": Miklos Maranyi, "Zwischen ʿ*aṣr* and *mağrib* in Mekka – Ein Augenzeugenbericht von al-Laiṯ b. Saʿd aus dem Jahr 113/732", *Die Welt des Orients: wissenschaftliche Beiträge zur Kunde des Morgenlandes* (Göttingen: Vandenhoek & Ruprecht) 23 (1992), pp. 101-128.

al-Marrākushī, *A-Z of Astronomical Timekeeping*: Fuat Sezgin *et al*., *Comprehensive Collection of Principles and Objectives in the Science of Timekeeping: Jāmiʿ al-mabādiʾ wa-'l-ghāyāt by al-Marrākushī,* (facsimile "edition" of MS Topkap1 Ahmet III 3343), 2 vols., Frankfurt am Main: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1984. [See also Sédillot-*père*, *Traité*, and Sédillot-*fils*, "Mémoire".]

Martí & Viladrich, "Tratados de astrolabio": Ramon Martí and Mercè Viladrich, "En torno a los tratados de uso del astrolabio hasta el siglo XIII en al-Andalus, la Marca Hispánica y Castilla", in Vernet, ed., *Nuevos estudios*, pp. 9-74.

Matvievskaya & Rosenfeld, *MAMS*: Galina P. Matvievskaya and Boris A. Rosenfeld, *Matematiki i astronomi musulmanskogo srednevekovya i ikh trudi*, 3 vols., Moscow: Nauk, 1983.

S. & K. Maurice, "Counting the Hours": Sigrid and Klaus Maurice, "Counting the Hours in Community Life of the 16th and 17th Centuries", in Maurice & Mayr, eds., *The Clockwork Universe*, pp. 144-156.

Maurice & Mayr, eds., *The Clockwork Universe*: *The Clockwork Universe – German Clocks and Automata 1550-1650*, Klaus Maurice and Otto Mayr, eds., New York: Neale Watson, 1980.

Mayer, *Islamic Astrolabists*: Leo A. Mayer, *Islamic Astrolabists and Their Works*, Geneva: Albert Kundig, 1956, with a supplement in *Aus der Welt der islamischen Kunst*, Richard Ettinghausen, ed., Berlin: Gebrüder Mann, 1959, pp. 293-296.

McCluskey, *Medieval Astronomy*: Stephen C. McCluskey, *Astronomies and Cultures in Early Medieval Europe*, Cambridge, *etc*.: Cambridge University Press, 1998.

McKay, "How Far to Mecca?": J. McKay, "How Far to Mecca?", *Mercator's World* 5:5 (2002), p. 58.

Meliconi, "Brechte Plate" and "Oxford *Quadrans vetus*": see EPACT.

Mercier, "Astronomical Tables in the 12th Century": Raymond Mercier, "Astronomical Tables in the Twelfth Century", in Burnett, ed, *Adelard of Bath*, pp. 87-118.

— , "English Orientalists and Islamic Astronomy": *idem*, "English Orientalists and Mathematical Astronomy", in Russell, ed., *'Arabick' Interest in 17th-Century England*, pp. 158-214.

— , "Ibn al-Aʿlam's Parameters": *idem*, "The Parameters of the *Zīj* of Ibn al-Aʿlam", *AIHS* 39 (1989), pp. 22-50.

Mestres, "Hyderabad MS of the *Zīj* of Ibn Isḥāq": Àngel Mestres, "Maghribī Astronomy in the 13th Century: a Description of Manuscript Hyderabad Andra Pradesh State Library 298", in *Vernet Festschrift*, I, pp. 383-443.

— , Zīj *of Ibn Isḥāq*: *idem*, *Materials andalusins en el Zīj d'Ibn Isḥāq al-Tūnisī* (edited text and tables, with introduction and commentary in English), doctoral thesis, University of Barcelona, 2000.

Meucci, *Globo celeste arabico*: Ferdinando Meucci, *Il globo celeste arabico del secolo XI nel Gabinetto degli strumenti antichi di astronomia, di fisica e di matematica del R. Istituto di studi superiori*, Florence: Successori Le Monnier, 1878, repr. in *AIOS*, II, pp. 126-144 plus pullout.

Meyer, *Istanbul Sundials*: Wolfgang Meyer, *İstanbul'daki güneş saatleri*, *Sandoz kültür yayınları* (Istanbul) 7 (1985). [See also *idem*, "Sundials of the Osmanic Era in Istanbul", in *Istanbul 1977 Symposium Proceedings*, pp. 193-202.]

Meyerhof: see Frank & Meyerhof.

Michel, "L'astrolabe linéaire": Henri Michel, "L'astrolabe linéaire d'al-Tusi", *Ciel et terre* 59 (1943), pp. 101-107, repr. in *IAM*, vol. 94.

— , *Images des sciences*: *idem*, *Images des sciences: les anciens instruments scientifiques vus par les artisites de leur temps*, Rhode-St-Genèse (B): Albert de Visscher, 1977.

— , "Construction des astrolabes persans": *idem*, "Méthodes de trace et d'exécution des astrolabes persans", *Ciel et terre* 57 (1941), pp. 481-496.

— , *Scientific Instruments in Art and History*: *idem*, *Scientific Instruments in Art and History*, London: Barrie & Rockliff, 1967.

— , *Traité de l'astrolabe*: *idem*, *Traité de l'astrolabe*, Paris: Gauthier-Villars, 1947, reprinted with a new preface by Francis R. Maddison, Paris: Librarie Alain Brieux, 1989.

— & Ben-Eli, "Cadran solaire polaire": *idem* and A. Ben-Eli, "Un cadran solaire remarquable", *Ciel et terre* 81 (1965), pp. 214-216.

Mielgo, "Analyzing Mean Motion Tables": Honorino Mielgo, "A Method of Analysis for Mean Motion Astronomical Tables", in *Vernet Festschrift*, I, pp. 159-180".

Mielgo: see also Samsó & Mielgo.

Milenio, "Cuadrante Vetustissimus": M. Milenio, "Cuadrante Vetustissimus", *Analema* no. 30 (Septiembre-Diciembre 2000), pp. 20-24.

— , “Relojes de altura Regiomontano”: *idem*, “Sobre el diseño de la forma de los relojes de altura Regiomontano y Capuchino”, *Analema* no. 27 (Septiembre-Diciembre 1999), pp. 11-14.

Millás Vallicrosa, *Assaig*: José M. Millás Vallicrosa, *Assaig d’història de les idees físiques i matemàtiques a la Catalunya medieval*, vol. I, (Estudis Universitaris Catalans, sèrie monogràfica I), Barcelona: 1931.

— , *Azarquiel*: *idem*, *Estudios sobre Azarquiel*, Madrid & Granada: Consejo Superior de Investigaciones Científicas and Instituto Miguel Asín, 1943-50.

— , “Cuadrante con cursor”: *idem*, “La introducción del cuadrante con cursor en Europa”, *Isis* 17 (1932), pp. 218-258, repr. in *idem*, *Estudios*, I, pp. 65-110.

— , *Don Profeit Tibbón sobre l’assafea*: *idem*, *Don Profeit Tibbón*: *Tractat de l’assafea d’Azarquiel*, Barcelona: Universitat de Barcelona, 1933.

— , “Un ejemplar de azafea árabe de Azarquiel”, *Al-Andalus* 9 (1944), pp. 11-119.

— , *Estudios* and *Nuevos estudios*: *idem*, *Estudios sobre historia de la ciencia española*, Barcelona: Consejo Superior de Investigaciones Científicas, Instituto “Luis Vives” de Filosofía, 1949, and *idem*, *Nuevos estudios sobre historia de la ciencia española*, Barcelona: Consejo Superior de Investigaciones Científicas, Instituto “Luis Vives” de Filosofía, and Asociacíon para la Historia de la Ciencia Española, 1960, both repr., 2 vols., Madrid: Consejo Superior de Investigaciones Científicas, 1987.

— , “Estudios sobre Azarquiel: El tratado de la azafea”, *Archeion* 14 (1932), pp. 393-419.

Millás-Vallicrosa Festschrift: *Homenaje a Millás-Vallicrosa*, 2 vols., Barcelona: Consejo Superior de Investigaciones Cientificas, 1959.

Millás Vendrell, “Apogeos planetarios en el ecuatorio de Azarquiel”: Eduardo Millás Vendrell, “Las posiciones de los apogeos planetarios en el ecuatorio de Azarquiel”, in Vernet, ed., *Nuevos estudios*, pp. 119-123.

Millás: see also Samsó & Millás.

Mills, “Canterbury Dial”: Allan Mills, “The Canterbury Pendant: A Saxon Seasonal-Hour Altitude Dial”, *Bulletin of the British Sundial Society* 95.2 (June 1995), pp. 39-44.

Miniati Festschrift: *MUSA MUSAEI: Studies on Scientific Instruments and Collections in Honour of Mara Miniati*, Marco Beretta, Paolo Galluzzi and Carlo Triarico, eds., Florence: Leo S. Olschki, 2003.

Monzawi, *Persian Manuscripts*: Aḥmad Monzawi, *A Catalogue of Persian Manuscripts - Fihrist-i nuskhahā-yi kha-i fārisī* (in Persian), 6 vols., Tehran: Regional Cultural Institute, 1348-1353 HS = 1969-1974.

Morelon, “Early Eastern Arabic Astronomy”: Régis Morelon, “Eastern Arabic Astronomy between the Eighth and Eleventh Centuries”, in *EHAS*, I, pp. 20-57.

— , *Thābit ibn Qurra*: *idem*, *Thābit ibn Qurra – oeuvres d’astronomie*, Paris: Les Belles Lettres, 1987.

Morelon: see also *EHAS*.

Morley, “Arabic Quadrant”: William H. Morley, “Description of an Arabic Quadrant”, *Journal of the Royal Asiatic Society* 17 (1860), pp. 322-330 and two figs., repr. in *AIOS*, I, pp. 326-336.

— , “Astrolabe of Shāh Ḥusayn”: *idem*, *Description of a Planispheric Astrolabe Constructed for Sháh Sultán Husain Safawí ...* , London: Williams and Norgate, 1856, repr. as the introduction to Gunther, *Astrolabes*, I, and again in *AIOS*, I, pp. 249-325.

Mortillaro, “Astrolabio arabo”: Vincenzo Mortillaro, *Illustrazione di un astrolabio arabo-sicula* [*sic*], Palermo: Stamperia Oretea, 1848, also appeared as “Illustrazione di un astrolabio arabo del nono secolo”, in *idem*, *Opere*, IV, Palermo, 1848, pp. 110-135, repr. from the latter in *AIOS*, I, pp. 162-191.

Müller, *Tables des hauteurs du soleil*: Fréderic Christophe Müller, *Tables des hauteurs du soleil pour toute l’Allemagne et les pays qui la bornent au levant et au couchant*, Leipzig: Siegfried Lebereclt Crusius, 1791.

Munich Catalogue: Burkhard Stautz, *Die Astrolabiensammlungen des Deutschen Museums und des Bayerischen Nationalmuseums*, Munich: Oldenbourg, 1999.

Murdoch, John E., *Album of Science – Antiquity and the Middle Ages*, New York: Charles Scribner’s Sons, 1984.

Murray, Alexander, *Reason and Society in the Middle Ages*, (first published 1978), Oxford: Clarendon Press, 1991 edn.

Nadir, Nadi, “Abū al-Wafāʾ on the Solar Altitude”, *The Mathematics Teacher* 53 (1960), pp. 460-463, repr. in Kennedy *et al*., *Studies*, pp., 280-283.

Naffah, “Cadran cylindrique”: Christiane Naffah, “Un cadran cylindrique ottoman du XVIII$^{\text{ème}}$ siècle”, *Astrolabica* 5 (1989), pp. 37-52.

Nallino, *al-Battānī*: Carlo Alfonso Nallino, ed., transl. and comm., *al-Battani sive Albatenii opus astronomicum*, 3 pts., Pubblicazioni del Real Osservatorio di Brera in Milano, no. XL, pts. I-III, Milan, 1899-1907, repr. in 1 vol., Hildesheim & New York: Georg Olms, 1977. [Pts. I-II were also reprinted Frankfurt am Main: Minerva, 1969; pt. III (the Arabic text) was reprinted in an undated, unlabelled pirate edition by [Muthannā, Baghdad], in the 1960s.]

— , “Islamic Astronomy”: *idem*, “Sun, Moon, and Stars (Muhammadan)”, in *ERE*, vol. XII (1921), pp. 88-101.

— , *Scritti*, V: *idem*, *Raccolta di scritti editi e inediti* (ed. Maria Nallino), vol. V: *Astrologia – Astronomia – Geografia*, Rome: Istituto per l’Oriente, 1944.

Nasr, Seyyed Hossein, *Islamic Science: An Illustrated Study*, London: World of Islam Festival Publishing Company Ltd., 1976, repr. Istanbul: "nsan Yayınları, 1989. [A Persian translation *ʿIlm dar Islām* was published in Tehran, 1366 HS [= 1988]. [This work is to be used with caution as far as any technical aspects of Islamic science are concerned.]

Necipoglu, *The Topkapı Scroll*: Gülrü Necipoglu, *The Topkapı Scroll – Geometry and Ornament in Islamic Architecture ...* , Santa Monica, Ca.: The Getty Center for the History of Art and the Humanities, 1995.

Neugebauer, *Abu Shaker's Chronography*: Otto Neugebauer, *Abu Shaker's "Chronography" – Treatise of the 13th Century on Chronological, Calendrical, and Astronomical Matters, written by a Christian Arab, preserved in Ethiopic*, Vienna: Österreichische Akademie der Wissenschaften, 1988.

— . "Astronomical and Calendrical Data in the Très Riches Heures": *idem*, "Astronomical and Calendrical Data in the Très Riches Heures", (originally published 1974), repr. in *idem*, *Astronomy and History*, pp. 507-520.

— , "Astronomische Papyri": *idem*, "Astronomische Papyri aus Wiener Sammlungen: II. Über griechische Wetterzeichen und Schattentafeln", *Sitzungsberichte der philosophisch-historischen Klasse der Österreichischen Akademie der Wissenschaften* 240:2 (1962), pp. 29-44.

— , "Early Greek Astronomy": *idem*, "On Some Aspects of Early Greek Astronomy", *Journal of the American Philosophical Society* 116 (1972), pp. 243-251, repr. in *idem*, *Essays*, pp. 361-369.

— , "The Early History of the Astrolabe": *idem*, "The Early History of the Astrolabe", *Isis* 40 (1949), pp. 240-256, repr. in *idem*, *Essays*, pp. 278-294.

— , *ESA*: *idem*, *The Exact Sciences in Antiquity*, reprint of the 1957 2nd edn., New York: Dover, 1969.

— , *Essays*: *idem*, *Astronomy and History – Selected Essays*, New York, *etc*.: Springer, 1983. Contains:
- Neugebauer & Olaf Schmidt, "Hindu Astronomy at Newminster in 1428", *AS* 8 (1952), pp. 221-228, repr. here pp. 425-432.

— , *Ethiopic Astronomy and Computus*: *idem*, *Ethiopic Astronomy and Computus*, (Österreichische Akademie der Wissenschaften, Philosophisch-Historische Klasse, Sitzungsberichte, 347. Band, Veröffentlichungen der Kommission für Geschichte der Mathematik, Naturwissenschaften und Medizin, Heft 22), Vienna, Verlag der Österreichischen Akademie der Wissenschaften, 1979.

— , *HAMA*: *idem*, *A History of Ancient Mathematical Astronomy*, Berlin, Heidelberg & New York: Springer, 1975.

— , *al-Khwārizmī*: *idem*, *The Astronomical Tables of al-Khwārizmī*, in *Kgl. Danske Vidensk. hist.-fil. Skrifter*, 4:2 (1962).

— , "Studies in Byzantine Astronomy": *idem*, "Commentary on the Astronomical Treatise Par. gr. 2425", *Mémoires de l'Académie Royale de Belgique* 49:4 (1969), pp. 3-45.

— & Schmidt, "Hindu Astronomy at Newminster": *idem* and Olaf Schmidt, "Hindu Astronomy at Newminster in 1428", *AS* 8 (1952), pp. 221-228, repr. in Neugebauer, *Essays*, pp. 425-432.

New Delhi 1985 Colloquium Proceedings: *History of Oriental Astronomy – Proceedings of an International Astronomical Colloquium No. 91, New Delhi, India, 13-16 November, 1985*, G. Swaru, A. K. Bag and K. Shukla, eds., Cambridge, *etc*.: Cambridge University Press, 1987.

Nicholas of Lynn, *Kalendarium*: *see Eisner*.

Nolte, "Die Armillarsphäre": Friedrich Nolte, "Die Armillarsphäre", *Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin* (Erlangen: M. Mencke), Heft 2, 1922.

North, "Astrolabe": John D. North, "The Astrolabe", *Scientific American* 230 (1974), pp. 96-106, repr. in *idem*, *Essays*, no. 14.

— , *Chaucer's Universe*: *idem*, *Chaucer's Universe*, Oxford: Clarendon Press, 1988.

— , *Essays*, A-B:, *idem*, *Stars, Minds and Fate – Essays in Ancient and Medieval Cosmology* (A) and *The Universal Frame – Historical Essays in Astronomy, Natural Philosophy and Scientific Method* (B), London & Ronceverte, W.V.: Hambledon, 1989.

— , *Horoscopes and History*: *idem*, *Horoscopes and History*, (Warburg Institute Surveys and Texts, vol. 12), London: The Warburg Institute, 1986.

— , "Hour-Line Ritual": *idem*, "Astrolabes and the Hour-Line Ritual", *JHAS* 5 (1981), pp. 113-114, repr. in *idem*, *Essays*, A, no. 15.

— , "Meteoroscope": *idem*, "Werner, Apian, Blagrave and the Meteoroscope", *British Journal for the History of Science* 3 (1966-67), pp. 57-65 and 1 pl.

— , "*Opus quarundam rotarum mirabilium*", *Physis* 8 (1966), pp. 337-372, repr. in *idem*, *Essays*, I, pp. 135-169 (no. 11).]

— , "Review of King, *Mecca-Centred World-Maps*": *idem*, in *Bibliotheca Orientalis* (Leiden), 57 (2000), cols. 747-750.

— , *Richard of Wallingford*: *idem*, *Richard of Wallingford*: *An edition of his writings with introductions, English translation and commentary*, 3 vols., Oxford: The Clarendon Press, 1976.

North Festschrift: *Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North*, Arjo Vanderjagt and Lodi Nauta, eds., Leiden: E. J. Brill, 1999.

Part I

A survey of tables for timekeeping by the sun and stars

To Ted Kennedy

ACKNOWLEDGEMENTS AND NOTES TO THIS VERSION

The present study (**I-II**) was made possible by the cooperation of the directors of numerous manuscript libraries in Europe, the Near East and the United States, who allowed me access to their collections of Arabic scientific manuscripts or provided me with microfilms of specific manuscripts. I should like to express my particular gratitude to the Süleymaniye Library in Istanbul and the Egyptian National Library in Cairo.

In 1972 I received a two-year research grant to catalogue the scientific manuscripts in the Egyptian National Library. When I arrived at the Library I did not know how many manuscripts there were or that, for security reasons, it would take me two years to obtain unlimited access to them. There was a sense in which I had to justify my existence. Whilst preparing my doctoral thesis on the astronomical works of Ibn Yūnus during 1969-72 I had written a description of the tables for timekeeping attributed to him in a Dublin manuscript, which was published in 1973. Even before that paper was published I realized that the tables were not all computed by Ibn Yūnus. It seemed like a good idea at the time to try to figure out just who had computed them. In the course of this pursuit, and especially when I found the tables of al-Khalīlī and Najm al-Dīn al-Miṣrī and realized their scope, I decided to write an article on the development of all such tables from the Islamic Middle Ages. Fortunately, my grant was extended for an additional five years, and the catalogue was indeed prepared and eventually published. Slowly but surely, my article on tables for timekeeping also grew and grew, and by the time I left Cairo for New York an extensive monograph was finally finished. But in New York my research interests changed, and I started to work on the qibla, on the notion of the world centred on the Kaʿba, and on mosque orientations. In fact, when I left New York for Frankfurt two monographs were completed, one from Cairo on timekeeping and one more recent from New York on the sacred geography of Islam. At least summaries of these were published in two articles ("Mīḳāt. ii" and "Makka. iv") in the *Encyclopedia of Islam*.

The research on medieval Islamic science conducted at the American Research Center in Egypt during the years 1972-79 was supported by the Smithsonian Institution and National Science Foundation, Washington, D.C. (1972-79), and by grants from the Penrose and Johnson Funds of the American Philosophical Society (1972-74) and the Ford Foundation (1977-79). The support of each of these five institutions is gratefully acknowledged. Professor Owen Gingerich of the Smithsonian Astrophysical Observatories and Harvard University generously sponsored this research. All of the various drafts of the text were typed by Nabila Hajj, who was the mainstay of the project from its inception right through to 1985, when the Arabic catalogue of the scientific manuscripts was finally published. Kurt Maier of Frankfurt typed much of the new text for the computer in the early 1990s. François Charette controlled the penultimate version for consistency but should not in any way be held responsible for a text penned by someone who at the time he wrote it was younger than Charette is now.

Two sets of tables have been investigated by the next generation of researchers and some

of their results are noted in this book. First, Glen Van Brummelen attacked the tables of al-Khalīlī with statistical techniques that I could not hope to understand. Second, François Charette has produced masterly studies of the monumental triple-argument universal auxiliary table of Najm al-Dīn al-Miṣrī and the same author's remarkable treatise on instruments. He also helped me insert new materials by Najm al-Dīn in the final version of the present study.

This first part of the main study (**I**) is dedicated to the one man who influenced me most in my adult life. Ted Kennedy is a man for whom I have an undying feeling of gratitude. He is a man whom I can both love and respect and try – if in vain – in some ways to emulate. His achievements were to no small extent the result of the support of his wife, Mary Helen. I first met Ted Kennedy at Brown University in the autumn of 1969 when I was a graduate student at Yale. Asger Aaboe and Bernie Goldstein organized monthly expeditions there to gather with the Brown crowd, which then consisted of Otto Neugebauer, David Pingree, Abe Sachs and Gerald Toomer, with Ted Kennedy on sabbatical leave from the American University of Beirut. I had already encountered Ted Kennedy on paper, and I was fascinated by the fact that he was active in Beirut. It also seemed to me absurd that an Englishman, *manqué* Near East man, most at home in Dārfūr or Cairo, should be languishing in New Haven, Connecticut, when he too could be in Beirut. Sure enough, my wife and I spent the academic year 1970-71 there, getting to know the Kennedys and their world. I also worked on my dissertation, and enjoyed unlimited access to Ted's library, extensive microfilm collection and unpublished materials. Ted's generosity was boundless: one day George Saliba, the other of Ted's "terrible twins", discovered that Kodak had a special offer; we taped all of Ted's microfilms together and had a copy made for each of us, on several enormous spools.

In our early years in Cairo my wife and I went to Beirut regularly to see the Kennedys. During 1976-77 the Kennedys were in Cairo, living in an apartment next to ours. Ted and I braved the rigours of the Dār al-Kutub together; even with a sense of humour and a few provisions against the cold, this was not easy. When we moved to Frankfurt in 1985, the Kennedys were there too, thanks to the generosity of Professor Fuat Sezgin, for a couple of years. So our paths have crossed many times – in Providence, Beirut, Aynab, Aleppo, Cairo, Frankfurt, Barcelona, Princeton, New York City, and Doylestown, Pa., as well as various conference-sites such as Bucharest, Hamburg, Liège and Cambridge, Ma.

In November, 1999, I spoke at a dinner in Pittsburgh organised by a splinter group of the (U.S.) History of Science Society and attended by colleagues in the history of ancient and medieval science. I was waxing enthusiastically about the exciting materials available to historians of Islamic science. I happened to mention Ted Kennedy. A certain distinguished female colleague, versed in medieval European science of the Aristotelian variety and in American personalities, voiced the question: "You mean *the* Ted Kennedy?". The only answer was: "Of course!".

In this new version I have not added many new materials, since, after I stopped looking in the 1980s, I did not find any save those that more or less fell into my lap.[1] The few new

[1] But notice that few new materials of consequence appear to be documented in the monumental, newly-published volumes listed as İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, and *Ottoman Mathematical Literature*. For details see n. 1:14 below.

materials are obvious from the table of contents in **Parts I-II** for they bear an asterisc appended to the section number. Several of these are due to Najm al-Dīn al-Miṣrī and have now been studied by François Charette.[2] Others have been found in two manuscripts which I saw for the first time in 2001, namely, Bursa Haraççioğlu 1177,4 and the precious Istanbul U.L. A314. On the other hand, much more could be written on aspects of timekeeping using astronomical instruments, a topic which I have pursued since arriving in Frankfurt and which has more recently been taken up by Charette. Also I did not add much more information about the manuscripts themselves. I wrote at least **Parts I-II** of this book when I was young and innocent; with increasing years I realize that the lack of information such as date and location of copying is unfortunate, but I am not inclined to add it now since I no longer trust my own notes and the majority of the manuscripts are not properly catalogued anyway. The reader may assume that the manuscripts were copied in the locations for which the tables were compiled; if they were copied elsewhere, unless they were universal, the tables would be useless anyway, apart from their inherent academic interest. It was in Cairo that a very high proportion of the tables were compiled, and it is not an accident that it was in Cairo that I wrote most of this book. Most researchers will find my English survey of the Cairo scientific manuscripts more accessible than the manuscripts themselves. However, the survey is full of references to a certain "**SATMI**". Here, finally, it is.

---

[2] See the paragraph associated with n. 16 to the main preface.

This page intentionally left blank

TABLE OF CONTENTS

CHAPTER 1

ON ISLAMIC TABLES FOR TIMEKEEPING

"That cultural collaboration had ... become possible for the most different nations Al-Bîrûnî appreciates with full consciousness ... . For him Islam was culture rather than religion, and the Arabic language the language of Science rather than that of the Qurʾân." A. Z. Validi, "Islam and Geography" (1934), p. 519.

"Wherever in the medieval world there were tables, real astronomy was practiced; where tables were lacking there were only dilettantes and dabblers." James Evans, *Ancient Astronomy* (1998), p. viii.

1.0 Introductory remarks

In this study I present a survey of all known examples of a category of Islamic astronomical tables[1] preserved in scientific manuscripts located mainly in libraries in Europe and the Near East. Since many of these precious sources are uncatalogued or not properly catalogued, and also since many of the tables are not clearly associated with an author or do not boast a clear title, the vast majority of the tables have not been investigated previously in modern times.[2]

The tables were intended as aids to the solution of the basic problems of spherical astronomy: given the altitude of the sun or any star, to find the time of day or night, the azimuth of the celestial body, and the longitude of the ascendant or horoscopus. Such tables, many of which contain several thousand entries, are generally not contained in the Islamic astronomical handbooks known as *zīj*es, the profusion and diversity of which were shown by Ted Kennedy already in 1950s.[3] The *zīj*es generally tend to have no tables for timekeeping beyond a set of

---

[1] In the context of the tables and Islamic science in general the term "medieval" refers to the period from *ca*. 750 to *ca*. 1900. The "renaissance" of the mathematical sciences took place in the Islamic world in the late 8th, 9th and 10th centuries. Muslim interest in these sciences was progressive until the 15th century, and thereafter and until the 19th century the Muslims continued to practice them but without further original input of consequence. This is nowhere more evident than in the tables discussed in this study. See also n. **II**-1:1.

[2] Prior to 1970 the only study of an Islamic table for astronomical timekeeping was Goldstein, "Medieval Table for Reckoning Time from Solar Altitude", which contains an analysis of the table described in **2.3.4**. King, "Astronomical Timekeeping in Medieval Cairo", includes an analysis of the tables from the Cairo corpus described in **2.1.1**, **4.7.1**, and **5.1.1**; *idem*, "al-Khwārizmī", pp. 7-11, a discussion of the earliest prayer-tables for Baghdad (**II-3.1**); *idem*, "Astronomical Timekeeping in Medieval Syria", and "Astronomical Timekeeping in Ottoman Turkey", and "Astronomy in the Maghrib", pp. 37-40, for three regional surveys. An overview is in the article "Mīḳāt, ii. Astronomical aspects" in EI_2, repr. in King, *Studies*, C-V. An overview in Arabic – the only one of its kind – is in the Arabic edition of *EHAS*, I, pp. 173-238. Several of the tables listed in **7.1** and the auxiliary tables discussed in **9.1**, **9.3**, **9.5** and **9.6** have been analyzed before: references are given *ad loc*. The existence of the tables described in **2.3.1** and **2.3.2** was noted in Suter, *MAA*, p. 50, and Kennedy, "*Zīj* Survey", p. 161, respectively.

[3] Kennedy, "*Zīj* Survey", published in 1956 and reprinted *ca*. 1990, contains a survey of about 120 Islamic *zīj*es and a partial analysis of several notable examples. The number of individual *zīj*es compiled by Muslim

standard spherical astronomical tables for the latitude for which the work was compiled: with a few exceptions I have not treated these standard tables here. I shall, however, have occasional recourse to the data-base of coordinates of the extensive geographical tables in Islamic *zīj*es prepared by Ted and Mary Helen Kennedy, which is useful for identifying the localities associated with latitudes underlying tables when these are not noted in the sources.[4]

Timekeeping – called *ʿilm al-mīqāt* in Arabic – was an important branch of Islamic astronomy (see **Fig. 1.0a**).[5] In addition to the obvious need for the professional astronomer to be able to determine the time of day or night accurately, the times of the five daily prayers in Islam are astronomically defined.[6] In a parallel study (**II**) I discuss numerous medieval tables for determining the prayer times in different localities, some of which supplement the tables for timekeeping considered in this study (here occasionally referred to as **I**).

Previous investigations by Otto Neugebauer have called attention to the simple arithmetical schemes known from Hellenistic, Byzantine and Ethiopic sources, which display midday shadows throughout the year or, less frequently, the shadow lengths for each seasonal hour of daylight.[7] In another study (**III**) I have analyzed over 50 Islamic shadow-schemes of one kind or another; these are characteristic of the tradition of non-mathematical folk astronomy which flourished in the Muslim world throughout medieval times.[8] Likewise, David Pingree has drawn attention to tables compiled by Indian astronomers displaying the length of daylight as a function of solar longitude.[9] The Islamic tables investigated in this study represent a far

---

astronomers between the 8th and the 19th century is closer to 200. King, "Islamic Astronomical Tables", first published in 1975, contains a preliminary account of various categories of Islamic astronomical tables which are *not* generally found in *zīj*es. See now the interim report in King & Samsó, "Islamic Astronomical Handbooks and Tables", with a summary in the article "Zīdj" in EI_2. New insights are to be anticipated from the new survey of *zīj*es currently in preparation by Benno van Dalen of Frankfurt. On the calendars used by Muslim astronomers see van Dalen's article "Taʾrīkh. 2. Era chronology in astronomical handbooks" in EI_2.

[4] Kennedy & Kennedy, *Islamic Geographical Coordinates*, presents *ca*. 15,000 sets of coordinates for *ca*. 2,500 localities from some 75 medieval works, organised according to locality, source, and increasing longitudes, and increasing latitudes. This important reference work is currently being updated by Mercè Comes in Barcelona.

For the purposes of the identification of localities that could be served by a given medieval value of a latitude, modern latitudes are often irrelevant and, more often than not, misleading.

[5] The best basic survey of Islamic astronomy remains that of Nallino, "Islamic Astronomy", published in 1921, which must be read in the light of the later contributions of E. S. Kennedy and his school. For surveys of "Islamic astronomy" and "astronomy in the service of Islam" written for a general audience, see Saliba, "Islamic Astronomy and Astrology"; and King, "Islamic Astronomy", A-B, and *idem*, "Science in the Service of ... Islam" (first published in 1991), repr. in *idem*, *Studies*, C-I. For the broader picture see Hill, *Islamic Science and Technology*, and Dallal, "Islamic Science, Medicine, and Technology". A useful survey of Islamic scientific literature in general is Endreß, "Wissenschaftliche Literatur". A more popular overview in French by a specialist is Djebbar, *Une histoire de la science arabe*. It has taken an interested layman to produce the first overview of Islamic science that could be used in schools or colleges in the English-speaking world: see H. Turner, *Science in Medieval Islam*.

[6] See further **II-1.1** on these definitions, and **IV** on the motivation behind those for the daylight prayers.

I use the expression "timekeeping" rather than, say "time-measurement" (as in *mésure du temps* or *mesura del tiempo*) or "time-reckoning" (as in *Zeitrechnung*), because, at least as far as the prayers were concerned, the times were pre-defined (*mawāqīt al-ṣalāt*), and Muslims have always endeavoured to "keep" those times.

[7] See Neugebauer, *HAMA*, II , pp. 736-747, for an overview.

[8] See King, "Islamic Folk Astronomy", and, most recently, Varisco, "Islamic Folk Astronomy".

[9] See Schmidt, "Daylight in Hindu Astronomy", for a detailed description of one such table, and Pingree,

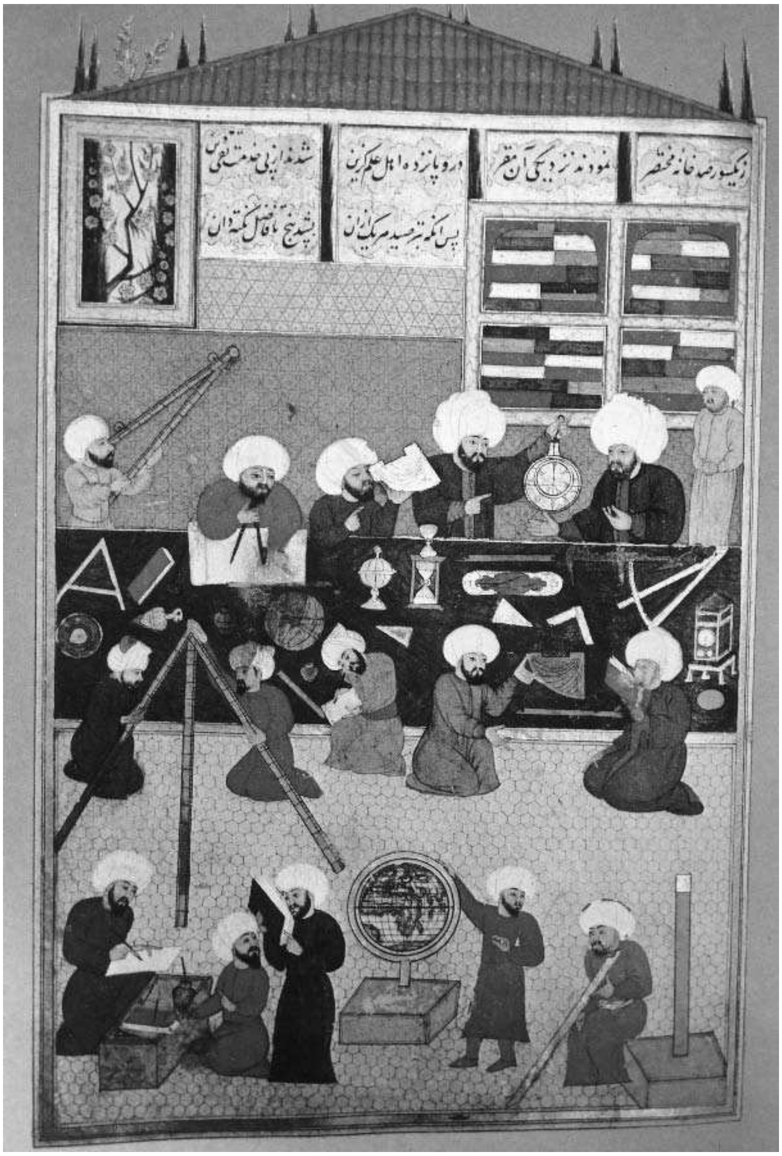


Fig. 1.0a: A beautiful miniature of the astronomers of the Istanbul Observatory *ca*. 1577. The men are depicted with various instruments (see **X**), none of which are known to have survived the vicissitudes of time, but some of the manuscripts on the bookshelf behind the men holding the astrolabe are now in the University Library, Leiden. Several manuscripts in that collection bear the mark of ownership of Taqi 'l-Dīn ibn Maʿrūf (see **Fig. II-5.1**), who is one of those two men. His compilations on astronomical timekeeping are surveyed in the present work (see **I-2.3.6** and **II-14.9**). [From MS Istanbul UL Yıldız F-1404, fol. 57r, courtesy of Istanbul University Library.]

"Sanskrit Astronomical Tables in the US", pp. 72-73, for references to several others. Most Islamic methods of determining time and azimuth are developments of methods originally adopted from Indian sources. See, for example, Davidian, "Al-Bīrūnī on the Time of Day"; E. S. Kennedy's commentary to al-Bīrūnī, *Shadows*; and, most recently, Lorch, "Sine Quadrant".

more sophisticated tradition of mathematical astronomy, and although they have no direct precursors in Greek astronomy, some of them in a sense represent a development of Ptolemaic tables for spherical astronomy, which included, for example, tables of ecliptic rising times for the seven "climates" of classical mathematical geography.[10] The influence of the climates in medieval astronomy and also astronomical instrumentation was far more widespread than has been recognized hitherto.[11] A significant part of the Islamic tradition which I investigate here was concerned with what I call "universal solutions", by which I mean solutions to problems of spherical astronomy which serve all latitudes; tables and instruments serving all of the climates are in a sense "universal" (see further **VIa-b**).

It has been possible to locate extensive corpuses of tables for timekeeping computed for each of Baghdad, Cairo, Taiz, Damascus, Jerusalem, Tunis, Alexandria and Istanbul. The earliest tables are those for Baghdad: some date from the 9th and 10th centuries, when Muslim astronomers had assimilated Indian and Ptolemaic astronomical ideas and were already making significant contributions of their own.[12] The principal centres of astronomical timekeeping from the 13th to the 15th century were Cairo and Damascus,[13] and from the 16th century onwards, Istanbul.[14] There was also impressive activity in Taiz and Tunis in the late 13th and 14th centuries. The growth of interest in timekeeping in Cairo lead to the establishment of the office of the *muwaqqit*, that is, the professional astronomer associated with a religious institution, one of whose functions it was to regulate the times of prayer. I have surveyed the activities of the *muwaqqit*s in another study (**V**). But as we shall see, the tables compiled by certain Syrian and Egyptian *muwaqqit*s between the 13th and 16th centuries relate not only to the times of prayer but also to astronomical timekeeping in general and they are particularly impressive from a mathematical point of view. The Yemeni and Tunisian tables form part of sophisticated traditions of mathematical astronomy in these regions which have only recently been

---

[10] In *Almagest*, II.8 (pp. 100-103) the individual and accumulated rising times of each 10° of the ecliptic are tabulated, and in the *Handy Tables* (Stahlman, *Handy Tables*, pp. 206-242) normed right ascensions and oblique ascensions are tabulated for each 1° of the ecliptic.

[11] See the *EI*$_2$ article "Iḳlīm" by A. Miquel, and on the importance of the climates in instrumentation see King, "Astronomical Instruments between East and West", esp. pp. 152, 168-169, and *idem*, "Geography of Astrolabes", pp. 6-9, as well as **VIa-b**.

The only Islamic table for timekeeping involving the climates is a 16th-century Egyptian table for twilight at each half-climate discussed in **II-8.2** (illustrated). A much earlier (Andalusī?) table for lunar crescent visibility computed for each of the climates is discussed in King, "Islamic Tables for Lunar Crescent Visibility", pp. 197-207, and also **VIa-7** (illustrated).

[12] On the foreign influences on early Islamic astronomy see Pingree, "Indian Influence", and *idem*, "Greek Influence", and the same author's article "*ʿIlm al-hayʾa*" in *EI*$_2$; and Saliba, "Arabic Science and the Greek Legacy". The only survey of early Islamic astronomy is Morelon, "Early Eastern Arabic Astronomy".

[13] Several Cairo and Damascus astronomers associated with timekeeping and their major works are listed in Suter, *MAA*; Brockelmann, *GAL*; and *Cairo ENL Survey*. For an overview of their activities see King, "Astronomy of the Mamluks".

[14] The Ottoman tradition in mathematical astronomy was until recently documented only in a few specialized studies by Aydın Sayılı and Sevim Tekeli and the art-historian Süheyl Ünver, and various symposium publications such as *Istanbul 1977 Symposium Proceedings* and *Istanbul 1991 and 1994 Symposia Proceedings*. An overview of Ottoman activity in timekeeping is in King, "Astronomical Timekeeping in Ottoman Turkey". We now have the six impressive volumes of references to hundreds of Ottoman scientific scholars and their works listed as İhsanoğlu *et al*., *Ottoman Astronomical / Mathematical / Geographical Literature*.

investigated for the first time.[15/16] We shall be constantly confronted with the fact that astronomy in Islamic civilization did not progress uniformly, and that eventually, when all the relevant problems had been solved, some many times over, all initiative dried up. What any given astronomer might know in the late period was to some extent a matter of chance. It depended not least on his location, and the authorities locally recognized. The works of astronomers in the period from the 8th to the 11th period were not generally available, and some of them are known now thanks only to the labours of a series of orientalists.

The reader will observe that I have been able to locate virtually no material from al-Andalus[17] and rather little from medieval Iran, Central Asia and India. In al-Andalus at least, mathematical astronomy was apparently directed towards other ends. Yet there must have been some activity in this field: how else can one explain the appearance in the early 15th century of a table of solar altitude as a function of the date and the time of day computed for the latitude of Baez (see **10.1**)? Were corpuses of tables for timekeeping available for Rayy and Nishapur, Bukhara and Samarqand? I have no evidence that they were, but find it hard to believe that they were not. The rich collections of astronomical manuscripts preserved in Iran remain virtually untouched by modern scholarship,[18] but those manuscripts of Iranian provenance which I have examined elsewhere (including the former Uzbek S.S.R. and India) contain little material on timekeeping.[19] However, I do not doubt that further investigation of the vast manuscript sources available for the study of the history of Islamic astronomy will bring to light other examples of the various categories of tables described in this part of my study. Indeed, no-one will be happier than myself to see a supplement to the present work.[20]

Another question that must be posed is: were these tables ever used? I have no evidence (like a sentence here or there in a historical source) that they were. The fact that some of them were copied dozens of times over several centuries is all the indirect evidence that we have.

Furthermore, the Muslim astronomers are generally silent about the methods used to compile their tables, the interpolation schemes they employed,[21] and the nature of the tables for sexagesimal arithmetic[22] as well as of the trigonometric tables which they had at their disposal.[23] Some examples are shown in **Figs. 1.0b-f**. All that we have is the tables for

[15] See King, *Astronomy in Yemen*, based on over 100 Yemeni astronomical manuscripts.

[16] For a first attempt to survey the Maghribī tradition of astronomy see the second version of King, "Astronomy in the Maghrib", parts of which are already superceded by various studies of the Barcelona School, including Samsó, "Maghribī *Zīj*es", and notably Mestres, Zīj *of Ibn Isḥāq*.

[17] I use this term to indicate that part of the Iberian Peninsular under Muslim domination at any given time.

[18] This is not to say that they are not catalogued, and much credit is due to Iranian scholars (especially al-ʿĀmilī, *Aʿyān al-Shīʿa* (56 vols., published 1935-1962), and Āghā Buzurg, *al-Dharīʿa* (25 vols., published 1965-1978), who have surveyed vast quantities of manuscripts mainly in Iranian collections. But tables of the kind studied here seldom bear titles that a cataloguer would pick up.

[19] The standard bibliographical reference work on Iranian and Indian astronomy in Persian, listed as Storey, *PL*, II:1, contains no references to any works of consequence dealing with timekeeping. Likewise the *Aʿyān al-Shīʿa* and *al-Dharīʿa ilā taṣānīf al-Shīʿa* (see previous note), contain innumerable references to astronomical works in Persian.

[20] See the text to n. 1:51.

[21] On Islamic interpolation schemes see Hamadanizadeh, "Medieval Islamic Interpolation Schemes", and the references there cited, as well as Rashed, "Méthodes d'interpolation". On interpolation in tables of timekeeping see **II-5.8**. Some primitive schemes are discussed in **II-8.7** and **11.9**. The short notes in MS Cairo Zakiyya 917,4 would be worth investigating: see n. **II**-10:41.

[22] See King, "Medieval Islamic Multiplication Tables", A-B.

[23] On Islamic trigonometric tables see the ground-breaking studies of Carl Schoy listed as "Beiträge zur

Fig. 1.0b: An extract from a sexagesimal multiplication table displaying products m × n for m = 0;1, 0;2, ... , 59;59, 60;0 and n = 1 , 2, ... , 60. This extract shows products for m = 14;21, 14;22, ... , 14;40, and n = 1, 2, ... , 60. This manuscript was copied *ca*. 1700. Smaller multiplication tables with integral m are attested from the 10th century onwards. [From MS Paris BNF ar. 2552, fols. 44v-45r, courtesy of the Bibliothèque Nationale de France.]

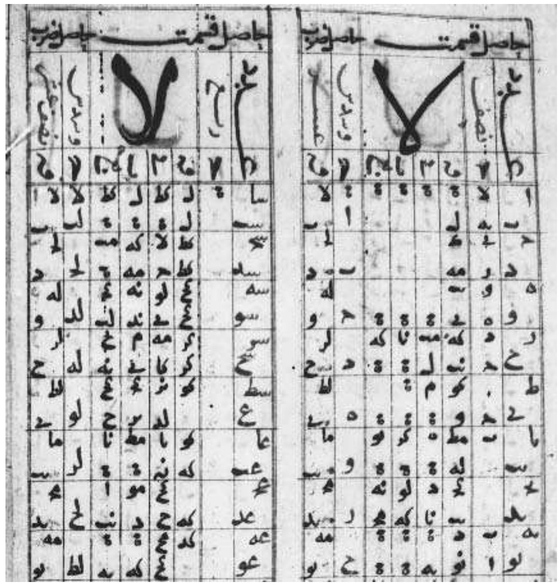


Fig. 1.0c: An extract from a table of sexagesimal quotients displaying quotients m ÷ n for m = 1, 2, ... , 60 and n = 1, 2, ... , 120. This extract shows quotients for m = 31 and 32. This table is unique of its kind. [From MS Cairo DMF 8,1, fols. 23v-24r, courtesy of the Egyptian National Library.]

Fig. 1.0d: An extract from the sine tables attributed to the 10th-century Cairene astronomer Ibn Yūnus. The table gives values of the sine to base 60 for each minute of each degree, with first differences for each second. Entries in the main table are to five sexagesimal digits, but they are based on the values for each 0;10°, which are taken from his major work, the *Ḥākimī Zīj*. Their attribution to Ibn Yūnus cannot be considered certain, given the penchant of Mamluk astronomers for "stretching" tables in this way (see **II-7.2**). The extract shows the entries for 22° and 23°. [From MS Berlin 5752, fols. 13r-14r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

Fig. 1.0e: An extract from some trigonometric tables from Mamluk Egypt. This extract is from a table of cotangents to base 12 and values are given to three sexagesimal figures for each minute. This double-page serves arguments 42°-47°. The tables were copied by the Cairo muwaqqit Sayf al-Dīn Sāṭilmish *ca*. 1450. [From MS Cairo ṬM 80, fols. 23v-24r, courtesy of the Egyptian National Library.]

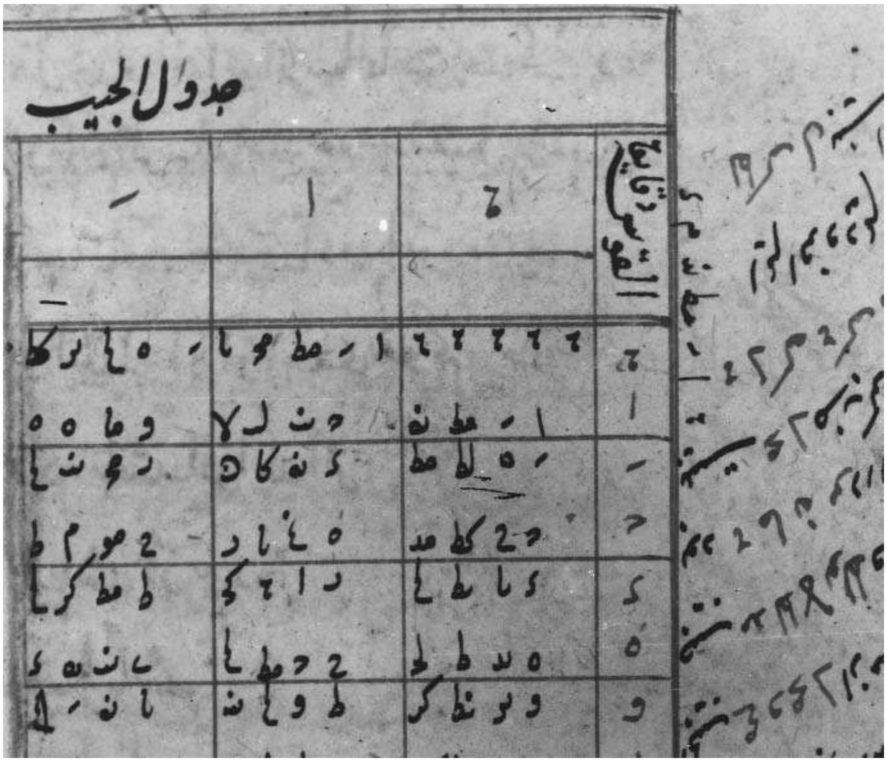


Fig. 1.0f: An extract from the sine tables compiled by Ulugh Beg in Samarqand *ca*. 1430. Entries are to five sexagesimal digits, with values for each minute of arc. This extract shows the first few entries in the table, serving 0-13 minutes over 0°, 1° and 2°. These tables were often copied separately from the *zīj*. [From an unidentified Cairo manuscript (one of those listed in *Cairo ENL Survey*, no. G49, 2.2.1), courtesy of the Egyptian National Library.]

timekeeping themselves, and detailed analysis of error patterns would doubtless furnish additional information on medieval computational techniques. The analysis of medieval tables has made vast strides in recent years.[24] See further **1.5** and **II-1.5**, also **II-10.10**.

Until a survey has been made of medieval and Renaissance European tables for spherical astronomy it will not be possible to ascertain whether any of the different categories of Islamic tables found their way to medieval Europe. There is, alas, no survey of medieval European astronomical tables in general, arranged either chronologically or regionally, nor has any

---

arabischen Trigonometrie", *Die trigonometrischen Lehren des al-Bîrûnî*, and *Schattentafeln* (on the cotangent and tangent functions), as well as numerous others reprinted in his *Beiträge*, and the following more recent studies: Kennedy, "Overview of the History of Trigonometry"; *idem*, "*Zīj* Survey", pp. 139-140; Berggren, *Islamic Mathematics*, pp. 127-156.

[24] On the first attempts the reader may consult Gingerich, "Applications of Computers to the History of Astronomy"; Kennedy, "The Digital Computer and the History of the Exact Sciences"; and King, "Astronomical Timekeeping in Medieval Cairo", p. 353.

On more recent activity see van Dalen, *Ancient and Mediaeval Astronomical Tables*, and *idem*, "Statistical Method for Recovering Parameters"; Van Brummelen, "Mathematical Tables in Ptolemy's *Almagest*"; *idem* & Butler, "Interdependence of Astronomical Tables"; Mielgo, "Analyzing Mean Motion Tables"; and King, *Mecca-Centred World-Maps*, pp. 163-168.

attempt been made to prepare one.[25] However, the very first medieval European manuscript that I looked at (in Gotha in 1973) and several others that I have seen in recent years (mainly during the 1990s) also contain tables for timekeeping, and it is clear that further research in this field would be worthwhile (see already **10.1-2**).

For inadequacies in the presentation of the material in **I-II**, I beg the indulgence of the reader. The problems associated with research on sources scattered between Princeton and Tashkent, and between Dublin and Sanaa, should be obvious. Not all of the libraries where the sources are housed have facilities for photographing, or even for photocopying manuscripts, and some of those that do have such facilities are administrated by bureaucrats, in-house or in-capital, who are too stupid to provide such services to researchers or who are so clever that they charge outrageous amounts for a single photo. I make no apologies for the organisation of the material presented: it seemed a good idea at the time to arrange the spherical astronomical tables according to their structure and the prayer-tables and the corpuses in which they are found chronologically according to provenance. Also, all this was done in the 1970s, when one worked with type-writers and revised texts using a technique known as "cut and paste". In this first part of my study the reader should bear in mind that the standard tables of spherical astronomy also occur in *zīj*es, and for these only unusual tables relating specifically to timekeeping have been included here.

This study owed its inception to the fact that the manuscripts in Berlin and Paris had long been catalogued in an excellent fashion, and that the descriptions of tables were sufficiently detailed to convey a good idea of the nature of the tables themselves. It owed its continuation and fruition to the fact that for several years I had unlimited access to manuscripts in Cairo and Istanbul. As I prepare the final version of this text I feel confident that most of the materials preserved in the libraries of those two cities, as well as the major European libraries, have been included. Libraries in the Maghrib, Iran and India have been exploited to a much lesser extent, and it may well be that new tables unknown to me now will one day be found there. Any future researcher would do well to start with some materials, mainly anonymous, listed by Ekmeleddin İhsanoğlu and his colleagues in Istanbul.[26] That researcher, if diligent, will find that some of these are indeed already treated in the present work, others maybe not. In any case, the present work should serve as a framework for controlling, classifying, and setting a context for any significant new sources.

---

[25] There are, of course, numerous valuable studies of particular corpuses of tables by scholars familiar with the manuscript sources. See, for example, Toomer, "Toledan Tables", and now F.S. Pedersen, *The Toledan Tables*; Poulle, *Studies*; Goldstein & Chabás, *Zacuto*. See also North, *Horoscopes and History*, on the way in which astronomical tables were used to compile horoscopes, and Chabás, "Cahier d'un croisier", on an astronomer's notebook.

[26] See İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 729, 793-795, 801, 802, 804, 805, 836, 869-870, for various tables relating to timekeeping. For one table mentioned by İhsanoğlu *et al.* which is highly important and of a type not attested elsewhere see **8.5.1*** and **9.4*** below.

1.1 Notation used in the analysis

In this text notation in the form "(**1.2.3**)" stands for "(see Section 1.2.3)". Occasionally here in **I** there are also cross-references to **II** in the form "(**II-1.2.3**)", and *vice versa*.

In the mathematical analysis of tables I use the notation f(x,y) to indicate that the function f is tabulated with x as the horizontal argument and y as the vertical argument. The following notation is used freely in the sequel, On the auxiliary functions for timekeeping (B, C and G), and for calculating ascensions (e) and the azimuth (k and L) see further **Chs. 6-9**.

Latin

a	azimuth (usually measured from the prime vertical)
d	equation of half-daylight (= D – 90°)
B	an auxiliary function for timekeeping, called "absolute base", related to cos δ cos ϕ
C	an auxiliary function for timekeeping related to sin δ sin ϕ
D	semi diurnal arc (sun) or semi nocturnal arc (stars)
e	the auxiliary function tan δ tan ϕ
G	an auxiliary function for timekeeping related sec δ sec ϕ
h	instantaneous solar or stellar altitude
$\breve{h}$	number of degrees in one seasonal day-hour (= $^{1}/_{12}$ 2D)
$\breve{h}'$	number of degrees in one seasonal night-hour (= $^{1}/_{12}$ 2N)
h_a	solar altitude at the beginning of the afternoon prayer
h_0	altitude in the prime vertical
$-^{h}$	equinoctial hours (also used in the representation of functions, thus: $2D^{h}(\lambda) = {}^{1}/_{15}\ D(\lambda)$)
H	meridian altitude
H′	the difference between the Sines of the meridian and instantaneous altitudes (H′ = Sin h - Sin h)
k	an auxiliary function for azimuth calculations, related to sin h tan ϕ
L	an auxiliary function for azimuth calculations, related to sin ψ = sin δ / cos ϕ
L	terrestrial longitude
N	semi nocturnal arc
q	qibla, measured from the meridian
$-_{q}$	relates to the qibla
R	base for trigonometric functions (usually 60, although also 12 and 7 are used for Cotangents and Tangents)
$-^{sdh}$	seasonal day-hours
$-^{snh}$	seasonal night-hours
t	hour-angle (actually an arc in equatorial degrees, rather than an angle)
T	for the sun: time since sunrise or time until sunset; for a star: time since rising or time remaining until setting
z	horizonal shadow of a vertical gnomon (base given in parentheses as subscript)

z_a	shadow at the beginning of the afternoon prayer
z'	vertical shadow
Z	shadow at midday

Greek

α	right ascensions
α'	normed right ascensions ($\alpha' = \alpha + 90°$)
α_ϕ	oblique ascensions for latitude ϕ
α_r	oblique ascensions of the ascendant at daybreak
α_s	oblique ascensions of the ascendant at nightfall
δ	solar declination
δ^*	solar declination augmented by 90° (useful for finding $H = \delta^*-\phi$)
Δ	stellar declination
ε	obliquity of the ecliptic
θ	independent variable
λ	solar longitude or ecliptic longitude
λ'	see below
λ^*	see below
λ_H	longitude of the ascendant (horoscopus)
ρ	longitude of the co-ascendant (point of the ecliptic which rises with a star)
ϕ	terrestrial latitude
ψ	rising amplitude

Miscellaneous

$\bar{\theta}$	complement of θ
λ'	longitude measured from the nearer equinox
λ^*	longitude of the point on the ecliptic opposite the point with longitude λ ($\lambda^* = \lambda + 180°$)
$[x]$	integral part of x

In virtually all of the tables inspected the entries are written sexagesimally in standard Arabic alphabetical notation (*abjad*).[27] In one important table dating from the earliest period of Islamic astronomy and in most which date from recent centuries the entries are given in Arabic numerical notation – see **Figs. II-3.1** on the former and **I-2.2.4** for one example of the latter. In the analysis the term "digit" means "sexagesimal digit",[28] unless otherwise stated, so that, for example, a number written 23;50,21 represents $23 + {}^{50}/_{60} + {}^{21}/_{3600}$. In the middle of an Arabic text this number would be written "23 50 21 seconds". In the tables the number would be written "23 50 51", and the headings would indicate that the three figures are degrees (abbreviated *j* for *daraj*), minutes (*q* for *daqāʾiq*) and seconds (*y* for *thawānī*).

---

[27] On this notation see Irani, "Arabic Numeral Forms", and Destombes, "Chiffres coufiques".
[28] See, for example, Kennedy, "*Zīj* Survey", p. 17, and Berggren, *Islamic Mathematics*, pp. 39-63.

The medieval trigonometric functions are to base R other than unity (see below). The functions are here denoted by the standard capital notation;[29] thus for a general arc θ:[30]

$$\mathrm{Sin}\,\theta = \mathrm{Sin}_R\,\theta = R\sin\theta\ ,$$
$$\mathrm{Cos}\,\theta = \mathrm{Sin}\,\bar{\theta} = R\cos\theta\ ,$$
$$\mathrm{Vers}\,\theta = R - \mathrm{Cos}\,\theta = R\,(1 - \cos\theta)\ ,\ etc.$$

The Cotangent and Tangent functions were generally, but not exclusively, used with the solar altitude as argument to represent the horizontal shadow of a vertical gnomon length n and the vertical shadow of a horizontal gnomon of the same length, respectively. Thus:

$$\mathrm{Cot}\,h = \mathrm{Cot}_n\,h = n\cot h\ ,$$
$$\mathrm{Tan}\,h = \mathrm{Tan}_n\,h = n\tan h\ .$$

I use Z for Cot H and z for Cot h and occasionally z′ for Tan h, with the base indicated in parentheses in the subscript, thus, for example, $Z_{(6;40)}$ stands for $\mathrm{Cot}_{6;40}$ H. I also use z_a, *etc*., for the shadow at the *ʿaṣr* prayer. The 'z', for Arabic *ẓill*, shadow", should serve as a reminder that the medieval Cotangent function is of a different nature from the medieval Sine function.

The base used for the Sine and related functions is generally 60, although some of the earliest sources use the Indian value 150.[31] The Cotangent and related functions are usually to base 12 or 7, although in some sources we find 60 (see further below). Most of the sources surveyed in this study use 12 (digits) or 7 (feet) for the gnomon length n. We shall also encounter occasional use of 6 or 6;40, intended as variants of 7;[32] as well as of 60, the last-mentioned intended to correspond to the tables of the Sine function. The use of base 5 is not to be compared with these, for the 5 here is simply 60÷12.[33] However, and this is of greater historical interest (if not significance), we shall also encounter the use of 1 and 10 in early sources, with entries expressed sexagesimally.[34] Very occasionally, we find 20 used as a base.[35]

In the analysis, it is to be assumed that all intervals of time are expressed in equatorial degrees unless otherwise indicated. These may be converted to equinoctial hours (denoted by superscript h) according to the relation:

$$T^{h} = T^{\circ} / 15 = 0;4\ T^{\circ}\ ,$$

or to seasonal day-hours (denoted by superscript sdh) according to:

$$T^{sdh} = 6\ T^{\circ} / D\ .$$

In certain Ottoman tables the time of day is given in equinoctial hours according to the convention that sunset is 12 o'clock.[36] If T represents the equinoctial hours that have elapsed

---

[29] On this see, for example, Kennedy, "*Zīj* Survey", pp. 139-140.

[30] The arguments of the medieval Sine and related functions were considered to be arcs of a base circle, as opposed to angles (a more modern notion in trigonometry).

[31] Base 150 is used, for example, in the Sine tables discussed in **4.1.1**, **4.2.2**, and also **7.1.1** and **7.2.1**.

[32] On these and other bases for the cotangent function see the text to n. **II**-1:17. For the use of 6 see **II-2.9** and **5.6**.

[33] See, for example, various tables discussed in **7.1.5** and **7-8**, and also **II-6.6-7**.

[34] For a Cotangent table to base 1 from the 9th century see **4.1.1**. For the use of 1 and 10 in the same 10th-century source see **4.2.3**. For the use of 10 (and 20) in a 14th-century source see **II-10.6**, especially n. 10:22. For the use of 100 in a 16th-century source see **7.1.12**.

[35] For 20 see **II-10.6**, especially nn. 10:22 and 39a.

[36] On this convention see Würschmidt, "Osmanische Zeitrechnung", and the last paragraphs of **II-1.2** and also **II-14.0**.

since sunrise on a day when daylight is 2D equinoctial hours long, then the time of day T′ according to this convention is:

$$T' = 12 - 2D + T\ .$$

1.2 The standard medieval formulae for timekeeping

For the convenience of the reader I list in this section the formulae that underlie the major tables described in the sequel. To simplify the formulae slightly I use modern notation for trigonometric functions, that is, base R = 1 rather than 60. Each of these formulae is numbered, and I refer to them in the analysis by these numbers preceded by **F**. The formulae underlying the minor tables discussed in the sequel are presented *ad loc*. Likewise approximate methods are discussed only in the analysis. As we shall see, the Muslim astronomers used an array of auxiliary functions to facilitate their computations: these are introduced in the immediate sequel in special script, which is thereafter abandoned but used again in **XIIb**. The formulae for which they provide simplified solutions are presented in rectangular frames.

The formulae derived below represent methods that would be outlined *in words* in the astronomical handbooks known as *zīj*es or in the instructions to the tables themselves.[37] At least from the 13th century onwards lists of formulae of the form a : b = c : d, with four quantities a,b,c,d arranged in columns were available: see **Fig. 1.2a** and **Table 10.3b**. Two examples are:

(1) the Sine of instantaneous altitude : the Cosine of the instantaneous altitude = the length of the gnomon : the horizontal shadow,

that is, $\text{Sin } h : \text{Cos } h = n : z_n\ (= n : \text{Cot}_n\ h)\ .$

(8) the absolute base : the difference between the Sine of the meridian altitude and the Sine of the instantaneous altitude = unity : the Versed Sine of the hour-angle,

that is, $B : (\text{Sin } H - \text{Sin } h) = 1 : \text{Vers } t\ .$

It is perhaps appropriate that for technical reasons the formulae in this study are reproduced on a single line: in the first computer-generated version of this study they were more visually impressive, with large curly brackets and multi-level division bars, but with updated hard- and soft-ware these behemoths actually disappeared, their previous existence being indicated on my computer-screen by the message "**Fehler!**".[38] The present mode of rendition of the formulae should constantly remind the reader that the equivalent procedures in medieval Arabic texts

---

[37] On Islamic methods in spherical astronomy, see, for example, Neugebauer, *al-Khwārizmī* (al-Khwārizmī/al-Majrīṭī), and Goldstein, *Ibn al-Muthannā on al-Khwārizmī*; Nallino, *al-Battānī*; King, *Ibn Yūnus*; Samsó, *Estudios sobre Abū Naṣr*; Berggren, "Spherical Trigonometry in the *Jāmiʿ Zīj*" (Kūshyār); Schoy, *Die trigonometrischen Lehren des al-Bīrūnī*; Debarnot, *al-Bīrūnī's* Maqālīd; Villuendas, *Trigonometría de Ibn Muʿāḏ*; Sédillot-*père*, *Traité* (al-Marrākushī); Vernet, *Ibn al-Bannāʾ*; Kennedy, "Spherical Astronomy in Kāshī's *Khāqānī Zīj*"; and Sédillot-*fils*, *Prolégomènes* (Ulugh Beg).

On methods for timekeeping in particular see especially Nadir, "Abū al-Wafāʾ on the Solar Altitude"; and Davidian, "al-Bīrūnī on the Time of Day", and E. S. Kennedy's commentary on al-Bīrūnī, *Shadows*. See also the articles "Maṭāliʿ" [= ascensions] and "Mayl" [= declination] in EI_2.

For a modern discussion of spherical astronomy see Smart, *Spherical Astronomy*, esp. Ch. II.

[38] See n. 14 to the main preface.

Fig. 1.2a: A list of 32 sets of four quantities in spherical astronomy such that the ratio of the first to the second equals the ratio of the third to the fourth. Notice that all expressions are in words, without any recourse to symbols of any kind. This list is anonymous but was taken from the *magnum opus* of the late-13th-century Cairo astronomer Abū ʿAlī al-Marrākushī. [From MS Cairo DM 291,3, fols. 4v-5r, courtesy of the Egyptian National Library.]

involved sentences of several lines. The language of the astronomers of the late medieval period (13th-15th century) considerably facilitated the formulation of complicated mathematical procedures: the crisp, highly technical Arabic of al-Khalīlī (Damascus, *ca.* 1360) and al-Māridīnī (Cairo and/or Damascus, *ca.* 1375), let alone the garbled colloquial of Najm al-Dīn al-Miṣrī (Cairo, *ca.* 1325), is quite different in vocabulary and style from the Arabic of earlier scholars such as al-Battānī (Raqqa, *ca.* 910) and Ibn Yūnus (Cairo, *ca.* 990).[39]

All of the formulae are easily derived either from trivial considerations of the celestial sphere or from not-so-trivial orthogonal projections of the sphere, using the elegant procedure called the analemma that was known since classical Antiquity.[40] Certain Muslim astronomers preferred solutions to problems in spherical astronomy using the rules of spherical trigonometry,[41] but not those who specialised in timekeeping.

The celestial sphere

Fig. 1.2b shows a sphere of unit radius representing the celestial sphere about the observer at O on a fixed earth, with the zenith at Z and the local horizon NWSE. As a result of the rotation of the earth on its axis, the celestial sphere appears to the observer to rotate about

[39] The text of al-Battānī's *Zīj* is published in Nallino, *al-Battānī*, III. For an example of this late medieval scientific Arabic see the texts of al-Māridīnī's treatises on a quadrant of his own invention and on the use of his auxiliary tables published in King, "al-Māridīnī's Universal Quadrant", and al-Khalīlī's instructions on the use of his various tables translated in **II-10.4**, **10.7** and **10.8**). For the Arabic of Najm al-Dīn see Charette, *Mamluk Instrumentation*.

[40] On the analemma, see, for example, Paul Luckey, "Das Analemma von Ptolemäus"; Id, "Analemma for Ascensions"; Kennedy & Id, "Ḥabash's Analemma for the Qibla"; Carandell, "Analemma for the Qibla"; E. S. Kennedy's commentary to al-Bīrūnī, *Shadows*; King, *Ibn Yūnus*, *passim*; Berggren, "Four Analemmas for the Qibla"; and, most recently, King, *Mecca-Centred World-Maps*, pp. 15-16.

[41] See, for example, Samsó, *Estudios sobre Abū Naṣr*; and al-Bīrūnī, *Maqālīd*.

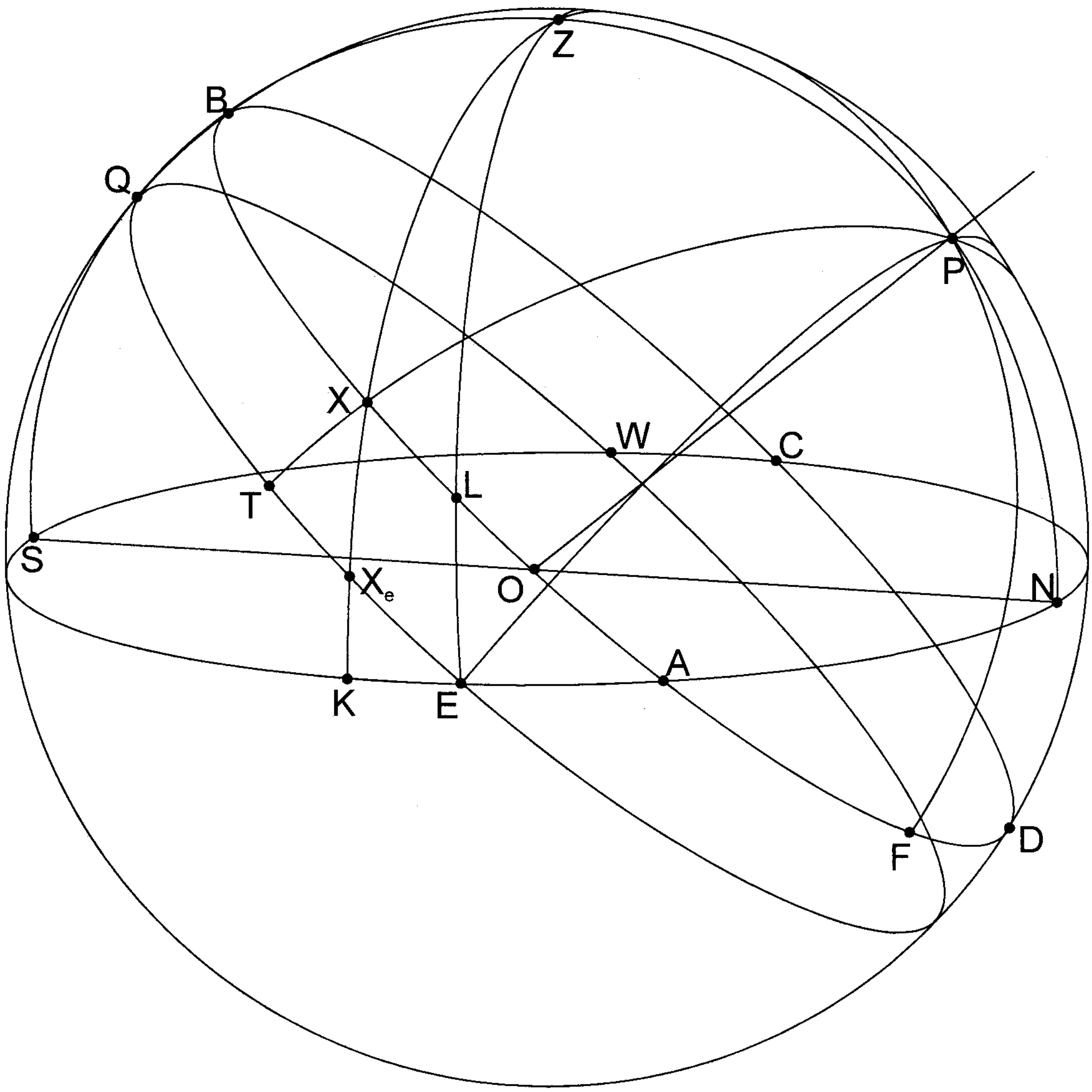


Fig. 1.2b. The celestial sphere. Compare **Fig. VIIc-4a**.

the axis OP once every 24 hours, where P is the celestial pole. The segment ZP defines the local meridian, intersecting the horizon at N and S, the north- and south-points, and the arc PN measures the local latitude ϕ (Arabic, *ʿarḍ al-balad*). The perpendicular plane EZW is the prime vertical, intersecting the horizon at E and W, the east- and west-points. The great circle EQW with its pole at P is the celestial equator.

The sun and stars participate in the apparent motion of the celestial sphere and are observed to traverse small circles parallel to the celestial equator, called declination circles or day-circles (*madārāt*). A given celestial body rises at A, culminates at B on the meridian, sets at C, and crosses the meridian below the horizon at D; its day-circle is ABCD.

The position of the day-circle relative to the celestial equator is determined by the declination of the celestial body, measured by arc BQ. I use Δ to denote declination in general or stellar declination in particular (*al-buʿd*) and δ only to denote solar declination (*al-mayl*). Observe that the meridian altitude H (*ghāyat al-irtifāʿ*), measured by arc SB, and the meridian altitude below the horizon H* are related to the local latitude and the declination by:

F1 $$H = \bar{\phi} + \Delta \quad \text{and} \quad H^* = \bar{\phi} - \Delta.$$

Some Muslim astronomers tabulated $\delta^*(\lambda) = \delta(\lambda) + 90°$, with which $H(\lambda) = \delta^*(\lambda) - \phi$.

Two other quantities related to the latitude and declination are the rising amplitude ψ (*saʿat al-mashriq*), measured by arc EA on the horizon, and the altitude in the prime vertical h_0 (*al-irtifāʿ alladhī lā samt lahu*), measured by arc EL on the prime vertical or east-west colure, which is the great-circle ELZW perpendicular to the meridian.

The most important problem of spherical astronomy is, given the instantaneous altitude h (*al-irtifāʿ*) of a celestial body at X, to determine the time since rising T (*al-dāʾir*), measured by the arcs AX on the day-circle or FT on the celestial equator, or the hour-angle t (*faḍl al-dāʾir*), measured by the arcs XB or TQ. These times are clearly related by the simple formula:

F2 $$T + t = D ,$$

where D is half the diurnal arc (*niṣf qaws al-nahār*), measured by arcs AB or FQ. The arc FE is the equation of half-daylight d (*taʿdīl niṣf qaws al-nahār*) and clearly:

F3 $$D = 90° + d .$$

When the sun or star is at G, the intersection of arcs AB and PE, the time since rising is d and the hour-angle is 90°. Another important problem of spherical astronomy is to determine the azimuth a of the celestial body (*samt*), measured by the arc EK on the horizon, where ZXK is the altitude circle of the celestial body.

Solar coordinates

As a result of the motion of the earth about the sun, the sun appears to move against the background of fixed stars on a great circle called the ecliptic which is inclined to the celestial equator at an angle $\varepsilon \approx 23^1/_2°$ called the obliquity (*al-mayl al-aʿẓam*). The sun moves along the ecliptic in a direction opposite to that of the apparent daily rotation and completes its journey around the ecliptic in a year.

When the sun is at one or other of the points of intersection of the ecliptic and celestial equator it rises at the east-point and sets at the west-point, and day is equal to night: these solar positions are the vernal and autumnal equinoxes. General solar positions on the ecliptic are determined by the sun's longitude λ (*darajat al-shams*), which is measured from the vernal equinox V. In **Fig. 1.2c** the longitude of the sun at X is measured by the arc VX.

The solar longitude at a given time can be determined either from the tables standard in *zīj*es, or from an ephemeris showing solar and other planetary positions for each day over a period of several years.[42] Another kind of table (called in Arabic *shabaka*) was common in tables for timekeeping: these show the solar longitude for each day of a period of four Syrian

[42] On the solar tables in *zīj*es see Kennedy, "*Zīj* Survey", p. 141. On Islamic ephemerides see the EI_2 article "Taḳwīm. i" by Michael Hofelich.

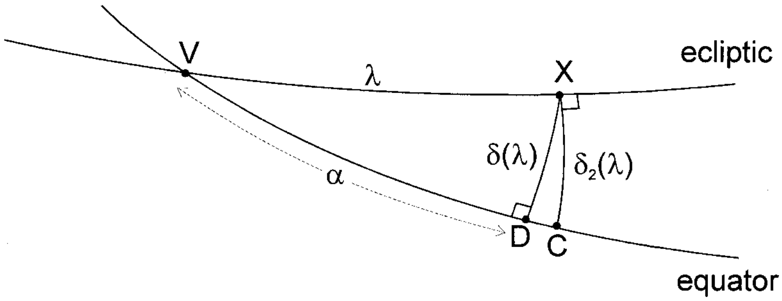


Fig. 1.2c.

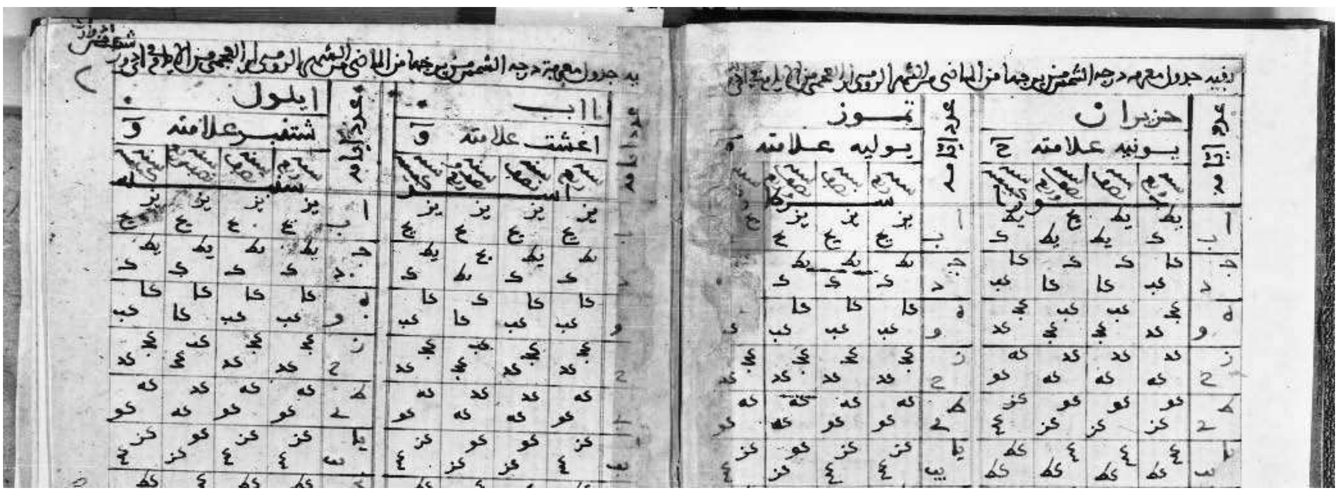


Fig. 1.2d: An extract from a Tunisian table showing the solar longitude as a function of the date in the Syrian (solar) calendar over a four-year cycle. This precedes the set of auxiliary tables for timekeeping described and illustrated in **9.7**. The tables were compiled about 1400 and the manuscript copied about 1600. [From MS Cairo DM 689, fols. 1v-2r, courtesy of the Egyptian National Library.]

or Coptic years (to cover the leap year) at a particular epoch.[43] For an example, see **Fig. 1.2d**.

As the sun moves around the ecliptic from the vernal equinox its declination to the north of the celestial equator increases and the length of daylight also increases, reaching a maximum when $\lambda = 90°$ and $\delta = +\varepsilon$ at the summer solstice. Likewise, as the sun moves past the autumnal equinox its declination to the south of the celestial equator increases and the length of daylight decreases, reaching a minimum when $\lambda = 270°$ and $\delta = -\varepsilon$ at the winter solstice. The declination is measured by the arc XD drawn perpendicular to the equator. The ecliptic is divided into twelve zodiacal signs of 30° each, six northerly and six southerly. Tables of $\delta(\lambda)$ are standard in *zīj*es.

---

[43] The only published example of a *shabaka* table is that of al-Marrākushī computed for Cairo, 992 Diocletian [= 1275/76], reproduced in Sédillot-*père*, *Traité*, I, pp. 136-137. See also nn. **I**-9:26 and **II**-10:19.

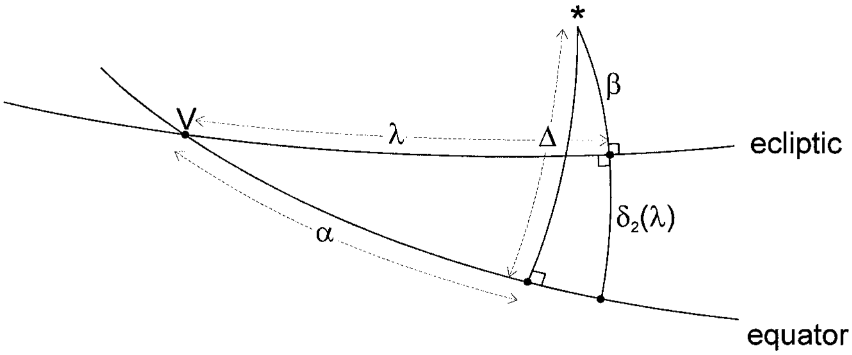


Fig. 1.2e.

Fig. 1.2f: The star table of Ibn Yūnus, missing from the surviving fragments of his *zīj*, was discovered during in the 1970s in an Egyptian manuscript copied *ca*. 1475 by Aḥmad ibn Timurbāy. It displays longitudes and latitudes for 59 stars for epoch 400 Yazdigird, that is 1032 C.E., some 30 years after the death of Ibn Yūnus. (His solar, lunar and planetary tables run up to 1800 Yazdigird, that is, 2432 C.E., so he clearly was in the habit of planning ahead.) The table has been edited but has not yet been published. [From MS Cairo MM 188,1, fol. 72v, courtesy of the Egyptian National Library.]

The position of the sun relative to the celestial equator is measured by the declination perpendicular to the equator and a coordinate called right ascension α (*maṭāliʿ darajat al-shams*) measured as VD along the equator from the vernal equinox V. Tables of α(λ) are also standard in *zīj*es. Another useful notion in spherical astronomy is the "second declination" δ_2 (*al-mayl al-thānī*), which is measured perpendicular to the ecliptic and represented by the arc XC. This quantity, occasionally tabulated in *zīj*es, is of use in the conversion of ecliptic and equatorial coordinates – see below.

Stellar coordinates

The positions of the stars in the heavens are conveniently measured with respect to the ecliptic or the celestial equator by two systems of orthogonal coordinates. In the first the coordinates

are longitude and latitude, λ and β, and in the second they are right ascension and declination, α and Δ – see **Fig. 1.2e**. The phenomenon of precession causes stellar longitudes slowly to increase but does not affect the latitudes; both the right ascension and declination of the stars vary slowly with time. Islamic star catalogues display coordinates of prominent stars in one or other system, sometimes both, for a particular epoch: for an example see **Fig. 1.2f.**[44] *Zīj*es generally contain instructions on methods of converting from the one coordinate system to the other.[45] For calculating the time by night it is necessary to know the coordinates of the stars in the equatorial system.

Solar declination and right ascensions

We shall now derive δ, δ_2 and α as functions of λ by means of an analemma. Firstly we project the celestial sphere into the plane whose pole is the vernal equinox V – see **Fig. 1.2g**. AB and CD represent the projections of the celestial equator and the ecliptic so that:

$$\text{arc BD} = \text{arc AC} = \varepsilon \ .$$

O and X_1 represent the projections of V and X, with X_1Y perpendicular to AB, and EGF with midpoint at G represents the projection of the declination circle through X, so that:

$$\text{arc BF} = \delta\ ,\ X_1Y = GO = \sin\delta \text{ and } GF = \cos\delta\ .$$

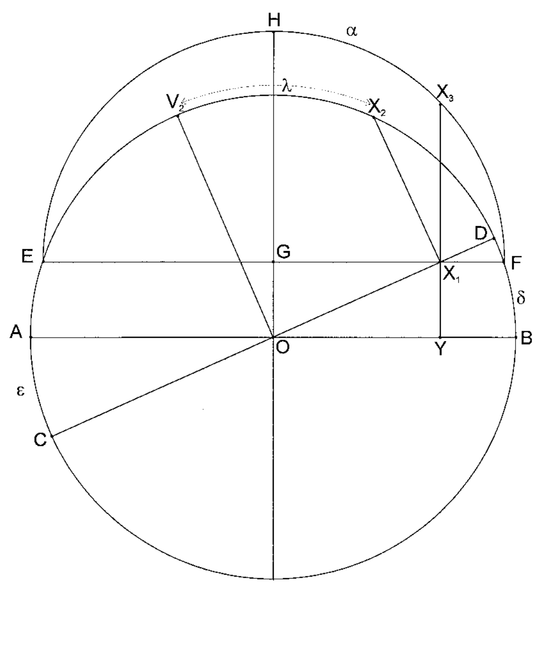


Fig. 1.2g.

[44] Kennedy, "*Zīj* Survey", p. 144a. The earliest Islamic star-catalogues from 9th- and 10th-century Baghdad are investigated in Girke, "Die frühesten islamischen Sternkataloge", alas not yet published. On the most significant star-catalogue of late Islamic astronomy see Knobel, *Ulughbeg's Catalogue of Stars*.

[45] The conversion of stellar coordinates from one system to the other can also be effected using an instrument such as the *ṣafīḥa* of the 11th-century Andalusī astronomer Ibn al-Zarqālluh (Azarquiel), which was specifically designed for that purpose. See the article "Shakkāziyya" in EI_2 and the references there cited, and also **X-5.2**.

Now we rotate the ecliptic into the working plane so that V and X move to V_2 and X_2. Clearly:

$$\text{arc } V_2X_2 = \lambda \quad \text{and} \quad OX_1 = \sin\lambda \ .$$

Next we rotate the declination circle about its axis EF into the semicircle EHF with midpoint at H on the working plane so that X moves to X_3. Clearly:

$$\text{arc } HX_3 = \alpha \text{ and } OY = GX_1 = \cos\delta \sin\alpha \ .$$

Now in ΔX_1OY we have:

$$\angle O = \varepsilon,\ OX_1 = \sin\lambda\ ,\ X_1Y = \sin\delta \text{ and } OY = \cos\delta\sin\alpha \ .$$

It follows immediately that:

F4 $$\delta = \text{arc sin } (\sin\varepsilon \sin\lambda)$$

and:

F5 $$\alpha = \text{arc sin } \{ \tan\delta / \tan\varepsilon \} \ .$$

Note that the second declination $\delta_2(\lambda)$ equals the declination corresponding to a longitude λ'' whose right ascension is λ, that is:

$$\delta_2(\lambda) = \delta(\lambda'') \text{ where } \alpha(\lambda'') = \lambda \ .$$

It follows from **F5** that:

$$\lambda = \text{arc sin } \{ \tan\delta(\lambda'') / \tan\varepsilon \} = \text{arc sin } \{ \tan\delta_2(\lambda) / \tan\varepsilon \} \ ,$$

whence:

F6 $$\delta_2(\lambda) = \text{arc tan } \{ \tan\varepsilon \sin\lambda \} \ .$$

Hour-angle and time since rising

To derive the time as a function of h, D and ϕ, we first project the celestial sphere onto the meridian plane – see **Fig. 1.2h**. The segments OP, OQ and A_1B represent the projections of the celestial axis, the celestial equator and the day-circle. Also X_1 and L_1 represent the projections of X and L. Observe that:

$$\text{arc } PN = \phi\ ,\ \text{arc } BQ = \Delta\ ,\ \text{arc } BS = H = \bar{\phi} + \Delta \ .$$

and:

$$OG_1 = \sin\Delta\ ,\ G_1B = \cos\Delta \ .$$

Now we draw the perpendiculars BB_2 and DD_2 from B and D to NS. Then:

$$BB_2 = \sin H \text{ and } DD_2 = \sin H^* \ .$$

Now the segment A_1D below the horizon measures the projection of the day-circle above the horizon for declination $-\Delta$. For the sun A_1D represents the projection of the day-circle for longitude $\lambda^* = \lambda + 180°$ (*naẓīr darajat al-shams*), so that:

$$H^*(\lambda) = H(\lambda^*) \ .$$

Next we rotate the celestial sphere about its axis ZO until X crosses the meridian at Y_1 and draw the perpendiculars X_1X_2 and Y_1Y_2 from X_1 and Y_1 to NS. Then since:

$$\text{arc } Y_1S = h \ ,$$

we have:

$$X_1X_2 = Y_1Y_2 = \sin h \text{ and } Y_2O = \cos h \ .$$

Similarly:

$$L_1O = \sin h_0 \ .$$

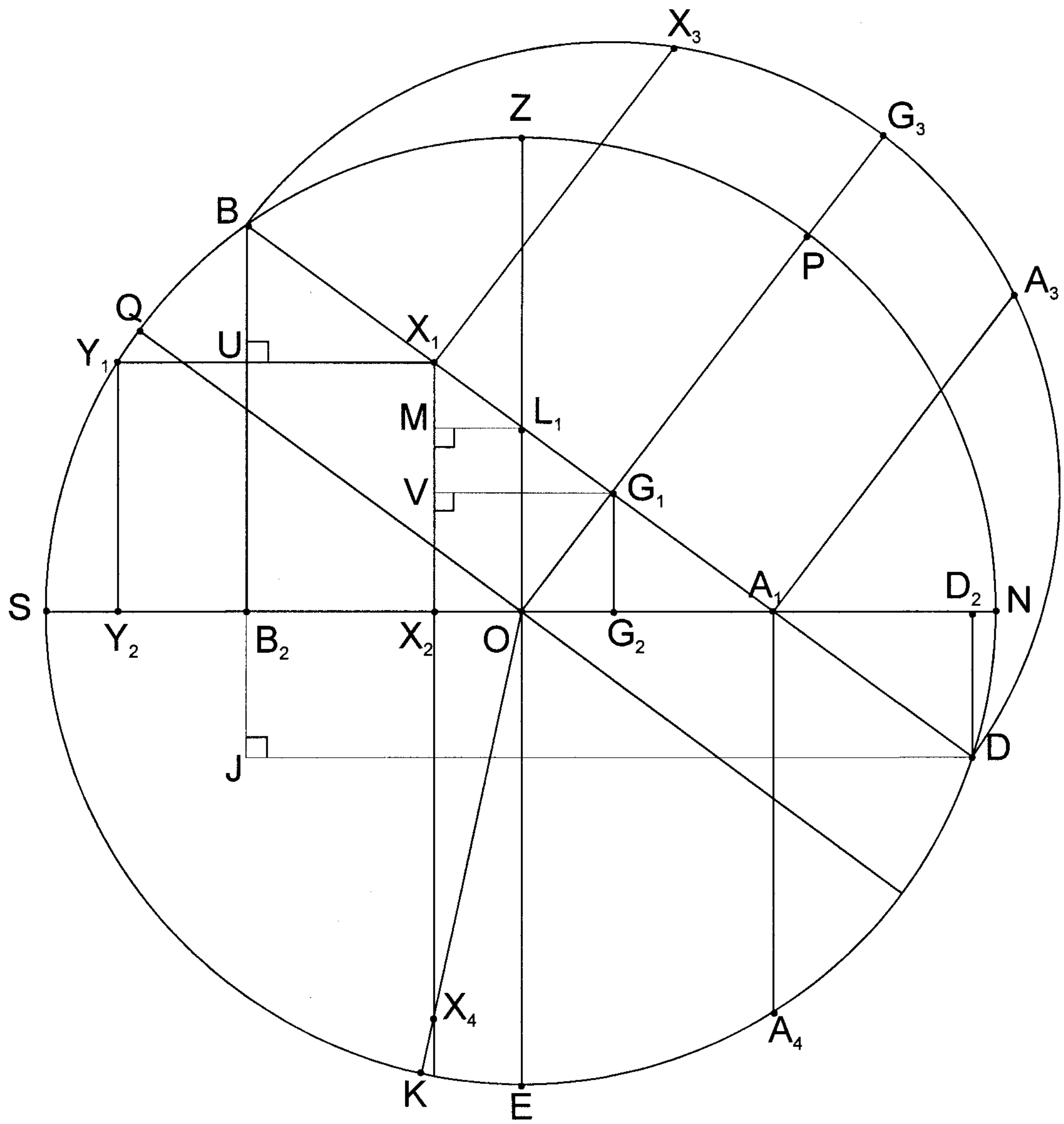


Fig. 1.2h. The celestial sphere reduced to two dimensions. Compare **Fig. VIIa-3.5**.

We now rotate the day-circle about its axis G_1B into the working plane so that A, G and X move to A_3, G_3 and X_3. Then:

$$\text{arc } A_3B = D, \quad \text{arc } A_3G_3 = d, \quad \text{arc } A_3X_3 = T, \quad \text{and} \quad \text{arc } X_3B = t\ ,$$

and since:

$$G_1B = \cos \Delta\ ,$$

we have:

$$A_1B = \text{vers } D \cos \Delta\ ,$$
$$A_1G_1 = \sin d \cos \Delta\ ,$$

$$A_1X_1 = \sin d \cos \Delta + \sin (T - d) \cos \Delta ,$$
$$G_1X_1 = \cos t \cos \Delta ,$$
$$X_1B = \text{vers } t \cos \Delta .$$

Now in ΔOA_1G_1, with G_1G_2 perpendicular to OA, we have:

$$\angle O = \phi,\ OG_1 = \sin \Delta ,\ A_1G_1 = \sin d \cos \Delta , \text{ and also } G_1G_2 = \sin \Delta \sin \phi ,$$

whence it follows that:

F7 $$d = \text{arc sin } (\tan \Delta \tan \phi) .$$

Further, since in ΔBB_2A_1:

$$\angle B = \phi,\ BB_2 = \sin H,\ A_1B = \text{vers } D \cos \Delta ,$$

and in ΔBJD, where J is defined by the perpendicular DJ onto BB_2 produced:

$$\angle B = \phi,\ BJ = BB_2 + D_2D = \sin H + \sin H^* ,$$
$$DB = 2\ G_1B = 2 \cos \Delta ,$$

it follows that:

$$\text{vers } D \cos \Delta / \sin H = 1 / \cos \phi = 2 \cos \Delta / (\sin H + \sin H^*)$$

F8 $$\text{vers } D / \sin H = 1 / (\cos \Delta \cos \phi) = 1 / [{}^1/_2 (\sin H + \sin H^*)] .$$

The product $\cos \Delta \cos \phi$ and the ratio vers D / sin H are of considerable importance in medieval timekeeping. I denote:

$$\boldsymbol{\mathcal{B}} = \cos \Delta \cos \phi \quad \text{and} \quad \boldsymbol{\mathcal{G}} = 1 / \boldsymbol{\mathcal{B}} = 1 / (\cos \Delta \cos \phi) .$$

Also of importance is the segment G_1G_2, the perpendicular from G_1 to SN, whose length we denote by $\boldsymbol{\mathcal{C}}$. Clearly:

$$\boldsymbol{\mathcal{C}} = \sin \Delta \sin \phi .$$

We can now derive four equivalent medieval formulae for t and T. Firstly, in ΔBUX_1, where X_1U is perpendicular to BB_2:

$$\angle B = \phi \text{ and } X_1B = \text{vers } t \cos \Delta ,$$

so that:

$$BU = \text{vers } t \cos \Delta \cos \phi .$$

But:

$$BU = BB_2 - X_1X_2 = \sin H - \sin h ,$$

so that:

F9 $$t = \text{arc vers } \{ (\sin H - \sin h) / (\cos \Delta \cos \phi) \}$$
$$= \text{arc vers } \{ (\sin H - \sin h) \bullet \text{vers } D / \sin H \} .$$

Note that using the auxiliary functions defined above, this reduces to:

$$t = \text{arc vers } \{ (\sin H - \sin h) / \boldsymbol{\mathcal{B}} \} = \text{arc vers } \{ (\sin H - \sin h) \bullet \boldsymbol{\mathcal{G}} \} .$$

Secondly, in $\Delta X_1X_2A_1$,

$$\angle X_1 = \phi \text{ and } A_1X_1 = A_1B - X_1B = \text{vers } D - \text{vers } t ,$$

so that:

$$X_1X_2 = (\text{vers } D - \text{vers } t) \cos \Delta \cos \phi .$$

But:

$$X_1X_2 = \sin h ,$$

so that:

F10 $$t = \text{arc vers}\ \{ \text{vers}\ D - \sin h / (\cos \Delta \cos \phi) \}$$
$$= \text{arc vers}\ \{ \text{vers}\ D - \sin h\ \text{vers}\ D / \sin H \} .$$

Alternatively, since also:

$$A_1X_1 = \{ \sin d + \sin (T-d) \} \cdot \cos \Delta ,$$

we have:

F11 $$T = d + \text{arc sin}\ \{ \sin h / (\cos \Delta \cos \phi) - \sin d \}$$
$$= d + \text{arc sin}\ \{ \sin h\ \text{vers}\ D / \sin H - \sin d \} .$$

Likewise in ΔX_1VG_1, where G_1V is perpendicular to X_1X_2:

$$\angle X_1 = \phi \text{ and } X_1G_1 = \cos t \cos \Delta ,$$

so that:

$$X_1V = \cos t \cos \Delta \cos \phi .$$

But:

$$X_1V = X_1X_2 - G_1G_2 = \sin h - \sin \Delta \sin \phi ,$$

so that:

F12 $$t = \text{arc cos}\ \{ [\sin h - \sin \Delta \sin \phi] / (\cos \Delta \cos \phi) \} .$$

This last is the modern formula for $t(h,\Delta,\phi)$. Notice that:

$$t = \text{arc cos}\ \{ [\sin h - \boldsymbol{\mathcal{C}}] / \boldsymbol{\mathcal{B}} \}$$

Finally, in ΔL_1OG_1 we have:

$$\angle L_1 = \phi,\ LO = \sin h_0, \text{ and } OG_1 = \sin \delta ,$$

whence:

F13 $$h_0 = \text{arc sin}\ \{ \sin \delta / \sin \phi \} .$$

Azimuth

To derive the azimuth a as a function of h, Δ and ϕ, we project the celestial sphere onto the horizon plane and rotate the projection about its axis SN into the semicircle SKEDN in the working plane. Points X and A move to X_4 on OK and A_4 on the semicircle SEN, respectively, and clearly:

$$\text{arc } EA_4 = \psi \text{ and } OA_1 = \sin \psi .$$

(For convenience we adopt the convention that $\psi \gtreqless O$ as $\delta \gtreqless 0$.) But in ΔOA_1G_1 we have also:

$$\angle O = \phi \text{ and } OG_1 = \sin \delta ,$$

whence:

F14 $$\psi = \text{arc sin}\ \{ \sin \delta / \cos \phi \} .$$

Furthermore, arc EK = a (now with the convention that $a \gtrless 0$ as X is north or south of E), and since:

$$OX_4 = OY_2 = \cos h ,$$

we have:

$$OX_2 = \cos h \sin a .$$

Now in $\Delta X_1X_2A_1$:

$$\angle X_1 = \phi \text{ and } X_1X_2 = \sin h ,$$

so that:

$$A_1X_2 = \sin h \tan \phi .$$

This quantity is of importance in medieval azimuth calculations and I denote it by k. With

$$k = \sin h \tan \phi ,$$

we have further:

$$OX_2 = \cos h \sin a = A_1X_2 - OA_1 = k - \sin \psi ,$$

whence we derive the standard medieval formula:

F15 $$a = \arc \sin \{ [\sin h \tan \phi - \sin \Delta / \cos \phi] / \cos h \} ,$$

or, in terms of the auxiliary function:

$$a = \arc \sin \{ [k - \sin \psi] / \cos h \}$$

which is equivalent to the modern formula:

$$a = \arc \sin \{ [\sin h \sin \phi - \sin \delta] / (\cos h \cos \phi) \} .$$

Finally, note that in ΔX_1ML_1, where L_1M is perpendicular to X_1X_2:

$$\angle X_1 = \phi \text{ and } X_1M = X_1X_2 - L_1O = \sin h - \sin h_0 ,$$

so that:

$$OX_2 = L_1M = (\sin h - \sin h_0) \cdot \tan \phi .$$

But:

$$OX_2 = \cos h \sin a ,$$

so that:

F16 $$a = \arc \sin \{ [\sin h - \sin h_0] \tan \phi / \cos h \} ,$$

as given in some medieval sources (see **5.3**).

Right and oblique ascensions

The rising times of segments of the ecliptic over the local horizon are measured by the corresponding segments of the celestial equator.[46] In **Fig. 1.2i** the arc VH of the ecliptic rises over the horizon at the equator (so that the celestial pole P is on the horizon) with arc VE of the celestial equator. The arc VE measures the right ascension α (*al-maṭāliʿ fī 'l-falak al-mustaqīm*) corresponding to the ecliptic arc VH. In **Fig. 1.2j** the arc VH rises over the horizon of a locality with latitude ϕ with arc VE of the celestial equator (so that the celestial pole P is at ϕ above the north point). This arc VE measures the oblique ascension α_ϕ (*al-maṭāliʿ al-baladiyya*) corresponding to the ecliptic arc VH. In this figure, if the longitude of H is λ and HF is drawn perpendicular to VE produced, then since VE = VF - EF, we have:

F18 $$\alpha_\phi(\lambda) = \alpha(\lambda) - d(\lambda,\phi) ,$$

where α and d are determined as in **F5** and **F7** above. Note also the important relation:

F19 $$\alpha_\phi(\lambda) - \alpha_\phi(\lambda^*) = 2D(\lambda) .$$

Horoscopus

The point of the ecliptic instantaneously rising over the local horizon (H in **Fig. 1.2h**) is called the ascendant or horoscopus (*al-ṭāliʿ*). The point where the ecliptic intersects the meridian is

[46] See the EI_2 article "Maṭāliʿ".

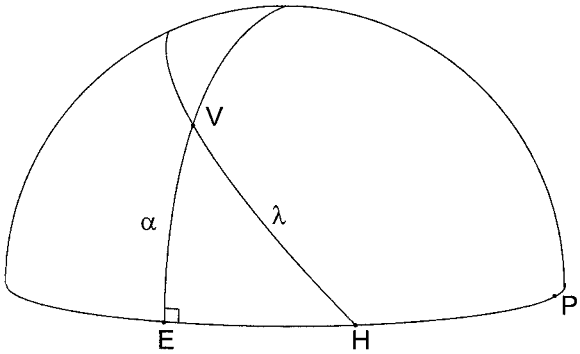


Fig. 1.2i.

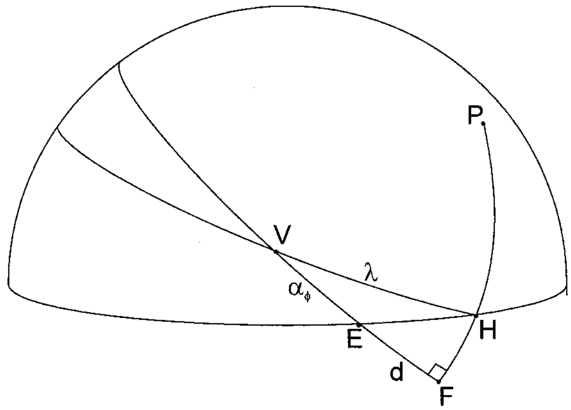


Fig. 1.2j.

called midheaven (*wasaṭ al-samāʾ*). The position of the ecliptic relative to the horizon and meridian is of prime importance in astrology, but also, needless to say, in timekeeping.

If we now denote the longitude of horoscopus H by λ_H when the sun at X has altitude h and longitude λ, it follows from a simple consideration of the celestial sphere that:

F20 $$\alpha_\phi(\lambda_H) = \alpha_\phi(\lambda) + T(h,\lambda) \ .$$

Likewise for a star with co-ascendant ρ and declination Δ:

F21 $$\alpha_\phi(\lambda_H) = \alpha_\phi(\rho) + T(h,\Delta) \ .$$

Note further that if λ_M denotes the longitude of upper midheaven M, then:

F22 $$\alpha_\phi(\lambda_H) = \alpha(\lambda_M) + 90^\circ = \alpha'(\lambda_M) \ ,$$

where α' denotes normed right ascensions, measured from the winter solstice rather than the vernal equinox, and more commonly used in Islamic timekeeping than α. See further **IX**.

The sacred direction

Another problem discussed by the Muslim astronomers is the determination of the qibla or the direction of Mecca from a given locality.[47] The reader is referred to **VIIa-c** for more information on this subject.

Notes on technical terminology

The Arabic expressions for the various astronomical concepts are either mentioned already above or are presented *ad loc*. It may suffice here to add that the arguments in the various tables are invariably denoted as *ʿadad*, literally "number". The term *ḥiṣṣa*, pl. *ḥiṣaṣ*, literally "share" or "portion", means "duration", as in, for example, *ḥiṣṣat al-fajr*, "duration of morning twilight".[48] A special function may be indicated by the expression *maḥfūẓ*, literally, "that which is kept in mind".[49] On the kind of table denoted as *ṭaylasān* see the next Section.

[47] See already the EI_2 articles "Ḳibla. i. ritual and legal aspects" by A. J. Wensinck, and "Ḳibla. ii. astronomical aspects" and "Makka. iv. As centre of the world" by D. A. King, the last two repr. in *idem*, *Studies*, C-IX-X; and the survey of Muslim approaches to the qibla-problem in *idem*, *Mecca-Centred World-Maps*, pp. 47-127.

[48] See, for example, **I-8.0**, **8.2.1**, **8.4.2**, **9.4***, **9.6**, **9.9**, and **II-4.10**, **6.1**, **10.3**, **10.6**, **10.7**, **11.5**, **13.8***, **14.3** and **14.13**.

[49] As, for example, in **II-10.7** and **10.11**.

1.3 Classification of subject matter

In the sequel the reader should bear in mind that the tables of:

$\delta(\lambda)$, $\delta_2(\lambda)$ and $\alpha(\lambda)$ or $\alpha'(\lambda)$,

as well as tables of:

$H(\lambda)$, $d(\lambda)$, $D(\lambda)$ and $\alpha_\phi(\lambda)$ for some particular latitude,

were standard in Islamic *zīj*es.[50] In these tables values were usually given to two sexagesimal digits, occasionally three, for each degree of λ. As noted already, I do not include such common (not in the sense of vulgar) tables in this survey. I have endeavoured to arrange the analysis of other Islamic tables for astronomical timekeeping in such a way that new material can be easily classified and included in any future supplement.[51]

In **Ch. 2** I present examples of five kinds of tables for finding the time of day from the solar altitude and two for finding the time of night by the stars:

1. $T(h,\lambda)$ and $t(h,\lambda)$ for specific ϕ
2. $T(\lambda,h)$ and $t(\lambda,h)$ for specific ϕ
3. $T(H,h)$ and $t(H,h)$ for specific ϕ
4. $t(h)$ for various stars and specific ϕ
5. $T(H,h)$ for all ϕ
6. $T(H,h,D)$ for all ϕ
7. $T(\alpha',\lambda)$ for specific ϕ

In **Ch. 3** I present examples of four kinds of related tables for finding the longitude of the horoscopus from solar or stellar altitudes or from the time of day or night:

1. $\lambda_H(h,\lambda)$ for specific ϕ
2. $\lambda_H(h)$ for various stars and specific ϕ
3. $\lambda_H(T,\lambda)$ for specific ϕ
4. $\lambda_H(T)$ for various stars and specific ϕ

In **Ch. 4** we consider examples of eight different kinds of tables for finding the altitude of the sun or fixed stars, given the time, or the longitude of the horoscopus, or the azimuth:

1. $h(T)$ at the equinoxes and solstices for specific ϕ
2. $h(T,\lambda)$ for specific ϕ
3. $h(T,H)$ for all ϕ
4. $h(D,T)$ for specific ϕ
5. $h(\lambda_H)$ for various stars and specific ϕ
6. $h(T)$ for various stars and specific ϕ
7. $h(a,\lambda)$ for specific ϕ
8. $h_0(\lambda)$ for specific ϕ

Ch. 5 contains a discussion of examples of seven different kinds of tables for finding the azimuth of the sun or the stars:

1. $a(h,\lambda)$ for specific ϕ
2. $a(\lambda,h)$ for specific ϕ

---

[50] See Kennedy, "*Zīj* Survey", pp. 140-141, and King, *Ibn Yūnus*, II.3.

[51] See the text to n. 1:20.

3. a(H,h) for specific ϕ
4. a(T,λ) for specific ϕ
5. a(T) at the equinoxes for specific ϕ
6. $\psi(\lambda)$ for specific ϕ
7. $\psi(\Delta)$ for specific ϕ

In **Chs. 6, 7 and 8** I note the existence of numerous examples of minor auxiliary functions for timekeeping and azimuth calculations. **Ch. 9** concludes the study with a discussion of some 15 examples of more extensive auxiliary tables for solving problems of spherical astronomy for all latitudes. In **Ch. 10** I survey a few medieval European and Renaissance tables in the same spirit that have come to my attention.

1.4 The format of the tables

In general the Muslim astronomers took advantage of the symmetry of the functions they were tabulating, in order to avoid pointless duplication of entries. On the behaviour of the various functions see my previous study of the Cairo corpus, where various graphs are presented.[52]

For functions such as $\delta(\lambda)$, as well as $d(\lambda)$ and $\psi(\lambda)$ for particular latitudes, whose absolute values are symmetrical about the equinoxes and solstices, 90 entries suffice to display the function for each degree of λ. Tables of these functions were usually arranged in three columns of 30 entries, often with a single vertical argument running from 1 to 30, understood to represent 1°-30°, or 31°-60° or 61°-90° as necessary.

For functions such as $H(\lambda)$ and $D(\lambda)$ which are symmetrical about the solstices but not the equinoxes, two sets of 90 entries and an additional (non-zero) equinoctial value display the function for each degree of λ. Tables of these functions were usually arranged in six columns of 30 entries, the first three serving the northern signs and the second three the southern ones. A vertical argument running from 1 to 30 downwards serves the first and third quarters of the ecliptic, and a second one running from 0 to 29 upwards serves the second and fourth quadrants. The equinoctial value is thus omitted (but is generally obvious anyway). Alternatively, since both H and D increase monotonically between the winter and summer solstice, the tables might be arranged so that the first three columns serve the southern signs and the second three columns the northern ones. In this case, if the arguments run from 1 to 30 downwards and 0 to 29 upwards, the value of the function at the winter solstice will be omitted.

The functions $t(h,\lambda)$, $T(h,\lambda)$, $a(h,\lambda)$ and $h(a,\lambda)$ display the same symmetry as $H(\lambda)$ and $D(\lambda)$, and for a given value of the first argument a page or double page of tables serves to display the function for each degree of λ. See, for example, **Figs. 2.1.1a** and **II-5.6a**. If either of the first three functions are tabulated with λ as the horizontal argument, a given page or double page might serve each degree of λ from, say, the winter solstice to the summer solstice, but now the maximum vertical argument is H or [H] and hence varies with λ. See, for example, **Figs. 2.2.1-3**.

[52] See King, "Astronomical Timekeeping in Medieval Cairo" (n. 1:2), esp. pp. 361, 363, 365, 367 and 370.

The functions $\alpha(\lambda)$ and $\alpha_\phi(\lambda)$ are not symmetrical about the equinoxes or the solstices. Tables of these functions were usually arranged in 12 columns of 30 entries, with a single vertical argument running from 1 to 30 or from 0 to 29, with each column essentially serving a single zodiacal sign. The tables of $\lambda_H(h,\lambda)$ are of the same kind, and for a given value of h a double page of tables with 12 columns of 30 entries serves to display the function for each degree of λ. See, for example, **Fig. 3.1.1**.

The various functions f(H,h) discussed in the sequel are computed for a given range of H and each degree of h up to H. The resulting table, which is usually spread over several pages, is trapezoidal in shape. Such tables were generally called in medieval Arabic *ṭaylasān*, which means "shawl".[53] See, for example, **Figs. 2.3.1-2**.

For other large tables the format is devised to best suit the function tabulated, often with astounding ingenuity. See, for example, **Figs. 2.6.1** and **4.4.1**.

1.5 Recomputation of tables

"One man's error is another man's data." Berman's corollary to Robert's axiom.

"... (the) non-existence (of al-Khāzinī's qibla-table) does not prevent King from specifying its characteristics in considerable detail. This level of theoretical construction makes a reader seriously question imposing on data an order that might not be fully justified by the medieval texts (some of which are not even extant)." E. Savage-Smith, "Review of King, *Mecca-Centred World-Maps*", p. 34c.

Most of the tables discussed below were recomputed during the period 1970-74 with the electronic computers and myriads of punched cards at the American University of Beirut, Yale University or the American University in Cairo.[54] In certain cases, the computer was useful in facilitating the analysis of the tables and the determination of the underlying parameters. Very occasionally, one may be able to reconstruct a table that does not survive in its original form, or at least comment on its structure and the accuracy of its entries, using in both cases, other derivative tables that have survived.[55] Such techniques may inevitably be beyond the

[53] On this term see Goldstein, "Medieval Table for Reckoning Time", p. 61; and Albert Arazi, "Noms des vêtements d'après *al-Aḥādīṯ al-ḥisān fī faḍl al-ṭaylasān* d'al-Suyūṭī", *Arabica* 23 (1976), pp. 109-155. In Yedida Kalfon Stillman, *Arab Dress from the Dawn of Islam to Modern Times – A Short History*, Norman A. Stillman, ed., Leiden, *etc.*: E. J. Brill, 2000, pp. 18, 39, 44, 48, 51-52, 71, 73-74, 91, 96 and 179, there no mention of a *trapezoidal-shaped* shawl.

The term is uncommon in the medieval astronomical literature, apart from the titles of this kind of table. In fact, I know of only two other attestations, both by al-Bīrūnī (see **II-2.2**). In his treatise on the construction of astrolabes entitled *al-Istīʿāb* (MS London BL Or. 5593, fol. 56v), al-Bīrūnī poses the question whether a particular variety of non-standard astrolabe "leads to the knowledge of the hours and the ascendants in the same way as the *ṭaylasān* table and different kinds of sundials do". (This statement is recorded in Wiedemann, *Aufsätze*, II, pp. 523-524; see also his n. 10 on p. 528.) Again, al-Bīrūnī in his *Chronology* (p. 132 of Sachau's translation) uses the term *ṭaylasān* to describe a table for calendar conversion. (This reference is already recorded in Kennedy, "*Zīj* Survey", p. 137, *ad* no. X201.)

[54] Computer time was made available in Beirut by the Mathematics Department, American University of Beirut; in New Haven by the Department of Near Eastern Languages and Literatures, Yale University; and in Cairo by the Smithsonian Institution, Washington, D.C.

[55] See, for example, Hogendijk, "al-Khwārizmī's Sine Table", and King, *Mecca-Centred World-Maps*, pp. 71-75.

comprehension of colleagues innocent of mathematics and not well-versed in the history of mathematical astronomy.

I have refrained from giving sample entries from the tables but occasionally comment on their accuracy. Several individual tables deserve more detailed analysis than I present in this survey, and now computer-assisted techniques are available than I cannot hope to understand, let alone to employ myself.[56]

[56] See various recent studies by Glen Van Brummelen and Benno van Dalen listed in n. 1:24.

CHAPTER 2

TABLES OF TIME AS A FUNCTION OF SOLAR AND STELLAR ALTITUDE

2.0 Introductory remarks

The functions tabulated in the tables described in this chapter are either the time since rising of the sun or star T (for altitudes in the east), or the hour-angle t. These are related by the simple formula (*cf.* **F2**):

$$T + t = D ,$$

where D is the semi diurnal arc. The Arabic terms for the time since rising and hour-angle are *al-dāʾir* and *faḍl al-dāʾir*, respectively.

In those tables for the sun in **2.1** and **2.2** the arguments are h and λ. The tables discussed in **2.3** and **2.5** have H and h as arguments, and in the case of those for the stars in **2.4** the argument is h, but a value of H underlies the table for each star. The tables of **2.5** display the time approximately in seasonal day-hours for all latitudes. The table described in **2.6** has three arguments – H, h and D – and serves both the sun and stars and works for any latitude. The tables for timekeeping by the stars discussed in **2.7** are of a different kind, giving the time as a function of the ascension of the star that is culminating and the solar longitude λ. Those in **2.8** take advantage of a convenient temporary celestial phenomenon. In most of the tables time is measured in equatorial degrees.

See **II** for the other, smaller tables which accompany such tables as these for Cairo, Mecca, Damietta, Jerusalem, Damascus, Aleppo, Anatolia and Istanbul.

2.1 Tables of time as a function of instantaneous solar altitude and solar longitude, for a specific latitude

In this section I note examples of tables of t(h,λ) or T(h,λ) for particular latitudes. Since both of these functions are dependent on δ, and since two points on the ecliptic equidistant from the solstices have the same declination, for a given altitude 181 values suffice to display either function for each integral degree of solar longitude ($\lambda = 0°$ and $\lambda' = 1°, 2°, \ldots, 90°$, $\delta \gtrless 0$). In the tables described below, full advantage is taken of the symmetry of the functions t(h,λ) and T(h,λ). For given h, generally six columns of 30 entries display the tabulated function. When for certain longitudes the sun does not attain the altitude in question, no entry is given in the table, and columns that would otherwise be devoid of entries are omitted. This format results in the omission of either the equinoctial value or a solstitial value. The format of the tables described in **2.2** obviates the need to prepare additional tables for either the equinoxes or the solstices (**2.1.1** and **5.1.1**). Each of the tables of t or T described in this section and in the next contains over 10,000 entries.

A few Muslim astronomers used tables of t(h,λ) or T(h,λ) to compile other tables for the duration of twilight,[1] based on the assumption that this phenomenon begins or ends when the sun is at a particular angle of depression below the horizon – see **Figs. 2.1.1a-b** and **II**, *passim*.

2.1.1 Ibn Yūnus / al-Maqsī / Ibn al-Kattānī / al-Bakhāniqī: Cairo

In the main corpus of tables for timekeeping that was used in medieval Cairo[2] the functions:

$$t(h,\lambda) \text{ and } T(h,\lambda)$$

are tabulated for the domains:

$$h = 1°, 2°, \ldots , 83° \text{ and } \lambda = 1°, 2°, \ldots , 90° \text{ and } 181°, 182°, \ldots , 270° .$$

Entries are computed to two digits and are based on the parameters:

$$\phi = 30;0° \text{ (Cairo-Fustat) and } \varepsilon = 23;35° .$$

No entry is given for the equinoxes. These tables are rather accurately computed, although for higher altitudes there are errors that are sometimes as high as ±5 in the second digit, and the interpolation system used is inadequate.

The tables exist in numerous manuscript sources, which are discussed in greater detail in **II-4** and **5**. (See also **I-4.7.1** and **5.1.1** on certain other tables in the corpus.) The two sets of tables of T(h,λ) and t(h,λ) may be copied separately or both functions may be displayed on facing pages of the manuscript – see **Figs. 2.1.1a-b**. In some manuscripts the two functions are tabulated side by side with the solar azimuth, a triplet of values (T,t,a) being given for each pair of arguments (h,λ) – see **Figs. II-5.6a-b**.

In several manuscripts the tables are attributed to the late-10th-century Egyptian astronomer Ibn Yūnus,[3] who is certainly the compiler of the tables of a(h,λ) (**5.1.1**), but it seems that the tables of T(h,λ) in their present form were compiled by the late-13th-century Cairo astronomer al-Maqsī.[4] This is confirmed by the mid-14th-century Jerusalem astronomer al-Karakī (**2.2.1**) and also by his contemporary, the Cairo astronomer al-Bakhāniqī (see below).[5] Nevertheless, I suspect that al-Maqsī based his tables of T(h,λ) on an earlier set of tables by Ibn Yūnus which has not survived (see, however, **II-5.3-4**). About the middle of the 14th century the Egyptian astronomer Ibn al-Kattānī[6] prepared a set of tables of t(h,λ) using al-Maqsī's tables of T(h,λ): Ibn al-Kattānī's tables exist in a unique copy in his own hand, namely MS Istanbul Kılıç Ali Paşa 684. Not long thereafter, the astronomer al-Bakhāniqī rearranged the tables so that al-

---

[1] See the article "Shafaḳ" on twilight in EI_2.

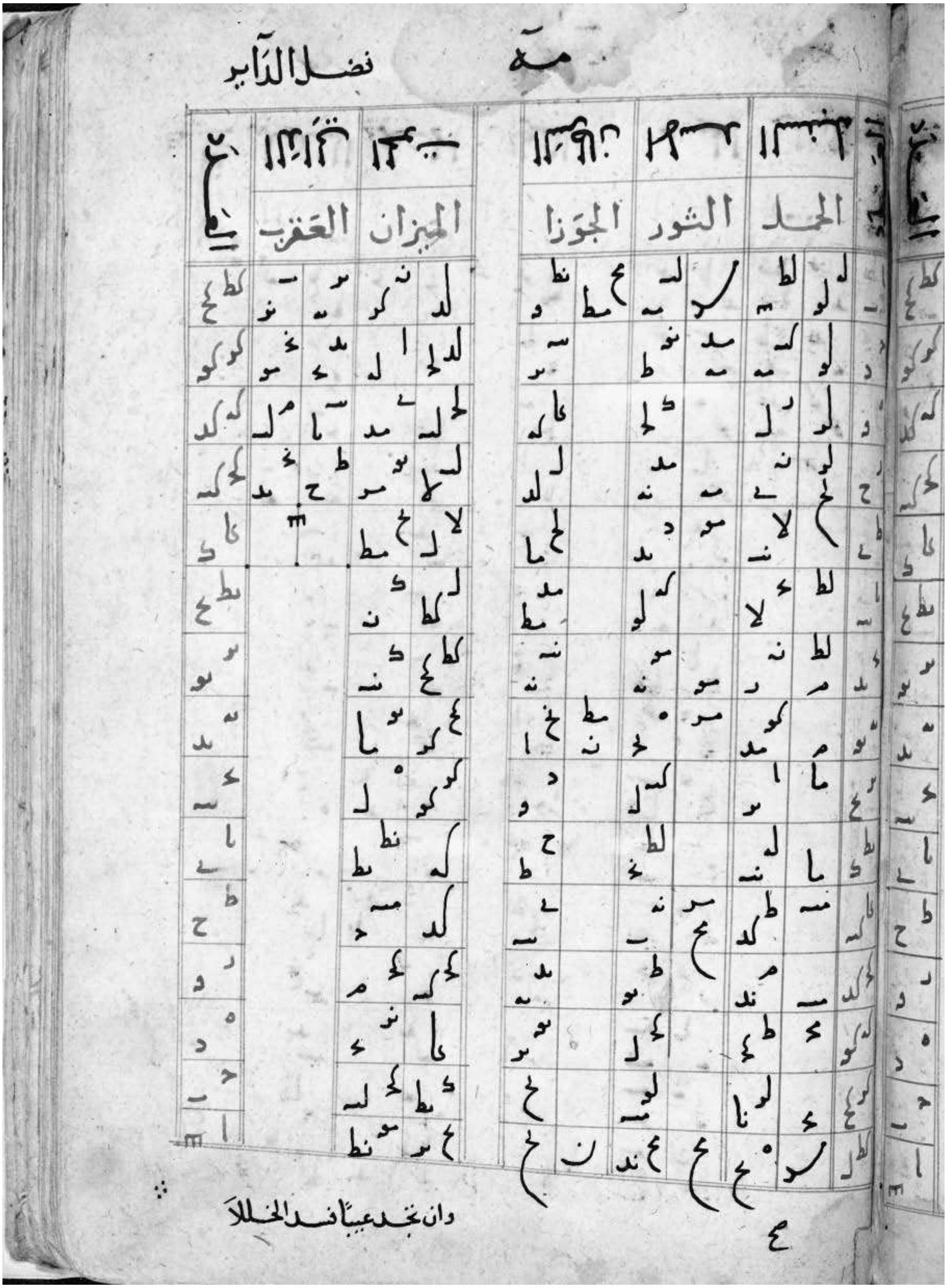
[2] This corpus was analyzed for the first time in King, "Ibn Yūnus' *Very Useful Tables* for Reckoning Time by the Sun", *AHES* 10 (1973), pp. 342-394, repr. in *idem*, *Studies*, A-IX (hereafter "Astronomical Timekeeping in Medieval Cairo"). However, that early study was based on a set of manuscripts in which the tables are mainly attributed to Ibn Yūnus (see next note). The corpus is investigated anew in **II-4-5**.

[3] On Ibn Yūnus see the article in *DSB*. The introduction and first few chapters of his major work are published with French translation in Caussin, "Table Hakémite", and his treatment of spherical astronomy is analyzed in King, *Ibn Yūnus*. On his influence in later Egyptian astronomy see *idem*, "Astronomy in Fatimid Egypt", and *idem*, "Astronomy of the Mamluks".

[4] On al-Maqsī (Suter, *MAA*, no. 383) see *Cairo ENL Survey*, no. C15, and King, "Astronomy of the Mamluks", pp. 540, 541 and 547.

[5] On al-Bakhāniqī see Brockelmann, *GAL*, SII, pp. 158 and 1019; *Cairo ENL Survey*, no. C28; and King, *Astronomy in Yemen*, no. 13.

[6] On Ibn al-Kattānī (Suter, *MAA*, no. 411) see *Cairo ENL Survey*, no. C32.


مه
فضل الدائر
الحمل
الثور
الجوزا
الميزان
العقرب
وان تجد عيبا فسد الخللا

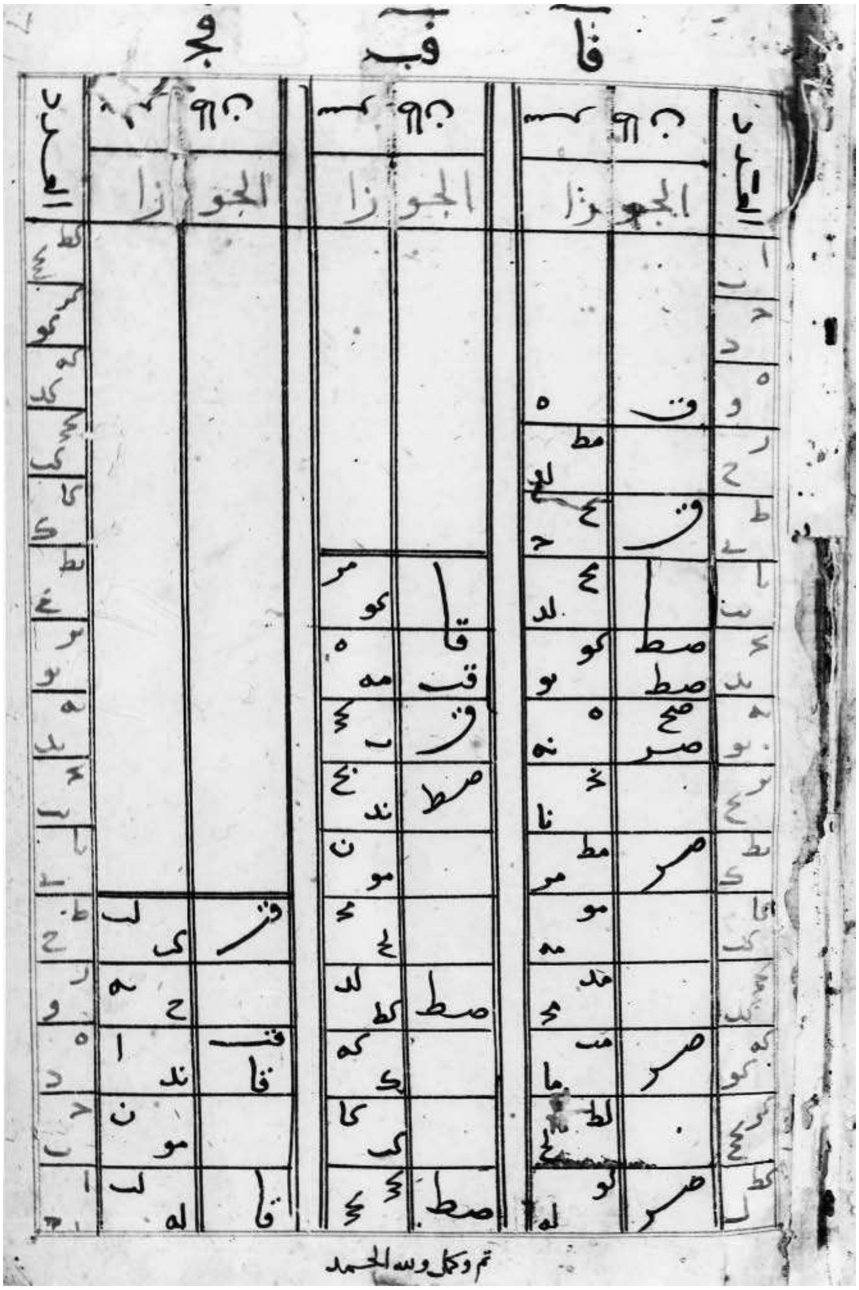


Fig. 2.1.1b: An extract from the tables of the time since sunrise for altitudes 81°-83°. See also **Figs. II-5.4a** and **5.6b**. [From MS Cairo DM 444, courtesy of the Egyptian National Library.]

←
Fig. 2.1.1a: An extract from the tables of the hour-angle serving altitude 45°. [MS Dublin CB 3673, courtesy of the Chester Beatty Library.]

Fig. 2.1.1c: The table of time since sunrise for solar altitude 16° with the instruction that the table serves to find the duration of evening twilight if one enters with the opposite point of the ecliptic from the longitude of the sun. [From MS Cairo DM 444, courtesy of the Egyptian National Library.]

Maqsī's values of T(h,λ), Ibn al-Kattānī's values of t(h,λ) and Ibn Yūnus' values of a(h,λ) were displayed side by side.

The following list indicates the manuscript sources for the tables of T, t and a for Cairo and their attribution. (The tables in MS Cairo TR 354 falsely attributed to Ibn Yūnus are computed for Alexandria – see **2.1.6**.) The notation T/t means that T and t are tabulated on facing pages, and the notation T/t/a means that T, t and a are tabulated together, with triplets of values for each pair of arguments. Otherwise the functions listed are tabulated separately. An asterisk indicates that tables are preceded by an introduction on their use prepared by al-Maqsī, and a double asterisk indicates that this introduction is accompanied by some notes by al-Bakhāniqī describing the way in which he had rearranged the tables (see further **II-5.6**).

Berlin Ahlwardt 5753	T/t, and a	Ibn Yūnus
Cairo TR 191	t	Ibn Yūnus
Dublin CB 3673	T/t and a	Ibn Yūnus*
Escorial ár. 924,7	a	Ibn Yūnus
Cairo DM 108	T/t/a	Ibn Yūnus**
Cairo Azhar *falak* 4382	t and a	Ibn Yūnus
Cairo MM 137	a	Ibn Yūnus
Dublin CB 4078	T/t	anon.
Gotha A 1410	a	Ibn Yūnus
Cairo DM 53	T/t/a	al-Maqsī**
Cairo DM 45	T/t/a	anon.
Gotha A1402	T	al-Maqsī
Istanbul Kılıç Ali Paşa 684	t	Ibn al-Kattānī
Leipzig 817	T/t	anon.
Istanbul Nuruosmaniye 2903	T/t/a	anon.*

Istanbul Nuruosmaniye 2925	T/t/a	al-Maqsī**
Cairo TR 354	T/t/a (ϕ=31°)	Ibn Yūnus
Cairo MM 64	a	anon.
Cairo DM 444	T	al-Maqsī*
Cairo DM 690	T/t/a	anon.
Cairo DM 777	T/t	anon.
Cairo DM 616	T/t/a	al-Maqsī**
Cairo DM 739	T/t/a	al-Maqsī**
Cairo DM 776,1	T	anon.
Cairo DM 778	t	anon.
Cairo DM 786	T/t/a	anon.
Cairo DM 1101	a	anon.
Cairo DM 1108,9	a	anon. (**5.2.1**)
Cairo DM 1109	T/t/a	Ibn Yūnus
Cairo DM 1224	T/t/a	Ibn Yūnus
Cairo K 4044	t and a	al-Maqsī (t)/Ibn Yūnus (a)

There is no indication in any of the sources investigated of the way in which the tables of $T(h,\lambda)$ were compiled. The simplest method would be to compile tables of $t(h,\lambda)$ first. A formula for $t(h,\lambda)$ that is not given in Ibn Yūnus' major work, the *Ḥākimī Zīj*, but which is outlined in MS Paris BNF ar. 2513, fol. 62v, of a recension of the 13th-century Egyptian *Muṣṭalaḥ Zīj* based mainly on his writings (**6.7.1**), is the following (*cf.* **F10**):

$$\text{Vers } t(h,\lambda) = \text{Vers } D(\lambda) - \text{Sin } h \bullet G(\lambda) = \text{Vers } D(\lambda) - p(h,\lambda),$$

where p is an auxiliary function not used elsewhere and G is defined by (see **6.0**):

$$G(\lambda) = G_1(\phi) \bullet G_2(\lambda) = R \,/\, \text{Cos } \phi \bullet R \,/\, \text{Cos } \delta(\lambda) \ .$$

A table of $G_2(\lambda)$ accompanies the theoretical discussion in MS Paris BNF ar. 2513 (**6.7.1**). In the *Ḥākimī Zīj* itself, Ibn Yūnus describes the computation of $T(h,\lambda)$ rather than $t(h,\lambda)$.[7] His method is the following. Firstly form the product:

$$p(h,\lambda) = \text{Sin } h \bullet G(\lambda) \ ,$$

where $G(\lambda)$ is defined by any one of three equivalent expressions (*cf.* **F8**):

$$G(\lambda) = R^2 \,/\, (\text{ Cos } \delta(\lambda) \text{ Cos } \phi \ ) = \text{Vers } D(\lambda) \,/\, \text{Sin } H(\lambda)$$
$$= R \,/\, \{ \ ^1/_2 \ [\text{ Sin } H(\lambda) - \text{Sin } H(\lambda^*) \ ] \ \}$$

and then (*cf.* **F11**):

$$T(h,\lambda) = d(\lambda) - \text{arc Sin } \{ \ p(h,\lambda) - \text{Sin } d(\lambda) \ \} \ ,$$

where d is the equation of half-daylight (= D - 90°). In the *Ḥākimī Zīj* Ibn Yūnus tabulated both $d(\lambda)$ and Sin $d(\lambda)$ to three digits for each degree of λ, giving no reason for tabulating the latter.

In passing we note that since:

$$H(\lambda) \ = \ \bar{\phi} + \delta(\lambda) \quad \text{and} \quad H(\lambda^*) \ = \ \bar{\phi} - \delta(\lambda) \ ,$$

the identity:

$$\cos \delta \cos \phi \ = \ ^1/_2 \ \{ \cos (\phi\text{-}\delta) - \cos (\phi\text{+}\delta) \ \}$$

[7] *Cf.* King, *Ibn Yūnus*, III.15.3.

underlies the equivalence of the first and the third of the above expressions for G(λ). This was noted already by Jean-Baptiste Delambre in his *Histoire de l'astronomie du moyen âge*, published in Paris in 1820.[8] His pronouncement:

> "Ebn Jounis a changé depuis cosϕ cosδ en $^1/_2$ {cos(ϕ-δ) - cos(ϕ+δ)}, et c'est le premier exemple qu'on trouve de cette pratique connue sous le nom de prostaphérèse,"

led to the notion that Ibn Yūnus was:[9]

> "the first to propound the prostapherical formula ... which before the invention of logarithms was of great value to astronomers as it transformed the complicated multiplication of trigonometric functions expressed in sexagesimal fractions into an addition."

This has percolated through both the scholarly and popular literature on Islamic mathematics.[10] The fact that Ibn Yūnus did not propound *any* prostaphaeretical formulae does not, however, detract from his remarkable achievements in mathematical astronomy.

2.1.2 Abu 'l-ʿUqūl: Taiz

MSS Berlin Ahlwardt 5720 (Mq. 733,3), copied in Mukhkha in 1209 H [= 1795], and Milan Ambrosiana C84 are two fragments of an extensive corpus of spherical astronomical tables for the latitude of Taiz, computed by the Yemeni astronomer Abu 'l-ʿUqūl about 1300.[11] The corpus is entitled *Mirʾāt al-zamān*, "Mirror of Time", and more information on it is con-tained in **II-12.1**. A manuscript in a private collection in Sanaa of an astronomical miscellany by the Yemeni ruler al-Sulṭān al-Afḍal, compiled about 1375 (**II-12.4**),[12] contains other tables from this corpus, no longer extant in other two sources. Abu 'l-ʿUqūl also compiled a *zīj* called the *Mukhtār Zīj*, extant in MS London BL Or. 3624 and based mainly on the work of Ibn Yūnus.

The Berlin manuscript (fols. 128v-166r) contains a complete set of tables of the functions:

$$T(h,\lambda) \text{ and } t(h,\lambda)$$

computed to two digits for the domains:

$$h = 1°, 2°, \ldots , 90° \text{ and } \lambda = 1°, 2°, \ldots , 90° \text{ and } 181°, 182°, \ldots , 270°$$

and based on the parameters:

$$\phi = 13;40° \text{ (Taiz) and } \varepsilon = 23;35°.$$

The tables are less accurately computed than those for Cairo and Damascus (**2.1.1** and **2.1.4**). An extract is shown in **Fig. 2.1.2**. On the first page of the tables it is stated that they are for the town of Zabid, which is incorrect. On fols. 88r-92v of the Milan manuscript there is an incomplete set of tables of T(h,λ) only, for arguments h = 52°, 53°, ... , 75°. In both the Berlin and Milan copies there are also tables of $\lambda_H(h,\lambda)$ (**3.1.1**), which were doubtless compiled using

8 Delambre, *HAMA*, pp. 112 and 164.

9 Quoted from Heinrich Suter's article "Ibn Yūnus" in EI_1. See also, for example, Sezgin, *GAS*, V, p. 342.

10 For some comments on this see Al-Daffa & Stroyls, *Studies,* pp. 27-28. (This work is otherwise to be used with extreme caution.) See also King, *Ibn Yūnus*, pp. 7 and 149; *idem*, "Astronomical Timekeeping in Egypt", p. 360; and *idem*, "Astronomy in Fatimid Egypt", p. 508.

11 On Abu 'l-ʿUqūl see King, *Astronomy in Yemen*, no. 9. In Varisco, *Yemeni Almanac*, p. 13, he is identified as Muḥammad ibn Aḥmad al-Ṭabarī, the first teacher appointed by al-Malik al-Muʾayyad Dāʾūd, the brother and successor of al-Ashraf, to the Muʾayyadiyya *madrasa* in Taiz.

12 On al-Afḍal see King, *Astronomy in Yemen*, no. 18. The manuscript is now available in facsimile in Varisco & Smith, eds., *al-Afḍal's Anthology*.

Fig. 2.1.2: The tables of Abu 'l-ʿUqūl displaying the time since sunrise and the hour-angle for altitude 35°. [From MS Berlin Ahlwardt 5720, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

these tables of T(h,λ) and t(h,λ). However, I know of no auxiliary tables of any kind which Abu 'l-ʿUqūl might have used to compile his main tables for timekeeping.

2.1.3 al-Mizzī: Damascus

The early-14th-century Syrian astronomer al-Mizzī[13] is known to have compiled a set of tables of t(h,λ) for the latitude of Damascus, taken as 33;27°. These tables do not exist in the known manuscript sources, but are mentioned in MS Leipzig 808, fol. 2v, by al-Karakī (**2.2.1**). MS Cairo MM 62, dating from *ca*. 1400, contains a set of prayer-tables compiled by al-Mizzī for parameters ϕ = 33;27° and ε = 23;33°, having the same format as the prayer-tables in the main Cairo corpus. See further **II-9.2**.

2.1.4 al-Khalīlī: Damascus

Shams al-Dīn al-Khalīlī, who worked in Damascus about 1360,[14] compiled a set of tables of t(h,λ) based on the parameters newly derived by his better-known colleague Ibn al-Shāṭir,[15] namely:

$$\phi = 33;30° \text{ (Damascus) and } \varepsilon = 23;31° .$$

These tables, which form part of an extensive corpus compiled by al-Khalīlī, are extant in

[13] On al-Mizzī see Suter, *MAA*, no. 406; Mayer, *Islamic Astrolabists*, pp. 61-62; *Cairo ENL Survey*, no. C34; and King, "Astronomical Timekeeping in Syria", pp. 78-79.

[14] On al-Khalīlī (Suter, *MAA*, no. 418) see my article in *DSB*, supplement.

[15] On Ibn al-Shāṭir (Suter, *MAA*, no. 416, and Kennedy, "*Zīj* Survey", no. 11) see my article in *DSB* and

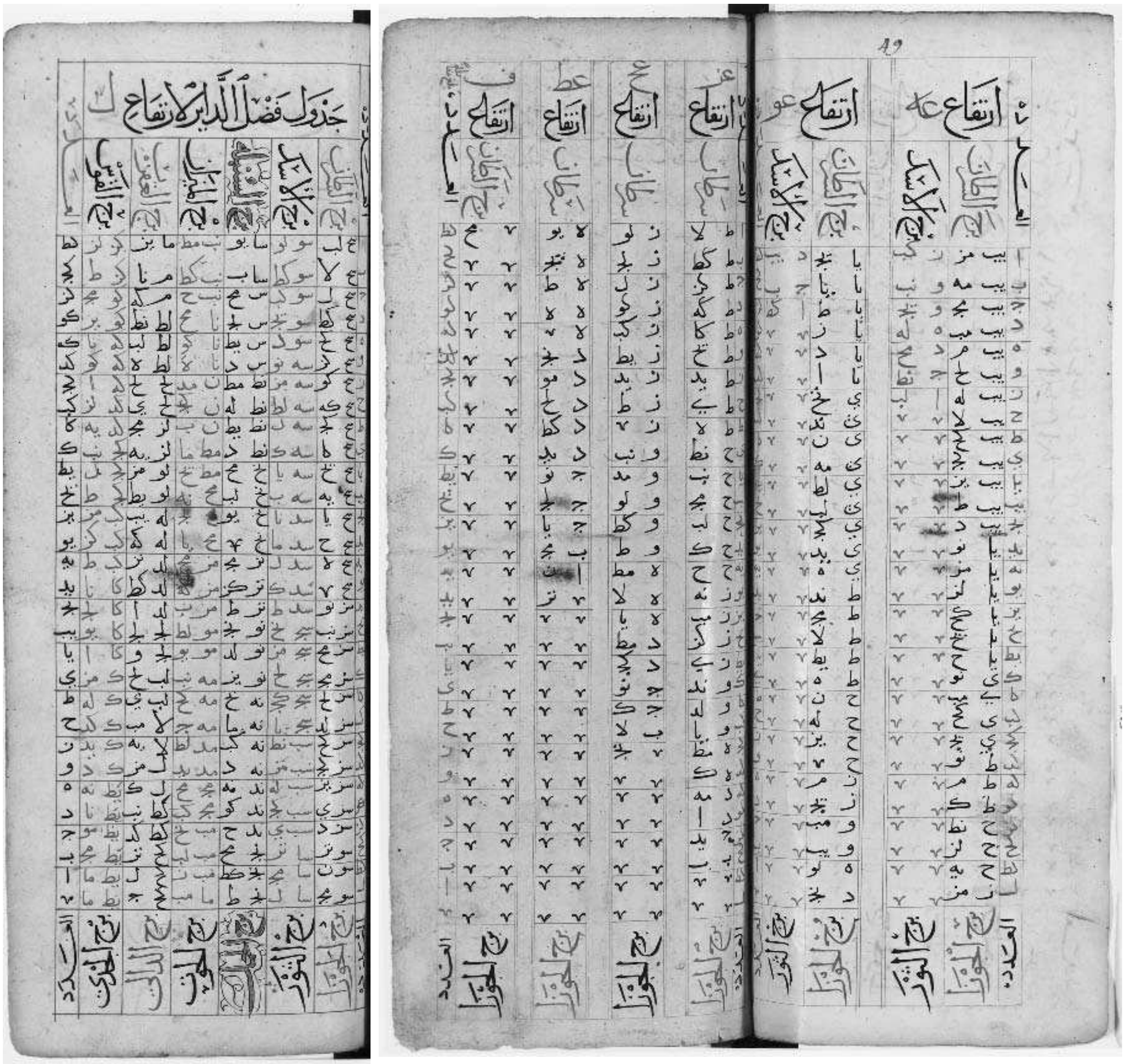


Fig. 2.1.4a-b: al-Khalīlī's hour-angle tables for altitudes 30° (a) and 75°-80° (b). Note the different format from the tables of the Cairo corpus shown in **Fig. 2.1.1**. [From MS Paris BNF ar. 2558, courtesy of the Bibliothèque Nationale de France.]

---

the references there cited, to which add Kennedy & Ghanem, eds., *Ibn al-Shāṭir*, and King, "Astronomy of the Mamluks", pp. 538-539 and 547-548. Several more recent papers reprinted in Saliba, *Studies*, deal with his contributions to theoretical astronomy. At the time I wrote the *DSB* article it was not known that Muslim astronomers concerned themselves with non-Ptolemaic models for several centuries after the time of Ibn al-Shāṭir. Several new sources from later centuries, albeit none yet from Syria, have been located and described in numerous papers by George Saliba: see, for example, his "The Astronomical Tradition of Maragha" (1991), "Arabic Planetary Theories after the 11th Century" (1996), "Al-Qūshjī's Reform of the Ptolemaic Model for Mercury" (1993), "A 16th-Century Arabic Critique of Ptolemy" (1994), and "Arabic Planetary Theories after the 11th Century" (1996).

several copies including MSS Paris BNF ar. 2558, Cairo MM 71, Damascus Ẓāhiriyya 3116 and 9233, Oxford Seld. Supp. 100, Oxford Marsh 39 (Uri 1042), fols. 113v-142v, Cairo K 8525 and ṬM 228,5, and Mosul al-Muḥammadiyya 129. Of these, MS Paris BNF ar. 2558 is the finest copy, dated 811 H [= 1408] and containing all of the tables of the corpus. See further **4.8.3**, **5.6.6**, **6.1.2**, **6.3.2**, **6.5.1**, **8.3.2**, **9.5** and **II-10**, especially **10.5**.

The format of al-Khalīlī's hour-angle table differs from that used in the earlier Egyptian tables (**2.1.1**), the argument domains being:

$$h = 1°, 2°, \ldots, 80° \quad \text{and} \quad \lambda = 271°, 272°, \ldots, 359°, 0°, 1°, \ldots, 90° .$$

Note that no entry is given for the winter solstice. Extracts from these tables are shown in **Figs. 2.1.4a-b**.

al-Khalīlī's computational accuracy is remarkably high. He may have used his first set of auxiliary tables (**9.4**) to compile the hour-angle tables for the latitude of Damascus, but he may also have tabulated the functions B(λ) and C(λ) for this latitude (**6.0**) and then used the simple formula (*cf.* **F12**):

$$t(h,\lambda) = \text{arc Cos } \{ R \bullet [\text{Sin } h - C(\lambda)] / B(\lambda) \} .$$

In MS Cairo MM 71 triplets of entries (t,T,a) for Damascus are given for each pair of arguments (h,λ). The values of both t and T are attributed to al-Khalīlī, and the values of a are attributed to Shihāb al-Dīn al-Ḥalabī: see **5.1.2** (illustrated).

In MS Damascus Ẓāhiriyya 7387, fols. 57v-60r, of an abridgement of the *Zīj* of Ibn al-Shāṭir by ʿAbd al-Raḥīm al-Qazwīnī of Damascus (*fl. ca.* 1610),[16] there is a set of tables displaying the time remaining till moonset for lunar altitudes h = 7°, 8°, ... , 16° and each degree of lunar longitude (symmetrically arranged). The entries are those of al-Khalīlī's tables of T(h,λ), and al-Qazwīnī's tables, as stated in the title, can be used to find the arc of visibility (*qaws al-ruʾya*) from the arc of tarrying (*qaws al-makth*) for calculations relating to lunar crescent visibility.[17]

2.1.5 Ibn al-Rashīdī (?): Jerusalem

Two disordered copies of tables for timekeeping for Cairo, MSS Cairo DM 45 and DM 153, copied in the same hand *ca*. 1650, contain odd folios from one and the same set of tables of the function t(h,λ) computed for:

$$\phi = 32;0° \text{ (Jerusalem) and } \varepsilon = 23;35° ,$$

with the same format as the Cairo tables. The altitude arguments 22°-23° (MS DM 153, fols. 11r-11v), 24°-25° (MS DM 45, fols. 27r-27v), and 34°-35° (MS DM 153, fols. 13r-13v) head the fragments of the tables which have found their way into these two manuscripts. The six pages of tables are copied in the same hand. There is no indication of the parameters underlying the tables, which were determined by inspection.

The entries in these fragments are reasonably accurately computed and are identical to the corresponding entries in al-Karakī's tables of t(λ,h) – note the change in format – for Jerusalem (**2.2.1**). Since al-Karakī states in his introduction to his tables that (the 14th-century Egyptian astronomer) Ibn al-Rashīdī also compiled a set of tables of t(h,λ) (latitude unspecified), I am

---

16 On al-Qazwīnī see *Cairo ENL Survey*, no. D38.

17 On these concepts see my article "Ruʾyat al-hilāl" [= lunar crescent visibility] in EI_2.

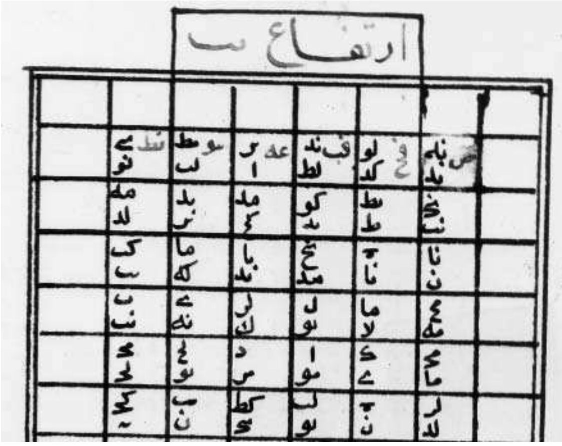


Fig. 2.1.5: An extract for solar altitude 12° from an anonymous corpus of hour-angle tables for Jerusalem. The corpus was copied *ca*. 1900, possibly from a manuscript in the Khālidiyya Library in Jerusalem. [From MS Cairo ṬM 81,1, courtesy of the Egyptian National Library.]

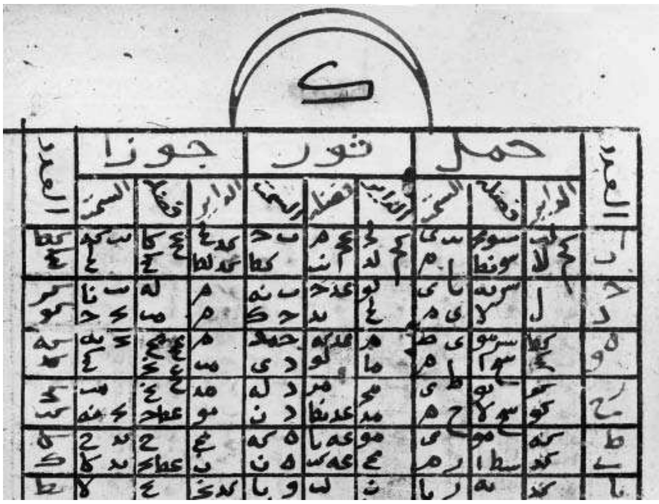


Fig. 2.1.6: An extract from the anonymous tables of the time since sunrise, the hour-angle and the azimuth for Alexandria showing the entries for solar altitude 20°. [From MS Cairo TR 354, courtesy of the Egyptian National Library.]

inclined to think that these fragments are from Ibn al-Rashīdī's hour-angle tables.[18] Ibn al-Rashīdī also made some corrections to Ibn Yūnus' azimuth tables where these had been incorrectly copied (**5.1.1**).

In MS ṬM 81,1, copied *ca*. 1900, there is a complete set of tables of the function $t(h,\lambda)$ with values in degrees and minutes, for the same parameters:

$$\phi = 32;0° \text{ (Jerusalem) and } \varepsilon = 23;35° .$$

These tables are copied without the solar longitude arguments, and without the degrees of the entries, except at the head of each column. I have not been able to check that the entries in various sub-tables are the same as those in the fragments mentioned above, but it seems unlikely that there would be two different sets in existence. See also **I-2.2.6** and **II-6.12** and **II-11.12**.

2.1.6 Anonymous: Alexandria

The apparently unique source MS Cairo TR 354, copied *ca*. 1700,[19] contains a set of tables of the functions:

$$T(h,\lambda),\ t(h,\lambda) \text{ and } a(h,\lambda) ,$$

computed to two digits for the domains:

$$h = 1°, 2°, \ldots , 82°, \text{ and } \lambda = 1°, 2°, \ldots , 90° \text{ and } 181°, 182°, \ldots , 270°$$

and the parameters:

$$\phi = 31;0° \text{ (Alexandria) and } \varepsilon = 23;35° .$$

These tables, which contain over 30,000 entries, are falsely attributed to Ibn Yūnus on the title folio and they are less carefully computed than the corresponding tables for Cairo (**2.1.1** and **5.1.1**). Values of the three functions are displayed side by side for each pair of arguments –

[18] On Ibn al-Rashīdī, whose full name was Shams al-Dīn Abū ʿAbdallāh Muḥammad ibn Burhān al-Dīn Ibrāhīm al-Rashīdī, see *Cairo ENL Survey*, no. C39, and **II-4.1.3** and **6.11**.

[19] See *Cairo ENL Survey*, no. C143.

see **Fig. 2.1.6** – as in al-Bakhāniqī's edition of the Cairo corpus – see **Figs. II-5.6a-b**. The errors in the sum of corresponding values of t and T are related to those in a table of D(λ) for these parameters in MSS Oxford Marsh 676 (Uri 944 = 995) and Princeton Yahuda 861,1, originally compiled by Najm al-Dīn al-Miṣrī (**2.6.1**). I know of no tables of auxiliary functions based on latitude 31;0° which might have been used to compile the larger set. Several manuscripts exist of a corpus of prayer-tables based on the same parameters: see further **II-8.5**. However, the identity of the compiler of these tables remains obscure.

MS Cairo ṬM 216, copied in 1003 H [= 1594/95], contains a different set of tables of the functions T(h,λ) and t(h,λ) based on the parameters ϕ = 31;0° and ε = 23;35° (?). No compiler is associated with this set either.

2.1.7 Aḥmad Efendī: Istanbul

MS Istanbul Kandilli 196 contains a set of tables of the function t(h,λ) computed to two digits for the domains:

h = 1°, 2°, ... , 72° and λ = 1°, 2°, ... , 90° and 181°, 182°, ... , 270° .

The underlying parameters are found by inspection to be:

ϕ = 41;0° (Istanbul) and ε = 23;30° .

The tables are attributed to "Aḥmed Efendī known as al-Miṣrī al-Islāmbūlī", which double epithet suggests that he was an Egyptian who had taken up permanent residence in Istanbul,[20] and they are dated 1095 H [= 1684]. They are rather accurately computed.

2.1.8 al-Kutubī: Cairo

MSS Cairo DM 1104, ṬJ 811,8, DM 149, ṬM 88 and ṬM 142, DM 812 and 1103 contain a set of tables for timekeeping prepared in the year 1150 H [= 1737/38] by ʿAbd al-Laṭīf al-Dimashqī, also known as al-Kutubī.[21] The tables, which were prepared for the latitude of Cairo, display the times before midday and after midday when the sun has a given altitude and longitude, for each degree of both arguments – see **Fig. 2.1.8**. The entries are given in equinoctial hours and minutes, according to the Ottoman convention that sunset is 12 o'clock (**1.1**). The format of the tables is identical with that of the earlier tables of T(h,λ) and t(h,λ) in the main Cairo corpus (**2.1.1**), from which these tables were no doubt derived. If we denote the two times tabulated by al-Kutubī as t_1 and t_2, then:

$$t_1(h,\lambda) = {}^1/_{15}\ \{180° - D(\lambda) - t(h,\lambda)\} \text{ and}$$
$$t_2(h,\lambda) = {}^1/_{15}\ \{180° - D(\lambda) + t(h,\lambda)\} = {}^1/_{15}\ \{180° - T(h,\lambda)\} .$$

al-Kutubī states that he also included a correction for refraction at the horizon. See further **II-7.14**.

2.2 Tables of time as a function of solar longitude and instantaneous altitude for a given latitude

These tables differ from those discussed in **2.1** only in that the arguments h and λ are interchanged. For each degree of λ a given table displays entries for each degree of h up to

[20] On Aḥmad Efendī see *Cairo ENL Survey*, nos. H28. He does not appear to be listed in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*.

[21] On al-Kutubī see *Cairo ENL Survey*, no. D76; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 427-429, no. 281.

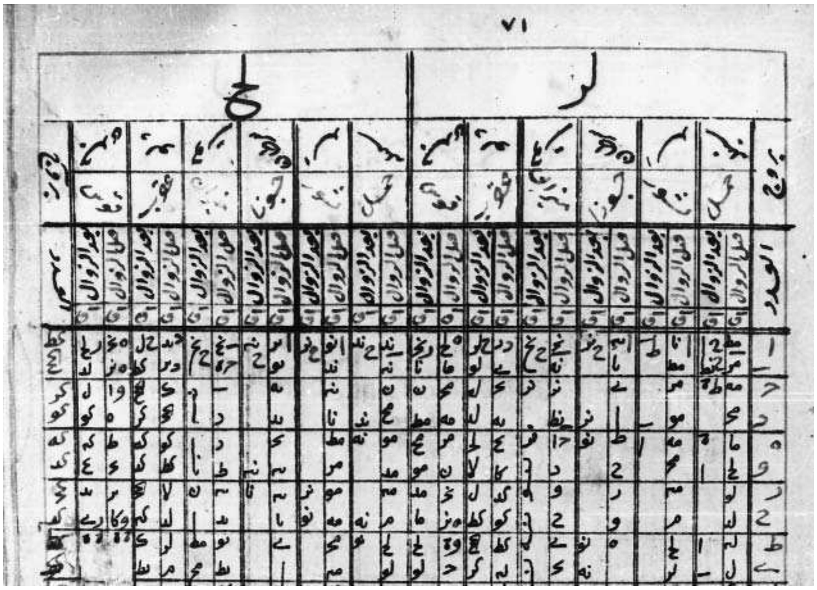


Fig. 2.1.8: al-Kutubī's tables for solar altitudes 37°-38°. These are based on the earlier tables of the Cairo corpus, although the author gives no indication of this. [From MS Cairo ṬJ 811,8, fol. 71r, courtesy of the Egyptian National Library.]

the maximum for the solar longitude in question. that is, H(λ) or the smaller integral value [H(λ)]. This arrangement has the advantage that entries can be included both for the equinoxes and for the solstices.

2.2.1 al-Karakī: Jerusalem

MS Leipzig UB 808, fols. 3r-93r, copied in 805 H [= 1402], contains a set of tables for timekeeping computed for the latitude of Jerusalem. The tables are introduced in the name of the 14th-century astronomer al-Karakī[22] and display, separately in two different hands, the functions:

$$T(\lambda,h) \text{ and } t(\lambda,h)$$

with entries computed to two digits for the domains:

$$\lambda = 270°, 271°, \dots, 359°, 0°, 1°, \dots, 90° \text{ and } h = 1°, 2°, \dots, [H(\lambda)], H(\lambda)\ ,$$

and based on the parameters:

$$\phi = 32;0° \text{ (Jerusalem) and } \varepsilon = 23;35°\ .$$

The values are rather accurately computed. **Figs. 2.2.1a-b** show sample sub-tables from the Leipzig manuscript. See further **II-9.4** on the Jerusalem corpus.

In the introduction to his tables (fol. 2v) al-Karakī states that he wished to emulate al-Maqsī, who had computed T(h,λ) for ϕ = 30° (**2.1.1**), his own teacher al-Mizzī, who had computed t(h,λ) for ϕ = 33;27° (**2.1.2**), and Ibn al-Rashīdī, who had computed t(h,λ) for an unspecified latitude. None of these scholars had prepared tables showing both T and t, and al-Karakī states that there he saw his opportunity to join their ranks by compiling tables of both functions for

[22] On al-Karakī see *Cairo ENL Survey*, no. C35.

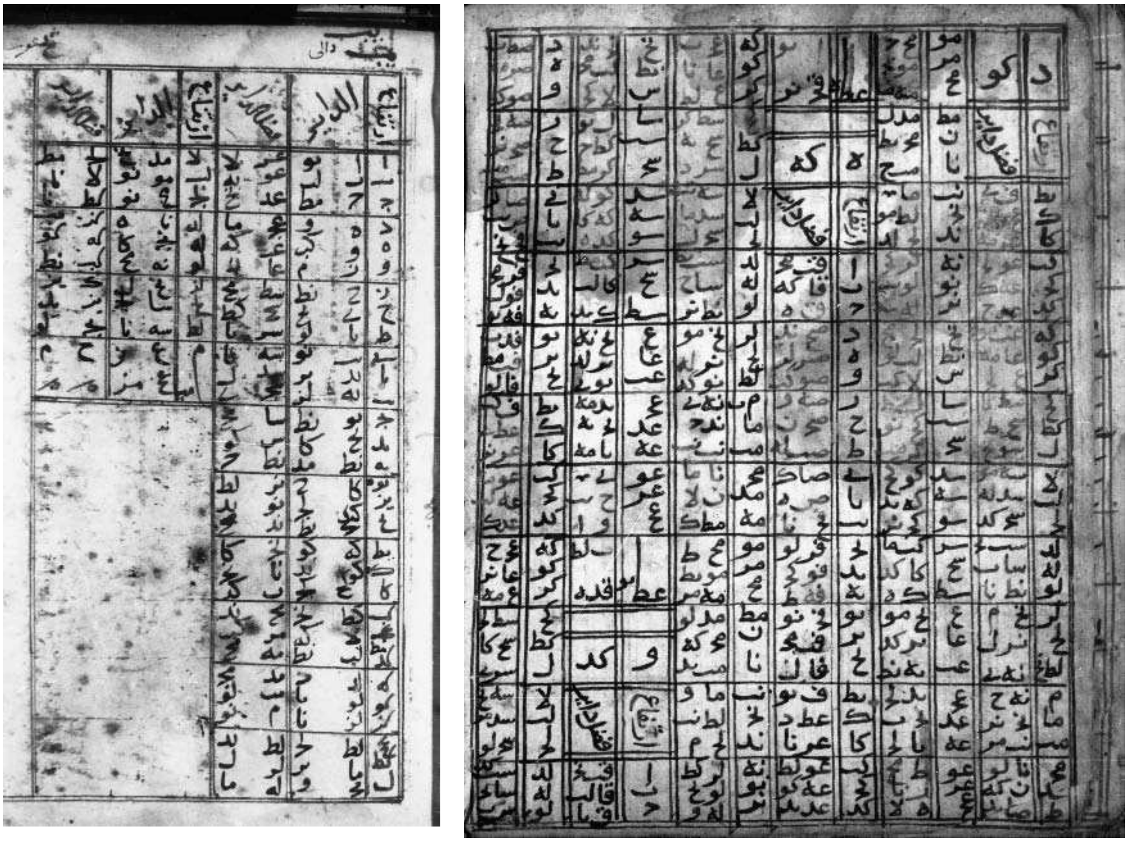


Fig. 2.2.1a-b: The tables of the time since sunrise for solar longitude Aquarius 12° / Scorpio 18° (a) and the hour-angle for longitudes 4°-6° of a certain sign (b) in the Jerusalem corpus. [From MS Leipzig UB 808, fol. 23v and 119v, courtesy of the Universitätsbibliothek.]

Jerusalem. With due respect to al-Karakī, it seems to me probable that Ibn al-Rashīdī's tables of t(h,λ) were computed for Jerusalem and that al-Karakī simply changed the format and added the values of T(λ,h). The entries for t(λ,h) in the Leipzig manuscript are identical with the corresponding entries in the tables of t(h,λ) for Jerusalem in MSS Cairo DM 45 and Cairo DM 153, which I suspect were computed by al-Rashīdī (**2.1.5**).

In fols. 94v-123v of the Leipzig manuscript there is another set of hour-angle tables in a different hand – see **Fig. 2.2.1b**. It is stated at the beginning and at the end of the tables that they are for latitude 32;40° (Ramla), but in fact the tables are simply those of al-Karakī for latitude 32;0°.

In MS Princeton Yahuda 861,1, copied *ca.* 1600, amidst a set of anonymous prayer-tables for latitude 32°, there is an odd table of T(h) and t(h) computed for the equinoxes, which apart from copyist's errors has the same entries as al-Karakī's tables of T(λ,h) and t(λ,h) for λ = 0°. Likewise the entries in the twilight tables for latitude 32° in MS Princeton Yahuda 861,1 are related to the entries for solar altitudes 20° and 16° in al-Karakī's set. See further **II-8.1** and **9.5.**

2.2.2 Anonymous: Edirne

MS Oxford arab. e. 93, fols. 3v-25r, contains a set of tables of t(λ,h) and T(λ,h) for the domains:

λ = 270°, 271°, ... , 90° and h = 1°, 2°, ... , [H(λ)], H(λ)

based on the parameters:

ϕ = 41;30° (Edirne) and ε = 23;30°.

An extract is shown in **Fig. 2.2.2**. The manuscript is undated and the tables are anonymous and without introduction, but the city of Edirne is mentioned at the head of some of the tables. I have not investigated their accuracy.

2.2.3 Ṣāliḥ Efendī: Istanbul

The tables for timekeeping compiled in the late 18th century by Ṣāliḥ Efendī Miʿmārī "the architect",[23] display values of the functions T(λ,h) and t(λ,h) computed to three digits with remarkable accuracy for the domains:

λ = 270°, 271°, ... , 90° and h = 1°, 2°, ... , [H(λ)], H(λ)

and based on the parameters:

ϕ = 41;0° (Istanbul) and ε = 23;28,54°.

An extract is shown in **Fig. 2.2.3**. The distinctive value of ε used by Ṣāliḥ Efendī is due to his predecessor Taqi 'l-Dīn, who worked in Istanbul two centuries previously (see, for example, **6.4.8**).

MSS Princeton Yahuda 353, Istanbul Aşir Efendi 224, Istanbul Kandilli 219, Vienna 2379 (Mixt. 989), Cairo K 18199, Cairo ṬM 151 and 215, and perhaps also Baghdad Awqāf 324/ 12230, are complete copies of these tables, which contain over 80,000 entries. An extract from the main tables for timekeeping in the superior Princeton manuscript is shown in **Fig. 2.2.3**. A given sub-table displays entries for one particular solar longitude, with the times for solar altitudes in the east and west expressed in equatorial degrees, equinoctial hours and seasonal day-hours. The times in equinoctial hours are also given (and now labelled *al-muwāfiqa*) according to the Ottoman convention that sunset is 12 o'clock (**1.2** and **II-14.0**). Various functions relating to timekeeping are displayed around each sub-table. Those relating to the afternoon prayers and twilight are also tabulated separately at the end of the main tables. For more details see **II-14.13**.

The method by which the values of t(λ,h) were computed is clear from some of the auxiliary functions displayed on each sub-table. For each value of λ Ṣāliḥ Efendī tabulates B(λ) and C(λ), as well as, for each degree of h, the function b(λ,h). These three functions are labelled *al-aṣl al-muṭlaq*, *buʿd al-quṭr* and *al-aṣl al-muʿaddal*, respectively (see further **6.0**). The hour-angle is then given by the simple formula (*cf.* **F12**):

$$\mathrm{Cos}\ t(\lambda,h) = R \bullet b(\lambda,h) \ / \ B(\lambda) \ .$$

Numerous other sources contain smaller sets of tables for Istanbul that are based on those of Ṣāliḥ Efendī. For example, MSS Istanbul Bağdatli Vehbi Efendi 990, Istanbul Lala Ismail 287, Istanbul Husrev Paşa 232, Istanbul Kiliç Ali Paşa 677, Istanbul Kandilli 220, Istanbul

---

[23] On Ṣāliḥ Efendī see *ibid.*, no. H36; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 453-458, no. 303.

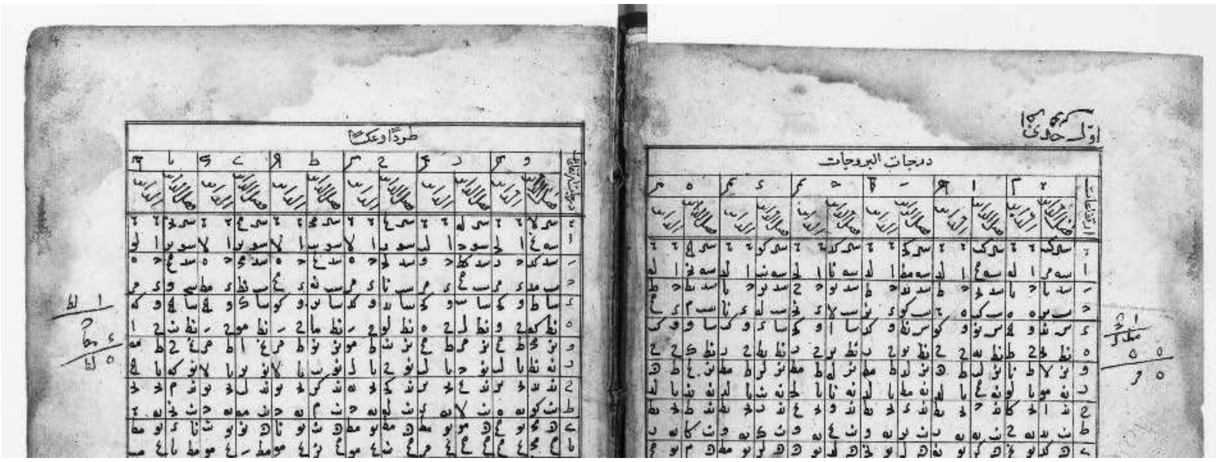


Fig. 2.2.2: The tables for Capricorn 0°-11° / Sagittarius 30°-19° in the Edirne corpus. [From MS Oxford arab. e. 93, fols. 3v-4r, courtesy of the Bodleian Library.]

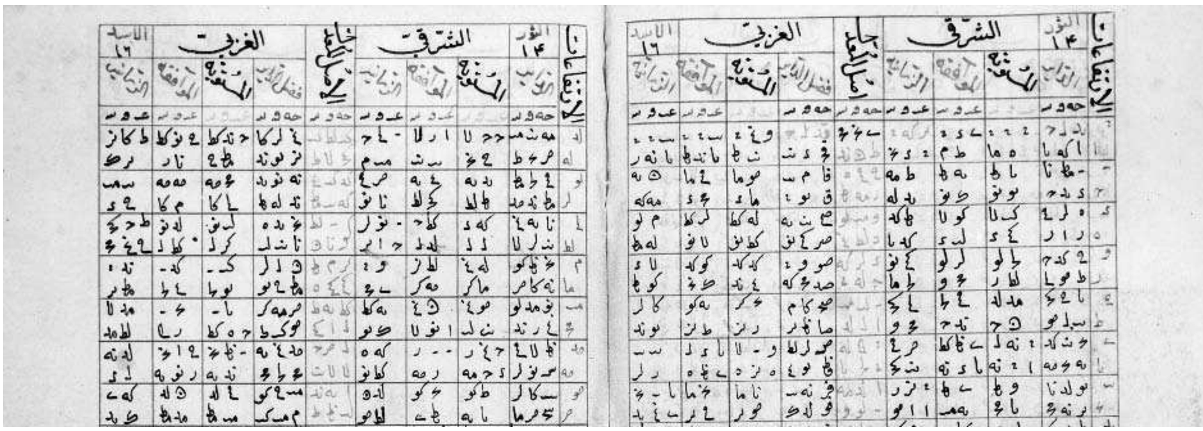


Fig. 2.2.3: The tables for Taurus 14° / Leo 16° in the corpus of Ṣāliḥ Efendī. [From MS Princeton Yahuda 353, courtesy of the Special Collections, Princeton University Library.]

Kandilli 440 and 441, and Cairo ṬM 120 contain tables displaying the times before and after midday when the sun has the given longitude and altitude, with entries in equinoctial hours, minutes and seconds, expressed according to the Ottoman convention. Likewise the tables in MSS Istanbul Esat Efendi 1979, Istanbul UL T1963 and T1964 display these same times in equinoctial hours and minutes. Most of these sets of tables are anonymous and most of them are labelled in Turkish *irtifā cedveli* < *irtifāʿ jadwali*, “altitude tables”. In MS Istanbul Husrev Paşa 232, also apparently MS Baghdad Awqāf 325/12248, the tables are attributed to the *muwaqqit* Muḥammad Ṣādiq Jihāngīrī. An abridged version of Ṣāliḥ Efendī’s tables appears on an Ottoman quadrant.[24]

[24] See *Khalili Collection Catalogue*, II, p. 268, no. 157, and King, “Review”, col. 256.

2.2.4 Ḥusayn Ḥusnī: Mecca

MS Cairo K 4002 contains a set of tables for timekeeping for Mecca prepared in 1239 H [= 1823/24] by the Ottoman astronomer Ḥusayn Ḥusnī, also known for his translation of the "*zīj*" of the French astronomer Lalande.[25] The function tabulated is:

$$T'(\lambda,h)$$

for arguments:

$$\lambda = 270°, 271°, \dots, 359°, 0°, \dots, 90° \quad \text{and} \quad h = 0°, 1°, \dots, [H(\lambda)] .$$

The times are expressed according to the Ottoman convention and values are given in equinoctial hours, minutes and seconds for solar altitudes in both east and west: see the extract in **Fig. 2.2.4**. The entries are in Hindu-Arabic numerals. The underlying latitude is stated to be ϕ = 21;45° (Mecca). I have not investigated the accuracy of these tables. al-Ḥusnī's value of the latitude of Mecca is not used in any other Islamic astronomical work known to me. The accurate value is 21;26°, so that Ḥusnī's value was no improvement over the standard medieval values 21°, 21;20°, 21;30° and 21;40°.[26] On the prayer-tables accompanying Ḥusnī's main tables for timekeeping see **II-12.9**.

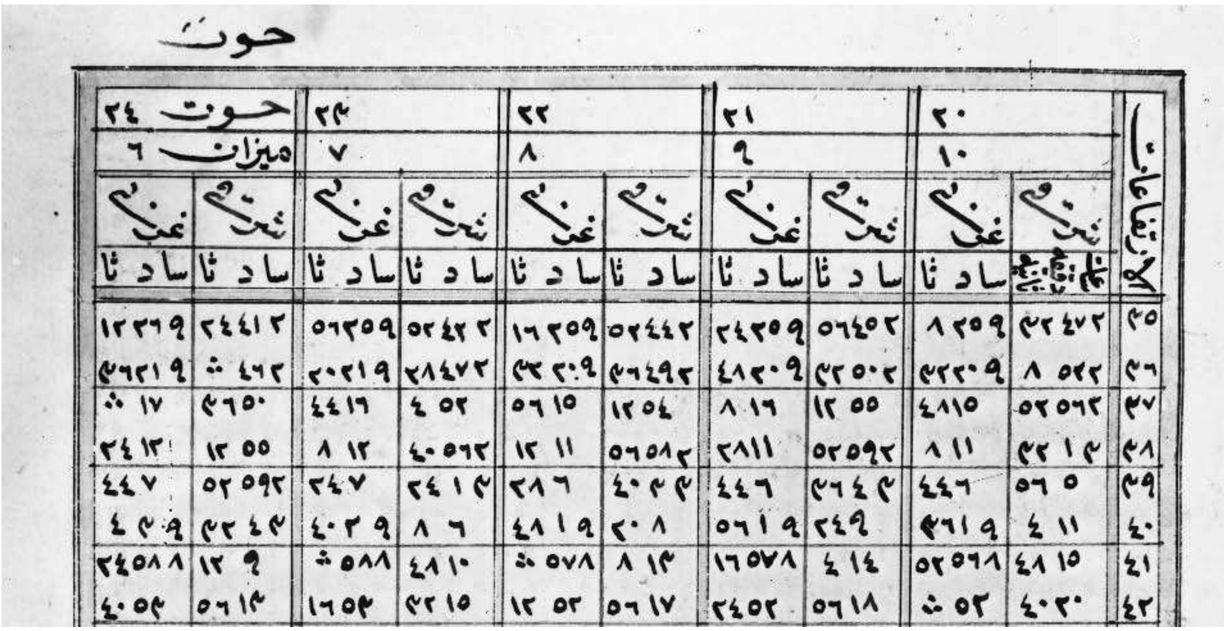


Fig. 2.2.4: This extract from Ḥusayn Ḥusnī's tables for Mecca shows the time in hours, minutes and seconds for solar altitudes 35°-65° in the east and west and for solar longitudes Pisces 20°-24° and Libra 10°-6°. [From MS Cairo K 4002, fol. 10v, courtesy of the Egyptian National Library.]

---

[25] On Ḥusayn Ḥusnī see Azzawi, *History of Astronomy in Iraq*, pp. 268, 284 and 285; *Cairo ENL Survey*, no. H47; and Ihsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 581-584, no. 419.

[26] On Islamic values for the latitude of Mecca see King, "Earliest Muslim Geodetic Measurements", pp. 225-226.

2.2.5 al-Ṭanṭāwī: Damascus

MSS Cairo TR 129 and DM 1007 and an unnumbered manuscript in *al-Maktaba al-ʿArabiyya* bookshop in Damascus in 1973 contain a set of tables of the function:

$$[T(\lambda,h)]^{h}$$

attributed to the Damascus astronomer al-Ṭanṭāwī (d. 1889).[27] The values are computed for Damascus, are expressed in equinoctial hours and minutes and are probably derived from al-Khalīlī's tables of t(h,λ) or T(h,λ) (**2.1.4**), which, however, have a different format. Other tables to which al-Ṭanṭāwī put his name were likewise computed by his predecessor of five centuries.

2.2.6 Anonymous: Jerusalem

MS Cairo ṬM 81,1, copied *ca*. 1900, contains some very late tables displaying values of the function:

$$T(\lambda,h)$$

expressed in hours and minutes, with entries for altitude in both east and west – see **Fig. 2.2.6**. They are based on the parameter λ = 32;0° (Jerusalem). I have not tried to establish whether or not these tables were derived from those described in **2.1.5** or **2.2.1**. See further **II-11.12**.

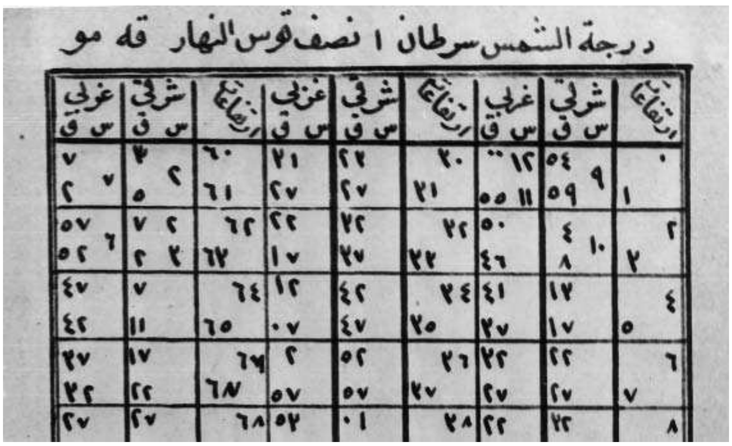


Fig. 2.2.6: An extract from some tables for Jerusalem preserved in a late manuscript. These are probably based on those illustrated in **Fig. 2.1.5**. [From MS Cairo ṬM 81,1, fols. 1v-2r, courtesy of the Egyptian National Library.]

2.3 Tables of time as a function of meridian altitude and instantaneous altitude, for a specific latitude

The tables listed in this section display the function T(H,h) and can in principle be used for either the sun or the stars. Since h ≤ H the tables are trapezoidal in shape, and in medieval

[27] On al-Ṭanṭāwī see *Cairo ENL Survey*, no. D123; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 646-649, no. 496.

Arabic these tables are generally refered to by the term *ṭaylasān* (**1.4**). In such tables only the entries for $\bar{\phi}$-ε ≤ H ≤ $\bar{\phi}$+ε can be used for the sun. The entries for general arguments H can be used to find the time elapsed since the rising of a star with declination Δ such that Δ = H - $\bar{\phi}$. Tables of T(H,h) computed for a given latitude for timekeeping by the sun require about one-third as many entries as tables of T(h,λ), if entries are given for each degree of argument in both cases.

No indication is given in the sources of the method of computation of the entries in these tables, but it seems reasonable to assume that one would first compute the value of D corresponding to H and then apply the standard formula (*cf.* **F9**):[28]

$$\text{Vers } t(H,h) = [\ \text{Sin } H - \text{Sin } h\ ] \cdot \text{Vers } D\ /\ \text{Sin } H\ .$$

Given D and t the computation of T is trivial. Note that the function T(H,h) is independent of the obliquity ε.

Other tables of this kind are attested in some copies of the *Toledan Tables* and in a medieval Hebrew source. In the former, values are given for each degree of both arguments, and the meridian altitude runs up to about 80°. In the latter, the horizontal arguments are periods of days of the year (during which the meridian altitude is assumed constant) and the vertical argument is the solar altitude in 5°-intervals. See further **10.1**.

2.3.1 ʿAlī ibn Amājūr: Baghdad

Some of the earliest known Islamic tables for reckoning time from solar or stellar altitudes are due to the Baghdad astronomer ʿAlī ibn Amājūr (*fl. ca.* 950).[29] One of these is contained in MS Paris BNF ar. 2486, fols. 239r-255r, a unique manuscript of the *Zīj* of al-Baghdādī copied by the author for himself in 684 H [= 1285].[30] (On the other table attributed to Ibn Amājūr see **2.5.1**.) The function tabulated here is:

$$T(H,h)\ ,$$

with values computed to two digits for the domains:

$$H = 21°, 22°, \ldots, 84°, \text{ and } h = 1°, 2°, \ldots, H\ ,$$

and based on the parameter:

$$\phi = 33;25° \text{ (Baghdad)}\ .$$

The entries in this table, which number about 3,300, are computed with remarkable accuracy. The lower and upper limits for H indicate that the table was intended for timekeeping by the stars as well as by the sun. An extract from this table is shown in **Fig. 2.3.1**.

2.3.2 al-Ṭūsī: Maragha

Another such table is contained in some copies of the *Īlkhānī Zīj*, prepared under the celebrated scholar Naṣīr al-Dīn al-Ṭūsī (d. 1274)[31] at the observatory in Marāgha in N.W. Iran. I have

[28] See, for example, Nallino, *al-Battānī*, I, pp. 189-191, King, *Ibn Yūnus*, III.15.3 (pp. 149-150), and Nadir, “Abu ’l-Wafāʾ on the Solar Altitude”, on this formula as used by al-Battānī (*ca.* 910), Ibn Yūnus (*ca.* 980), and Abu ’l-Wafāʾ (*ca.* 1000), respectively. See also Goldstein, *Ibn al-Muthannā on al-Khwārizmī*, pp. 208-209; and Davidian, “al-Bīrūnī on the Time of Day”, p. 333.

[29] On Ibn Amājūr see Suter, *MAA*, no. 99, and Sezgin, *GAS*, V, p. 282, where this *ṭaylasān* table is mentioned.

[30] On al-Baghdādī and his *zīj* see Kennedy, “*Zīj* Survey”, no. 3; Kennedy, “Comets in Islamic Astronomy and Astrology”, p. 48; and Jensen, “The Lunar Theories of al-Baghdādī”, p. 322.

[31] On al-Ṭūsī and the *Īlkhānī Zīj* see Suter, *MAA*, no. 368; Kennedy, “*Zīj* Survey”, no. 6; and Storey, *PL*,

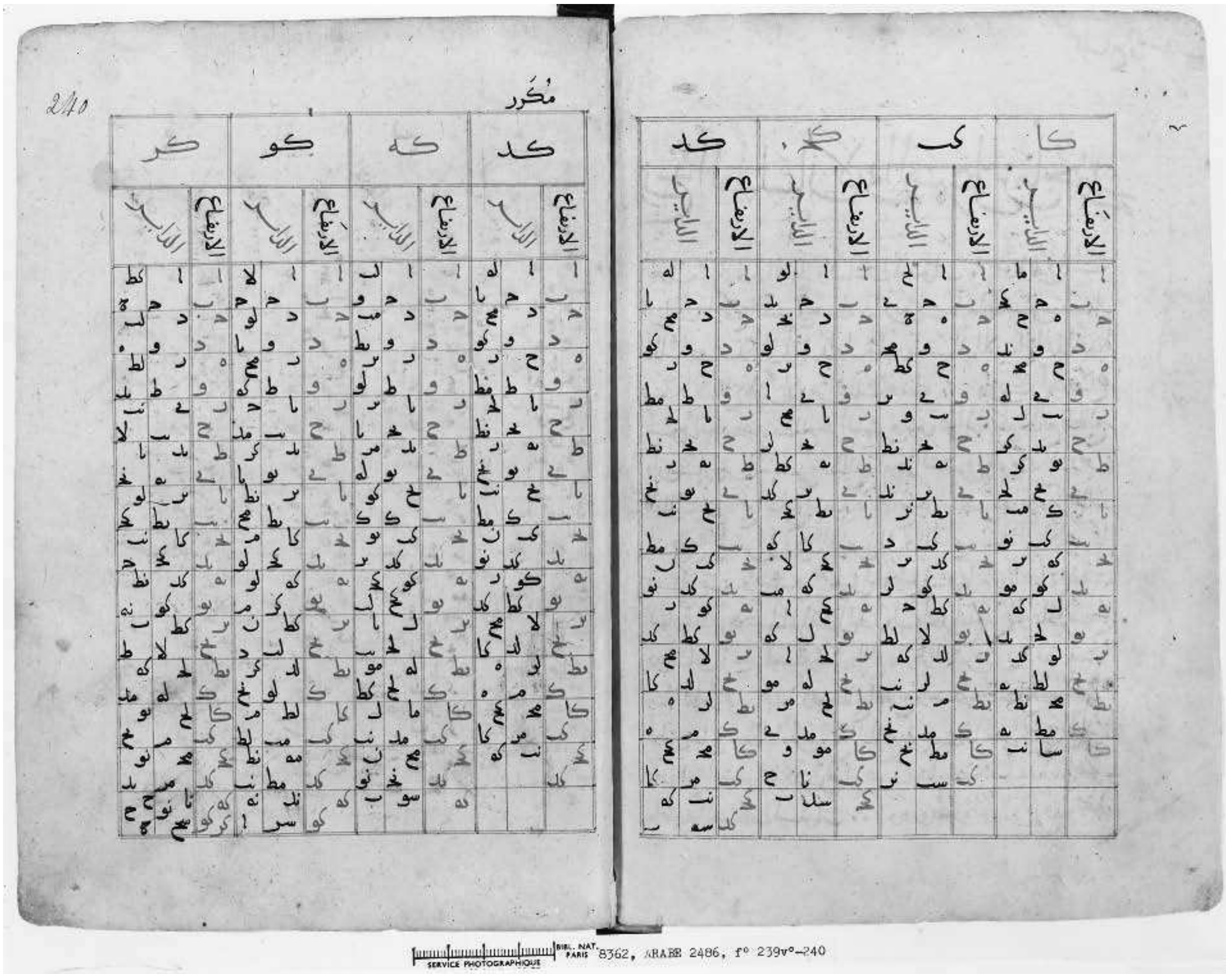


Fig. 2.3.1: The sub-tables for solar altitudes 21°-27° in the Baghdad corpus of ʿAlī ibn Amājūr. Note that the scribe copied the column for 24° twice, realising his error just before he finished the second set and marking it *mukarrar*, "repeated". [From MS Paris BNF ar. 2486, 239v-240r, courtesy of the Bibliothèque Nationale de France.]

examined MS Cairo DMF 1, 136 fols., copied at Maragha in 692 H [= 1293], as well as MSS Florence Medici 269, fols. 150r-152v, Istanbul UL P1418,2, fols. 234r-237v, and Oxford Hunt. 143, fols. 155v-160v, of this table. There the function:

$$T(H,h)$$

is tabulated to three digits for the domains:

$$H = 29;10°, 30°, 31°, \ldots , 75°, 76;10°, \text{ and } h = 1°, 2°, \ldots , H ,$$

based on the parameter:

$$\phi = 37;20° \text{ (Maragha).}$$

II:1, pp. 58-60; and, most recently, Ragep, *al-Ṭūsī's Memoir on Astronomy*, and the same author's article "al-Ṭūsī. 3. As scientist" in EI_2. The existence of the *Īlkhānī ṭaylasān* table was noted in Kennedy, *op. cit*., p. 161.

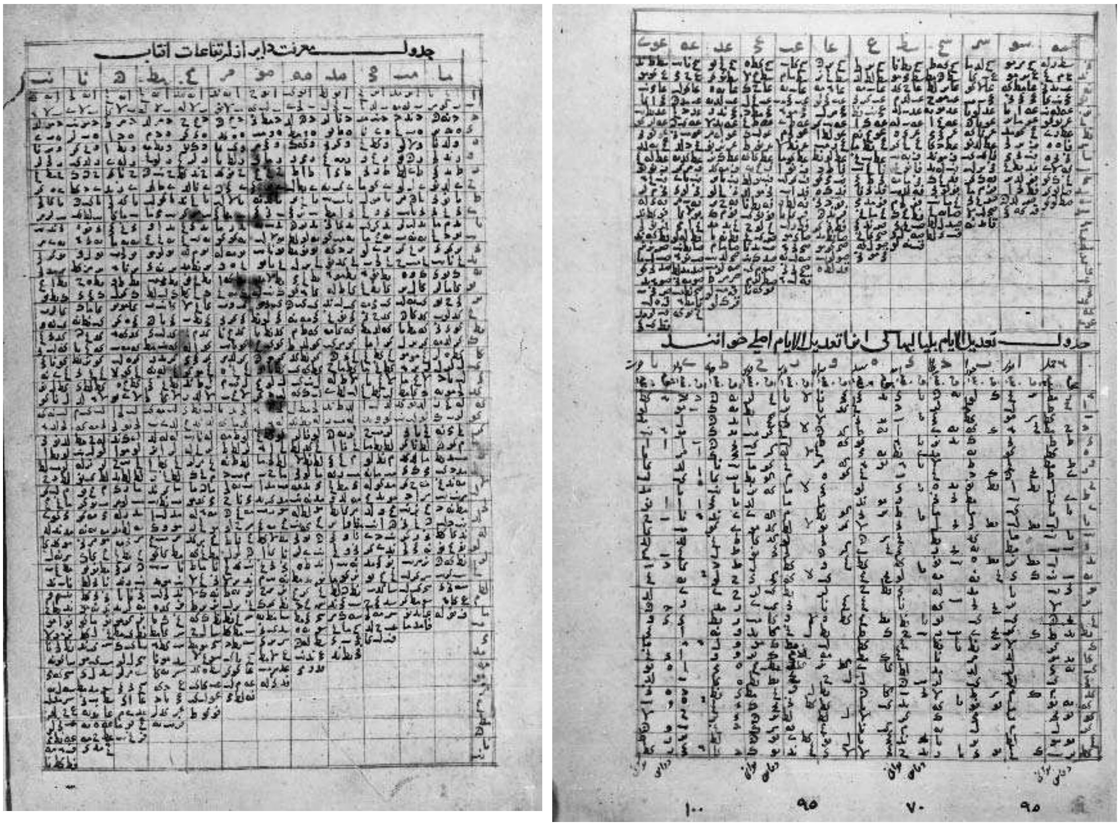


Fig. 2.3.2a-b: Some of the entries in a splendid late 13th-century copy of al-Ṭūsī's tables for meridian altitudes 41°-64° (a) and 65°-75° and then 76;10° (b). After this *ṭaylasān* table there is a table for the equation of time. From MS Cairo DMF 1, courtesy of the Egyptian National Library.]

Two extracts are displayed in **Figs. 2.3.2a-b**. The minimum and maximum values of H indicate that the value assumed for ε was the *Īlkhānī* value 23;30°, although as noted above the values of T(H,h) are independent of this parameter. Since the second digits in the entries are often in error, the third ones are meaningless. Values are also given in equinoctial hours, minutes and seconds, and these are generally accurate to the nearest minute.

2.3.3 Anonymous: Shiraz

MS Paris BNF supp. pers. 1488 is a unique copy from *ca*. 1400 of the Persian *Zīj-i Ashrafī*, compiled in Shiraz by Sanjar al-Kamālī in the year 702 H [= 1302/03].[32] This *zīj* contains several tables relevant to the present study. One of these is a table (fols. 205r-207r) displaying:

T(H,h)

with entries to two sexagesimal digits for each degree of both arguments. Values of D(H) are also given. The underlying parameters are:

[32] On Sanjar al-Kamālī and the *Ashrafī Zīj* see Kennedy, "*Zīj* Survey", no. 4, and Storey, *PL*, II:1, p. 64.

$$\phi = 29;30° \text{ (Shiraz) (and } \varepsilon = 23;35°) .$$

See also **2.5.1**, **5.8.3**, **6.4.5**, **7.2.2** and **8.1.2** for other tables in the *Zīj-i Ashrafī*.

2.3.4 Anonymous: unspecified locality (Iran)

An anonymous table in MS Leiden Or. 199, fols. 21v-27v, copied *ca*. 1400, displays the same function computed to two digits for the domains:

$$H = 30°, 31°, \ldots , 78°, \text{ and } h = 1°, 2°, \ldots , H ,$$

and:

$$\phi = 36;0° .$$

The limits for H suggest that the author accepted the Indian value 24;0° for ε. The table is preceded by a short introduction in Persian. Immediately before this on fol. 21r of the Leiden manuscript there is a table of $\alpha_\phi(\lambda)$ for $\phi = 37;40°$ (locality unspecified). The *ṭaylasān* table, which contains many errors difficult to attribute to copyists, has been studied previously by Bernard Goldstein,[33] and its structure is now explained (**XI-4.2**). In brief, the entries were computed using the approximate formula presented in **2.5** and then converted to equatorial degrees for latitude 36°.

2.3.5 Anonymous: Tunis

MS Berlin Ahlwardt 5724 (Wetzstein 1150), copied *ca*. 1700, contains an anonymous corpus of tables for timekeeping computed for the latitude of Tunis *ca*. 1400.[34] One of these (fols. 16r-28v) displays the hour-angle t(H,h), with entries computed to two digits for arguments:

$$H = 29;25°, 30°, 31°, \ldots , 76°, 76;35°, \text{ and } h = 1°, 2°, \ldots , H .$$

The underlying latitude is 37;0° (Tunis), and the limits for H imply that the value of ε was taken as 23;35°. The entries are rather accurately computed, and the format of the tables differs from that used in those discussed in **2.3.1-4** above – see the extract in **Fig. 2.3.5**. For other tables in the Tunis corpus see **2.4.1**, **8.1.3**, **8.3.3** and **8.4.1** and also **II-13.2**.

2.3.6 Taqi ’l-Dīn: Istanbul

MS Istanbul Kandilli 208 is a collection of works by the 16th-century Ottoman astronomer Taqi ’l-Dīn copied in his own hand.[35] Fols. 96r-100v contain a *ṭaylasān* table displaying t(H,h) and T(H,h) with values computed to two digits for each degree of both arguments: see **Fig. 2.3.6** for an extract. The underlying parameters (ε underlies the argument H) are:

$$\phi = 41° \text{ (Istanbul) (and } \varepsilon = 23;30°).$$

2.3.7 Anonymous: Edirne (?)

MS Cairo KhMT 2,1 (fol. 1r), copied *ca*. 1750, contains part of an anonymous *ṭaylasān* table displaying the function t(H,h) with values computed to two digits for each degree of both

---

33 See Goldstein, “Medieval Table for Reckoning Time”, the first modern study of a large medieval table for timekeeping.

34 On the manuscript see *Berlin Catalogue*, Ahlwardt no. 5724. *Cf.* King, “Astronomy in the Maghrib”, p. 38, for brief notes on this manuscript and others reflecting the level of astronomy in medieval Tunis.

35 On Taqi ’l-Dīn (Suter, *MAA*, no. 471 and *Cairo ENL Survey*, no. H12) see Sayılı, The *Observatory in Islam*, pp. 289-305, the article “Taḳī al-Dīn” in EI_2, and now also İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 199-217, no. 96.

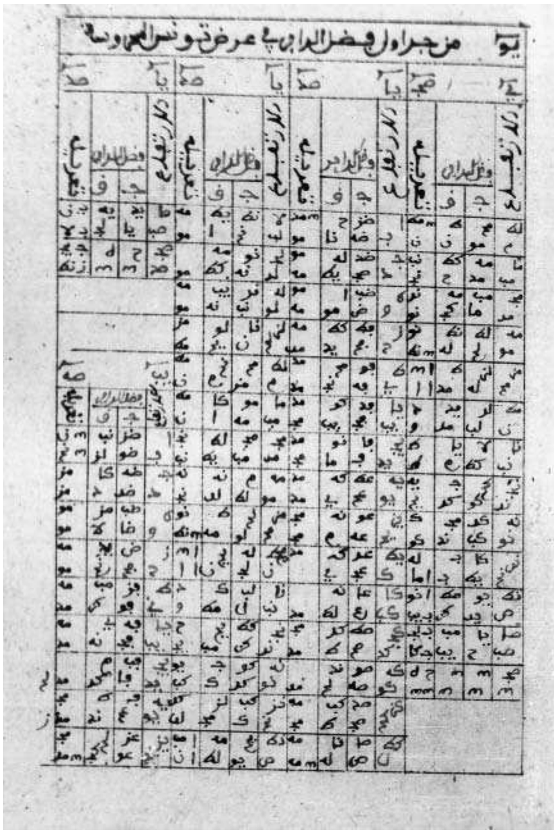



Fig. 2.3.5: The sub-tables for solar meridian altitudes 63°-64° in the Tunis corpus. [From MS Berlin Ahlwardt 5724, fol. 24r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

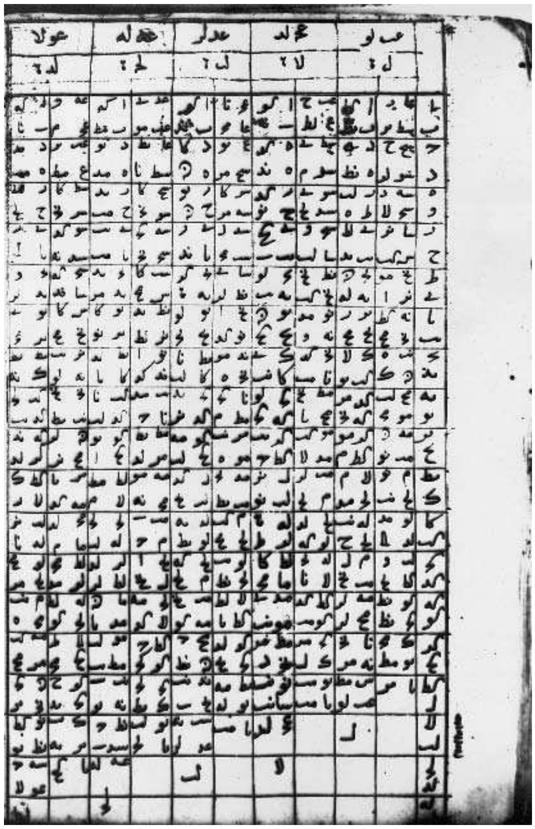


Fig. 2.3.6: The sub-tables for solar meridian altitudes 30°-34° from the *ṭaylasān* table of Taqi 'l-Dīn. [From MS Istanbul Kandilli 208, courtesy of Kandilli Observatory.]

arguments. The underlying parameters are ϕ = 42° and ε = 23;39°, and the table may have been intended to serve Edirne.

2.4 Tables of time as a function of stellar altitude, for a specific latitude

Such tables are clearly analogous to those described in **2.3** since for each star there is an underlying value of H. See also **6.16**.

2.4.1 Anonymous: Tunis

MS Berlin Ahlwardt 5724, fols. 55r-56r, of the anonymous corpus for Tunis (**2.3.5**) contains some simple tables displaying the hour-angle from stellar altitudes. For some 21 stars the function t(h) is tabulated for each degree of h up to the maximum for the star in question, computed for the latitude of Tunis (36;40° or maybe 37;0°). Entries are given to one digit only – see **Fig. 2.4.1**. The table may have been computed using the auxiliary table for latitude 36;40° on fol. 42v of the Berlin manuscript (on which see **6.8.2**).

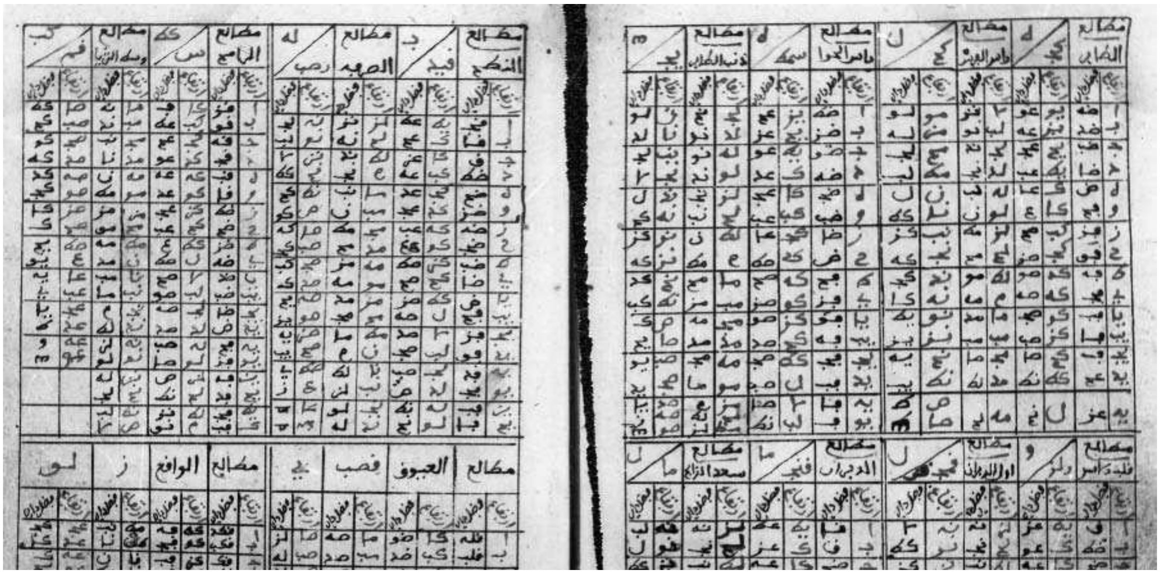


Fig. 2.4.1: Tables from the Tunis corpus for 14 stars. For each star the ascensions at culmination are also given. [From MS Berlin Ahlwardt 5724. fols. 55v-56r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

2.5 Tables of time since sunrise as a function of meridian altitude and instantaneous altitude, for all latitudes

The tables discussed in this section are also usually called by the name *ṭaylasān* (**2.3**) and are based on an approximate formula:

$$T(H,h) \approx {}^{1}/_{15} \text{ arc Sin } \{ R \text{ Sin } h \,/\, \text{Sin } H \} \text{ ḥ} .$$

This is a good approximation to the actual value of T(H,h). The expression is accurate for the equinoxes (and, of course, when h = 0° or H). I have computed the error incurred by using the equivalent expression in degrees, namely:

$$T(H,h) \approx D \,/\, 90 \bullet \text{ arc Sin } \{ R \text{ Sin } h \,/\, \text{Sin } H \} ,$$

in the domain:

$$\bar{\phi} - \varepsilon \leq H \leq \bar{\phi} + \varepsilon ,$$

corresponding to solar altitudes for various values of ϕ. The approximate values are less than the accurate values, but the maximum errors for latitudes 15° (Yemen) and 30° (Cairo) are only about 1° and $1^{1}/_{2}$°, respectively. The tables could also be used for timekeeping by the stars.

This approximate formula for reckoning time from solar altitude was used in the early 9th century by astronomers such as al-Khwārizmī and Ḥabash.[36] Certain later Muslim astronomers

---

[36] Ibn al-Muthannā (10th century?) attributes this method to al-Khwārizmī (Goldstein, *Ibn al-Muthannā on al-Khwārizmī*, pp. 81-82 and 207-208), and states that in addition al-Khwārizmī proposed an exact formula (*ibid.*, pp. 83). However, the approximate method is not included in the Latin translation of the recension of al-Khwārizmī's *Zīj* by al-Majrīṭī (*ca.* 1000) (Kennedy, "*Zīj* Survey", no. 21).

The duration of twilight is determined by an equivalent method in MS Berlin Ahlwardt 5750, fol. 153r, of a recension of the *Zīj* of Ḥabash (Kennedy, *op. cit.*, no. 15) but the attribution of the twilight theory in the Berlin

used an equivalent formula to compile tables of the duration of twilight: see **II-3.3** and **3.7** for tables of this kind for Baghdad and Alamut (?). Others tabulated the solar altitude as a function of time using this formula: see **4.3**. The approximation is further attested in a Byzantine astronomical treatise,[37] and also underlies a table in an English manuscript of the *Toledan Tables* (**10.1**). See also **II-1.4** and **XI**.

It is also worth noting that certain Muslim astronomers in early Abbasid times as well as later astronomers concerned with timekeeping in Cairo and Damascus used an approximation ultimately of Indian origin[38] involving the increase of a gnomon shadow over its midday minimum, $\Delta z = \mathrm{Cot}_n\, h - \mathrm{Cot}_n\, H$, namely:

$$T(h,H) \approx \{\ 6n\ /\ (\Delta z + n)\ \}^{\mathrm{sdh}}$$

for various values of the base n. No tables based on this formula are known to me.

2.5.1 ʿAlī ibn Amājūr (Baghdad)

MS Paris BNF supp. pers. 1488 of the early 14th-century Persian *Ashrafī Zīj* contains the table of T(H,h) for Shiraz discussed in **2.3.3** above, preceded by another table (fols. 201v-204v) of the function $T(H,h)^{\mathrm{sdh}}$ with entries computed to two digits for the domains:

$$H = 1°, 2°, \ldots , 90° \quad \text{and} \quad h = 1°, 2°, \ldots , H\ .$$

A note by this table states that it is due to ʿAlī ibn Amājūr and that he included it in his *zīj* called *Zīj al-Ṭaylasān* (**2.3.1**): see **Fig. 2.5.1** for an extract. There is no mention of the fact that the table is intended to serve all latitudes.

2.5.2 Anonymous (Baghdad / Damascus)

The anonymous MS Paris BNF ar. 2514, copied in 612 H [= 1215], bears the title *Kitāb fīhi maʿrifat Zīj al-Ṭaylasān al-laylī wa-'l-nahārī fī sāʿāt khāṣṣa*^{tan}. "Book containing instructions for the *Ṭaylasān Zīj* for timekeeping by day and night in hours".[39] The first part of this work (fols. 1v-26r), which may be all that the title refers to, consists of a set of tables of:

$$T(H,h)^{\mathrm{sdh}},\ T(H,h)^{\mathrm{h}},\ \mathrm{Cot}_{12}\ h,\ \mathrm{Cot}_{6;40}\ h,\ h_a(H)$$

for each degree of H up to 90° and/or each degree of h (≤ H). An extract is illustrated in **Fig. 2.5.2**.

The values of the first function differ from those attributed to Ibn Amājūr in the *Ashrafī Zīj* (**2.5.1**). Values of the second function are given only for 33° ≤ H ≤ 80° (the pages for 15° ≤ H ≤ 32° are missing from the manuscript), and these relate to timekeeping by day to a specific latitude, whereas the values of the first function are intended to serve any latitude. The entries for the third and fourth functions are independent of H and the same values are tabulated for each value of H. The entries for the fifth function are independent of h, and only one value is tabulated for each value of H. The value of ϕ underlying the table is about 33;30°, which would serve either Baghdad or Damascus. A smaller table (fols. 26v-27r) displays functions:

---

manuscript to Ḥabash is not certain, since it is not contained in MS Istanbul Yeni Cami 784 of his *Zīj* (*ibid.*, no. 16, and Debarnot, "Zīj of Ḥabash"). See further **XI-5.0**.

37 See Neugebauer, "Studies in Byzantine Astronomy", pp. 11-12, and Jones, *Byzantine Astronomical Manual*, pp. 154-155.

38 On this formula see Pingree, "Indian Influence", pp. 121-122; Davidian, "Bīrūnī on the Time of Day", pp. 331-332 and 334; and now **II-1.4**, **III-1.3**, and **XI-1.2**.

39 *Paris Catalogue*, pp. 446-447.

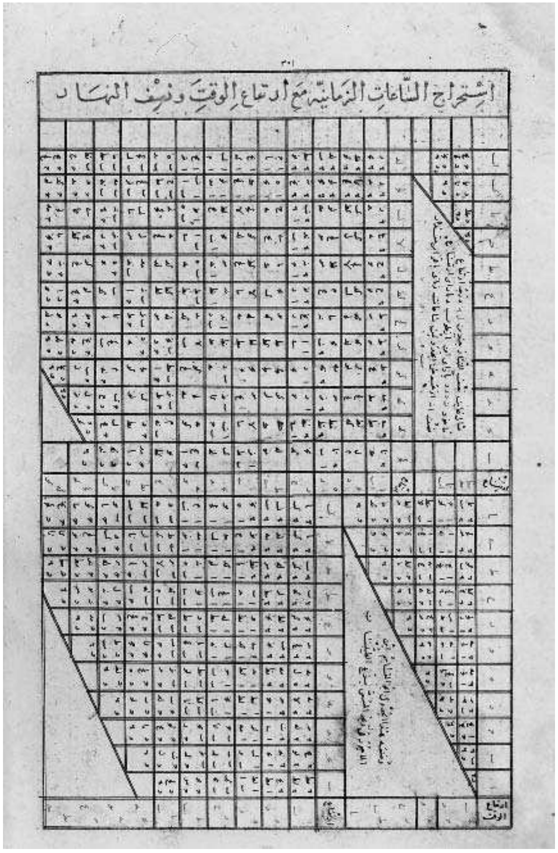


Fig. 2.5.1: Part of ʿAlī ibn Amājūr's *ṭaylasān* table. [From MS Paris BNF supp. pers. 1488, fol. 201v, courtesy of the Bibliothèque Nationale de France.]

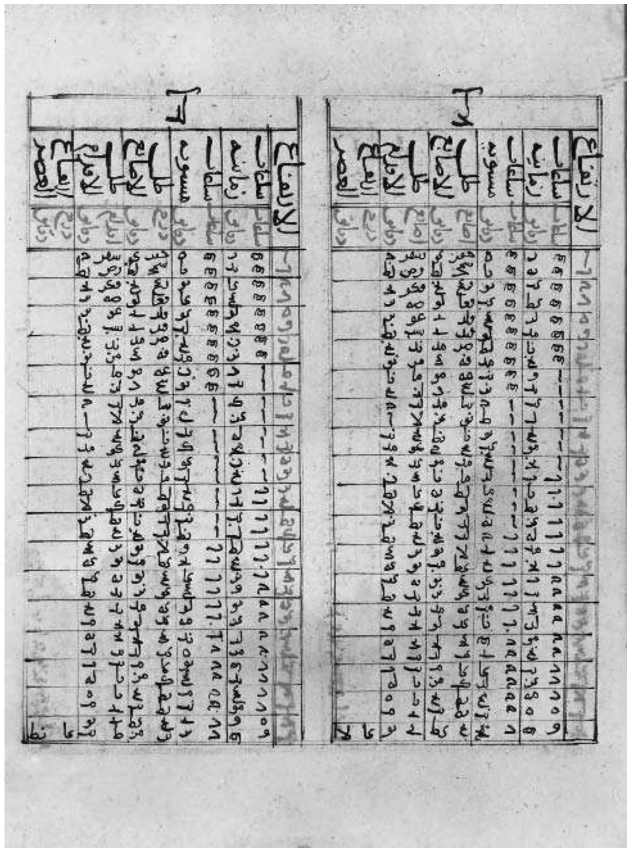


Fig. 2.5.2: The part of an anonymous *ṭaylasān* table serving solar meridian altitudes 33°-34°. [From MS Paris BNF ar. 2514, fol. 3r, courtesy of the Bibliothèque Nationale de France.]

$$f_1(h),\ f_2(h),\ \mathrm{Cot}_{12}\ h,\ \mathrm{Cot}_{6;40}\ h,\ h_a(H) \text{ and } \mathrm{Sin}\ h$$

to two digits for each degree of h (*al-irtifāʿ*). The first two functions are labelled *sāʿāt muʿwajja*, seasonal hours, and *sāʿāt mustawiya*, equinoctial hours, and appear to display the time since sunrise in hours and minutes for altitude h when the meridian altitude is 90°. The remainder of the tables in this manuscript relate to timekeeping to the stars (**3.2.1**).

2.5.3 Anonymous (Yemen)

The anonymous Yemeni *Zīj* preserved in MS Paris BNF ar. 2523, which was compiled *ca.* 1375,[40] contains a table of T(H,h) (fols. 97v-100v). Entries are computed to two digits for the domains:

$$H = 46°, 47°, \ldots, 90°, \text{ and } h = 1°, 2°, \ldots, H\ .$$

Note that the table is adequate for use in the Yemen where the minimum value of H for the sun is about 50°. See also **4.3.3** for a related table preserved in the same manuscript.

2.6 Tables of the time since rising of the sun or fixed stars as a function of their altitude, for all latitudes

To prepare a table which will give the time since rising of a celestial body for all latitudes, one can either tabulate T(H,h) for each degree of latitude or generate the function T in terms of h and pairs of arguments such as (δ,ϕ), (δ,d) or (H,D). The last pair has the advantage that – in modern terms – both arguments are always positive.

2.6.1 Najm al-Dīn al-Miṣrī (Cairo)

The apparently unique sources MSS Cairo MM 132 and Oxford Marsh 676 (Uri 944 = 995), now preserved in Cairo and Oxford, are the two halves of a single manuscript copied *ca*. 1325 which contains a table for timekeeping serving all latitudes. With a total of over 420,000 entries this is the largest table known from the medieval period.

A note on the flyleaf of the Oxford manuscript attributes the tables to Najm al-Dīn Abū ʿAbdallāh Muḥammad ibn Muḥammad al-Miṣrī, an Egyptian astronomer of the early 14th century before the 1970s virtually unknown to the scholarly literature. Thanks to the labours of François Charette, he is now well established as one of the most important astronomers of the Mamluk period.[41] No compiler is mentioned in the Cairo manuscript, but the title page is missing. Najm al-Dīn also wrote a treatise on spherical astronomy, extant in MS Milan Ambrosiana 227a (C49), fols. 85v-97r, copied in 1386, and the tables and treatises are clearly the work of the same person. It may be that the tables in MS Cairo MM 72, which were copied in 747 H [= 1346/47], are also by Najm al-Dīn (**4.4.1**).

The main function tabulated is:

$$T(H,h,D)$$

for the domains:

$$H = 2°, 3°, \ldots , 73° \text{ (Cairo MS) and } 74°, 75°, \ldots , 90° \text{ (Oxford MS) },$$
$$h = 1°, 2°, \ldots , (H-1°) \quad \text{and} \quad D = D^*(H), [D^*(H)+1°], \ldots , 180° .$$

For each value of H there are several pages of tables, in which the arguments h and D are entered horizontally and vertically, respectively. The lower limit D* for D is discussed below. **Fig. 2.6.1** shows an extract from the Cairo manuscript.

Clearly the two arguments H and D depend on the local latitude ϕ and the declination of the celestial body in question. There is no table of δ(λ) with which one could find H(λ) for the sun, but the work does contain a star table giving the equatorial coordinates of about 370 stars (fols. 148v-151r of the Oxford MS). On the other hand, tables of D(ϕ,λ) for each degree of ϕ from 1° to 90° and each degree of λ are provided (fols. 123r-142v of the Oxford manuscript), as well as tables of d(ϕ,Δ) (fols. 152r-166v) for each degree of ϕ and each degree of Δ from 1° to $\bar{\phi}$ (see further **7.0**). With such tables the appropriate arguments H and D can be found very easily for given ϕ and δ or Δ. The main table then gives immediately the time

---

[40] On this *zīj* see King, *Astronomy in Yemen*, no. 17.

[41] On Najm al-Dīn al-Miṣrī (Suter, *MAA*, no. 460, where two different individuals with this name are listed together; *Cairo ENL Survey*, no. C16; King, "Astronomy of the Mamluks", pp. 540-541) see now Charette, "Najm al-Dīn's Monumental Table", and *idem*, *Mamluk Instrumentation*. I have not updated my text to incorporate Charette's findings.

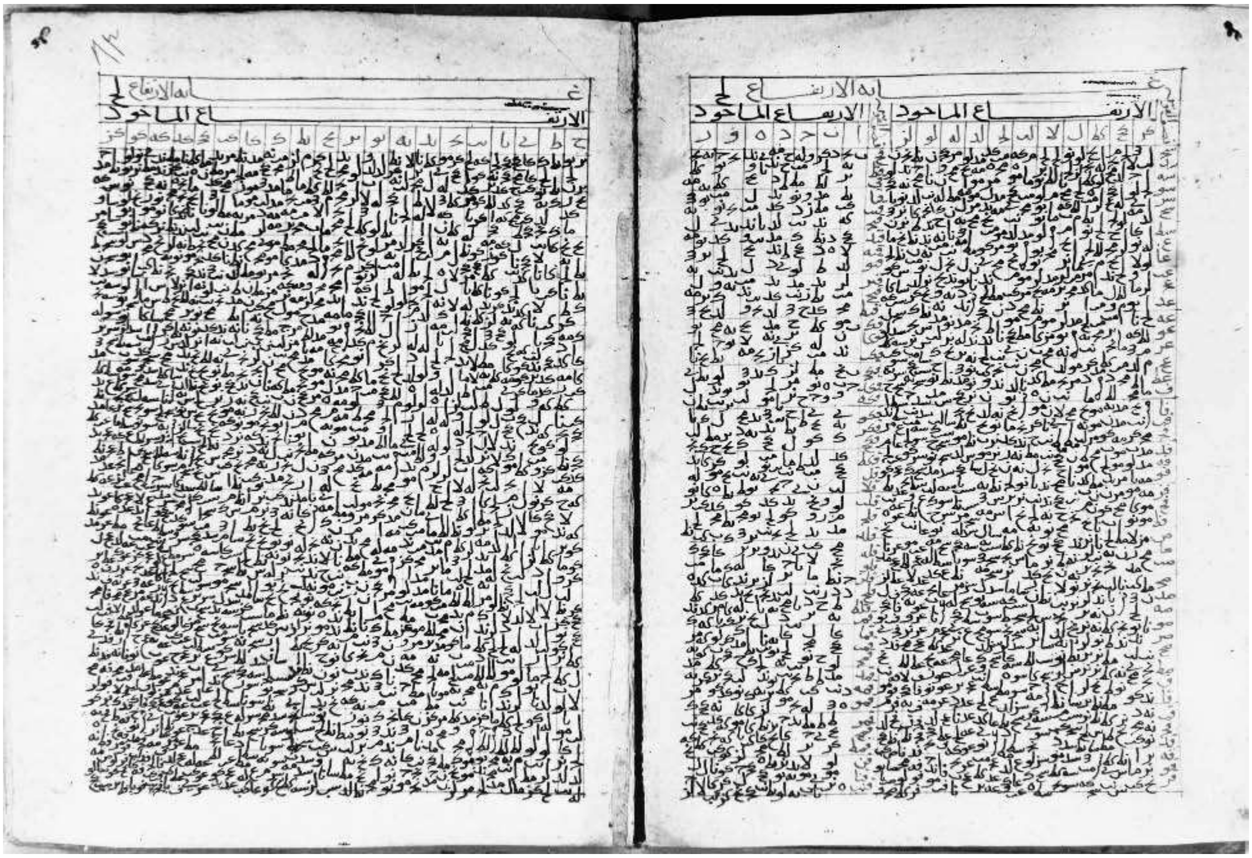


Fig. 2.6.1: An extract from Najm al-Dīn's monumental table serving meridian altitude 38°. Values are shown for altitudes 27°-37° and half arc of visibility 63°-87°, and then altitudes 1°-27° and half arc 108°-152°. [From MS Cairo MM 132, fols. 82v-83r, courtesy of the Egyptian National Library.]

since sunrise or since the rising of the star in question. For given H, it is not difficult to show that D varies between a certain minimum, defined by $\phi = -\delta = {}^1/_2\ \bar{H}$, and 180°. The lower limit D* for D in each table is simply the smallest integer greater than this minimum, *i.e.*:

$$D^*(H) = [\min D(H,\phi)] + 1\ .$$

The compilation of the table is in fact extremely straightforward. The formula for finding T (= D - t) from H, h and D outlined in Ch. 12 of Najm al-Dīn's treatise on spherical astronomy (on which see **II-2.5**) is the following (*cf.* **F8** and **F10**):

$$\text{Vers } t(H,h,D) = \text{Vers } D - \text{Sin } h \bullet \text{Vers } D \,/\, \text{Sin } H\ .$$

Note that this is the equivalent to:

$$\text{Vers } t = \text{Vers } D\ \{\ 1\ -\ \text{Sin } h \,/\, \text{Sin } H\ \}\ ,$$

and that if one had first compiled a table of the function:

$$H''(H,h) = 1\ -\ \text{Sin } h \,/\, \text{Sin } H\ ,$$

the table of t(H,h,D) could be generated with considerable facility. Ibn al-Mushrif (**6.13.1**) tabulated the related function:

$$H'(H,h) = \text{Sin } H - \text{Sin } h\ ,$$

but no Islamic tables of H″ have been located.

In a sample of 100 random entries in Najm al-Dīn's main table about 55 showed errors of ±3 or less in the second digit, 35 showed errors between ±4 and ±9, and there were 10 errors between ±10 and ±15. It should be borne in mind that 0;15° corresponds to one minute of time, and I make bold to say that Najm al-Dīn's table shows the time for any solar or stellar altitude with *the vast majority of entries accurate to the nearest minute*.

Only in 1982 with the discovery of an introduction by him in MS Dublin CB Persian 102,1, did it become clear that Najm al-Dīn also intended this table as a universal auxiliary table: see further **9.3***. For a complete discussion of all aspects of this table, the reader is referred to the publications of François Charette.

2.7 Tables of the time of night as a function of the normed right ascensions of culminating stars and the solar longitude, for a specific latitude

2.7.1 Shihāb al-Dīn al-Ḥalabī: Damascus

MS Cairo K 8525 is a copy of part of al-Khalīlī's corpus of tables for Damascus (**2.1.4**) in the hand of the Damascus astronomer Shihāb al-Dīn al-Ḥalabī (d. 1455).[42] Certain additional tables are stated to have been computed by al-Ḥalabī himself, including a table of the oblique ascensions of the ascendant at daybreak (based on a value 20° for the solar depression rather than al-Khalīlī's 19°), and a complete set of azimuth tables (**5.1.2**), as well as a table (fols. 14r-20r) displaying the time remaining until daybreak as a function of solar longitude for seven culminating stars which vary for each zodiacal sign – see **Fig. 2.7.1** for an extract. Values are given in equatorial degrees and minutes and are based on the relation:

$$f(\lambda) = \alpha' - \alpha_r(\lambda) \ ,$$

where α' is the normed oblique ascension of the particular star (= $\alpha_\phi(\lambda_H)$, where λ_H is the longitude of the horoscopus when the star is culminating) and $\alpha_r(\lambda)$ is the "ascensions at daybreak" (= $\alpha_\phi(\lambda_H)$, where λ_H is the longitude of the horoscopus at daybreak). These values are easily derived from the entries for α' in the star catalogue in MS Cairo K 8525 and al-Ḥalabī's table of $\alpha_r(\lambda)$ in the same manuscript, which is derived from al-Khalīlī's tables of $\alpha_\phi(\lambda)$ and $T(h,\lambda)$ for Damascus using $\alpha_r(\lambda) = \alpha_\phi(\lambda) - T(h,\lambda^*)$ with $h_r = -20°$.

2.7.2 Muḥammad ibn Kātib Sinān: Istanbul

MSS Istanbul Ayasofya 2710 and Istanbul Topkapı T 3046 (Ahmet III 3515) are the only known copies of an extensive set of tables for timekeeping by the stars by the late-15th-/ early-16th-century Ottoman astronomer Muḥammad ibn (Kātib) Sinān.[43] According to the introduction, the compiler was a *muwaqqit* at *al-ʿataba al-ʿulya 'l-sulṭāniyya*, "the Sublime Porte", in Istanbul. The work is entitled *Mīzān al-kawākib*, "Balance of the Stars", and is dedicated to the Sultan Sulaymān ibn Sālim (*reg*. 1520-1566).

The main tables are preceded by a star catalogue displaying the declination and normed right ascension ($\alpha' = \alpha + 90°$) for about 510 stars, apparently for Ramaḍān, 916 H [= December,

---

[42] On al-Ḥalabī (Suter, *MAA*, no. 434) see *Cairo ENL Survey*, no. C69.

[43] On Muḥammad ibn Kātib Sinān (Suter, *MAA*, no. 455) see *Cairo ENL Survey*, no. H8; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 84-90, no. 46.

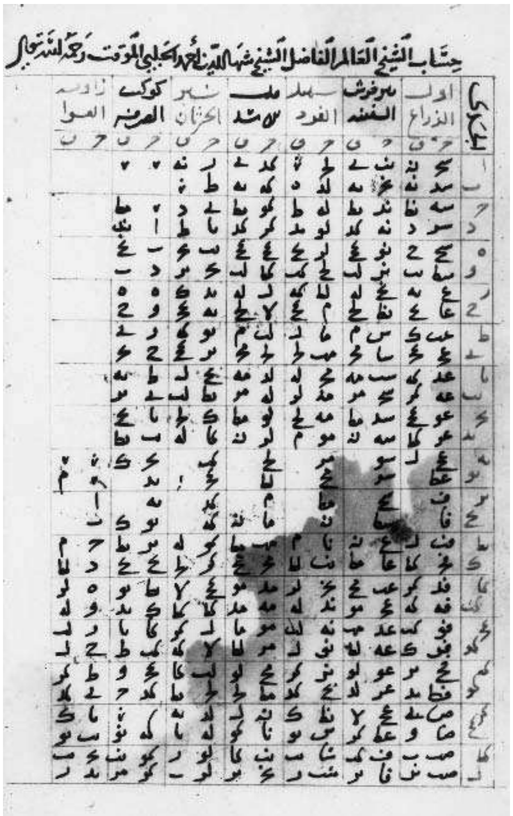


Fig. 2.7.1: The part of al-Ḥalabī's table serving solar longitudes in each degree of Capricorn and Aquarius. [From MS Cairo K 8525, fols. 14v-15r, courtesy of the Egyptian National Library.]

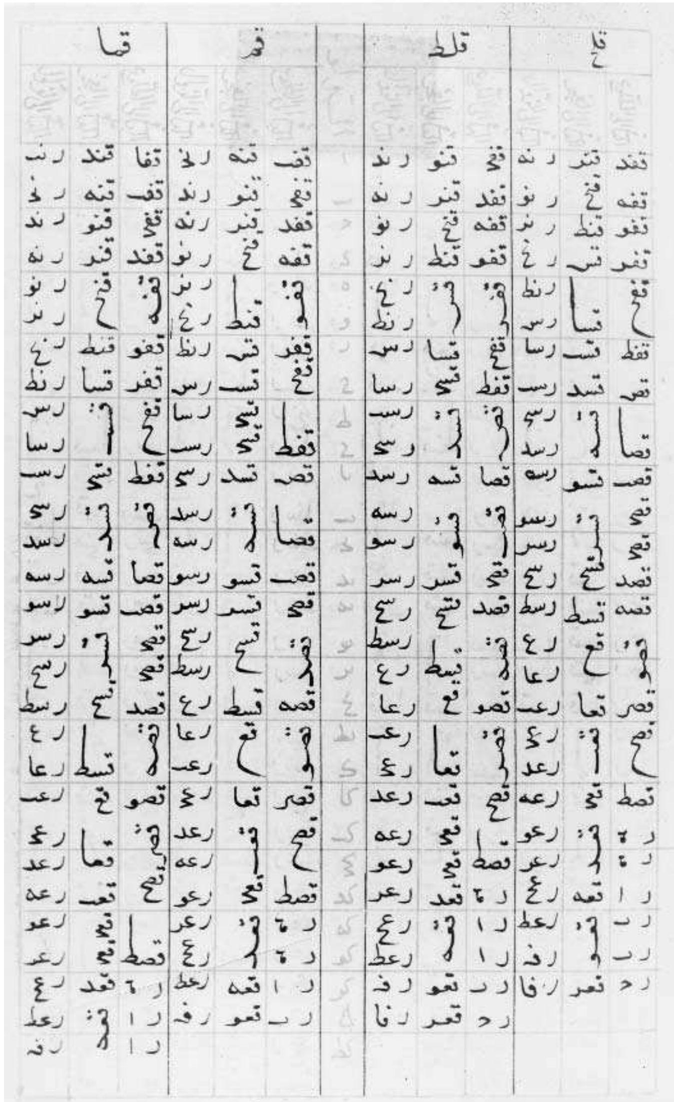


Fig. 2.7.2: The section of Ibn Kātib Sinān's enormous table showing the time of night for each degree of the ascensions of culminating stars from 138° to 141°, entered horizontally, for each degree of solar longitude in Aquarius, entered vertically. [From MS Istanbul Topkapı T 3046 (Ahmet III 3515), courtesy of the Topkapı Library.]

1510]. All of the right ascensions have been increased by 2;0° in the margin of the star catalogue. The main tables themselves occupy about 500 pages in each of the manuscripts and contain about 240,000 entries. The horizontal argument is intended to be the normed right ascension α', and the vertical argument is the solar longitude λ; the argument increments are 1° for both. Several dozen consecutive pages serve each 30° of solar longitude. Four functions are tabulated side by side for each pair of arguments:

t_1 the time since sunset,

t_2 the time remaining until sunrise,

t_3 the time remaining until daybreak, and
t_4 the time remaining until midday.

Entries are given in equatorial degrees to one digit only. See the extract in **Fig. 2.7.2**. To use the table one simply feeds in the normed right ascensions of the star which is culminating instantaneously and the solar longitude and reads off the appropriate time. The four functions tabulated are defined by the simple relations:

$$t_1(\alpha',\lambda) = \alpha' - \alpha_\phi(\lambda)$$
$$t_2(\alpha',\lambda) = 2N(\lambda) - t_1$$
$$t_3(\alpha',\lambda) = t_2 - r(\lambda)$$
$$t_4(\alpha',\lambda) = t_2 + D(\lambda)$$

where r is the duration of morning twilight. For each zodiacal sign (I-XII) of solar longitude, an appropriate range of arguments is given for the ascensions, namely:

I: 204°-259°; II: 235-20; III: 274-47; IV: 315-84; V: 330-115; VI: 5-140;
VII: 5-168; VIII: 33-196; IX: 50-249; X: 78-293; XI: 100-321; XII: 190-345

The entries in the tables correspond roughly to the latitude $\phi = 41;0°$ (Istanbul), and in MS Istanbul Ayasofya 2708 of Muḥammad ibn Kātib Sinān's treatise on timekeeping he uses parameters 20° and 16° for morning and evening twilight, although in MS Istanbul Ayasofya 2590 of his Turkish introduction to al-Khalīlī's universal auxiliary tables (**9.5**), written in 897 H [= 1491], he uses al-Khalīlī's parameters 19° and 17°. I have not investigated the accuracy of the tables.

2.8 Tables of the time of night as a function of the longitude of the horoscopus, for a specific latitude

The star *al-kaff al-khaḍīb* (= β Cassiopeiae)[44] was used by Ottoman and Safavid astronomers for timekeeping by night because its right ascension was close to zero, that is, it culminated with the vernal equinox. These tables represent a special case of the theory underlying those treated in **2.7**.

2.8.1 Anonymous: Istanbul

MS Cairo ZK 782,5 (fols. 25v-28r), copied *ca*. 1700, contains an isolated table of a function for timekeeping, which according to information in the margin is computed for the star *al-kaff al-khaḍīb* using coordinates $\alpha' = 86;50°$ and $\Delta = 56;37°$ for the year 950 H [= 1543/44] – see the extract in **Fig. 2.8.1**. The accompanying instructions are in Turkish. The tabulated function is computed to two digits for each degree of ecliptic longitude. It displays discontinuities at arguments 106° and 286° and in the three argument domains 0°-105°, 106°-285° and 286°-360° is labelled *al-māḍī mina 'l-ṭulūʿ*, "time since rising", *al-māḍī mina 'l-ghurūb*, "time since setting", and again *al-māḍī mina 'l-ṭulūʿ*. The function tabulated is simply:

$$f(\lambda) = \alpha' - \alpha_\phi(\lambda) \quad \text{for } \lambda < 106° \text{ and } \lambda > 286° \quad \text{or}$$
$$\alpha' - [\alpha_\phi(\lambda) - 180°] \text{ for } 106° \leq \lambda \leq 285° ,$$

for the latitude of Istanbul, taken as 41°.

---

[44] See Kunitzsch, *Sternnomenklatur der Araber*, p. 72, nos. 136a-c. This star was considered as one of the hands associated with the female face identified as *al-thurayyā*, the Pleiades (*ibid*., pp. 114-115, no. 306).

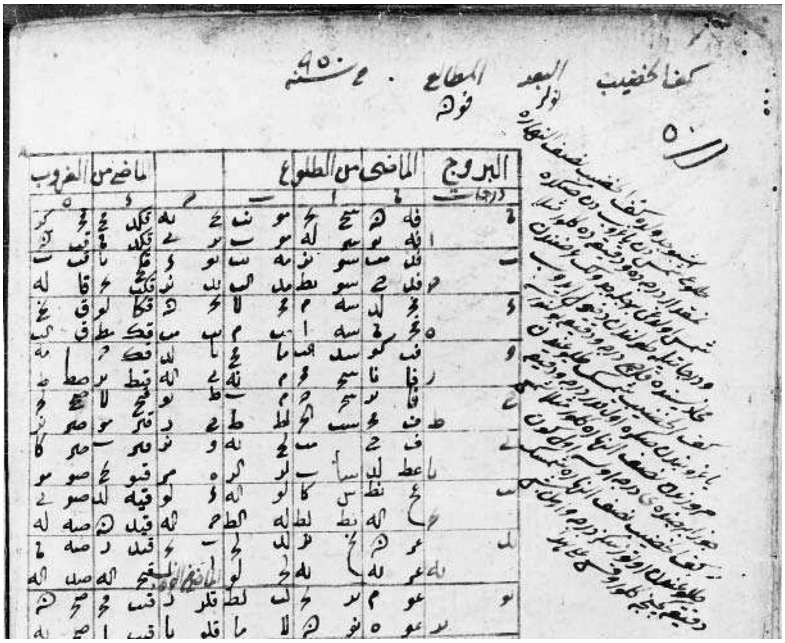


Fig. 2.8.1: An extract from a table for timekeeping by the star *al-kaff al-khaḍīb*, chosen because of its fortuitous position relative to the vernal equinox in the 16th century. [From MS Cairo ZK 782,5, fols. 25v-26r, courtesy of the Egyptian National Library.]

2.8.2 Muḥammad Zamān Mashhadī: Meshed

MS Cairo ṬFF 14 (111 fols.), penned *ca*. 1700, is a unique copy of a Persian astronomical work entitled *Tuḥfa-yi Sulaymānī* [*sic*] compiled by the astronomer and instrument-maker Muḥammad Zamān ibn Sharaf al-Dīn al-Mashhadī.[45] The treatise was compiled in 1078 H [= 1667/68] and it contains some spherical astronomical tables computed for the latitude of Meshed, taken as 37°. On fol. 42v there is a table entitled *jadwal wuṣūl kaff al-khaḍīb bi-niṣf al-nahār li-ṭūl Mashhad-i muqaddas bi-taʾrīkh gh-ʿ-ḥ*, "table of the culmination of *al-kaff al-khaḍīb* for the longitude of Meshed in the year 1078 (H)". Entries are given in hours and minutes for each degree of solar longitude. In the star catalogue on fol. 102r the right ascension of the star is given as 359;0°. See **II-3.14** on the prayer-tables in this work.

2.8.3 Anonymous: unspecified locality

In MS Aleppo Awqāf 948, p. 20, copied about 1800, there is a similar table following a larger set of tables of the astrological houses for an unspecified latitude,[46] probably that of Aleppo, where the manuscript is now preserved.

---

45 On Muḥammad Zamān al-Mashhadī (**I-2.8.2**) see Mayer, *Islamic Astrolabists*, pp. 78-79 and 87; *Cairo ENL Survey*, no. G76; and King, *Mecca-Centred World-Maps*, pp. 172-175.

46 See n. 3:2.

CHAPTER 3

TABLES OF THE LONGITUDE OF THE HOROSCOPUS AS A FUNCTION OF SOLAR AND STELLAR ALTITUDE

3.0 Introductory remarks

Tables displaying the longitude of the horoscopus are related to the categories described in **Ch. 2** by virtue of the simple formulae (*cf.* **F20**):

$$\alpha_\phi\{\lambda_H(h,\lambda)\} = \alpha_\phi(\lambda) + T(h,\lambda)$$

for the sun and:

$$\alpha_\phi\{\lambda_H(h)\} = \alpha_\phi(\rho) + T(h)$$

for a fixed star. It should be remembered that tables of oblique ascensions $\alpha_\phi(\lambda)$ for different latitudes were standard equipment of the Muslim astronomers.[1]

The horoscopus is of prime importance in astrology as well as in timekeeping and the Muslim astronomers also compiled extensive tables displaying the longitudes of the astrological houses and the projection of the rays.[2] These tables are not discussed in the present study.[3] See **10.1** on a table displaying the function $\lambda_H(T,\lambda)$ for a particular latitude found in an early-15th-century manuscript of the *Alphonsine Tables* written in Germany.

3.1 Tables of the longitude of the horoscopus as a function of the instantaneous solar altitude and solar longitude, for a specific latitude

3.1.1 Abu 'l-ʿUqūl: Taiz

In the corpus of tables of Taiz by Abu 'l-ʿUqūl (**2.1.2**) there is a set displaying the function:

$$\lambda_H(h,\lambda)$$

for:

$$h = 1°, 2°, \ldots, 90° \text{ and } \lambda = 1°, 2°, \ldots, 360°$$

and based on parameters:

$$\phi = 13;40° \text{ (Taiz) and } \varepsilon = 23;35° .$$

Entries are given to degrees and minutes for both eastern and western altitudes. An extract is shown in **Fig. 3.1.1**. There are over 40,000 entries in the entire set, which is complete in both the Berlin and Milan copies. They were probably computed by Abu 'l-ʿUqūl using his tables of $T(h,\lambda)$ and the table of $\alpha_\phi(\lambda)$ for these parameters in the *Mukhtār Zīj* (MS London BL Or. 3624, fols. 178v-179r). With such tables at hand the compilation of a table of $\lambda_H(h,\lambda)$

---

[1] See the article "Maṭāliʿ" in EI_2.2

[2] On these concepts see now Hogendijk, "Mathematical Astrology".

[3] For examples of such tables see those of al-Majrīṭī (*fl. ca.* 1000 – see the article in *DSB* by Juan Vernet) published in Suter, *al-Khwārizmī*, pp. 195-229; analyzed in Kennedy & Krikorian-Preisler, "Projecting the Rays"; and explained in Hogendijk, "Tables for Casting the Rays".

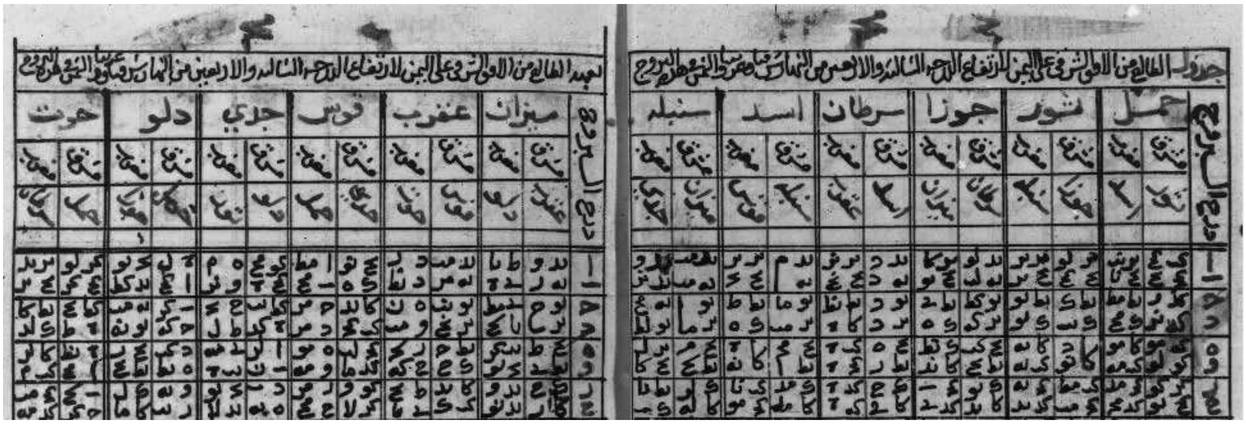


Fig. 3.1.1: The part of Abu 'l-'Uqūl's enormous table displaying the longitude of the horoscopus for solar altitude 43° in the east and west. [From MS Berlin Ahlwardt 5720, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

is tedious but very straightforward. Given also a table of inverse oblique ascensions $\lambda(\alpha_\phi)$, the compilation of the larger table is made even easier.

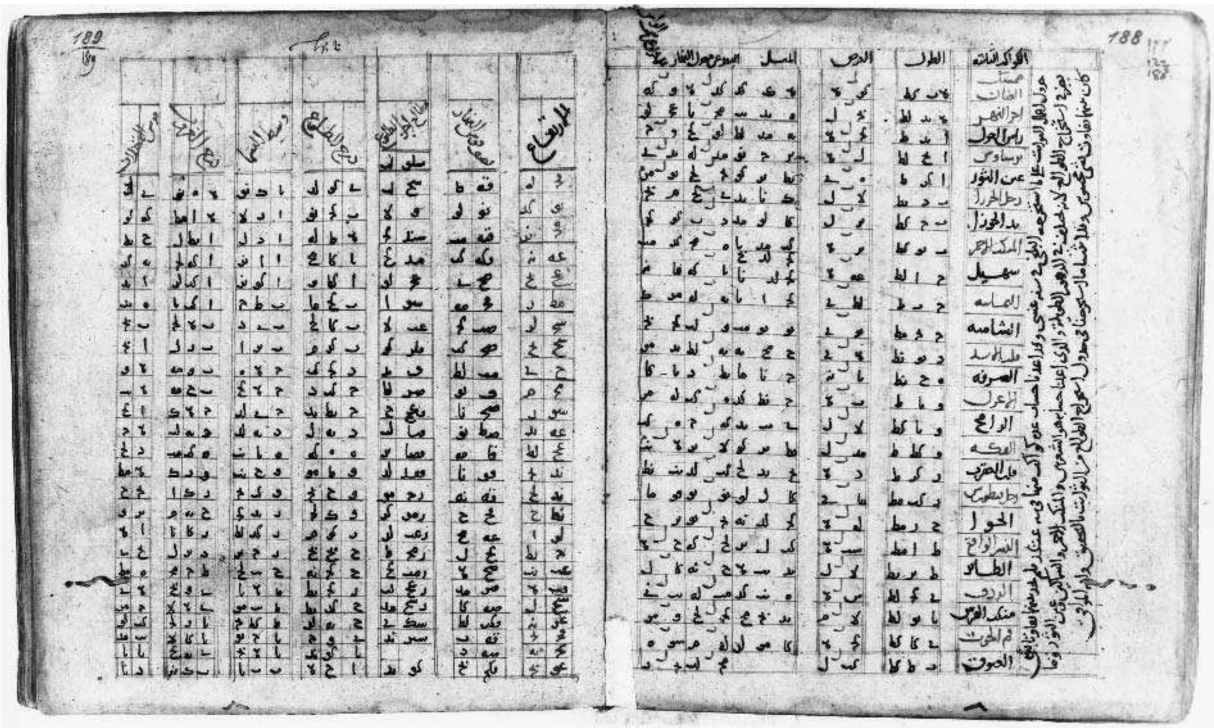
3.2 Tables of the longitude of the horoscopus as a function of stellar altitude, for a specific latitude

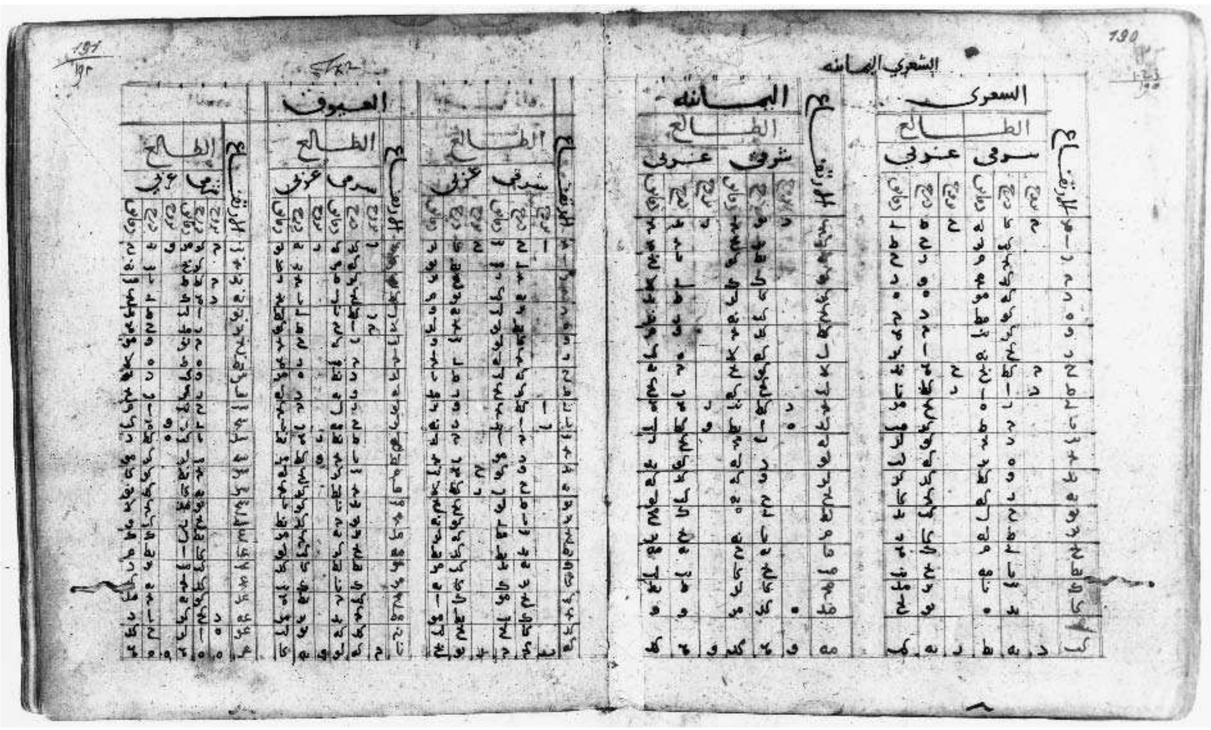
3.2.1 Anonymous: Qandahar

At the end of MS Berlin Ahlwardt 5751 (Mq. 101,1), copied *ca*. 1300, of the *Zīj* of Kūshyār ibn Labbān, who worked in Iran *ca*. 1000,[4] there are various tables of considerable historical interest that have been taken from other early Islamic sources. All of these tables merit detailed study. One such is a star catalogue (pp. 188-189) displaying values of twelve functions for 25 stars: see **Fig. 3.2.1a**. These functions are:

λ	*al-ṭūl*	longitude
β	*al-ʿarḍ*	latitude
$\delta_2(\lambda)$	*al-mayl*	second declination
Δ	*al-buʿd ʿan muʿaddil al-nahār*	declination
$\bar{H}$	*al-inḥirāf ʿan samt al-raʾs*	zenith distance when culminating
H	*al-irtifāʿ*	altitude when culminating
D	*niṣf qaws al-nahār*	half arc of visibility
$\alpha_\phi(\rho)$	*maṭāliʿ ajzāʾ al-ṭulūʿ*	ascensions of the co-ascendant
ρ	*daraj al-ṭulūʿ*	co-ascendant (degree of the ecliptic which rises with the star)
σ	*daraj al-ghurūb*	co-descendant

[4] On Kūshyār (Suter, *MAA*, no. 192) see Kennedy, "*Zīj* Survey", no. 9, and Sezgin, *GAS*, V, pp. 343-345, VI, pp. 246-249 and VII, pp. 182-183. On the theoretical treatment of spherical astronomy in his *Zīj* see Berggren, "Spherical Trigonometry in the *Jāmiʿ Zīj*".





Figs. 3.2.1a-b: The star catalogue with additional information for timekeeping by the stars (a), and the sub-tables for timekeeping by Sirius and Capella (b). [From MS Berlin Ahlwardt 5751, pp. 188-189 and 190-191, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

τ	*wasaṭ al-samā*ʾ	co-culminant (degree of the ecliptic which culminates with the star)
ζ	*qaws al-ikhtilāf*	unidentified function

The underlying latitude is found by inspection to be 30;25°. A note by the side of these tables states:

> "(This is) a table for operations (of timekeeping) with the fixed stars according to the results of al-Balkhī in the year 1310. We recomputed (the entries for) several stars for the year 1424 and did not find any difference which would affect the determination of the (co)ascendants, because they (?) do not vary over long periods. We recomputed values for Sirius and Procyon, Betelgeuze, Spica and Arcturus, and Aldebaran (*al-shiʿrayn wa-'l-mankib al-ayman wa-'l-simākayn wa-ʿayn al-thawr*),[5] and for these there was no perceptible difference. We used what we derived in the table for finding the ascendants accurately from the fixed stars. God grants success."

The dates 1310 and 1424, which are written in the *abjad* notation in the text, must relate to the calendar of Alexander, and thus correspond to 1000 and 1125 in the Julian calendar. Also from the table itself (the longitude of Regulus is 136;39°) a date of *ca*. 1050 is suggested.[6] At the time of writing this in the 1970s I was not aware of any astronomer named al-Balkhī who was active in Southern al-ʿIrāq or Iran *ca*. 1000, although ʿAlī ibn Aḥmad al-Nasawī compiled a *zīj* in central Iran in the early 11th century. The latitude 30;25° was used for Qandahar (now in Afghanistan) in a recension of the geographical tables of al-Khwārizmī (**7.1.1**),[7] and we can be certain that the table was intended for use there.

The next four pages in the Berlin manuscript (pp. 190-194) contain the only remaining part of these ascendant tables: see **Fig. 3.2.1b**. The function tabulated is $\lambda_H(h)$ for each degree of eastern and western altitudes for Sirius, Capella, Vega and Arcturus. Entries are given in zodiacal signs, degrees and minutes, and the underlying latitude appears to be 30;25°. Both star catalogue and ascendant tables seem to be unrelated to the auxiliary tables that immediately precede them (on pp. 188-189) in the Berlin manuscript (**6.16.1**), although these auxiliary tables do indeed serve to compute $\lambda_H(h)$ for different stars. See **5.8.1** on another table from this collection in the same manuscript. Both tables are featured again in **IX**.

3.2.2 Anonymous: Baghdad / Damascus

MS Paris BNF ar. 2514, fols. 28v-48v, (**2.5.2**), copied in the year 612 H [= 1215], contains some anonymous tables of the functions $\lambda_H(h)$ and $\alpha_\phi\{\lambda_H(h)\}$ computed for 21 stars. Values are given in signs, degrees and minutes for each degree of eastern and western altitude up to the maximum for each star: see **Fig. 3.2.2** for an extract. The underlying latitude is about 33;30°, which would serve either Baghdad or Damascus.

---

[5] On these six stars see Kunitzsch, *Sternnomenklatur der Araber*, pp. 111-112, nos. 289-290; p. 57, no. 158; p. 105, nos. 269-270; and p. 51, no. 69, respectively.

[6] For example Ibn Yūnus has Regulus at Leo 16° *ca*. 1000, and the rate of precession is 1° in *ca*. 70 years.

[7] Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. 261-262.

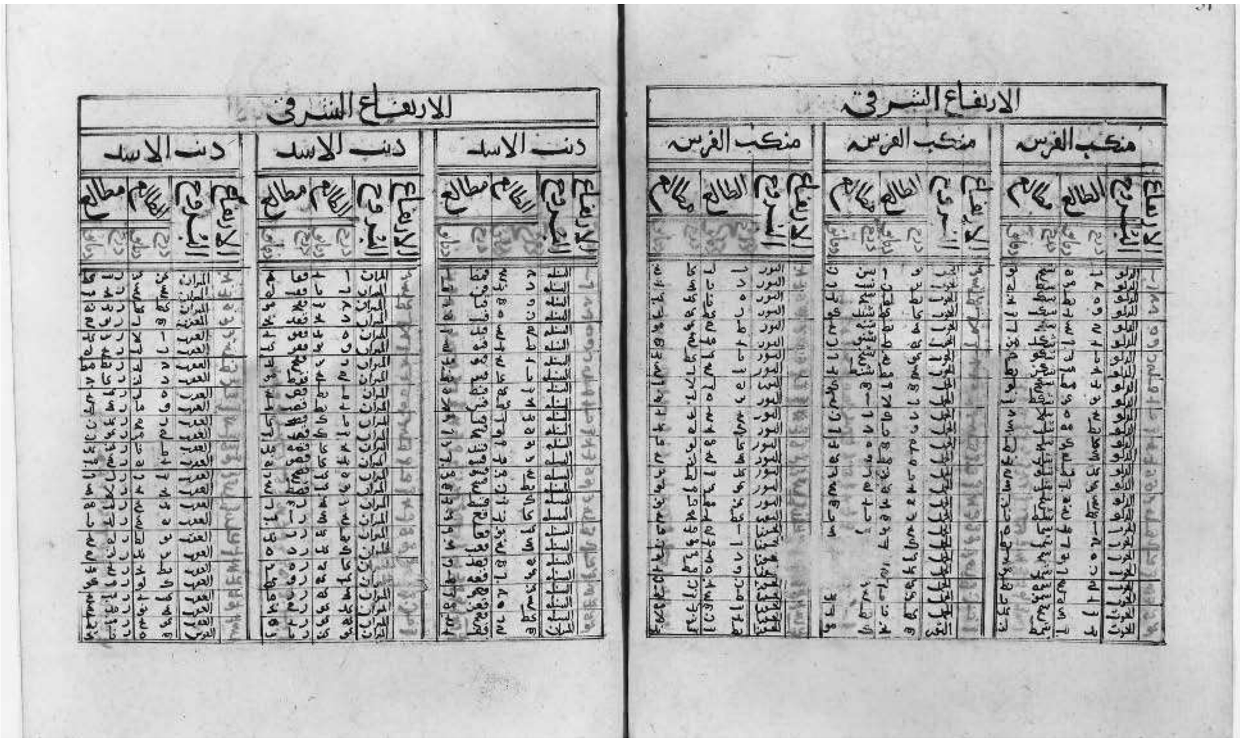


Fig. 3.2.2: An extract from the anonymous tables for timekeeping by the stars, serving *mankib al-faras* and *dhanab al-asad*. [From MS Paris BNF ar. 2514, fols. 31v-32r, courtesy of the Bibliothèque Nationale de France.]

3.2.3 Ibn Dāʿir: Sanaa

MS Berlin Ahlwardt 5720 of Abu 'l-ʿUqūl's corpus of tables for Taiz (**2.1.2**) contains three tables (fols. 121v, 124r-127r) computed for the latitude of Sanaa and taken from another source. The tables are all for timekeeping by the stars, and it is stated (fol. 121v) that the stellar coordinates were based on the observations of Ibn Yūnus, but the name of the compiler of the tables and the date for which he computed the stellar coordinates have been deliberately erased. However, it is just possible to read the name ʿAbdallāh ibn Ṣalāḥ Dāʿir,[8] which also appears elsewhere (fol. 121r) in reference to some instructions on how to use tables of stellar altitude at the seasonal hours. Ibn Dāʿir (d. 1013 [= 1604] is otherwise known as the author of a geographical treatise compiled for the Ottoman Sultan Murad III. See **4.5.1** on these altitude tables and **3.4.1** on the other tables for Sanaa in this set, and also **II-12.6**.

The first of the three tables (fols. 121v, 124r-126r), which is also the largest, displays the longitude of the horoscopus as a function of the altitude of 16 stars. Entries are given in signs, degrees and minutes and for each degree of altitude up to the maximum for each star, and there are two sub-tables for altitudes in the east and west.

---

[8] On Ibn Dāʿir see King, *Astronomy in Yemen*, no. 31, and now Ihsanoğlu *et al*., *Ottoman Geographical Literature*, I, pp. 76-78, no. 39. The word Dāʿir means "profligate": see the article "Zuʿʿār" by Thierry Bianquis in EI_2.

3.3 Tables of the longitude of the horoscopus as a function of time and solar longitude, for a specific latitude

3.3.1 al-Sulṭān al-Ashraf: Sanaa

MS Oxford Hunt. 233 is an apparently unique copy of a monumental work on astrology compiled by the Yemeni Sultan al-Ashraf (*fl. ca.* 1295).[9] The work contains considerable material on timekeeping, including several tables. One of the latter (fols. 63v-69r) displays the functions:

$$H(\lambda) \text{ and } h_a(\lambda) \ ,$$

that is, the solar altitudes at midday and at the beginning of the afternoon prayer (when $z_a = Z + n$). Values are given to two digits and are based on the parameters:

$$\phi = 14;30° \text{ (Sanaa) and } \varepsilon = 24° \ .$$

Note the use of the Indian value of ε! Alongside these values are also displayed the corresponding values of:

$$\lambda_H(\lambda)$$

at both these times. Values are given in zodiacal signs, degrees and minutes – see **Fig. 3.3.1a**. These tables are based on the formulae:

$$\alpha_\phi\{\lambda_H(\lambda)\} = \alpha_\phi(\lambda) + D(\lambda) \quad \text{or} \quad \alpha_\phi(\lambda) + D(\lambda) + t_a(\lambda) \ ,$$

where $D(\lambda)$ and $t_a(\lambda)$ are the half diurnal arc and hour-angle at the beginning of the afternoon prayer. No tables of $\alpha_\phi(\lambda)$, $D(\lambda)$, or $t_a(\lambda)$ based on the above parameters are contained in al-Ashraf's treatise or in any other known Yemeni source.

Elsewhere in al-Ashraf's treatise (fols. 80v-92r) there are tables of the function:

$$\lambda_H(T,\lambda) \ ,$$

with values expressed in zodiacal signs, degrees and minutes for the domains:

$$T = 1, 2, \ldots ,12^{sdh} \text{ and } \lambda = 1°, 2°, \ldots , 360° \ .$$

The function is called *ṭāliʿ sāʿāt (al-)nahār*, "the ascendant of the hours of daylight" – see **Fig. 3.1.1b**.

See **4.2.3** on another table in MS Oxford Hunt. 233 and **4.1.4** and **5.4.4** on tables in a second work by al-Ashraf, and also **II-12.2**.

3.3.2 Abu 'l-ʿUqūl: Taiz

In the 14th-century Yemeni miscellany preserved in a manuscript in a private collection in Sanaa (**2.1.2**) there is an incomplete set of tables of the function:

$$\lambda_H(T,\lambda)$$

computed to signs, degrees and minutes, for the domains:

$$T = 1, 2, \ldots , 12^{sdh} \text{ and } \lambda = 1°, 2°, \ldots , 360°.$$

The underlying parameters are:

$$\phi = 13;37° \text{ (Taiz) and } \varepsilon = 23;35° \ .$$

---

[9] On the astronomical works of al-Ashraf (Suter, *MAA*, no. 394 and Mayer, *Islamic Astrolabists*, pp. 83-84) see King, *Astronomy in Yemen*, no. 8.

Fig. 3.3.1a: An extract from al-Ashraf's tables giving the altitude of the sun at midday and the beginning of the ʿ*aṣr* and the corresponding ascendant. This sub-table serves solar longitudes in Aries. [From MS Oxford Hunt. 233, fol. 63v, courtesy of the Bodleian Library.]

Fig. 3.3.1b: An extract from al-Ashraf's tables showing the ascendants at the hours as a function of solar longitude, here Gemini. The same entries are for the night hours for solar longitudes in Sagittarius. [From MS Oxford Hunt. 233, fol. 82v, courtesy of the Bodleian Library.]

An extract is shown in **Fig. 3.3.2**. The tables are unattributed, but the smaller table described in **4.2.5** is based on the same distinctive latitude and is attributed to Abu 'l-ʿUqūl. It seems that this latitude for Taiz was derived by Abu 'l-ʿUqūl after he had computed his corpus of tables for latitude 13;40° (**2.1.2**). A table of $\alpha_\phi(\lambda)$ for the same parameters is preserved in the anonymous late-14th-century Yemeni *zīj* MS Paris BNF ar. 2523, fol. 95v (**2.5.3**). Likewise the tables for marking astrolabes and sundials for Taiz compiled by the Rasulid Sultan al-Ashraf about 1295 which are extant in MS Cairo TR 105 (**4.1.4**) are also based on ϕ = 13;37°, but the obliquity used is the *Īlkhānī* value 23;30° (**2.3.2**).

The function $\lambda_H(T,\lambda)$ is called *ṭawāliʿ al-sāʿāt al-zamāniyya*, "the ascendants of the seasonal

Fig. 3.3.2: An extract from the anonymous tables of the horoscopus at the hours of day and night for the latitude of Taiz. [From a manuscript in a private library in Sanaa; taken from the facsimile in Varisco & Smith, eds., *al-Afḍal's Anthology*, p. 114, with permission of Professor Daniel Varisco.]

hours", and the existence of a similar table in a 15th-century German manuscript has been noted in **3.0** (see also **10.1**). Implicit in the headings of the Yemeni table is the fact that the horoscopus is the same at the nth seasonal hour of the day for solar longitude λ as it is at the nth seasonal hour of the night for solar longitude $\lambda^* = \lambda + 180°$. A given sub-table on one page serves one zodiacal sign, but there are only five sub-tables in the manuscript now. The compilation of such a table is quite straightforward, since if $T = n^{sdh}$,

$$\alpha_\phi\{\lambda_H(T,\lambda)\} \;=\; \alpha_\phi(\lambda) + n/12 \bullet \{2D(\lambda,\phi)\} \;.$$

3.3.3 Anonymous: Anatolia

MS Istanbul Nuruosmaniye 2782, completed in Sivas in 774 H [= 1371/72], contains an anonymous set of calendrical and astrological tables perhaps of Seljuk origin. MS Cambridge Browne O.1, copied *ca*. 1400, is the only known copy of the *Zīj-i mufrad* by the 11th(?)-century astronomer Muḥammad ibn Ayyūb al-Ṭabarī. Both of these contain a remarkable table (fols. 53v-54r of the Nuruosmaniye manuscript and fol. 179r of the Cambridge one); the captions

are in Persian in the former and in Arabic in the latter (see **Fig. IV-5.3**). The table displays the function:

$$\lambda_H(T_n,\lambda)$$

for the domains:

$$\lambda = 0°, 6°, \ldots , 354° \quad \text{and} \quad T_n(\lambda) = n/8 \bullet \{2D(\lambda)\} \ (n = 1, 2, \ldots , 7)$$

as well as at daybreak and nightfall. The functions are called simply *ṭāliʿ-i awqāt*, "the ascendant at different times of day". The underlying latitude is found by inspection to be about $\phi = 38°$, and it is not possible to determine the value used for ε; this latitude perhaps corresponds to Konya. Entries are given in signs and degrees and it is stated that they were derived using an astrolabe. The underlying formula is trivial, namely:

$$\alpha_\phi \{\lambda_H(T,\lambda)\} = \alpha_\phi(\lambda) + T_n(\lambda) \ .$$

Using an astrolabe fitted with a plate for latitude ϕ one sets the ecliptic longitude λ on the rete over the local horizon on the plate and can immediately read off $\alpha_\phi(\lambda)$ and $D(\lambda)$. One then rotates the rete by the amount $T_n(\lambda)$ in the direction of the daily rotation and simply reads off the longitude of the new point where the ecliptic intersects the horizon, which is $\lambda_H(T_n,\lambda)$. See also **II-14.2** and **IV-5.3** for further information on this unusual table.

3.4 Tables of the longitude of the horoscopus as a function of the time since rising of the stars, for a specific latitude

3.4.1 Ibn Dāʿir: Sanaa

One of the anonymous tables for the latitude of Sanaa in MS Berlin Ahlwardt 5720 (**3.2.3**) displays the longitude of the horoscopus for each "seasonal hour" of visibility of 18 fixed stars (fol. 127r) – see **Fig. 3.4.1**. These "hours" are simply twelfths of the arcs of visibility of each particular star. The compilation of such a table is trivial since, if λ_n be the longitude of the horoscopus at the n[th] "hour" and 2N the arc of visibility of a star whose co-ascendant is ρ, then:

$$\alpha_\phi(\lambda_n) = \alpha_\phi(\rho) + n/12 \bullet (2N) \ .$$

See also **4.6.1** below on a related table by Ibn Dāʿir, and further **II-12.6**.

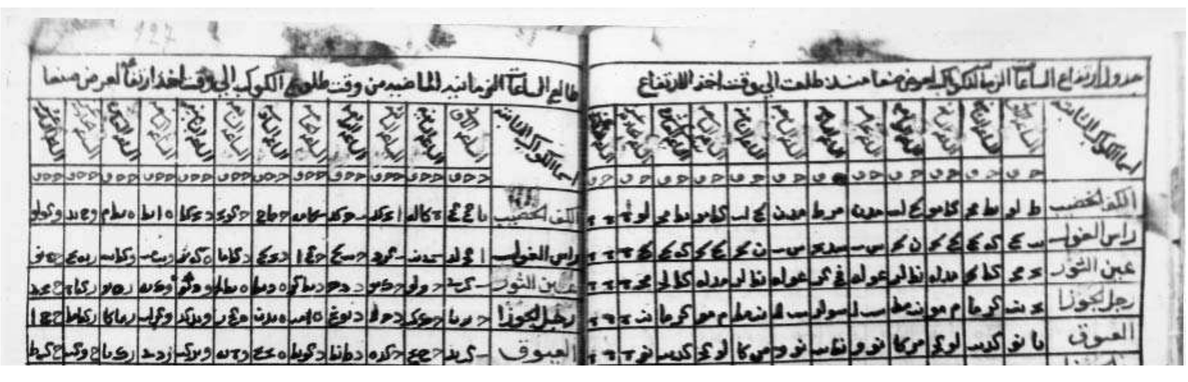


Fig. 3.4.1: The table by Ibn Dāʿir on the right displays the horoscopus at each "seasonal hour" of visibility for 18 stars. The one on the left shows the corresponding altitudes (**4.6.1**). [From MS Berlin Ahlwardt 5720, fols 126v-127r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

CHAPTER 4

TABLES OF SOLAR AND STELLAR ALTITUDE

4.0 Introductory remarks

The extensive stellar and solar altitude tables located in various Islamic sources are closely related to the tables described in **Chs. 2** and **3**. The numerous smaller solar altitude and azimuth tables for marking solstitial shadow traces on sundials belong to another category of Islamic astronomical astronomical tables which merit a separate study,[1] and I have intentionally limited examples of such altitude tables cited below to the tables of Ḥabash (Pseudo-al-Khwārizmī), al-Battānī, al-Marrākushī, al-Maqsī and the Yemeni Sultan al-Ashraf. Tables of solar meridian latitudes for specific latitudes were standard in *zīj*es, and already in the 9th century we find a small table of $H(\lambda,\phi)$ for each of the climates (see **Fig. VIa-1.1**).

4.1 Tables of solar altitude at the solstices as a function of time, for a specific latitude

Tables of the solar altitude for each seasonal hour at the solstices, and more especially the corresponding gnomon shadow lengths and azimuths, were used by Muslim astronomers for constructing the points of intersection of the hour lines with the solstitial shadow traces on horizontal sundials.[2]

4.1.1 Pseudo-al-Khwārizmī / Ḥabash: various latitudes

MS Istanbul Ayasofya 4830, fols. 231v-235r, copied in Damascus in 626 H [= 1228/29], contains a treatise on sundials attributed to the early-9th-century Baghdad astronomer al-Khwārizmī (**7.1.1**).[3] In fact, the treatise is more likely due to his late contemporary Ḥabash al-Ḥāsib (**9.1**).[4] The work consists mainly of tables for constructing horizontal sundials with markings for each seasonal hour. An extract is shown in **Fig. 4.1.1a**. A related set of tables, appended to the treatise on astrolabes and sundials by the late-10th-century scholar al-Sijzī,[5] is extant in MS Istanbul Topkapı A3342 (disordered, correctly: fols. 123r-153v then 114r-122v), copied also in Damascus in 634 H [= 1236/37]. An extract is shown in **Fig. 4.1.1b**. Each of these sources merits detailed study.

---

[1] See King, "Islamic Astronomical Tables", pp. 51-53, and *idem* & Samsó, Islamic Astronomical Tables", pp. 92-94, for preliminary discussions of the various kinds of tables used by the Muslim astronomers for marking curves on sundials, many of which are of considerable sophistication. A detailed survey of all extant Islamic sundial tables would be worthwhile.

[2] See my article "Mizwala" in EI_2, repr. in King, *Studies*, C-VIII, also **X-7**.

[3] On al-Khwārizmī see n. 7:5. On these tables see already King, "al-Khwārizmī", pp. 7-11. An edition of the tables and a Russian translation of the accompanying text is in Rosenfeld *et al*., eds., *Al-Khorezmi*, pp. 221-234.

[4] For the arguments see King, *Mecca-Centred World-Maps*, p. 350. In Charette, *Mamluk Instrumentation*, p. 181, on the other hand, it is argued that the tables are indeed by al-Khwārizmī.

[5] On al-Sijzī (Suter, *MAA*, no. 185) see Sezgin, *GAS*, V, pp. 329-334, VI, pp. 224-226, and VII, pp. 177-182.

Fig. 4.1.1a: On the right we see the sub-tables for latitudes 38° and 40° from the treatise on sundial construction attributed to al-Khwārizmī in the unique manuscript, but more probably due to Ḥabash. Part of the reason for the latter attribution is that many of Ḥabash's works include tables for Samarra, with latitude 34°, here seen on the left. See also **Fig. VIa-10.1**. [From MS Istanbul Ayasofya 4830, fol. 234r, courtesy of the Süleymaniye Library.]

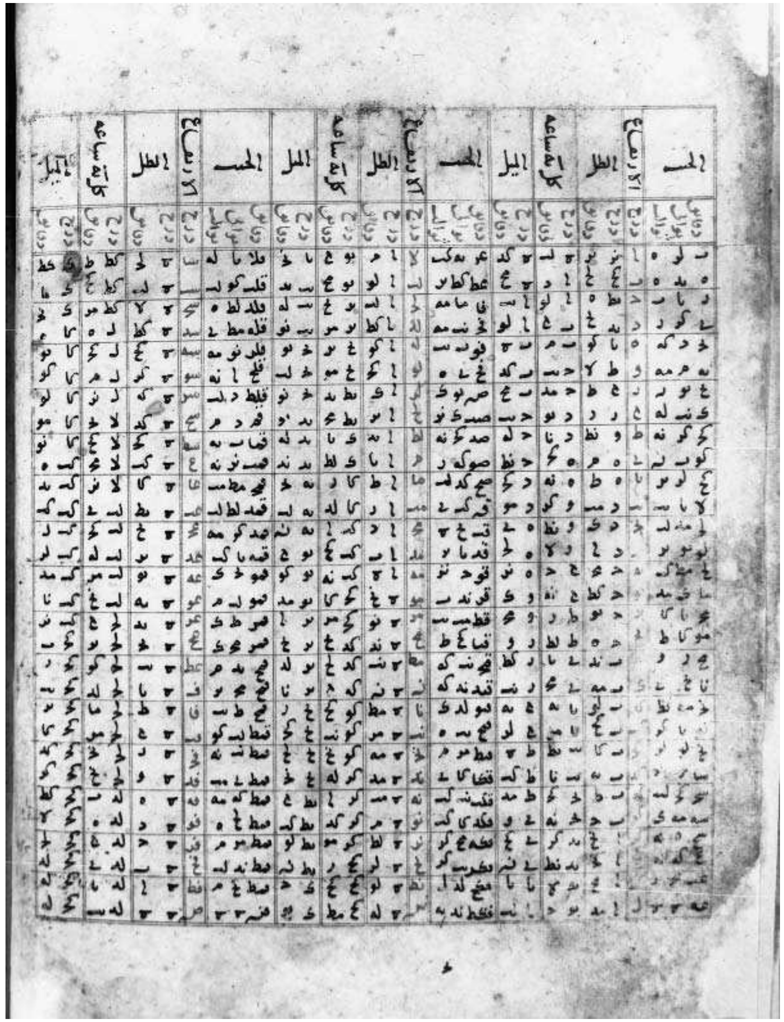



Fig. 4.1.1c: A set of basic tables probably to be associated with Ḥabash. Values are given of the sine to base 150 (*al-jayb*) and the cotangent to base 1 (*al-ẓill*), as well as the excess of daylight for an unspecified latitude – actually 34° – labelled "every 15° is one hour" (*kull y-h sāʿa*) and the solar declination (*al-mayl*). The argument labelled "altitude" (*al-irtifāʿ*) can serve all of the functions: a general arc for the sine, the solar altitude for the cotangent, and the solar longitude for the other two. [From MS Istanbul Ayasofya 4830, fol. 188v, courtesy of the Süleymaniye Library.]

Fig. 4.1.1b: Some sub-tables in the Topkapı manuscript, serving latitudes 21°, 28;30°, (33°, 40°, 35° and 30°). [From MS Istanbul Topkapı A3342, courtesy of the Topkapı Library.]

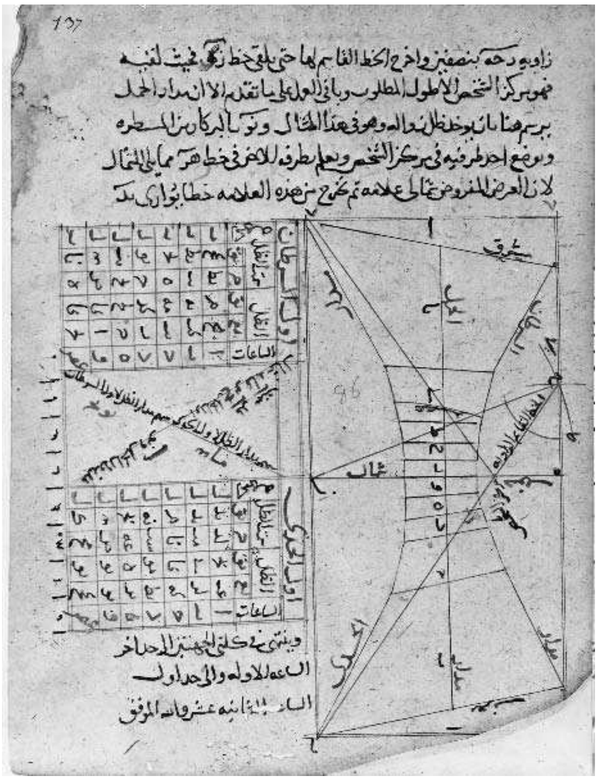


Fig. 4.1.3: The tables for constructing a horizontal sundial for latitude 30° in the treatise of Marrākushī. [From MS Paris BNF 2507, fol. 137r, courtesy of the Bibliothèque Nationale de Paris.]

Fig. 4.1.4: This extract from al-Ashraf's tables for sundial construction serves latitude 21°. [From MS Cairo TR 105, fols. 132v-133r, courtesy of the Egyptian National Library.]

The tables in the Ayasofya manuscript display the functions:

$$h(T),\ a(T) \text{ and } z_{(12)}(T),$$

giving values for arguments T = 1, 2, ... , 5^{sdh} at the summer and winter solstices for:

$$\phi = 0°, 15°, 18°, 20°, 24°, 27°, 30°, 33°, 34°, 35°, 38° \text{ and } 40°.$$

Entries are given to two digits and the underlying value of ε is 23;51°. For ϕ = 34° (Samarra) values are given for each $0;30^{sdh}$ and for ϕ = 33° (Baghdad) values of the second and third functions are given each $0;10^{sdh}$ in a separate table. The special attention paid to the latitude of Samarra is a further indication that the tables are by Ḥabash.[6]

The tables on fols. 119v-122v of the Topkapı manuscript are in some disorder. Those for:

$$\phi = 21°, 24°, 27°, 30°, 33°, 35°, 38° \text{ and } 40°$$

are based on ε = 23;51°, and those for:

$$\phi = 28;30° \text{ and } 36°$$

are based on ε = 23;35° and ε = 23;30°, respectively. Further investigation of the two manuscripts might establish whether any of these tables is due to al-Sijzī.

In the pages preceding the treatise attributed to al-Khwārizmī in the Ayasofya manuscript there are some other Abbasid treatises on astronomy containing a few simple tables of functions, such as (fol. 188v):

$$H(\lambda) \text{ for } \phi = 33;0° \text{ and } 34°\ ,$$

based on ε = 23;35°, called *al-mayl al-mumtaḥan*, by which may be meant "the obliquity of the *Mumtaḥan Zīj*" (**9.1**).[7] In any case, this is one of the two values used by Ḥabash (see again **9.1**).[8] Another set (fol. 188v – see **Fig. 4.1.1c**) displays the functions:

$$\mathrm{Sin}_{150}\ \theta,\ \mathrm{Cot}_1\ \theta,\ 2d(\lambda)\ (\phi = 34° \text{ and } \varepsilon = 23;35°), \text{ and } \delta(\lambda)\ (\varepsilon = 23;35°).$$

Values are given to two digits for each degree of argument.[9] The use of base 1 is remarkable.

Note that one of the tables discussed above displays h(T) for each $0;10^{sdh}$ of T for latitude 33° (Baghdad). In this connection it is worth mentioning that the 10^{th}-century bibliographer Ibn al-Nadīm attributes to Yaḥyā ibn Abī Manṣūr,[10] a leading astronomer of early-9^{th}-century Baghdad, a work entitled *Maqāla fī ʿamal irtifāʿ suds sāʿa li-ʿarḍ Madīnat al-salām*, "Treatise on the determination of the (solar) altitude for each one-sixth of an hour for the latitude of Baghdad". This work is no longer extant, but we can assume from the title that it must have been a set of tables displaying the solar altitude as a function of time of day, with argument increments of 0;10 hours, which might have been equinoctial or seasonal. The other argument was probably solar longitude λ or half daylight D. On the other hand, the work refered to by Ibn al-Nadīm may be identical with the table I have described above. However, a few spherical

---

6 See King, *Mecca-Centred World-Maps*, p. 350, for a list of his works in which this latitude is featured, usually to the exclusion of all others.

7 On Yaḥyā ibn Abī Manṣūr and the *Mumtaḥan Zīj* see Kennedy, "*Zīj* Survey", no. 51; Vernet, "Las Tabulae Probatae", the same author's article in *DSB*; and Sezgin, *GAS*, V, pp. 227-228, and VI, 136-137. A distorted facsimile of the Escorial manuscript was published in Frankfurt (IGAIW) in 1986.

8 See, in particular, n. 9:5.

9 On the sine tables see n. 7:10 to **7.1.1**.

10 See Sezgin, *GAS*, VI, p. 137, no 2. The only other early Islamic work mentioned by Ibn al-Nadīm which relates specifically to timekeeping is a treatise by Ibn Karnib (see Sezgin, *GAS*, V, p. 275), who lived in Baghdad probably in the latter half of the 9th century, on the determination of the time of day from the solar altitude.

astronomical tables for Baghdad are contained in the recension of Yaḥyā's *Mumtaḥan Zīj* preserved in MS Escorial ár. 927: these are based on parameters ϕ = 33;21° and ε = 23;33°.

4.1.2 al-Battānī: Raqqa

MS Escorial ár. 908, fol. 240r, of the *Zīj* of the celebrated early-10th-century Syrian astronomer al-Battānī contains two small tables of the functions h(T) and a(T) for arguments T = 1, 2, ... , 12^{sdh} and T = 1, 2, ... , 6^{sdh} at the summer and winter solstices.[11] The underlying parameters are:

$$\phi = 36;0° \text{ (Raqqa) and } \varepsilon = 23;35°$$

and the entries are to two and three digits respectively. See also **5.4.2** on another table associated with al-Battānī.

4.1.3 al-Maqsī and al-Marrākushī: various latitudes

The Egyptian astronomer al-Maqsī (**2.1.1**) was a contemporary of al-Marrākushī (**4.2.4**), but the relationship between the two scholars is as yet unclear. Both compiled tables for marking the shadow traces on sundials oriented in various planes, but al-Maqsī's tables, of which I have examined MSS Cairo Azhar *falak* 5528, Dublin CB 4090 and Cairo DM 103, are more extensive. His tables for horizontal sundials, to which I restrict attention here, display the solar altitude and azimuth, as well as the length of gnomon shadow, that is:

$$h(T),\ a(T) \text{ and } z_{(12)}(T)\ ,$$

computed to two digits for T = 1, 2, ... , 6^{sdh} at the summer and winter solstices for:

ϕ = 0° (Equator), 21;0° (Mecca), 24;0° (Medina), 30;0° (Cairo) and 32;0° (Jerusalem).

For Cairo the same functions are also tabulated for each equinoctial hour at the solstices. al-Marrākushī's treatise (I.492 and 494) also contains tables of a(T) and $z_{(12)}$(T) for latitudes 0° and 30°. An extract is shown in **Fig. 4.1.3**.

4.1.4 al-Sulṭān al-Ashraf: Yemen and Hejaz

MS Cairo TR 105 is a copy of the treatise on the astrolabe, horizontal sundial and magnetic compass by the late-13th-century Yemeni ruler Sultan al-Ashraf (**3.3.1**), apparently in the author's hand. The treatise includes tables of the functions:

$$z_{(12)}(T) \text{ and } a(T)$$

for each seasonal day-hour at the solstices and equinoxes, computed for latitudes:

13;0°, 13;37°, 14;0°, 14;30°, 15°, 21° (Mecca), and 24° (Medina) .

Note the use of the distinctive parameter 13;37°, which is intended for Taiz (**3.3.2**). Extracts are shown in **Fig. 4.1.4**.

4.1.5 Anonymous: Istanbul

In MS Cairo ṬM 255,10 (fol. 91r), copied *ca*. 1700, following a collection of prayer-tables and treatises on instruments, there is an isolated table displaying values of h(t) to two digits

[11] On al-Battānī see Willy Hartner's fine article in *DSB*. His *Zīj* is published in Nallino, *al-Battānī*, where the sundial tables occur in II, p. 188 (see also p. 296).

for each 5° of t at the equinoxes and both solstices, computed for $\phi = 41°$ (Istanbul). There is also a table displaying values of h(T) for each seasonal hour at the solstices.

4.2 Tables of solar altitude as a function of time since sunrise and solar longitude, for a specific latitude

The following tables display the solar altitude at the seasonal hours corresponding to a wider range of values of λ than in the tables discussed in **4.1**. On the computation of these tables see **4.2.2** below.

4.2.1 Anonymous: Baghdad

In MS Berlin Ahlwardt 5793 (Landberg 56,2), fol. 97v, copied 783 H [= 1381], at the end of two treatises on the construction and use of the astrolabe by al-Khwārizmī (**4.1.1**), there is a table of a function f(λ,T) computed to two digits (labelled *daqāʾiq* and *thawānī*, minutes and seconds) for each zodiacal sign and T = 1, 2, ... , 6^{sdh}. The function is labelled *jayb al-sāʿāt*, that is, "sine of the hours", and its structure, which escaped me at first encounter, has been explained by Jan Hogendijk.[12] See **I-4.3.1** and especially **XI-9.3**.

4.2.2 Anonymous: Baghdad

In MS Istanbul Ayasofya 4830, fol. 197r, at the end of a short anonymous treatise of Abbasid origin on the construction of an horary quadrant (fols. 196v-197r, copied in Damascus in 626 H [= 1228/29]), there is a table displaying the solar altitude to the nearest degree (the second sexagesimal digit of each entry is always zero) at each seasonal hour from 1 to 6 for each zodiacal sign.[13] From the entries for T = 6 it is immediately obvious that the table is based on the parameters:

$$\phi = 33° \text{ (Baghdad) and } \varepsilon = 24° \text{ (or } 23;51°) .$$

From the text we learn that the values $h_i(T)$ for the signs i = 1, 2, ... , 12 are derived using the approximate formula:

$$\mathrm{Sin}_R\, h_i(T) \approx \mathrm{Sin}_R\, (15T) \bullet \mathrm{Sin}_R\, H_i \,/\, R ,$$

where H_i is the meridian altitude for the i^{th} sign and R = 150. See further **XI-4.1** (illustrated).

4.2.3 Saʿīd ibn Khafīf al-Samarqandī or Ibn al-Ādāmī: Baghdad

MS Paris BNF ar. 2506,1 (fols. 1r-62r) is a unique copy from the late 15th century of a treatise on sundial theory by either Ibn al-Ādamī or Saʿīd ibn Khafīf al-Samarqandī, both of whom lived in the 10th century.[14] On fols. 31v-33v there are some tables for facilitating the

[12] On the treatises see now Charette & Schmidl, "al-Khwārizmī on the Astrolabe". On this table see already King, "al-Khwārizmī", pp. 25 and 29, and Hogendijk, "al-Khwārizmī's Sine of the Hours".

[13] On this treatise and table see *ibid*., pp. 30-31.

[14] On Ibn al-Ādamī see Sezgin, *GAS*, VI, pp. 179-180, and on al-Samarqandī see *ibid*., VI, pp. 216-217, and *Cairo ENL Survey*, no. B58. The title page and the first few folios of this manuscript are in the hand of the late-15th-century Egyptian astronomer Ibn Abi ʾl-Fatḥ al-Ṣūfī (**9.10**), and the main part is in an older hand. On the title folio al-Ṣūfī wrote that the work was by al-Ḥusayn ibn Muḥammad al-Ādamī but that in another manuscript that he had seen, the work was attributed to Saʿīd ibn Khafīf al-Samarqandī.

construction of vertical sundials. The functions tabulated are:

$$a(T,\lambda) \text{ and } h(T,\lambda)$$

for the domains:

$$T = 0;30,\ 1;0,\ \ldots,\ 12^{sdh} \quad \text{and} \quad \lambda = 0°, 30°, 60°, 90°, 210°, 240°, 270° \ .$$

The azimuths are measured “clockwise” from the north point (not south as stated). Values are given to three sexagesimal digits and are based on parameters:

$$\phi = 33° \text{ (Baghdad) and } \varepsilon = 24° \ .$$

Another set of tables entitled *jadwal al-inḥirāfāt*, “table of inclinations”, follows. Three functions are tabulated, namely:

$$\mathrm{Sin}_{10}\ \theta,\ \mathrm{Cot}_{10}\ \theta \text{ and } \mathrm{Cot}_1\ \theta\ ,$$

and values are given to three digits for each degree of argument. The fact that the author uses base 10 is extremely interesting.

4.2.4 al-Marrākushī: Cairo

The extensive treatise on spherical astronomy and instruments by the Cairo astronomer of Maghribi origin Abū ʿAlī al-Marrākushī, entitled *Jāmiʿ al-mabādiʾ wa-'l-ghāyāt fī ʿilm al-mīqāt*, which means “An A to Z of Astronomical Timekeeping”, was compiled in Cairo in the late 13th century. The first part of this work dealing with spherical astronomy and sundials (hereafter, I) was translated into French by Jean-Jacques Sédillot (*père*), and a summary of some of the material on other instruments in the second part (hereafter, II) was published by Louis-Amélie Sédillot (*fils*).[15] The treatise was rather popular amongst later astronomers in Egypt, Syria and Turkey, and exists in several manuscript copies: I have consulted MSS Istanbul Selim Ağa 866, and Paris BNF ar. 2507-2508, 14th century, of which the latter was used by the Sédillots. For more on the spherical astronomy and tables in al-Marrākushī's *opus* see **II-2.7** and **6.7**.

One of the various tables presented by al-Marrākushī (I.454) displays the function:

$$h(T,\lambda)$$

with entries to two sexagesimal digits for the domains:

$$T = 1,\ 2,\ \ldots,\ 6^{sdh} \text{ and } \lambda' = 0°, 30°, 60°, 90° \ (\delta \gtreqless 0)$$

for the parameters $\phi = 30;0°$ (Cairo) and $\varepsilon = 23;35°$.

To determine $h(T,\lambda)$ al-Marrākushī prescribes the following equivalent methods (I.266-267, *cf.* **F10**):

$$h = \text{arc Sin } \{\ [\text{Vers } D - \text{Vers } t] \bullet \text{Sin } H \ /\ \text{Vers } D\ \}$$
$$= \text{arc Sin } \{\ \text{Sin } H(\lambda)\ -\ \text{Vers } t \bullet B(\lambda)\ \}\ ,$$

and presents a numerical example for $T = 1^{sdh}$ and $\lambda = 90°$ using each method. He apparently did not tabulate $B(\lambda)$ for Cairo (**6.4.2** and **6.4.3**).

Elsewhere in the same work (I.428-429 and 436-437), al-Marrākushī tabulates $z(T,\lambda)$ and

---

[15] On al-Marrākushī see Suter, *MAA*, no. 363; *Cairo ENL Survey*, no. C17; and the article in *EI*$_2$. Sédillot-*père*, *Traité*, contains a translation of the first half of al-Marrākushī's treatise on spherical astronomy and sundials. Sédillot-*fils*, *Mémoire*, contains a summary of the second half dealing with instruments. A Topkapı manuscript of the *Jāmiʿ al-mabādiʾ wa-'l-ghāyāt* was published by photo-offset at the Institut für Geschichte der Arabisch-Islamischen Wissenschaften, Frankfurt, in 1984.

$z'(T,\lambda)$ to base 12 for each seasonal day-hour and each 10° interval of λ, as well as (II.76) the function $h(T,\lambda)$, giving values to two sexagesimal digits for the domains:

$$T = 1, 2, \ldots, 6^h \ (1^h = 15°) \quad \text{and} \quad \lambda' = 0°, 30°, 60°, 90° \ (\delta \gtrless 0°)$$

and for the same parameters. The corresponding solar altitudes in the prime vertical and in the azimuth of the qibla are also given.

4.2.5 al-Sulṭān al-Ashraf: Sanaa

In MS Oxford Hunt. 233, fols. 73r-78v, of the astrological treatise of the Yemeni Sultan al-Ashraf (**3.3.1** and also **4.1.4**), there is a set of tables of the function:

$$h(T,\lambda)$$

with values computed to two digits for the domains:

$$T = 1, 2, \ldots, 6^{sdh} \quad \text{and} \quad \lambda = 1°, 2°, \ldots, 360° \ .$$

The underlying parameters are:

$$\phi = 14;30° \text{ (Sanaa)} \quad \text{and} \quad \varepsilon = 24;0° \ ,$$

and the function is called *irtifāʿ sāʿāt (al-)nahār*, "the (solar) altitude for each hour of daylight".

4.2.6 Abu 'l-ʿUqūl: Taiz

In the astronomical miscellany of the Yemeni Sultan al-Afḍal preserved in a manuscript in a private collection in Sanaa (**2.1.2**), there is a set of tables of the function:

$$h(\lambda,T)$$

computed to two digits for the same domains, but based on the parameters:

$$\phi = 13;37° \text{ (Taiz)} \quad \text{and} \quad \varepsilon = 23;35° \ .$$

It is stated in the margin that this table is taken from the *Zīj* of Abu 'l-ʿUqūl, but the latitude 13;40° is used for Taiz in both his main tables for timekeeping and in the *Mukhtār Zīj*. This may mean that Abu 'l-ʿUqūl prepared two *zīj*es. (See also **3.1.1** and **3.3.2**.)

4.2.7 Najm al-Dīn al-Miṣrī: 4th climate (?)

For the many tables of solar altitude in the treatise of the Cairene astronomer Najm al-Dīn (**2.6.1**) on instruments I refer the reader to the work of François Charette.[16] Some of these tables, intended for the construction of horary markings on different kinds of instruments, are computed for latitude 36°, which is not for Aleppo, as I thought in the days when I (falsely) attributed the (anonymous) treatise to the Aleppo scholar Ibn al-Sarrāj (**VIb-5**) . Rather it seems that this latitude was for the 4th climate. It remains a mystery why Najm al-Dīn did not compute all of his tables for the latitude of Cairo.

4.3 Tables of solar altitude as a function of time since sunrise and meridian altitude, for all latitudes

A number of sources contain tables of the function $h(T,H)$, where T is measured in seasonal hours, based on the approximate formula:

$$h(T,H) \approx \text{arc Sin} \{ \text{Sin } H \cdot \text{Sin } (15\ T) / R \} \ .$$

[16] Charette, *Mamluk Instrumentation*.

These tables are intended to be valid for all latitudes and the underlying formula is a transformation of the one discussed in **2.5** above. On a similar table in a 14th-century English astronomical manuscript of both the *Toledan* and *Alphonsine Tables* see **10.1**.

4.3.1 Anonymous (Baghdad)

The anonymous tables in MS Berlin Ahlwardt 5793 (Landberg 56,2), fols. 93r-95v, copied 783 H [= 1381] (**4.2.1**), appended to al-Khwārizmī's two treatises on the construction and use of the astrolabe, clearly date from the earliest days of mathematical astronomy in Baghdad and can indeed be shown to be by al-Khwārizmī. Following a table of normed right ascensions $\alpha'(\lambda)$ (fol. 93v, see **7.1.1**, n. 7:11) and some tables for latitude 33° (fol. 94r, see **II-3.1**) there is a table[17] of the function:

$$h(T,H)$$

with entries to one digit (fol. 94v), computed for the domains:

$$T = 1, 2, \ldots, 6^{sdh} \text{ and } H = 25°, 26°, \ldots, 90° .$$

There is also a table (fols. 95r-95v) of a function:

$$f(T,H) ,$$

labelled *jayb al-sāʿāt*, "Sine of the hours", and with entries to two digits for the same domains. See already **4.2.1** and especially **XI-9.3**.

4.3.1* Muḥyi 'l-Dīn al-Maghribī (?) (Maragha)

Another table of the function:

$$h(T,H)$$

is contained in MS Medina Aref Hikmet *mīqāt* 1, fols. 196r-196v, of the *Zīj* for Maragha by the 13th-century astronomer Muḥyi 'l-Din al-Maghribī (**5.6.3**). This particular copy was completed in 691 H [= 1292], and the table is not contained in any of the other manuscripts of al-Maghribi's various *zīj*es (he wrote at least three).

4.3.2 al-Marrākushī (Cairo)

In al-Marrākushī's treatise on spherical astronomy (**4.2.4**) there is a table (I.457) of the function:

$$h(T,H)$$

with the entries to two digits for:

$$T = 1, 2, \ldots, 5^{sdh} \quad \text{and} \quad H = 5°, 10°, \ldots, 90° .$$

(Clearly h = H when $T = 6^{sdh}$.) The solar altitude h_a at the beginning of the interval for the afternoon prayer is also given for each value of H. Elsewhere in the same work (I.253) al-Marrākushī tabulates $z'_{(12)}(T,H)$ to two digits for the same domains and also $T = 6^{sdh}$).

4.3.3 Anonymous (Yemen)

The anonymous late-14th-century Yemeni *Zīj* preserved in MS Paris BNF ar. 2523 (**2.5.3**) contains a table (fol. 86r) of the function h(T,H) with values computed to two digits for the domains $T = 1, 2, \ldots, 11^{sdh}$ and H = 51°, 52°, ... , 90°. See **Fig. 4.3.3**. Note that the table

[17] On this table see King, "al-Khwārizmī", pp. 7 and 10-11.

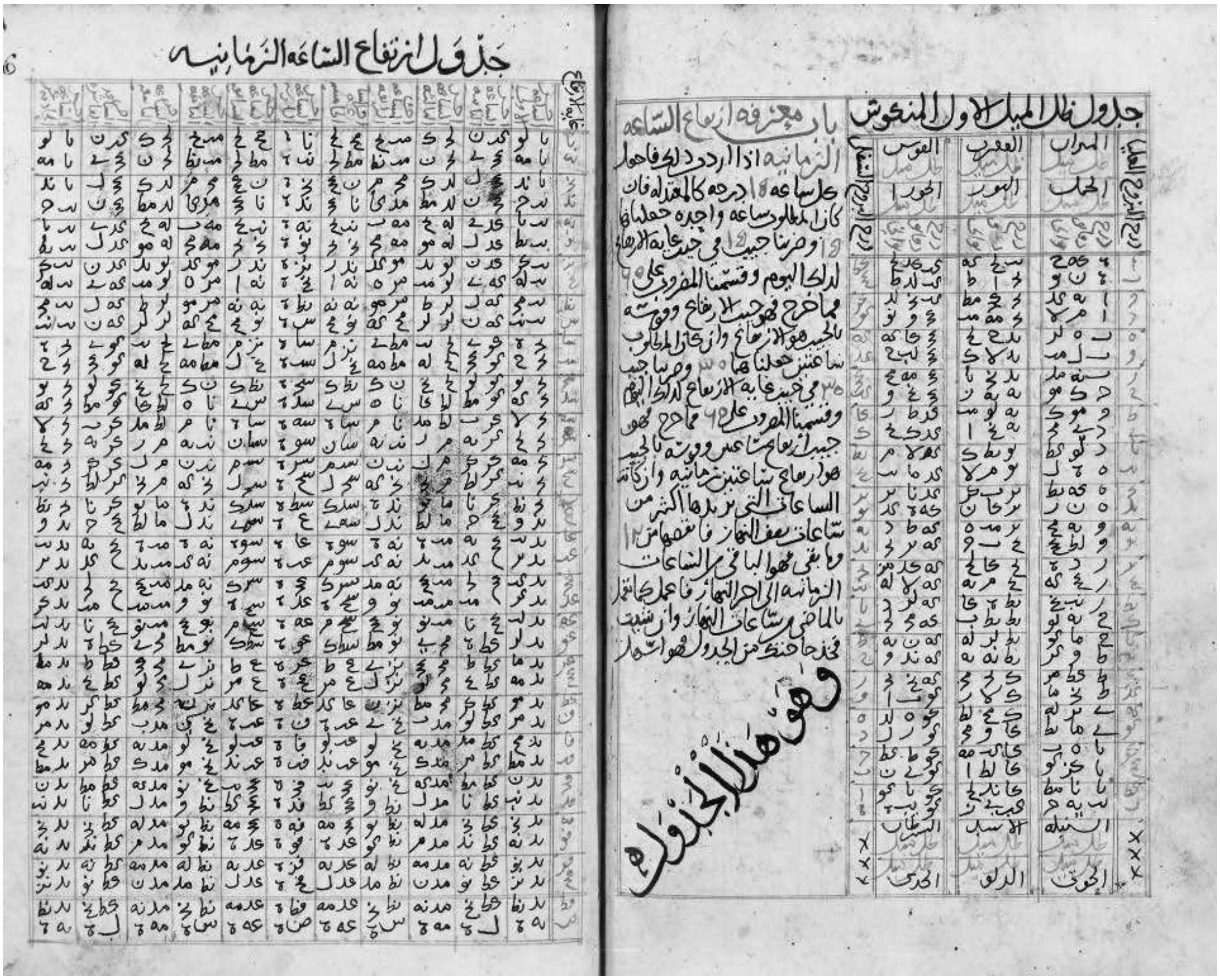


Fig. 4.3.3: A table of the tangent of the solar declination (**7.1.10**) and the anonymous Yemeni table serving solar meridian altitudes 11°-90°. [From MS Paris BNF ar. 2523, fol. 86r, courtesy of the Bibliothèque Nationale de France.]

is adequate for use in the Yemen, where the minimum value of the solar altitude is about 50°. See also **II-12.3** on this table.

4.3.4 Anonymous (Yemen)

A manuscript that was in the private collection of Sayyid Aḥmad ʿAbd al-Qādir al-Ahdal in Zabid in the 1980s contains a copy of the 16th-century Yemeni *zīj* compiled by Muḥammad al-Daylamī and entitled *Zād al-musāfir*.[18] This contains two similar tables but for vertical arguments H = 51°, 52°, ... , 90° and H = 40°, 41°, ... , 90°. The entries generally differ from those in MS Paris BNF ar. 2523 mentioned in **4.3.3**.

[18] On this work see further King, *Astronomy in Yemen*, no. 28.

4.3.5 al-Ṣāliḥī (?) (Damascus)

In MS Baghdad Awqāf 2966/6294, fol. 136r, of an anonymous *Zīj* containing tables due to Ibn al-Shāṭir (Damascus, *ca.* 1350), Ulugh Beg (Samarqand, *ca.* 1440) and ʿAbd al-Raḥmān ibn Banafshā al-Ṣāliḥī (Damascus, *ca.* 1500),[19] there is a table of the same function for arguments T = 1, 2, ... , 5^{sdh} and H = 1°, 2°, ... , 90°. In MS Leiden Or. 65, fol. 110r, of a *zīj* attributed to Ibn al-Shāṭir which purports to be entitled *Tuḥfat al-nāẓir* but which seems to be merely an early modification of his *Zīj*, there is a similar, if not identical, table. I have not come across such a table in the various manuscripts of Ibn al-Shāṭir's *Zīj*.

4.3.6 Ḥusayn Quṣʿa (?) (Tunis)

In MS Princeton Yahuda 147c of the *Zīj* for Tunis compiled by Ḥusayn Quṣʿa[20] *ca*. 1650 and based mainly on the *Zīj* of Ulugh Beg, there is a page ruled for a table with identical title and format to the one in MS Baghdad Awqāf 2966/6294 (see the previous section), but no entries have been copied.

4.3.7 Zacuto (?) (Salamanca) / Anonymous (Yemen)

In MS Milan Ambrosiana C82, a Yemeni copy from 1086 H [= 1675] of the Maghribi version of the perpetual almanac of the late-15th-century Jewish astronomer Abraham Zacuto of Salamanca[21] there are various additional tables (fols. 142r-148r), one of which is attributed to the 13th-century Tunisian astronomer Ibn Isḥāq (**6.9.1***). One anonymous table (fol. 144v) displays values of the function h(T,H) to one digit for arguments T = 1, 2, ... , 5^{sdh} and H = 1, 2, ..., 90°. This particular table is not contained in MS Hyderabad Āṣafiyya 298 of the Syrian recension of Ibn Isḥāq's *Zīj*. See further **6.9.1*** and **7.1.5***.

4.4 Tables of solar altitude as a function of the semi diurnal arc and the time since sunrise

4.4.1 Najm al-Dīn al-Miṣrī (?): Cairo

The unique source MS Cairo MM 72, copied in 747 H [= 1346/47], contains an anonymous set of tables of the function:

$$h(D,T)$$

with entries computed to two digits for the domains:

$$D = 104;36°, 104°, 103°, \ldots, 76°, 75;24° \quad \text{and} \quad T = 1°, 2°, \ldots, [2D]$$

and based on the parameters:

$$\phi = 30;0° \text{ (Cairo)} \quad \text{and} \quad \varepsilon = 23;35° .$$

The entries, which number over 5,500, are rather accurately computed. Note that 104;36° and 75;24° are the maximum and minimum values of D for these parameters. A given pair of facing

---

[19] On Ibn al-Shāṭir see n. 2:15. On Ulugh Beg see *Cairo ENL Survey*, no. G49, the articles by T. N. Kari-Niazov in *DSB* and by Beatrice F. Manz in EI_2. On al-Ṣāliḥi (Suter, *MAA*, no. 454) see *Cairo ENL Survey*, no. C87.

[20] This author is not listed in Suter, *MAA*; Brockelmann, *GAL*; or Ḳennedy, "*Zīj* Survey". See *Cairo ENL Survey*, no. F47 (also no. F53); Samsó, "Maghribī *Zīj*es", p. 98; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, p. 314.

[21] On this work see now Chabás & Goldstein, *Zacut*, where there is no mention of any tables for timekeeping in the original version. On this table see now Samsó, "Zacut in the Islamic East".

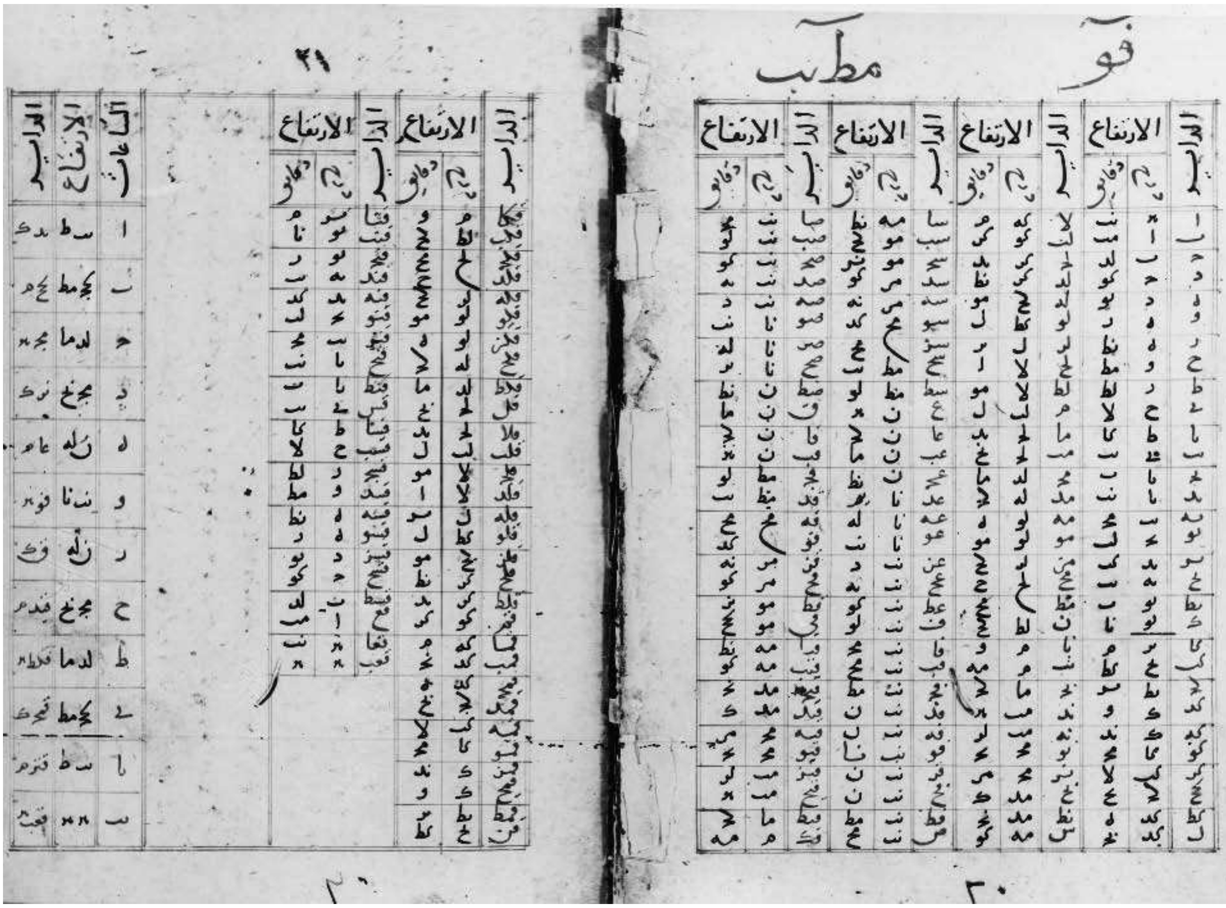


Fig. 4.4.1: An extract from the anonymous Mamluk tables of solar altitude serving half arc of daylight 86° (hour-angle at the ʿ*aṣr* 49;12°). [From MS Cairo MM 72, fols. 20v-21r, courtesy of the Egyptian National Library.]

pages in the manuscript serves a particular value of D, and the two unlabelled numbers at the head of each table are in fact the argument D and the associated time from midday to the beginning of the interval for the afternoon prayer t_a. By the side of each sub-table are the solar altitudes at each seasonal hour for the given value of D. **Fig. 4.4.1** shows a pair of facing pages of these tables.

The title folio and all but the last page of the introduction to these tables are missing from the manuscript. The copyist was Ibn al-Kattānī, who compiled the hour-angle tables for Cairo in MS Istanbul Kılıç Ali Paşa 684, also penned in his elegant hand (**2.1.1** and **II-4.1.**4).[22] I have previously suggested that these tables might have been computed by Ibn Yūnus,[23] but the possibility that the tables in MS Cairo MM 72 were computed by Ibn al-Kattānī himself cannot be entirely excluded. However, it now seems more likely that they were computed by

[22] On Ibn al-Kattānī (Suter, *MAA*, no. 411) see *Cairo ENL Survey*, no. C32.
[23] King, “Astronomical Timekeeping in Medieval Cairo”, pp. 365 and 391.

the early-14th-century Egyptian astronomer Najm al-Dīn al-Miṣrī (**2.6.1**). In his treatise on spherical astronomy preserved in MS Milan Ambrosiana 227a (C49), fols. 85v-97r, Najm al-Dīn discusses the determination of h from D and T. The method which he prescribes is equivalent to the following standard formula used, for example, by Ibn Yūnus[24] (*cf.* **F10**):

$$h(D,T) = \text{arc Sin } \{ \text{ [Vers } D - \text{Vers } (D\text{-}T) \text{] } \bullet \text{ Sin } H \text{ / Vers } D \text{ } \} \text{ .}$$

For each value of D one must first find the corresponding value of H. Najm al-Dīn then adds a note about a method for finding the solar altitude at the seasonal day-hours: for the n^{th} hour $T = n \bullet \tilde{h}$ where $\tilde{h}$ is the number of degrees corresponding to one seasonal day-hour. This is strong evidence that the tables were compiled by Najm al-Dīn himself. I see no particular advantage to tables of solar altitude as a function of the semi diurnal arc, but Najm al-Dīn may already have compiled his enormous table of T(H,h,D) for all latitudes and then thought it reasonable, and had energy and time, to compile a table of h(D,T) for Cairo.

4.5 Tables of stellar altitude as a function of the longitude of the horoscopus, for a specific latitude

4.5.1 Abu ’l-ʿUqūl: Taiz

The 14th-century Yemeni miscellany preserved in a manuscript in a private collection in Sanaa (see already **2.1.2**, **3.3.1** and **4.2.3**) contains an anonymous and incomplete set of tables (4 pages out of 12) of the altitudes of up to 12 stars as a function of the longitude of the horoscopus. Entries are given to two digits for each degree of ecliptic longitude, and a given page of the manuscript serves two zodiacal signs: see **Fig. 4.5.1**. Now there is no table of this kind in MS Berlin Ahlwardt 5720 of Abu ’l-ʿUqūl’s corpus for Taiz (**2.1.2**) but that copy is incomplete and such a table is indeed listed as part of the corpus on fols. 17v-18r of the Berlin manuscript in his introduction to the tables. Thus it is probable that this set of tables in the Sanaa manuscript is due to Abu ’l-ʿUqūl and is based on the value 13;40° for the latitude of Taiz.

Abu ’l-ʿUqūl also compiled an extensive set of tables displaying the altitudes of various prominent stars at daybreak as functions of the longitude of the horoscopus. These are extant in the Berlin manuscript mentioned above: see **Fig. 4.5.2** and also **II-12.1**.

4.6 Tables of stellar altitude as a function of time of night, for a specific latitude

4.6.1 Ibn Dāʿir: Sanaa

On fol. 126v of the Berlin copy of Ibn Dāʿir’s tables for Sanaa (**3.2.3** and **3.4.1**) there is a table displaying the altitudes of 18 fixed stars at each “seasonal hour” of visibility, with entries to two digits – see **Fig. 3.4.1**.

---

[24] King, *Ibn Yūnus*, III.15.4e.

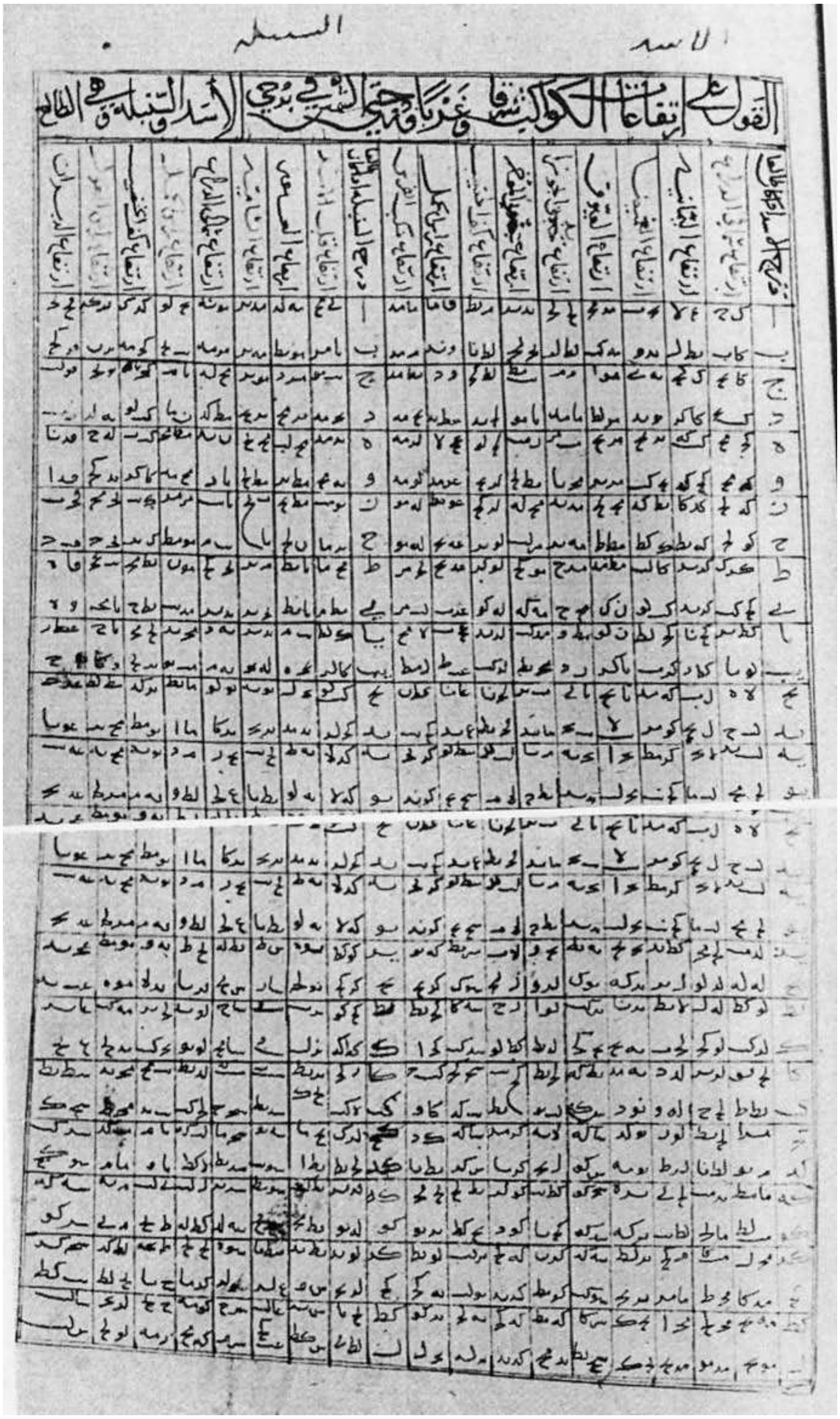



Fig. 4.5.1: An extract from Abu 'l-ʿUqūl's tables of the altitudes of various stars as a function of the horoscopus, here the first few degrees of the sign of Leo. [From Varisco & Smith, eds., *al-Afḍal's Anthology*, p. 173, with kind permission of Professor Daniel Varisco.]

Fig. 4.5.2: An extract from Abu ‘l-ʿUqūl’s tables of stellar altitudes in the east and west serving each degree of the horoscopus in Aquarius. [From MS Berlin Ahlwardt 5720, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

4.7 Tables of solar altitudes in certain azimuths as a function of solar longitude, for a specific latitude

The azimuths for which a knowledge of the corresponding solar altitude is of some importance in Islamic astronomy are defined by: (i) the meridian; (ii) the prime vertical; and (iii) the qibla or azimuth of Mecca. In Cairo a fourth direction was of interest to the astronomers because of the layout of the architecture of the city, namely, (iv) the direction for orienting the ventilators on the roofs of religious and secular buildings, these being a common feature of the medieval city.

Tables of the solar meridian altitude $H(\lambda)$ for a particular latitude are standard in *zīj*es, and their compilation is trivial since (*cf.* **F1**):

$$H(\lambda,\phi) = \bar{\phi} + \delta(\lambda) .$$

Numerous Islamic sources contain tables of the function:

$$\delta'(\lambda) = 90^\circ + \delta(\lambda) ,$$

to be used for finding the meridian altitude for any latitude. Clearly:

$$H(\lambda,\phi) = \delta'(\lambda) - \phi .$$

Tables of $h_0(\lambda)$, the solar altitude in the prime vertical, are less common. Those located thus far are discussed in **4.8** below.

Finally, tables of $h_q(\lambda)$, the solar altitude in the azimuth of Mecca, are attested for a wider variety of localities. These are discussed in **II-3.6, 3.9**, **4.7**, **10.6**, *etc.*[25]

To find $h(a,\lambda)$ for a particular azimuth, one can either use interpolation in tables of $a(h,\lambda)$ if these were available (**5.1**) or calculate the altitude directly. The method of Ibn Yūnus is as

[25] On Ibn Yūnus’ tables of $h_q(\lambda)$ for Cairo see already Schoy, *Schattentafeln*, p. 42; King, *Ibn Yūnus*, III.28.3, and *idem*, “Astronomical Timekeeping in Medieval Cairo”, p. 368. See also **II-4.7**.

follows.[26] First compute the altitude h_e of the sun at an equinox when it has azimuth a, thus:

$$h_e = \text{arc Sin} \{ R \text{ Sin } a / \sqrt{[(R \text{ Sin } \phi / \text{Cos } \phi)^2 + \text{Sin}^2 a]} \} .$$

Secondly compute a "correction arc" C for the solstitial points, thus:

$$C = \text{arc Sin} \{ \text{Cos } h_e \text{ Sin } \varepsilon / \text{Sin } \phi \} ,$$

and then a correction arc c(λ) for general solar longitudes, thus:

$$c(\lambda) = \text{arc Sin} \{ \text{Sin } C \text{ Sin } \lambda / R \} .$$

Finally the solar altitude is given by:

$$h(a,\lambda) = h_e(a) + c(\lambda) \qquad (c \gtreqless 0 \text{ as } \delta \gtreqless 0) .$$

Only one set of tables of h(a,λ) for general azimuths has come to light.

4.7.1 Ibn Yūnus: Cairo

In MS Oxford Hunt. 331, fols. 27v-42r, of the *Ḥākimī Zīj* Ibn Yūnus (**2.1.1**) presents a set of tables of the function:

$$h(a,\lambda)$$

for the domains:

a = 30°, 40°, 42°, 45°, 50°, 54°, 60°, 66°, 70°, 75° and λ = 1°, 2°, ... , 360°.

Values are carefully computed to two digits and are based on his parameters φ = 30;0° (Cairo) and ε = 23;35°.[27] The tables for a = 30° and 60° are also found in MSS Dublin CB 3673 and Cairo DM 153 of the Cairo corpus of tables for timekeeping (see further **II-4.6**).

Ibn Yūnus states in the *Zīj* that these tables are for finding the meridian. The azimuths are supposed to have been chosen so that the meridian could be determined using a simple geometrical construction once the direction corresponding to the azimuth a has been found by observation of the solar altitude. There are, of course, easier methods for finding the meridian, several of which are outlined elsewhere in the *Ḥākimī Zīj*.

Another table in the main Cairo corpus displays the altitude of the sun in the azimuth in which ventilators were oriented in popular Egyptian practice. This azimuth is that of the rising sun at the winter solstice, that is, about 27;30° S. of E. for Cairo, and it is understood that the base of the back of the ventilator should be aligned in this direction so that (a) the ventilator be aligned with the building on which it is place, and (b) the front be open to the favorable winds from the north. The table, which exists in numerous manuscripts, *e.g.* MS Dublin CB 3673, fol. 84v, may have been computed by Ibn Yūnus, but in MS Istanbul Nuruosmaniye 2925 one manifestation of it is attributed to Ibn al-Rashīdī (**2.1.5**).[28] See further **II-4.8** and **VIIb-3-4**.

4.8 Tables of solar altitude in the prime vertical, for a specific latitude

The solar altitude in the prime vertical, $h_0(\lambda)$, is called in Arabic *al-irtifāᶜ alladhī lā samt lahu* or *al-irtifāᶜ al-ᶜadīm al-samt*, that is, "the altitude with no azimuth". It is determined by the simple formula (*cf.* **F13**):

[26] See King, *Ibn Yūnus*, III.23.3, on the similar methods of Ibn Yūnus and al-Bīrūnī.
[27] See also *ibid.*, III.22.2, and *idem*, "Astronomical Timekeeping in Medieval Cairo", pp. 364-365.
[28] See also *ibid.*, pp. 371-373.

$$h_0(\lambda) = \text{arc Sin } \{ \text{ R Sin } \delta(\lambda) \ / \text{ Sin } \phi \ \} \ ,$$

and like the solar rising azimuth (**5.5** and **8.3**) is easily determined from Sin $\delta(\lambda)$ (**6.1**). The following tables of $h_0(\lambda)$ for particular latitudes have been located.

4.8.1 Ibn Yūnus: Cairo

In MS Leiden Or. 143, p. 360, of the *Ḥākimī Zīj*, Ibn Yūnus (**2.1.1**) tabulated $h_0(\lambda)$ to three digits for each degree of λ and parameters ϕ = 30;0° (Cairo) and ε = 23;35°.

4.8.2 al-Mizzī: Damascus

The prayer-tables of al-Mizzī preserved in MS Cairo MM 62 (**2.1.3**) display $h_0(\lambda)$ to two digits for each degree of λ and parameters ϕ = 33;27° (Damascus) and ε = 23;33°.

4.8.3 al-Khalīlī: Damascus

al-Mizzī's younger colleague al-Khalīlī (**2.1.4**) recomputed $h_0(\lambda)$ for parameters ϕ = 33;30° (Damascus) and ε = 23;31°. His values are also given to two digits for each degree of λ, and the table is contained, for example, in MS Paris BNF ar. 2558, fol. 51v.

4.8.4 Anonymous: Cairo

In several manuscripts of the main Cairo corpus (**2.1.1**) there occur tables of $h_0(\lambda)$ to two digits, ostensibly computed for Ibn Yūnus' parameters (**4.8.1**) but containing numerous garbled entries. Thus, for example, the 14th-century copy MS Dublin CB 3673, fol. 12v, contains a table where the entry for λ = 30° (which should be ε = 23;35° for ϕ = 30°) is 24;34°, and the entry for λ = 90° (which should be 53;9°) is 53;46°. Another such table in the same copy of the corpus (fol. 3v) has corresponding entries 24;34° and 53;9°. Again, the 16th-century Cairo astronomer al-Minūfī (**II-7.1**) recomputed some of the simpler tables in the corpus for ε = 23;30° Thus, for example, he prepared a new table of $\psi(\lambda)$ (**5.6.9**), but merely took over a corrupt table of $h_0(\lambda)$: his table of this function in MS Cairo DM 107 is the same as that in MS Dublin CB 3673, fol. 3v. Finally, the values of $h_0(\lambda)$ for λ = 30° and 90° in the prayer-tables of the 16th-century Egyptian astronomer al-Fawānīsī (**II-7.5**), preserved, for example, in MS Oxford Seld. Supp. 99, have been distorted by further copyist's errors to 23;47° and 53;21°.

4.8.5 al-Asyūṭī: Assiut

The late-15th-century Egyptian astronomer al-Asyūṭī (**II-6.16**) in his prayer-tables for Assiut extant in MS Cairo DM 188 tabulated $h_0(\lambda)$ to two digits for each degree of λ and parameters ϕ = 27;0° (Assiut) and ε = 23;35°.

4.8.6 al-Maḥallī: Damietta

In a manuscript formerly in the Institut für Geschichte der Medizin und der Naturwissenschaften in Berlin and apparently no longer extant (see **5.1.4**), there was a table of $h_0(\lambda)$ computed for ϕ = 31;25° (Damietta). This table is not contained in MS Cairo DM 106, an incomplete copy of the prayer-tables of Quṭb al-Dīn al-Maḥallī (**II-8.6** and also **I-5.1.4**) for parameters ϕ = 31;25° and ε = 23;35°, but may have originally been part of this set.

4.8.7 al-Dīsṭī: Lattakia

MSS Aleppo Awqāf 911 and Leiden Or. 2808(2) of a set of prayer-tables computed *ca*. 1700 for Lattakia by al-Dīsṭī (**II-11.8**) contain a table of $\psi(\lambda)$ with values to two digits for each degree of λ and parameters ϕ = 34;30° (Lattakia) and ε = 23;35°.

4.8.8 Anonymous: Alexandria

At the end of the tables for the latitude of Alexandria preserved in MS Cairo TR 354 (**2.1.6**) there is a single table of $h_0(\lambda)$, with entries to two digits for each degree of λ. The underlying parameters are ϕ = 31;0° (Alexandria) and ε = 23;30°. The value of the obliquity differs from that used for the main tables.

MS Cairo DM 1207 of the corpus of prayer-tables for Alexandria (**II-8.5**) contains a different table of $h_0(\lambda)$ with values to two digits for each degree of λ based on parameters ϕ = 31;0° (Alexandria) and ε = 23;35°.

4.8.9 Yūsuf Kilārjī: Crete

In MS Cairo DM 834 of a set of prayer-tables for Crete compiled by the early-18th-century Egyptian astronomer Yūsuf Kilārjī (**II-8.8**) there is a table of $h_0(\lambda)$ computed to two digits for each degree of λ and based on the parameters ϕ = 35;30° (Crete) and ε = 23;35°.

4.8.10 Anonymous: Nablus

In MS Cairo ṬM 81 at the end of some late timekeeping tables for Jerusalem (**2.2.6**) there are three tables computed for latitude ϕ = 32;10° (Nablus). The functions tabulated are $N(\lambda)$, $d(\lambda)$ and $h_0(\lambda)$.

CHAPTER 5

TABLES OF SOLAR AZIMUTH

5.0 Introductory remarks

The calculation of the azimuth of the sun and stars is a standard topic of Islamic spherical astronomy, and certain Muslim astronomers compiled tables of the solar azimuth as a function of the altitude. In **5.1** I describe tables having h and λ as arguments, and in **5.2** a table having arguments H and h. The tables of azimuth as a function of time discussed in **5.3** are samples of a category important in Islamic sundial theory.

The function a(h,λ) displays the same symmetry as t(h,λ) and T(h,λ) (**2.1**). It is generally defined in *zīj*es[1] in terms of two auxiliary functions k(h,ϕ) and Sin ψ(λ,ϕ), with which (*cf.* **F15**):

$$a(h,\lambda,\phi) = \text{arc Sin } \{ \ R \bullet [\ k(h,\phi) \ - \ \sin \psi(\lambda,\phi) \ ] \ / \ \cos h \ \} \ .$$

Certain Muslim astronomers tabulated k(h) and Sin ψ(λ) for specific latitudes, and at least one table of the secant function:

$$G_1(h) \ = R \ / \ \text{Cos } h \ ,$$

especially intended for azimuth calculations is attested in the Islamic sources. al-Khalīlī tabulated k(h) and Sin ψ(λ) for each degree of latitude, and another auxiliary function for deriving the azimuth from their difference. The determination of the azimuth with these three functions tabulated is very simple indeed: see further **Ch. 8** and especially **Sections 8.5** and **9.4a**.

In most Islamic sources the azimuth (Arabic, *samt*) is reckoned from the prime vertical. The azimuth of Mecca (Arabic, *inḥirāf al-qibla*), on the other hand, is generally reckoned from the meridian.

5.1 Tables of solar azimuth as a function of solar altitude and solar longitude, for a specific latitude

5.1.1 Ibn Yūnus: Cairo

Ibn Yūnus (**2.1.1**) compiled a set of tables called *Kitāb al-Samt*, "Azimuth Tables", displaying values of the function:

$$a(h,\lambda)$$

to two digits for the domains:

$$h = 1°, 2°, \dots , 83° \quad \text{and} \quad \lambda = 1°, 2°, \dots , 90° \text{ and } 181°, 182°, \dots , 270°$$

and based on the parameters:

$$\phi = 30° \text{ (Cairo-Fustat)} \quad \text{and} \quad \varepsilon = 23;35° \ .$$

[1] See, for example, Nallino, *al-Battānī*, I, pp. 183-184; Luckey, "Beiträge", I, pp. 500-501 (al-Māhānī); and King, *Ibn Yūnus*, III.20.

Fig. 5.1.1a: The additional values of the solar azimuth for solar altitudes from 84° to 88° in some early Mamluk copies of Ibn Yūnus' tables defy explanation. [From MS Dublin Chester Beatty 3673, fols. 86v-87r, courtesy of the Chester Beatty Library.]

These tables, extant in MSS Berlin Ahlwardt 5753 (incomplete), Dublin CB 3673, Escorial ár. 924,7, Cairo DM 108, Cairo Azhar *falak* 4382, Cairo MM 137, Gotha A 1410, Cairo DM 53, Istanbul Nuruosmaniye 2903 and 2925, Cairo MM 64, Cairo DM 690, DM 616, DM 739, DM 786, DM 1101, DM 1224 and Cairo K 4044, are very accurately computed. Tables for h = 30° and 35° are found in the *Ḥākimī Zīj*, MSS Leiden Or. 143, pp. 389-390, and Oxford Hunt. 331, fols. 124v-127v.[2] On the related tables of a(λ,h) in MS Cairo DM 1108,9 see **5.2.1** below.

It seems that the entries in Ibn Yūnus' tables for altitudes above 80° were either not complete or were not contained in any manuscripts that were available in the 14th century. Additional entries for altitudes 81° to 89° were added by some anonymous incompetent who was oblivious of the fact that the maximum solar altitude in Cairo is about $83^1/_2$°. Thus, for example, MS

[2] On Ibn Yūnus' azimuth tables see also King, *Ibn Yūnus*, III.22.2 (*Ḥākimī Zīj*); and *idem*, "Astronomical Timekeeping in Medieval Cairo", pp. 362-364 (*Kitāb al-Samt*). The tables for altitude 30° in the two sources are reproduced *ibid*., pp. 354-355.

Fig. 5.1.1b: Ibn Yūnus' table of the azimuth as a function of the solar altitude at the equinoxes and solstices. On the left is an auxiliary table for sundial construction, of the kind for which another detailed survey would be worthwhile. [From Cairo MM 57, fols. 1v-2r, courtesy of the Egyptian National Library.]

Gotha A 1410 contains entries for h running from 1° to 89°, MS Dublin CB 3673 entries to 88° (see **Fig. 5.1.1a**), MS Cairo MM 137 entries to 86°. The entries for 81° to 83° were recomputed properly by Ibn al-Rāshidī (**2.1.5**), and were incorporated in al-Bakhāniqī's edition of the Cairo corpus (**2.1.1**). Thus, for example, MS Cairo Azhar *falak* 4382 contains a corrected set of azimuth tables with entries to 83° for h, and in MSS Cairo DM 108 and Cairo DM 53 these corrected entries are tabulated alongside the corresponding values of T(h,λ) and t(h,λ). Further information on these tables is given in **II-4.4** and **II-5.2**.

Ibn Yūnus also tabulated the function a(h) at the equinoxes for h = 1°, 2°, ... , 60° (= $\bar{\phi}$) with additional values for the solstices. This table is extant in MSS Cairo Azhar *falak* 4382, fol. 71r, and Cairo MM 57, fol. 1v – see **Fig. 5.1.1b**. MS Vatican Borg. ar. 217,2 (fols. 6r-7r) of al-Khaṭāʾī's auxiliary tables (**6.15.1**) contains a table of this same function for the equinoxes with minor variants from Ibn Yūnus' table. Furthermore, al-Khaṭāʾī reiterates Ibn Yūnus' remark in the *Ḥākimī Zīj* to the effect that such a table also displays d(Δ).[3] Notice that if δ = 0 the formula for a(h) is (*cf.* **F15**):

$$a(h) = \text{arc Sin } \{ \text{ Tan } h \text{ Tan } \phi \ / \ R \ \} \ ,$$

[3] *Cf.* King, *Ibn Yūnus*, III.15.1b, and *idem*, "Astronomical Timekeeping in Medieval Cairo", pp. 389-390.

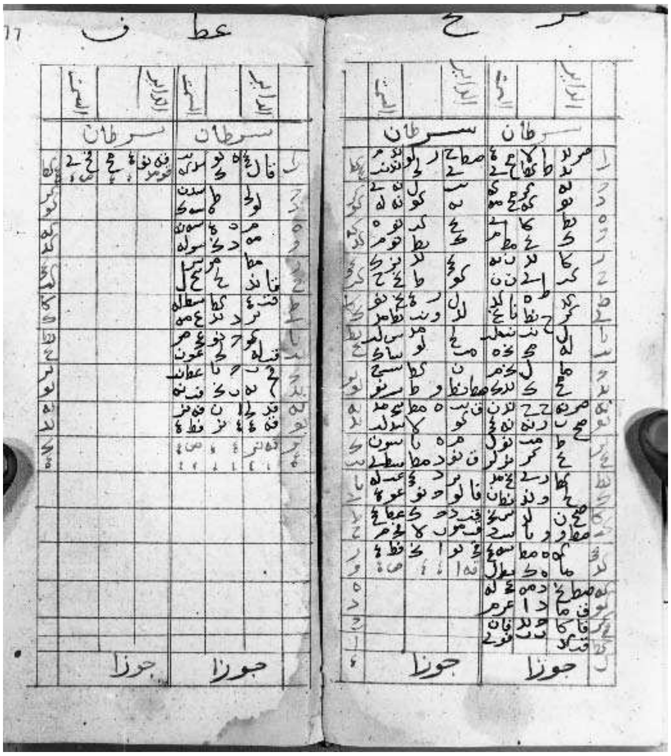


Fig. 5.1.2: The last sub-tables in the corpus for Damascus, in which al-Ḥalabī's values for the azimuth are tabulated alongside al-Khalīlī's values of the time since sunrise and time to midday. [From MS Cairo MM 71, fols. 65v-66r, courtesy of the Egyptian National Library.]

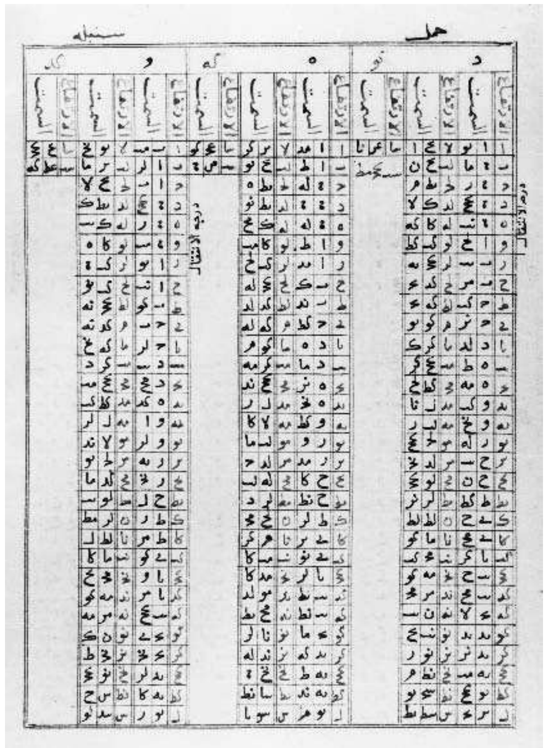


Fig. 5.2.1: An extract from the anonymous solar azimuth tables for Cairo serving solar longitudes Aries 4°-6° / Virgo 24°-24°. [From MS Cairo DM 1108,9, courtesy of the Egyptian National Library.]

and that d(Δ) is defined by (*cf.* **F7**):

$$d(\Delta) = \text{arc Sin } \{ \text{ Tan } \Delta \text{ Tan } \phi \text{ / R } \} .$$

On Ibn Yūnus' auxiliary tables for azimuth calculations see **8.1.1** and **8.3.1**.

5.1.2 Shihāb al-Dīn al-Ḥalabī: Damascus

al-Khalīlī's tables of t(h,λ) and T(h,λ) for Damascus (**2.1.3**) were supplemented by the 15^{th}-century astronomer Shihāb al-Dīn al-Ḥalabī (**2.8.1**) with a complete set of tables of a(h,λ) based on al-Khalīlī's parameters. These are contained in MS Cairo MM 71 where they are tabulated side by side with al-Khalīlī's functions (see **Fig. 5.1.2**), and in MSS Damascus Ẓāhiriyya 9227 and Cairo K 8525 where they are tabulated separately. See further **II-11.2**.

5.1.3 Anonymous: Alexandria

The tables for timekeeping in MS Cairo TR 354 computed for the latitude of Alexandria which are falsely attributed to Ibn Yūnus (**2.1.6**) also give values of a(h,λ) based on the same parameters as those of t(h,λ) and T(h,λ). See further **II-8.5**.

5.1.4 Anonymous: Damietta (?)

In a manuscript of late Egyptian provenance which in 1939 was in the Institut für Geschichte der Naturwissenschaften in Berlin, according to a description by Willy Hartner, there was a set of solar azimuth tables for an unspecified latitude occupying about 25 pages (fols. 259r-271r).[4] Since these tables were followed by two other spherical astronomical tables for the latitude of the Nile Delta town of Damietta, namely ϕ = 31;25°, it may be that the azimuth tables were also computed for this parameter. The other functions tabulated for the latitude of Damietta were the solar altitude in the prime vertical $h_0(\lambda)$ (**4.8.7**) and the (? solar altitude in the ?) azimuth of the qibla $h_q(\lambda)$. Elsewhere in the former Berlin manuscript (fols. 58v-61v) Hartner noted various other spherical astronomical tables for latitude 31;25° attributed to the mid-17th-century astronomer Quṭb al-Dīn al-Maḥallī. A more complete set of prayer-tables for Damietta by this individual is contained in MS Cairo DM 106: see **II-8.6**.

The possibility that the main azimuth tables were simply those of Ibn Yūnus for Cairo (**5.1.1**) cannot be excluded. In the same manuscript (fols. 71r-81r) Hartner noted some 20 pages of "*Tafeln des Vorübergangs*" for arguments 2 to 22 (degrees?) attributed to an individual named ʿAbd al-Rāziq al-Munāwī: perhaps these were an incomplete set of tables displaying the time since sunrise or the hour-angle.

5.2 Tables of solar azimuth as a function of solar longitude and solar altitude, for a specific latitude

5.2.1 Anonymous: Cairo

MS Cairo DM 1108,9 (fols. 35v-59r, copied 1053 H [=1642/43]) contains an anonymous set of tables of the function:

$$a(\lambda,h)$$

for Cairo. The tables begin with those for λ = 1° and go up to λ = 90° and then run from λ = 181° up to λ = 270°, so that no values are given for the equinoxes – see **Fig. 5.2.1**. The anonymous compiler overlooked the fact that the advantage in having separate pages of tables of a(h) for different longitudes enables the equinoctial values to be displayed conveniently (**2.2**) and he also overlooked Ibn Yūnus' special table of a(h) at the equinoxes. The entries in the tables in MS Cairo DM 1108,9 vary little from the corresponding entries in Ibn Yūnus' tables of a(h,λ), from which they were no doubt lifted.

5.3 Tables of solar azimuth as a function of meridian altitude and instantaneous altitude

The simplest method for calculating a(H,h) involves first finding the Sine of the altitude in the prime vertical (**4.8**) corresponding to each value of H, thus (*cf.* **F13**):

$$\mathrm{Sin}\ h_0(H) = R\ \mathrm{Sin}\ (\ H - \bar{\phi}\ )\ /\ \mathrm{Sin}\ \phi\ .$$

Then the azimuth is given by (*cf.* **F16**):

$$a(H,h) = \mathrm{arc\ Sin}\ \{\ [\ \mathrm{Sin}\ h - \mathrm{Sin}\ h_0(H)\ ] \cdot \mathrm{Tan}\ \phi\ /\ \mathrm{Cos}\ h\ \}\ .$$

[4] *Cf.* Hartner & Ruska, "Katalog", pp. 55-56.

Note that a(H,h) is independent of ε. Only one Islamic table of this function has come to my attention.

5.3.1 al-Khalīlī (Damascus)
MS Dublin CB 4091 of al-Khalīlī's set of minor auxiliary tables for timekeeping by the sun (**9.4**) also contains a set of tables of the function:

a(H,h)

computed to two digits for the domains:

H = 33°, 34°, ... , 80° and h = 1°, 2°, ... , H.

The underlying latitude is 33;30° (Damascus) and number of entries is about 2,700. Horizontal differences are also tabulated and labelled positive or negative by the Arabic letters *d* (for *zā'id*, increasing) or *ṣ* (for *nāqiṣ*, decreasing). See further **II-10.3** on al-Khalīlī's minor corpus.

5.4 Tables of solar azimuth as a function of time and solar longitude, for a specific latitude

The solar azimuth tables noted in this section were probably compiled from the corresponding solar altitude tables listed in **4.1**. The following list is not exhaustive.

5.4.1 Pseudo-al-Khwārizmī (Ḥabash): various latitudes
See **4.1.1** on the tables of a(T) for each seasonal hour at the solstices computed for various latitudes attributed to al-Khwārizmī but most probably due to Ḥabash.

5.4.2 al-Battānī: Raqqa
See **4.1.2** on the small table of the same kind computed for the longitude of Raqqa and contained in MS Escorial ár. 908 of the *Zīj* of al-Battānī.

5.4.3 Ibn al-Ādāmī or Saʿīd ibn Khafīf al-Samarqandī: Baghdad
See **4.2.3** on the tables of a(T,λ) for each seasonal hour and each zodiacal sign computed for Baghdad in the treatise on sundials by Ibn al-Adamī or al-Samarqandī.

5.4.4 al-Maqsī: Cairo
See **4.1.3** on the tables of al-Maqsī for Cairo.

5.4.5 al-Marrākushī: Cairo
See **4.2.4** on al-Marrākushī's tables of the same kind, also for Cairo.

5.4.6 al-Sulṭān al-Ashraf
See **4.1.4** on the tables of a(T) for each seasonal hour at the equinoxes and solstices computed for various latitudes in the Yemen and Hejaz by the Sultan al-Ashraf.

5.5 Tables of solar azimuth at the equinoxes as a function of time, for a specific latitude

Tables of solar azimuth at the equinoxes are useful in sundial construction. The azimuth is related to the time since sunrise by the simple formula:

$$a(T,\phi) = \text{arc Tan } \{ \text{ Sin } \phi \text{ Tan } T \text{ / } R \} \text{ .}$$

5.5.1 Anonymous: Cairo

MS Cairo MM 58,1, fol. 2r, copied *ca*. 1450, of an anonymous set of spherical astronomical tables for Cairo contains a table of the function a(T) at the equinoxes for T = 1°, 2°, ... , 90° based on ϕ = 30;0° (Cairo) – see **Fig. 5.1.1b**. Note that for this latitude:

$$a(T) = \text{arc Tan } \{ \ {}^1\!/_2 \text{ Tan } T \ \} \text{ .}$$

However, the table was probably not compiled using this formula. In the instructions the anonymous compiler notes that one can find the solar altitude at the equinox h(T) using a special table and then use the accompanying table in MS Cairo MM 58 of a(h) at the equinoxes (**5.1.1**) to compute a(T). He gives an example in which the value of h(T) is the same as that for h(T,90°) in the tables of h(T,D) preserved in MS Cairo MM 72 (**4.6.1**) and interpolates in the table of a(h) to derive the value of a(T) in his table.

5.5.2 Ibn Abi 'l-Fatḥ al-Ṣūfī: all latitudes

A more extensive table occurs in MSS Istanbul Hamidiye 874,4, copied 1104 H [= 1692/93], Istanbul Nuruosmaniye 2904, and Cairo DM 196 of a short treatise on sundials written in 878 H [= 1473] by the Egyptian astronomer Ibn Abi 'l-Fatḥ al-Ṣūfī (**9.10**). The treatise is entitled *Nuzhat al-nāẓir fī waḍʿ khuṭūṭ faḍl al-dāʾir*, "Diversion for those Concerned with Marking Hour-angle Curves on Sundials" and the table is called *al-jadwal al-mushtarak*, "the combined (?) table". Like al-Māridīnī's *shabaka* (**9.6**), al-Ṣūfī's table resembles a sexagesimal multiplication table in appearance.[5]

The function tabulated is found by inspection to be:

$$f(x,y) = \text{arc Tan } \{ \ x \cdot \text{Tan } y \text{ / } R \ \} \text{ ,}$$

and the table gives values to two digits for the domains:

$$x = 1, 2, \ldots , 60 \quad \text{and} \quad y = 1°, 2°, \ldots , 90° \text{ .}$$

It is intended to display the solar azimuth at the equinox as a function of time, since:

$$a(T,\phi) \ = \ f(\text{Sin } \phi, T) \text{ ,}$$

although the arguments are labelled *jayb al-ʿarḍ*, "Sine of the latitude", and *faḍl al-dāʾir*, "hour-angle". In MS Cairo MM 61 an incomplete anonymous copy of this table has been appended to an anonymous set of prayer-tables for Cairo, and in MS Cairo MM 67,1 the same table reappears with a set of instructions in Turkish, again without any mention of al-Ṣūfī. The entries in al-Ṣūfī's table for x = 30, which correspond to ϕ = 30°, differ from those in the anonymous table of a(T) for Cairo mentioned in **5.5.1** and are generally more accurate.

[5] See n. 1:22.

5.6 Tables of solar rising amplitude, for a specific latitude

The solar rising amplitude (*saʿat al-mashriq* in Arabic) is determined by the equivalent expressions (*cf.* **F4** and **F14**):

$$\psi(\lambda) = \text{arc Sin } \{ \text{ R sin } \delta(\lambda) / \text{Cos } \phi \} = \text{arc Sin } \{ \text{ Sin } \varepsilon \text{ Sin } \lambda / \text{Cos } \phi \}$$
$$= \text{arc Sin } \{ \text{ Sin [max } \psi(\lambda)] \cdot \text{Sin } \lambda / \text{R } \} .$$

Several tables of $\psi(\lambda)$ for specific latitudes are preserved, occasionally accompanying tables of Sin $\delta(\lambda)$ or Sin $\psi(\lambda)$ (**6.1** and **8.3**).

5.6.1 Anonymous: Baghdad

MS Alexandria 5577J, penned *ca*. 1400 (?), is a copy of the *Zīj* for the Yemen entitled *Taysīr al-maṭālib* ... , "Facilitating the Problems ... ", by the late-13th-century astronomer al-Kawāshī.[6] This work is of considerable interest because it contains material derived from a *zīj* by Ibn Yūnus (**2.1.1**) other than the *Ḥākimī Zīj* and an earlier *zīj* for Baghdad, as yet unidentified. al-Kawāshī also records some observations which he made in Egypt between 1273 and 1284.[7] MS London BL Or. 9116 is another copy of this work which lacks the title folio.

Alongside various simple spherical astronomical tables for the latitudes of Aden and Taiz, al-Kawāshī presents a table of $\psi(\lambda)$ for Baghdad. There are no other tables for Baghdad in his *Zīj*. Values of $\psi(\lambda)$ are given to three digits for each degree of λ and are based on the parameters ϕ = 33;0° (Baghdad) and ε = 23;33°. The same table occurs in MS Berlin Ahlwardt 5773/5776,1 (Mq. 98,16+18), fol. 47r, of a fragment of an unidentified *zīj*, probably of Egyptian origin (see also **5.6.4**). The value used for the obliquity and for the latitude of Baghdad suggests a 9th-century source (see **9.1**).

5.6.2 Ibn Yūnus: Cairo

In MS Leiden Or. 143, p. 356, of the *Ḥākimī Zīj* of Ibn Yūnus (**2.1.1**) there is a table of $\psi(\lambda)$ to three digits for the parameters ϕ = 30;0° (Cairo) and ε = 23;35°. Ibn Yūnus' values are extremely accurately computed. He also tabulated Sin $\psi(\lambda)$ for Cairo and Baghdad: see **8.3.1**. See also **5.6.9** on some later and less accurate tables of $\psi(\lambda)$ for Cairo.

5.6.3 Muḥyi 'l-Dīn al-Maghribī: Maragha

The mid-13th-century astronomer Muḥyi 'l-Dīn al-Maghribī[8] tabulated $\psi(\lambda)$ for the latitude of Maragha, where he worked (**4.3.1***). The table is preserved in MSS Dublin CB 3665, fol. 96v, and Meshed Shrine Library 332(103), fol. 99r, of his *Zīj*, and values are given to three digits for each 1° of λ and are based on the parameters ϕ = 37;20,30° (Maragha) and ε = 23;30°.

5.6.4 al-Marrākushī: Cairo

In MS Paris BNF ar. 2508, fol. 29r, al-Marrākushī (**4.2.4**) presents a table of $\psi(\lambda)$ for parameters

---

[6] The existence of this work is noted in Brockelmann, *GAL*, I, p. 625. On al-Kawāshī and his *zīj* see King, *Astronomy in Yemen*, no. 7.

[7] Published in King & Gingerich, "Astronomical Observations from 13th-Century Egypt".

[8] On al-Maghribī see Suter, *MAA*, no. 376; and Kennedy, "*Zīj* Survey", nos. 41 and 108. Another *zīj* for Maragha by al-Maghribī is preserved in Cairo: see *Cairo ENL Catalogue*, I, *sub* MM 188, *etc*.

ϕ = 30;0° (Cairo) and ε = 23;35°. The values are given to two digits for each 5° of λ and are intended to serve the construction of the markings on a horizontal dial (an orthogonal projection of the celestial sphere onto the plane of the horizon). They are not based on the corresponding values of Ibn Yūnus (**5.6.2**). Yet another table based on the same parameters but now with values to two digits for each 1° of λ is found in MS Berlin Ahlwardt 5773/5776,1 (Mq. 98,16+18), fol. 25v, of an Egyptian *zīj* (**5.6.1**).

5.6.5 al-Mizzī: Damascus

The prayer-tables of al-Mizzī, preserved in MS Cairo MM 62 (**2.1.3**), display $\psi(\lambda)$ for Damascus (fol. 7v). Values are carefully computed to two digits for each 1° of λ and are based on al-Mizzī's distinctive parameters ϕ = 33;27° (Damascus) and ε = 23;33°.

5.6.6 al-Khalīlī: Damascus

Several copies of the later prayer-tables for Damascus compiled by al-Khalīlī (**2.1.4**), for example MS Paris BNF ar. 2558, fol. 51v, contain a table of $\psi(\lambda)$ with values carefully computed to two digits for each degree of λ and parameters ϕ = 33;30° (Damascus) and ε = 23;31°. al-Khalīlī also tabulated Sin $\psi(\lambda)$ (**6.1.2** and **8.3.2**).

5.6.7 Anonymous: Tunis

MS Berlin Ahlwardt 5724, fols. 29v-30r, of the main corpus of tables for Tunis (**2.3.5**) contains tables of both $\psi(\lambda)$ and Sin $\psi(\lambda)$ carefully computed to two digits for each degree of λ and based on the parameters ϕ = 37;0° (Tunis) and ε = 23;35°. See also **8.3.3** on the latter.

5.6.8 Anonymous: Isfahan

MS Istanbul Topkapı B 411, an encyclopedia compiled *ca*. 1413 for Iskandar Sulṭān ibn ʿUmar Shaykh of Isfahan, contains an anonymous collection of astronomical and astrological tables. Amongst these tables is a set of prayer-tables for parameters ϕ = 32;25° (Isfahan) and ε = 23;30°, including a table displaying $\psi(\lambda)$ for each 1° of λ. See further **II-3.15** on the other tables in this set and also **VIIc-4c** on the latitude used for Isfahan, which was apparently derived by al-Khāzimī in the 11th century.

5.6.9 Miscellaneous: Cairo

In numerous sets of late Egyptian prayer-tables for Cairo dating from after the 16th century (**4.8.4**), such as MS Cairo MM 15 of the prayer-tables of al-Lādhiqī, or MS Alexandria 4441J of the prayer-tables of Ibn Abī Rāya, or MS Cairo MM 81 of the prayer-tables of al-Ikhṣāṣī, the function $\psi(\lambda)$ is tabulated to two digits for the parameters of Ibn Yūnus (**5.6.2**). However, most of these tables are corrupt and the errors seem to represent an attempt to recompute the function rather than to be mere copyists' errors. These tables are characterized by the entry 23;29° for λ = 60°. Note that:

$$\psi = \varepsilon \text{ for } \lambda = 60° \text{ and } \phi = 30° ,$$

so that the entry should be 23;35°. Also the entry for λ = 90° is 27;25° rather than the accurate value 27;31°. See **4.8.4** on some related tables of the solar altitude in the prime vertical.

5.6.10 al-Asyūṭī: Assiut

In MS Cairo DM 188 of the prayer-tables for Assiut prepared by al-Asyūṭī (**4.8.5**), there is a table of $\psi(\lambda)$ with values to two digits for each 1° of λ and parameters ϕ = 27;0° (Assiut) and ε = 23;35°.

5.6.11 al-Dīsṭī: Lattakia

MS Aleppo Awqāf 911 of al-Dīsṭī's prayer-tables for Lattakia (**4.8.7**) contains a table of $\psi(\lambda)$ with values to two digits for each 1° of λ and parameters ϕ = 34;30° (Lattakia) and ε = 23;30°.

5.6.12 Anonymous: Alexandria

MS Cairo DM 1207 of the corpus of tables for Alexandria (**4.8.8**) contains a table of $\psi(\lambda)$ for parameters ϕ = 31;0° (Alexandria) and ε = 23;35°.

5.6.13 Yūsuf Kilārjī: Crete

MS Cairo DM 834 of Yūsuf Kilārjī's prayer-tables for Crete (**4.8.9**) contains a table of $\psi(\lambda)$ for parameters ϕ = 35;30° (Crete) and ε = 23;35°.

5.7 Tables of rising amplitude as a function of declination, for a specific latitude

Only one Islamic table for finding the rising amplitude of non-circumpolar stars has come to light. The underlying formula is (*cf.* **F14**):

$$\psi(\ \Delta) = \text{arc Sin } \{ \text{ R Sin } \Delta \ / \text{ Cos } \phi \ \} \ .$$

Note that al-Khalīlī's auxiliary function $f_{\phi}(\theta)$ (**9.5**) displays Sin $\psi(\Delta,\phi)$ for each 1° of both arguments.

5.7.1 Anonymous: Cairo

MS Cairo MM 43, penned *ca*. 1450, is an Egyptian copy of al-Khalīlī's universal auxiliary tables (**6.4.3** and **9.5**), contains an anonymous table of the function $\psi(\Delta)$ computed for ϕ = 30° (Cairo). Values are given to two digits for each 1° of Δ up to 60° (= $\bar{\phi}$).

5.8 Tables of maximum solar rising amplitude as a function of latitude

In *Almagest* VI.12 Ptolemy presented a table of the maximum solar rising amplitude for the seven climates. The values for each zodiacal sign are given in degrees and minutes and are arranged in concentric circles for each climate, the so-called "horizon diagrams".[9] The underlying value of ε is 23;51,20°. I have not consulted any of the Arabic versions of the *Almagest* in which this diagram is surely reproduced, but in three sets of Islamic sources we find similar tables.

---

[9] See Ptolemy, *Almagest*, transl. Gerald Toomer, p. 320; and Neugebauer, *HAMA*, I, pp. 37-39. On the climates see n. 1:11.

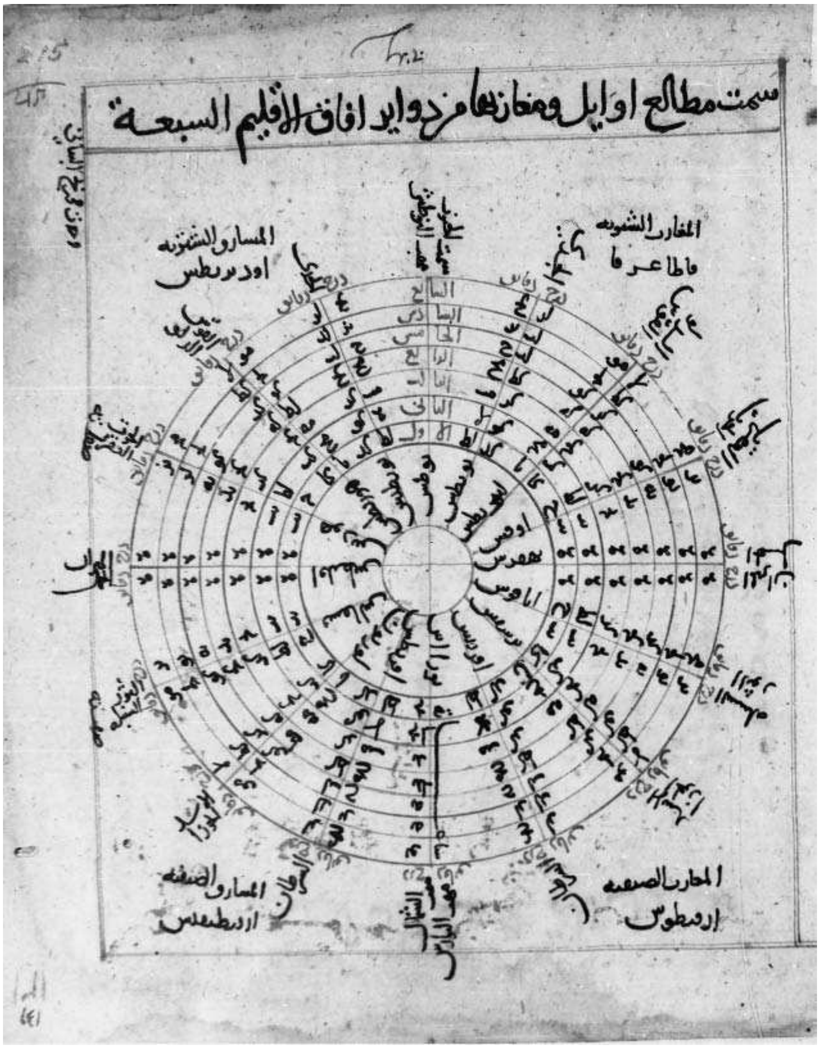


Fig. 5.8.1: A table of solar rising amplitudes in the Ptolemaic tradition. Values are given for each pair of zodiacal signs with the same amplitude (and declination), radially for the first climate out to the seventh. The (false) impression given is of directions around the horizon, not least because the 16 winds are displayed in the inner ring. [From MS Berlin Ahlwardt 5751, p. 215, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

5.8.1 al-Battānī (Raqqa)

In MS Escorial ár. 908, fol. 196v, of the *Zīj* of al-Battānī[10] (**4.1.2**), there is a horizon diagram displaying solar rising amplitudes for the seven climates as in the *Almagest* but with values based on ε = 23;35°. al-Battānī's table is reproduced in MS Berlin Ahlwardt 5751 (Mq. 101,1), p. 215, copied *ca*. 1300, amidst various tables appended to a copy of the *Zīj* of Kūshyār – see **3.2.1** and **Fig. 5.8.1** – and in MS Milan Ambrosiana C86, fol. 167v, of a small collection of extracts (on fols. 167r-168v) from al-Battānī's *Zīj* copied in the Yemen.

[10] Nallino, *al-Battānī*, II, p. 92.

5.8.2 Abu ’l-Wafāʾ (Baghdad)

MS Paris BNF ar. 2494, from the 12th century, is an incomplete copy of the *Zīj* by Abu ’l-Wafāʾ al-Būzajānī (*fl*. Baghdad *ca.* 970) entitled *al-Majisṭī* after Ptolemy’s *magnum opus*.[11] Unfortunately all of Abu ’l-Wafāʾ’s tables are missing from this manuscript. On fol. 28r he states that he computed a table of the maximum solar rising amplitude for each 0;15° of latitude up to 60°.

5.8.3 Sanjar al-Kamālī (Shiraz)

In MS Paris BNF supp. pers. 1488, fol. 100r, of the *Ashrafī Zīj* (**2.3.3**) there is a table of max $\psi(\phi)$ with entries to *four* sexagesimal digits for each degree of ϕ from 1° to 60°, based on ε = 23;35°.

---

[11] On Abu ’l-Wafāʾ (Suter, *MAA*, no. 167) see Sezgin, *GAS*, VI, pp. 222-224, and the references there cited, also the article by A. P. Youschkevitch in *DSB*.

CHAPTER 6

TABLES OF AUXILIARY FUNCTIONS FOR TIMEKEEPING

6.0 Introductory remarks

The computation of time from solar altitude (*cf.* **F9-12**) is considerably facilitated if the function:

$$B(\lambda) = \mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi\ /\ R$$

is tabulated first. This function B is called in Arabic *al-aṣl al-muṭlaq*, "the absolute base", or simply *al-aṣl*, "the base". Tables of the function:

$$G(\lambda) = R^2\ /\ [\mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi]$$

would be yet more useful, but Muslim astronomers generally prefered to use B(λ). To compute B(λ) it is necessary either to compute Cos δ(λ) or Sin H(λ). Given a table of Sin H(λ), B(λ) can be found using (*cf.* **F8**):

$$B(\lambda) = {}^1/_2\ \{\ \mathrm{Sin}\ H(\lambda) + \mathrm{Sin}\ H(\lambda^*)\ \}$$

This is clearly valid since:

$$H(\lambda) = \bar{\phi} + \delta(\lambda) \quad \text{and} \quad H(\lambda^*) = \bar{\phi} - \delta(\lambda)$$

and (in modern notation):

$$\cos\delta\ \cos\phi = {}^1/_2\ \{\ \sin(\ \bar{\phi} + \delta\ ) + \sin(\ \bar{\phi} - \delta\ )\ \}.$$

See also **2.1.1**.

Another important function of Islamic timekeeping is:

$$C(\lambda) = \mathrm{Sin}\ \delta(\lambda)\ \mathrm{Sin}\ \phi\ /\ R\ ,$$

called in Arabic *buʿd al-quṭr*, literally "the distance of the diameter". It measures the height of the day-circle centre above the horizon. To compute C(λ) one can either use Sin δ(λ) or B(λ) together with Sin H(λ) since:

$$C(\lambda) = B(\lambda) - \mathrm{Sin}\ H(\lambda^*)\ .$$

Given tables of B(λ) or G(λ) and C(λ) or Sin H(λ), the determination of the hour-angle reduces to an application of the equivalent formulae:

$$\mathrm{Vers}\ t(h,\lambda) = [\mathrm{Sin}\ H(\lambda) - \mathrm{Sin}\ h] \cdot R/B(\lambda) = [\mathrm{Sin}\ H(\lambda) - \mathrm{Sin}\ h] \cdot G(\lambda)$$

$$\mathrm{Cos}\ t(h,\lambda) = [\mathrm{Sin}\ h - C(\lambda)] \cdot R/B(\lambda) = [\mathrm{Sin}\ H(\lambda) - C(\lambda)] \cdot G(\lambda)\ .$$

If we now define the function:

$$H'(H,h) = \mathrm{Sin}\ H - \mathrm{Sin}\ h\ ,$$

called in Arabic *faḍl al-jaybayn*, "the difference between the two sines", the first formula for the hour-angle reduces to:

$$\mathrm{Vers}\ t = H' \cdot R/B = H' \cdot G\ .$$

Likewise if we define the function:

$$b(h,\lambda) = \mathrm{Sin}\ h - C(\lambda)\ ,$$

called in Arabic *al-aṣl al-muʿaddal*, "the modified base", the second formula for the hour-angle reduces to:

$$\text{Cos } t = b \cdot R/B = b \cdot G \ .$$

Similarly the functions:

$B(\Delta) = \text{Cos } \Delta \text{ Cos } \phi / R$, $C(\Delta) = \text{Sin } \Delta \text{ Sin } \phi / R$ and $G(\Delta) = R^2 / [\text{Cos } \Delta \text{ Cos } \phi]$ are useful in computing time from stellar altitudes.

I now present a description of various Islamic tables of Sin δ, Cos δ and Sin H (**6.1**. to **6.3**), B, C, G and related functions (**6.4** to **6.11**), b and H′ (**6.12** and **6.13**), and the inverse trigonometric functions necessary to find the hour-angle once one has used the functions B, C and G or Sin H (**6.14** and **6.15**). Finally I discuss certain star catalogues which present numerical information for each star to facilitate reckoning time by night (**6.16**).

6.1 Tables of the sine of the solar declination

The function Sin δ(λ) (Arabic, *jayb al-mayl*) is easily computed using:

$$\text{Sin } \delta(\lambda) = \text{Sin } \varepsilon \text{ Sin } \lambda / R \ .$$

At least three simple tables of the quantities:

$$n/R \cdot \text{Sin } \varepsilon \quad \text{and} \quad n/R \cdot \text{Cos } \varepsilon \qquad (n = 1, 2, \ldots, R)$$

are attested in the Islamic sources, two computed by Ibn Yūnus (MS Leiden Or. 143, pp. 236-237)[1] and Abu ’l-Wafāʾ (MS Escorial ár. 927, fol. 51r),[2] and another compiled several centuries later by al-Ḥamzāwī (MS Cairo TR 119, pp. 602-603).[3] Such tables for Sin ε can be used to facilitate multiplication by Sin ε in order to generate a table of Sin δ(λ).

In turn, a table of Sin δ(λ) can be used to prepare: (1) tables of δ(λ), which are standard in *zīj*es; (2) tables of the auxiliary function C(λ) for particular latitudes, which are fairly common in corpuses of tables for timekeeping (**6.10**); and (3) tables of $h_0(\lambda)$ and $\psi(\lambda)$ (**4.8**, **5.5** and **8.3**). Such a table can also be used to facilitate the transformation of ecliptic to equatorial coordinates by one of the standard Islamic methods.[4] The following tables of Sin δ(λ) have come to my attention.

6.1.1 Anonymous (Baghdad?)

In MS Berlin Ahlwardt 5750 (Wetzstein 90), copied *ca*. 1300, of the anonymous recension of one of the *zīj*es of Ḥabash (**9.1** and **9.2**) there is a set of tables of:

$$\delta(\lambda), \text{ Sin } \delta(\lambda) \text{ and Cos } \delta(\lambda) \ ,$$

computed to three digits for each degree of λ and based on the parameter ε = 23;35°. The identity of the compiler of these tables is uncertain.

6.1.2 al-Khalīlī (Damascus)

In MS Cairo DM 184, copied in 1171 H [= 1757/58], of the prayer-tables of the Syrian astronomer al-Manāshīrī (b. 1573, d. 1630),[5] the function Sin δ(λ) is tabulated alongside

[1] See King, *Ibn Yūnus*, III.11.4c; and *idem*, “Medieval Islamic Multiplication Tables”, B, pp. 409 and 411.
[2] These tables are mentioned in Kennedy, “*Zīj* Survey”, p. 145. On Abu ’l-Wafāʾ see n. 5:11.
[3] On al-Jamālī Yūsuf ibn Qurqmās al-Ḥamzāwī see *Cairo ENL Survey*, no. C91. On these tables of his see King, “Islamic Multiplication Tables”, B, pp. 409 and 410.
[4] See, for example, King, *Ibn Yūnus*, III.39.1a-b.
[5] On al-Manāshīrī see Brockelmann, *GAL*, I, p. 427; *Cairo ENL Survey*, no. D8; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. pp. 274-275, no. 138.

numerous other spherical astronomical tables for Damascus attributed to al-Khalīlī (**2.1.4**). This particular table, which is based on parameter ε = 23;31°, is also contained in MS Damascus Ẓāhiriyya 9233, p. 156, of the main Damascus corpus for timekeeping, there introduced in the name of al-Ṭanṭāwī (**2.2.5**). See also **6.3.2** and **8.3.2** and **II-11.10** and **11.13**.

6.1.3 Riḍwān Efendī (Cairo)

In MS Istanbul S. Esad Efendi Medresesi 119,1 of the prayer-tables of the late-17th-century Egyptian astronomer Riḍwān Efendī,[6] virtually all of which were lifted from earlier sources, there is a set of tables of:

Sin δ(λ), Cos δ(λ) and Tan δ(λ)

computed to three digits for each degree of λ, and based on the distinctive parameter of Ulugh Beg of Samarqand, namely ε = 23;30,17°. Although Riḍwān Efendī lifted various tables for ε = 23;35° from earlier sources, these for the new obliquity were probably computed by himself. On the other tables see **6.2.6** and **7.1.1**, and on the corpus associated with Riḍwān Efendī see **II-7.10**.

6.1.4 Taqi 'l-Dīn (Istanbul)

In MS Istanbul Nuruosmaniye 2930, fol. 23r, of the *zīj* entitled *Sidrat muntaha 'l-afkār*, "The Lotus Tree in the Seventh Heaven of Reflection", by Taqi 'l-Dīn (**2.3.6**), there is a set of tables of the functions:

δ(λ), Sin δ(λ) and Tan δ(λ)

computed to four sexagesimal digits and based on the parameter ε = 23;28,54° derived from his "new Murād Khān observations". See also **6.4.8** below.

6.2 Tables of the cosine of the solar declination

To compute Cos δ(λ) for given λ the easiest procedure is to compute δ(λ) first and then use a Sine table to find Cos δ(λ). In the Islamic sources we find tables of both Cos δ(λ) and Cos δ($\bar{\lambda}$). The former can be used to compute B(λ) for a particular latitude (**6.4**) and the latter is of limited use in certain operations in spherical astronomy, such as the conversion from ecliptic to equatorial coordinates.[7] The following tables of Cos δ(λ) or Cos δ($\bar{\lambda}$) are known.

6.2.1 Ḥabash (Baghdad / Damascus)

One of the auxiliary functions in the *Jadwal al-taqwīm* of Ḥabash (**9.1**) is simply Cos δ($\bar{\lambda}$). Values are given to three digits for each degree of λ, computed for ε = 23;35°. Ḥabash's table reappears in MS Paris BNF supp. pers. 1488, fol. 207v, of the early-14th-century Persian *Ashrafī Zīj* (**2.3.3**) where the function is referred to as *taʿdīl al-buʿd al-awwal ʿan muʿaddil al-nahār*

---

[6] On Riḍwān Efendī (**I-6.1.3**) see Dorn, "Drei arabische Instrumente", pp. 32-34; Brockelmann, *GAL*, II, p. 47l, and SII, p. 487, Azzawi, *History of Astronomy in Iraq*, pp. 319-320; Mayer, *Islamic Astrolabists*, pp. 81-82, Kennedy, "*Zīj* Survey", no. X209; and Ihsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 377-384, no. 246. See also **II-7.10**.

[7] *Cf.* Jensen, "Abū Naṣr's *Table of Minutes*", pp. 6-7 and 14.

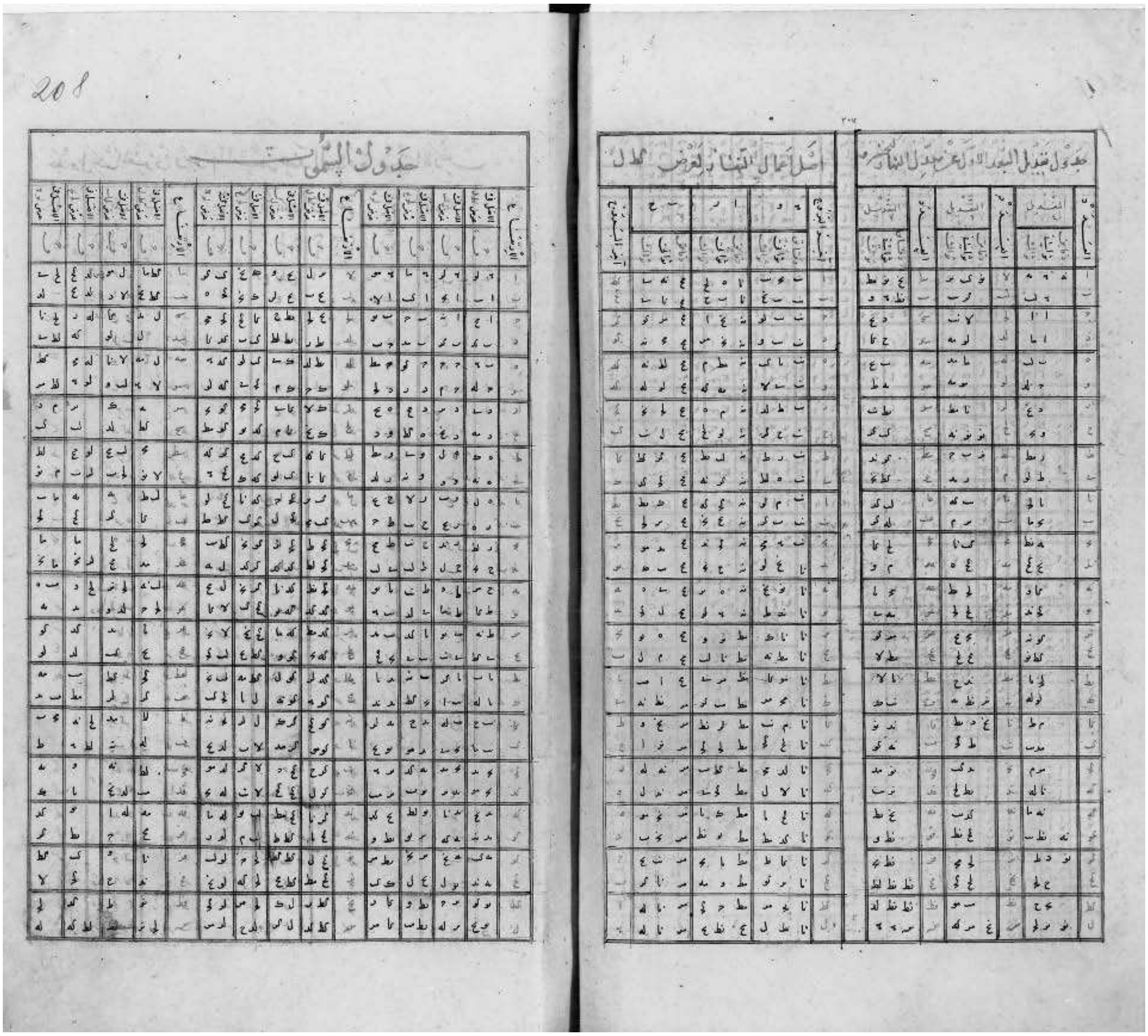


Fig. 6.2.1: Ḥabash's table of the cosine of the declination of the complement of the solar longitude, together with a table of the absolute base for latitude 29;30° (**6.4.5**), and tables of the auxiliary azimuth function for various latitudes in Iran (**8.1.2**). [From MS Paris BNF supp. pers. 1488, fol. 207v, courtesy of the Bibliothèque Nationale de France.]

and is attributed to *x-<u>kh</u>-s-r-w*, where *x* denotes a carrier which could serve as a *bāʾ*, *tāʾ* or *thāʾ*, *nūn* or *yāʾ*, the combination corrupted from *ḥ-b-<u>sh</u>* for Ḥabash! – see **Fig. 6.2.1**.

6.2.2 Anonymous (Baghdad ?)

In MS Berlin Ahlwardt 5750 of the anonymous recension of a *zīj* of Ḥabash (**6.1.1** and **6.2.1**) there are tables of Cos $\delta(\lambda)$ and Cos $\delta(\bar{\lambda})$ (**9.2**), both computed to three digits for each degree of λ and based on $\varepsilon = 23;35°$.

6.2.3 Abū Naṣr (Khwarizm)

One of the auxiliary functions tabulated by the late-10th-century scholar Abū Naṣr in his *Jadwal al-daqāʾiq* (**9.3**) is simply Cos $\delta(\bar{\lambda})$. Values are given to four digits for each degree of λ and are based on the Ptolemaic parameter ε = 23;51,20°.

6.2.4 Anonymous (Cairo)

In one copy of al-Bakhāniqī's edition of the Cairo corpus (**2.1.1**), namely MS Istanbul Nuruosmaniye 2925, the main tables for timekeeping are followed by tables of Cos $\delta(\lambda)$ (called *jayb tamām al-mayl*) and $C(\lambda)$ (**6.10.1**). Entries are given to two digits and the underlying value of ε is 23;35°. The same pair of tables occurs in MS Alexandria 4441J of the prayer-tables for Cairo of Ibn Abī Rāya (*fl. ca.* 1710 – see **II-7.9**), with many copyist's errors.

6.2.5 Anonymous (Tunis)

MS Cairo DM 689 of the anonymous Tunisian auxiliary tables for timekeeping (**9.7**) contains a set of tables of Sin $H(\lambda)$ computed for various latitudes (**6.3.3**). One of these is for latitude 0° (called *jayb al-ghāya fī kull balad lā ʿarḍ lahu*) and the tabulated function is thus Sin [90°-$\delta(\lambda)$] = Cos $\delta(\lambda)$. Values are given to two digits for each degree of λ and are based on ε = 23;35°.

6.2.6 Riḍwān Efendī (Cairo)

MS Istanbul S. Esad Efendi Medresesi 119, fol. 59v, of the prayer-tables of Riḍwān Efendī (**6.1.3**) contains a table of Cos $\delta(\lambda)$ for Cairo to three digits based on ε = 23;30,17°.

6.2.7 ʿAbdallāh al-Ḥalabī (?) (Aleppo)

MS Aleppo Awqāf 943 contains a set of prayer-tables for Aleppo attributed to ʿAbdallāh al-Ḥalabī (**II-11.14**), who was *muwaqqit* at the Umayyad mosque in Aleppo about 1750. One of the tables presented by al-Ḥalabī, but not necessarily computed by him, is of the function Cos $\delta(\lambda)$, with values displayed for each degree of λ and computed to three digits for ε = 23;30°.

6.3 Tables of the sine of the meridian altitude, for a specific latitude

Tables of $H(\lambda)$ for a particular latitude are standard in *zīj*es. The underlying formula is (*cf.* **F1**):

$$H(\lambda) \;=\; \bar{\phi} + \delta(\lambda) \;.$$

Tables of Sin $H(\lambda)$ (Arabic, *jayb al-ghāya*) are less common and can be used either to compute the function $B(\lambda)$ (**6.4**) or to find the hour-angle $t(h,\lambda)$ using the formula (*cf.* **F9**):

$$\text{Vers}\ t(h,\lambda) = [\ \text{Sin}\ H(\lambda) - \text{Sin}\ h\ ] \cdot R\ /\ B(\lambda)\ .$$

6.3.1 al-Khaṭāʾī (Cairo)

In MS Vatican Borg. ar. 217,2, of the auxiliary tables of al-Khaṭāʾī (**6.15.1**) the function Sin $H(\lambda)$ is tabulated to three digits for each degree of λ and parameters ϕ = 30;0° (Cairo) and ε = 23;35°. See also **6.4.2**.

6.3.2 al-Khalīlī: Damascus

In MS Cairo DM 184 of the prayer-tables of al-Munāshirī (**6.1.2**) the function Sin H(λ) is tabulated for φ = 33;30° (Damascus) and ε = 23;31°. These tables are attributed to al-Khalīlī (**2.1.4**).

6.3.3 Anonymous: Tunis

In MS Cairo DM 689 of the anonymous Tunisian auxiliary tables (**9.7**) Sin H(λ) is tabulated to two digits for each degree of λ and a total of 23 latitudes:

21;40° (Mecca), 25°(Medina), 0° (**6.2.5**), 30°, 30;30°, ... , 38°,
as well as 33;40° [Fez], 36;40° [Tunis] and 37;10° (locality ?) .

The latitude 37;10° is curious, for no locality to the north of Tunis comes to mind, and it may be intended as an alternative value for that city (for which 37;0° is used elsewhere). The underlying value of ε is 23;35°. The table for φ = 36;40° is also contained in MS Berlin Ahlwardt 5724 (**2.3.5**).

6.3.4 Anonymous: Tlemcen

On the table of Sin H(λ) for parameters φ = 35;0° (Tlemcen) and ε = 23;35° in MS London BL Or. 411,2 of an anonymous commentary on the astronomical poem of al-Jādarī see **6.15.2**.

6.3.5 Anonymous: Aleppo

One of the functions tabulated by ʿAbdallāh al-Ḥalabī (**6.2.7**) in his prayer-tables for parameters φ = 36;0° (Aleppo) and ε = 23;30° is Sin H(λ). al-Ḥalabī also displays values of the difference between Sin H(λ) and the Sines of the solar altitudes at various times of prayer – see **II-11.14**.

6.4 Tables of the absolute base for timekeeping by the sun, for a specific latitude

Several sources listed below contain tables of B(λ) for particular latitudes. The function is invariably called *al-aṣl al-muṭlaq* or simply *al-aṣl*. Note that in al-Khalīlī's first set of auxiliary tables (**9.4** and also **9.7** and **9.11**) the function $^1/_2$B(φ,λ) is tabulated for each degree of both arguments.

6.4.1 al-Nasawī: Rayy

MSS Cairo ṬFF 11, copied *ca*. 1400, and London BL Add. 27261, copied in 814 H [= 1411], contain a Persian treatise entitled *Rawḍat al-munajjimīn*, "The Garden of the Astronomers", compiled by Shāhmardān Rāzī,[8] a student of the astronomer-mathematician Abu 'l-Ḥasan ʿAlī ibn Aḥmad al-Nasawī who worked in Rayy in the late 10th century, but whose major astronomical work is lost.[9] One of the tables in the *Rawḍa* (fol. 78v of the Cairo manuscript) is specifically attributed to al-Nasawī: see **Fig. 6.4.1**. The function tabulated is simply B(λ)

---

[8] On Shāhmardān Rāzī see Storey, *PL*, II:1, p. 45, no. 81; Sezgin, *GAS*, VI, p. 246; and *Cairo ENL Survey*, no. B91.

[9] On al-Nasawī (Suter, *MAA*, no. 214) see Sezgin, *GAS*, V, pp. 345-348, VI, pp. 245-246, and VII, pp. 182, 402, and 410-411, also III, p. 311, and the article by A. S. Saidan in *DSB*.

Fig. 6.4.1: This table of the auxiliary function called the "base" (*aṣl*) or later the "absolute base" (*al-aṣl al-muṭlaq*), computed in the 10th century by a scholar well versed in mathematical astronomy as well as pure mathematics, rather than by a scholar specializing in astronomical timekeeping, shows that Muslim interest in practical astronomy started early. [From MS Cairo ṬFF 11, fol. 78v, courtesy of the Egyptian National Library.]

and is referred to as *aṣl* in the text. The values are computed to three digits for each degree of λ with first differences (*tafāḍul*) to two digits. The latitude 35;34,40° is mentioned in the accompanying text, and is attested for Rayy, being associated with the observations of al-Khujandī in the year 994.[10] However, the text further states that the Cosine of this latitude is 48;47,52, whereas the accurate value is 48;47,58,31. In fact, the value in the text was derived

[10] On the derivation of this value of the latitude of Rayy see Schirmer, "Studien zur Astronomie der Araber", pp. 63-79.

by *linear interpolation* (!) between the entries for 35° and 36° given to three (or perhaps four) significant digits. If we, not unreasonably, assume that this erroneous value of the Cosine was actually used to compile the table, Benno van Dalen has shown that the values used for Cos $\delta(\lambda)$ were extremely accurate for ε = 23;35,44 plus or minus 1 second. This unfortunately does not correspond to the obliquity associated with the first derivation of the latitude 35;34,40°, namely, 23;32,21°. We note in passing that 23;35,43,57,41 is the arc Sine of 24;1: this may somehow be relevant to explaining the makeup of this table.

Other spherical astronomical tables in the *Rawḍa*, for example, those for $\alpha_\phi(\lambda)$ (fol. 59v of the Cairo manuscript), are computed for ϕ = 36°, a popular medieval value for Rayy and also for the 4th climate.[11]

6.4.1* al-Baghdādī (?): Baghdad

In MS Paris BNF ar. 2486, fol. 236r, of the *Zīj* of al-Baghdādī (**2.3.1**), there is a table of $B(\lambda)$ computed to three digits for each degree of λ and based on the parameters ϕ = 33;25° (Baghdad) and ε = 23;35°.

6.4.1 Early anonymous (N. Iran)**

MS Istanbul U.B. A 314, copied *ca*. 1600 (?), is a precious collection of early Islamic treatises on mathematics, astronomy and astronomical instruments which has only in 2001 attracted the attention it merits, and a facsimile is now available in published form.[12] I refrain from commenting on the treatises at this point, confident that the next generation of scholars will occupy themselves with this manuscript, for there is something in it for everyone. In fact, there is one table for spherical astronomy in the collection, and it cannot be attributed or dated, but I suspect that it is very early. Certainly it is very corrupt. It occurs on fol. 47v, quite out of place, and shows values to three digits of three functions labelled *al-aṣl*, *jayb taʿdīl al-nahār*, and *taʿdīl al-nahār*, for each degree of the argument, which is labelled *buʿd muʿaddal al-nahār*. The "title " of the table translates: "Multiply the Cosine of the local latitude by the Cosine of the declination (lit., distance from the celestial equator) and divide it by 60 and the result is the base (*al-aṣl*)." The three functions:

$$B(\Delta), \quad \text{Sin } d(\Delta) \quad \text{and} \quad d(\Delta)$$

appear to be computed for a specific latitude of about 38°, which would serve a locality in Northern Iran. The treatise on planetary and spherical astronomy immediately preceding this table mentions Isfahan several times, so it is unrelated. There are no entries for the second and third functions beyond Δ = 55°, a feature which shows how corrupt the table is (see further **7.2.0***).

6.4.2 al-Khaṭāʾī: Cairo

In the auxiliary tables of al-Khaṭāʾī in MS Vatican Borg. ar. 217,2 (**6.15.1**), there is a table of $B(\lambda)$ to three digits for each degree of λ, based on the parameters ϕ = 30;0° (Cairo) and ε = 23;35°. A table of Sin $H(\lambda)$ to three digits accompanies this one of $B(\lambda)$ (**6.3.1**) and al-

[11] Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 284, and King, "Geography of Astrolabes", pp. 6-9.

[12] See Sezgin *et al*., *Scientific Manuscript*. On the importance of this see the introduction by Fuat Sezgin (pp. v-x) and also the remarks in **VIIc** below.

Khaṭāʾī points out the fact that the latter can be computed from the former using the relation noted in **6.0**. Indeed the errors in the table of B(λ) can be explained by the fact that the values of Sin H(λ) were derived from H(λ) by linear interpolation in an accompanying table of Sin θ which displays the function to three digits for each integral value of θ.

In MS Berlin Ahlwardt 5710, fol. 24r, of the prayer-tables of Riḍwān Efendī (**6.1.3**) the same table of B(λ) occurs with a few additional copyists' mistakes, alongside a more accurately computed table of C(λ). Note that C(λ) is easier to compute than B(λ) for φ = 30° (see further **6.10.1**).

6.4.3 Anonymous: Cairo

In MS Cairo MM 43, penned *ca*. 1450, an Egyptian copy of al-Khalīlī's universal auxiliary table (**9.5**), numerous other spherical astronomical functions are tabulated for the latitude of Cairo. There is a table of B(λ) computed to two digits for each 0;15° of λ, based on the same parameters as the one described in the previous section. Likewise in MS Cairo MM 241 of the auxiliary tables of Ibn al-Mushrif (**9.8**), there is a table of B(λ) for Cairo computed to two digits for each degree of λ.

6.4.4 Anonymous: Mecca

In MS Cairo MM 68, copied *ca*. 1500, of an anonymous set of prayer-tables for Mecca (**II-6.10**) the function B(Δ) is tabulated to two digits for each 0;5° of Δ up to Δ = ε = 23;35°. The underlying value of φ is 21;0°. See **6.6** below.

6.4.5 Sanjar al-Kamālī: Shiraz

In MS Paris BNF supp. pers. 1488, fol. 207v, of the *Ashrafī Zīj* (**2.3.3**) there is a table of B(λ) for each degree of λ computed to three digits for the parameters φ = 29;30° (Shiraz) and ε = 23;35°. The function is referred to as *aṣl aʿmāl al-nahār*, "the basic function for operations (of timekeeping) by day" – see **Fig. 6.2.1**.

6.4.6 Anonymous: Anatolia

MS Istanbul Süleymaniye 1037/32 (fols. 282v-285v) of an anonymous set of Seljuk prayer-tables for an unspecified locality in Anatolia (**3.3.3**) contains additional tables of B(λ) to two digits for each degree of λ. The underlying parameters are φ = 38;30° and ε = 23;35°.

6.4.7 Anonymous: Tunis

In MS Berlin Ahlwardt 5724, fol. 42r (**2.3.5**), and also in MS Princeton Yahuda 147c, p. 113, of the Tunisian *Quṣʿī Zīj* of Ḥusayn Quṣʿa (**4.3.6**) there is a table of B(λ) computed to two digits for each degree of λ and based on the parameters φ = 36;40° (Tunis) and ε = 23;35°. In the Berlin copy the function is referred to as *aṣl faḍl dāʾir al-shams*, "the base for (computing) the hour-angle of the sun". Both sources contain a table of Sin H(λ) for Tunis with which that of B(λ) might have been compiled (**6.3.3**).

6.4.8 Taqi 'l-Dīn: Istanbul

MS Istanbul Esat Efendi 1976, fol. 25r, of the *Zīj* of Taqi 'l-Dīn entitled *Jarīdat al-durar* (**6.1.4**) contains a table of B(λ) computed to four significant *decimal* digits. The latitude served is φ

= 40;58° (Istanbul), which is stated to underlie a table of $\alpha_\phi(\lambda)$ computed to four sexagesimal digits elsewhere in this particular manuscript. In his earlier works Taqi al-Dīn used 41;15° for the latitude of Istanbul (*e.g.*, in the geographical coordinates attributed to him in MS Istanbul Kandilli 441 and in his treatise *Rayḥānat al-rūḥ* on sundials in MS Vatican ar. 1424). The underlying value of ε is 23;28,54°, the distinctive parameter attributed to Taqi 'l-Dīn in other sources. See also **6.10.6** and **7.1.12** on other tables by the Istanbul astronomer.

6.4.9 Anonymous: Istanbul

MS Istanbul UL T1824,1 (fols. 3r-9r) of a set of anonymous prayer-tables for Istanbul contains tables of B(λ) computed to two digits for each degree of λ for the parameters φ = 41;15° (Istanbul) and ε = 23;30°. See also **6.5.2** and **6.10.7** and **II-12.8** for more information on these prayer-tables.

6.4.10 al-Dīsṭī: Lattakia

MSS Aleppo Awqāf 911 and Leiden Or. 2808(2) of al-Dīsṭī's tables for Lattakia (**4.8.7**) include tables of B(λ) and C(λ) with values to three digits for each degree of λ based on the parameters φ = 34;30° (Lattakia) and ε = 23;30°. See also **6.10.8**.

6.4.11 al-Jannād: various localities in the Western Maghrib

MS Cairo TR 338,2, penned *ca*. 1850, is the only known copy of a set of prayer-tables for various latitudes in the Western Maghrib (*i.e.*, what is now Morocco) compiled *ca.* 1775 by Abū ʿAbdallāh al-Jannād, an individual whose name is new to the modern literature. Details are given in **II-13.5**. Amongst the tables are some for B(λ) and C(λ) with entries to two digits for each degree of λ. The latitudes underlying al-Jannād's tables are 31° (Marrakesh, *etc*.); 34° (Meknes, Fez, *etc*.); 30° (Sijilmasa, *etc*.).

6.4.12 al-Maḥallī: Damietta

In the prayer-tables for Damietta compiled by Quṭb al-Dīn al-Maḥallī about 1650 (**5.1.4**) there are tables of B(λ) and C(λ) computed to two digits for each degree of λ and based on φ = 31;25° (Damietta) and ε = 23;35°. Of the various known copies of al-Maḥallī's prayer-tables only MS Cairo DM 1014, fol. 11r, contains these auxiliary tables.

6.4.13 Anonymous: Alexandria

In an anonymous set of prayer-tables preserved in MS Cairo DM 1207 (**4.8.8**) there are tables of B(λ) and C(λ) computed to two digits for each degree of λ and based on φ = 31;0° (Alexandria) and ε = 23;35°. The possiblity that these tables were intended for Maḥalla rather than Alexandria cannot be excluded.

6.4.14 Yūsuf Kilārjī: Crete

In MS Cairo DM 834 of the prayer-tables for Crete compiled by Yūsuf Kilārjī (**4.8.9**) there are tables of B(λ) and C(λ) computed to three digits for each degree of λ and based on φ = 35;30° (Crete) and ε = 23;35°.

6.4.15 Anonymous: Tlemcen

A table of the function $^1/_2$ B(λ) computed for parameters φ = 35;0° (Tlemcen) and ε = 23;35° (?) is contained in the anonymous tables for Tlemcen in MS London BL Or. 411,2. See further **6.15.2** on the main tables in this source.

6.4.16 Anonymous: Sfax

MS Cairo K 7584,1, copied in 1257 H [= 1841/42] in an elegant Maghribi hand, contains an extensive fragment of an anonymous treatise on spherical astronomy compiled in Sfax (see further **II-13.8**). The work was written after the 15th century since the obliquity used is that of Ulugh Beg and before 1257 H [= 1841/42] when the manuscript was copied. The work contains a few tables of various spherical astronomical functions including B(λ) and C(λ) for parameters:

φ = 34;48° (Sfax) and ε = 23;30,17° .

See further **6.10.13** and **8.4.2** on other tables in this source.

6.4.17 Ṣāliḥ Efendī: Istanbul

In the tables of Ṣāliḥ Efendī (**2.2.3**) the function B(λ) is tabulated to three digits for each 1° of λ. The underlying parameters are φ = 41;0° (Istanbul) and ε = 23;28,54°.

6.4.18 Anonymous: Istanbul

In several late copies of the simplified version of Ṣāliḥ Efendī's tables for timekeeping (**2.2.3**), *e.g.*, MS Istanbul Kandilli 441, there is a small set of tables appended which is entitled *jayb āfāqī*, literally, "universal Sine". In fact the tables display the four functions:

δ(λ), C(λ), B(λ) and d(λ) ,

each computed to two sexagesimal digits for each 1° of λ. The underlying parameters appear to be φ = 41;0° (Istanbul) and ε = 23;28°. The tables are arranged in three columns of thirty entries beginning with λ = 0°, so that there is no entry for λ = 90°. See also **6.10.15** on the table of C(λ).

6.4.19 ʿAbdallāh al-Ḥalabī (?): Aleppo

MS Aleppo Awqāf 943 of ʿAbdallāh al-Ḥalabī's prayer-tables for Aleppo (**6.2.7**) contains a table of B(λ) computed to three digits for each degree of λ and based on parameters φ = 35;50° (Aleppo) and ε = 23;30°.

6.4.20 al-ʿAlamī: Fez

MS Cairo TR 132,2 (pp. 2-139, *ca*. 1850) is the only known copy of a commentary by ʿAbd al-Salām al-ʿAlamī on an earlier work by ʿAbd al-ʿAzīz ibn ʿAbd al-Salām al-Warjānī entitled *Taḥrīr al-mawāqīt*, roughly "The Correct Times of Prayer", and compiled in 1142 H [= 1729/30]. Both individuals are new to the modern literature. The manuscript is datable to *ca.* 1875 and the treatment of spherical astronomy in the commentary is rather sophisticated. In particular the treatise contains tables (pp. 95 and 92) of the functions B(λ) and C(λ) computed to two digits for parameters:

φ = 34;10° (Fez) and ε = 23;29° .

See now **II-13.8*** for prayer-tables based on this latitude.

6.5 Tables for marking the absolute base on trigonometric quadrants

The purpose of the tables described below is to facilitate the marking of B(λ) on the instrument called the sine quadrant (Arabic, *al-rubʿ al-mujayyab*), a quadrant fitted with a trigonometric grid that was popular amongst Muslim astronomers from the 9th to the 19th century.[13] Complicated trigonometric problems can be solved with the sine quadrant without any calculation whatsoever. The grid consists of two orthogonal sets of equi-spaced lines drawn parallel to the axes of the grid and dividing each axis into 60 parts. The circumference is divided into 90 equal degree intervals. A thread attached to the centre of the quadrant bears a movable marker (Arabic, *murī* or *mūrī*). On the use of the instrument to compute the hour-angle from solar or stellar altitudes **6.16.4** below.

6.5.1 al-Khalīlī: Damascus

Tables of B(λ) for the latitude of Damascus do not occur in the known manuscripts of tables due to al-Khalīlī, although in his smaller set of auxiliary tables (**9.4**) he tabulates $^1/_2$ B(λ) for each degree of latitude and also ϕ = 33;30°.

However, MSS Oxford Seld. Supp. 100, fol. 16v, and Paris BNF ar. 2558, fol. 33r, contain two tables by al-Khalīlī, presented without instructions regarding their use. The two tabulated functions, having λ as argument, are called respectively *qaws al-aṣl bi-'l-mūrī*, "arc of the base (for a quadrant) with a marker" and *qaws al-aṣl bi-ghayr mūrī*", arc of the base (for a quadrant) without a marker". I denote these functions by b_1 and b_2 and find by inspection that:

$$b_1(\lambda) = \text{arc Sin } \{ B(\lambda) \} \quad \text{and} \quad b_2(\lambda) = \text{arc Tan } \{ B(\lambda) \}$$

for al-Khalīlī's values of ε and ϕ for Damascus.

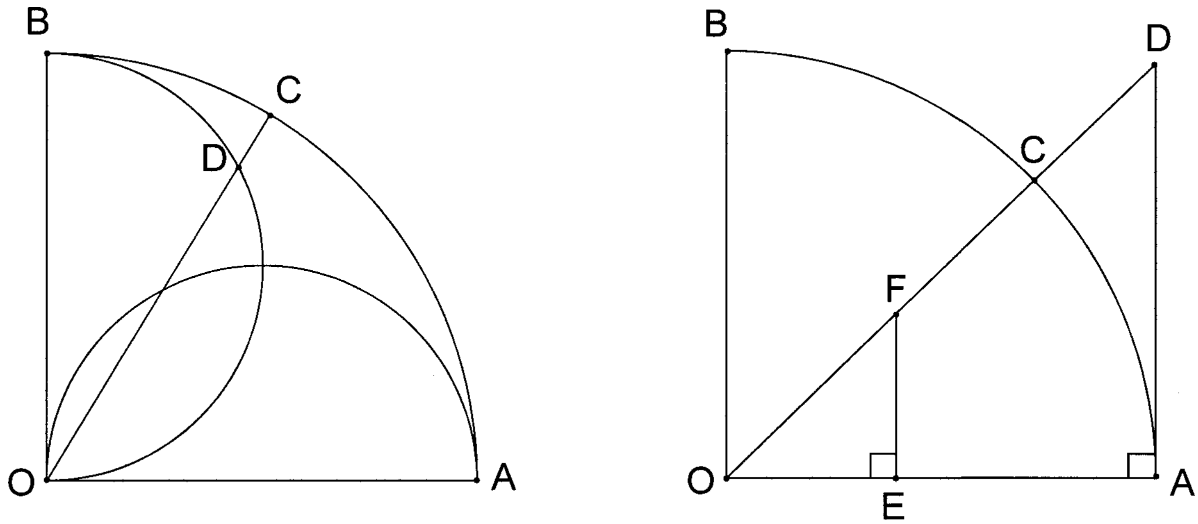


Fig. 6.5.1a-b: Diagrams illustrating the use of al-Khalīlī's tables for marking the absolute base on a sine quadrant.

[13] On the sine quadrant, see, for example, Schmalzl, *Geschichte des Quadranten*; Würschmidt, "Gedosi über den Quadranten"; and King, "al-Khwārizmī", pp. 28-29. The relatively complicated operation of finding the qibla with a sine quadrant is discussed in detail in King, "al-Khalīlī's Qibla Table", pp. 111-115.

The first function $b_1(\lambda)$ is intended to be used with a sine quadrant (radius R = 60) marked with two special semi-circles (Arabic, *dāʾirat al-tajyīb*) for finding Sines, drawn with the two axes of the quadrant as diameters. In **Fig. 6.5.1a** if the thread OC cuts off the arc AC equal to $b_1(\lambda)$ then OD equals Sin $b_1(\lambda)$, that is, $B(\lambda)$. Thus the marker can be moved to D and the quadrant is ready to be used for computations of time from solar altitude which involve a division by $B(\lambda)$.

The second function $b_2(\lambda)$ is intended to be used with a Sine quadrant bearing the usual grid and thread, but no marker. The use of this instrument is described by al-Khalīlī in MS Cairo MM 201, fols. 11v-12r, which also contains the table of $b_2(\lambda)$ (fol. 12r). His treatise on this quadrant is of considerable interest but I here restrict attention to the use of the table of $b_2(\lambda)$. In **Fig. 6.5.1b** the thread OC is moved to the position such that the arc AC equals $b_2(\lambda)$. To divide any quantity, say x, by $B(\lambda)$, simply measure EF equal x and then, because AD equals Tan $b_2(\lambda)$, that is, $B(\lambda)$, OE equals Rx / $B(\lambda)$. Such a simple operation is clearly extremely useful in timekeeping problems.

6.5.2 Anonymous: Istanbul

In MS Istanbul UL T1824,1 of a set of prayer-tables for Istanbul (**6.4.9**) the function called *qaws al-aṣl*, "arc of the base", is in fact $b_1(\lambda)$ as defined above. Values of $B(\lambda)$ and $b_1(\lambda)$ are given to two digits for each degree of λ and the underlying parameters are $\phi = 41;15°$ (Istanbul) and $\varepsilon = 23;30°$. There are no instructions accompanying the tables in the manuscript and I assume that the purpose behind the table of $b_1(\lambda)$ was to facilitate marking $B(\lambda)$ on a sine quadrant.

6.5.2* Anonymous: Maghrib

See **II-13.8*** for some tables of the same kind, probably for Fez.

6.6 Tables of the absolute base for timekeeping by the stars

The existence of a table of $B(\Delta)$ for the latitude of Mecca has been noted in **6.4.4**. Since the argument runs only as far as $\Delta = \varepsilon$ the table was clearly intended to be used for timekeeping by the sun. No other tables of $B(\Delta)$ for particular latitudes which would be suitable for timekeeping by the stars have been located. See, however, **6.16.4** below. Note also that al-Ṣūfī's second auxiliary function (**9.10**) actually defines $B(\phi,\Delta)$ for each degree of both arguments, and that al-Māridīnī's second auxiliary function M_2 (**9.6**) can be used to find $B(\phi,\Delta)$ with facility, namely, by simply using $\bar{\phi}$ and $\bar{\Delta}$ as arguments.

6.7 Tables of the reciprocal of the absolute base and related functions for timekeeping by the sun

It is rather curious that we find tables of the function $B(\lambda)$ rather than its reciprocal $G(\lambda)$ in the Islamic sources. As yet I have not come across any tables of G specifically intended for timekeeping by the sun – see, however, **6.8**. The following tables of functions related to $G(\lambda)$ have been located.

6.7.1 Anonymous: Cairo

In the *Ḥākimī Zīj* Ibn Yūnus suggests the method outlined in **2.1.1** above for finding t from h and λ. The function G he calls *al-nisba*, "the ratio", since:

$$G(\lambda) = R^2 / [\ \mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi\ ] = \mathrm{Vers}\ D(\lambda) / \mathrm{Sin}\ H\ (\lambda)\ .$$

I have not yet located a table of G(λ) for the latitude of Cairo in any of the manuscripts containing material ultimately due to Ibn Yūnus.

In one such source, however, containing a recension of the anonymous 13th-century Egyptian *Muṣṭalaḥ Zīj*,[14] MS Paris BNF ar. 2513, fols. 62r and 62v, copied in the 13th century, there is a table of at least one related function. The instructions in fact mention two related functions and indicate that the first, called *nisbat al-iʿtidāl*, "the ratio at the equinoxes" (δ = 0°), is:

$$G_1(\theta) = R / \mathrm{Cos}\ \theta\ .$$

However, the tabulated function is not $G_1(\theta)$ and I am unable to explain it. The table bears no title and the argument column is not visible on the microfilm which I have used. The 16 entries in the table are as follows:

13;11, 9	15;23,35	21;43,30	33;30,26
13;14,17	16;24,43	24; 0,30	42; 0,34
13;34,19	17;51,51	26;22,30	49;30,17
14;20,34	19;16, 0	28;45, 0	57; 0,53

This first table may be unrelated to spherical astronomy.

The second table accompanying these instructions is inappropriately entitled *sahm faḍl al-dāʾir*, "Versed Sine of the hour-angle". The function tabulated is in fact:

$$G_2(\lambda) = R / \mathrm{Cos}\ \delta(\lambda)\ .$$

Values are given to three digits but are badly garbled. It is not possible to determine the value of ε underlying the table. The instructions indicate that the "ratio" may be found from these tables using either:

$$G(\lambda,\phi) = G_1(\phi) \cdot G_1(\delta) \quad \text{or} \quad G(\lambda,\phi) = G_1(\phi) \cdot G_2(\lambda)\ .$$

6.7.2 al-Marrākushī: Cairo

In the treatise of al-Marrākushī (**4.2.4**), there is a set of values (I.264) of a function $G_3(\delta)$, as follows:

δ	8°	13°	16°	18°	20°	22°	23;35°
G_3	9	10	11	12	13	14	15

This function is simply:

$$G_3(\delta) = R^3 / [\ \mathrm{Cos}\ \delta\ \mathrm{Cos}\ \phi\ ] - R$$

for latitude 30° and its use is explained in **9.12**, where I describe a table of G_3 for all declinations and latitudes.

[14] On the *Muṣṭalaḥ Zīj* see Kennedy, "*Zīj* Survey", no. 47; *Cairo ENL Survey*, no. C12; King, "Lunar Equation Table", p. 135; and *idem*, "Astronomy of the Mamluks", pp. 535-536. See also **Fig. I-9.8a**.

6.8 Tables of the reciprocal of the absolute base for timekeeping by the sun and stars

The reader will by now have gained the impression that the Muslim astronomers tabulated virtually every function that was of any use in computing the hour-angle from solar or stellar altitudes. Yet I have not come across any Islamic tables of the function:

$$G = \text{Vers } D \;/\; \text{Sin } H = R^2 \;/\; [\ \text{Cos } \Delta \ \text{Cos } \phi\ ] \ ,$$

either for a specific latitude or for all latitudes, such as might have been used to facilitate the computation of a *ṭaylasān* table of T(H,h) (**2.3**) or a universal table of T(H,h,D) (**2.6**). Note that using such a table of G(H) for a particular latitude or G(H,D) for all latitudes, along with a table of the function:

$$H'(H,h) \ = \ \text{Sin } H - \text{Sin } h \ ,$$

such as compiled by Ibn al-Mushrif (**6.13.1**), the computation of the hour-angle reduces to (*cf.* **F9**):

$$\text{Vers } t \ = \ H' \cdot G \ .$$

On the other hand, two tables of the function G(Δ) have come to light and are described below. The first was compiled for Cairo, where, however, as far as we know, no *ṭaylasān* tables were prepared. The second was compiled for Tunis and may have been used to compute a set of tables for reckoning time by stellar altitudes. Note also that al-Wafāʾī's first auxiliary function (**9.9**) defines G(Δ,ϕ) for each degree of Δ and some 13 different values of ϕ.

6.8.1 Anonymous: Cairo

In the Egyptian copy of al-Khalīlī's *Universal Table* MS Cairo MM 43 (**6.4.3**) there is a table entitled *jadwal al-dāʾir*, "table for finding time". The function tabulated is simply G(Δ) computed to two digits for:

$$\phi = 30° \text{ (Cairo)} \quad \text{and} \quad \Delta = 1°, 2°, \ldots , 60° \ (= \bar{\phi}).$$

A page of anonymous tables in MS Oxford Seld. Supp. 99, fol. 63v, at the end of the prayer-tables for Cairo by al-Fawānīsī (**4.8.4**) contains two incomplete tables entitled *jadwal al-dāʾir min al-falak* and *jadwal al-dāʾir min al-sahm*. The first of these displays G(Δ) for latitude 30°, computed to two digits for each degree of argument (I have not been able to compare the entries with those in the Cairo manuscript) and the second displays arc Vers (x) (see further **6.14.2**).

6.8.2 Anonymous: Tunis

MS Berlin Ahlwardt 5724, fol. 42v, of the anonymous corpus of tables for Tunis based on ϕ = 36;40° (**2.3.5**) contains a table of G(Δ) computed to three digits for this value of ϕ and arguments Δ = 1°, 2°, ... , 60°. The table is entitled *jadwal aṣl faḍl dāʾir al-shams wa-'l-kawākib*, "table of the base (for calculating) the hour-angle of the sun and stars" – see **Fig. 6.8.2**. This table may have been used to compile the tables of t(h) for various stars that occur on fols. 55r-56r of the Berlin manuscript (**2.4.1**).

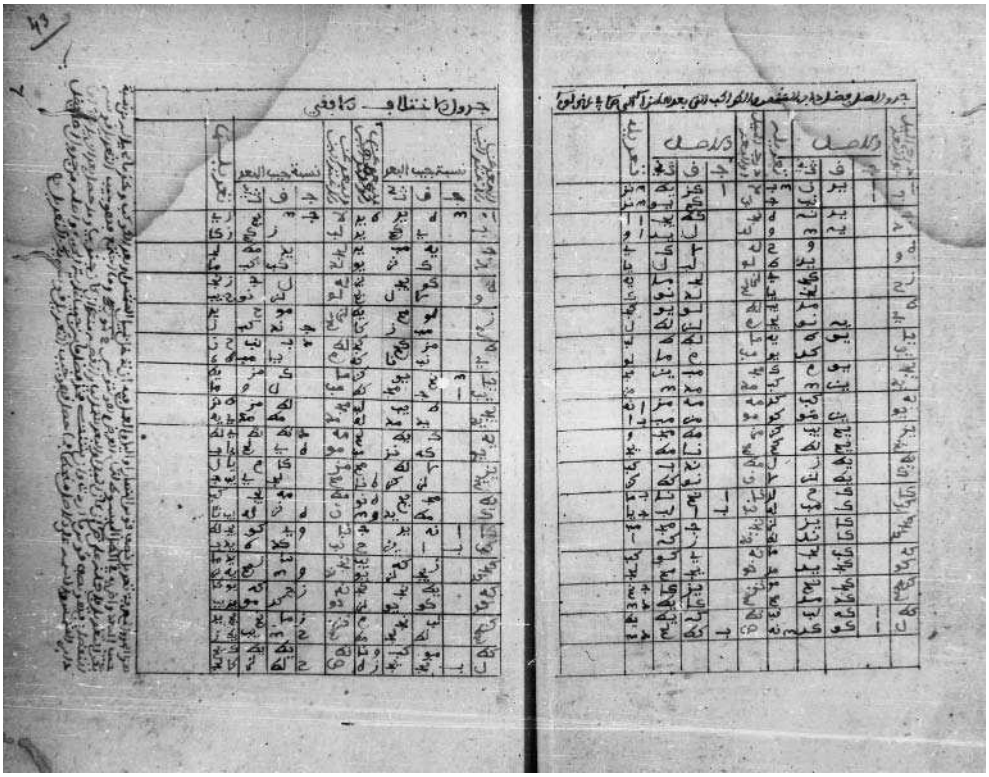
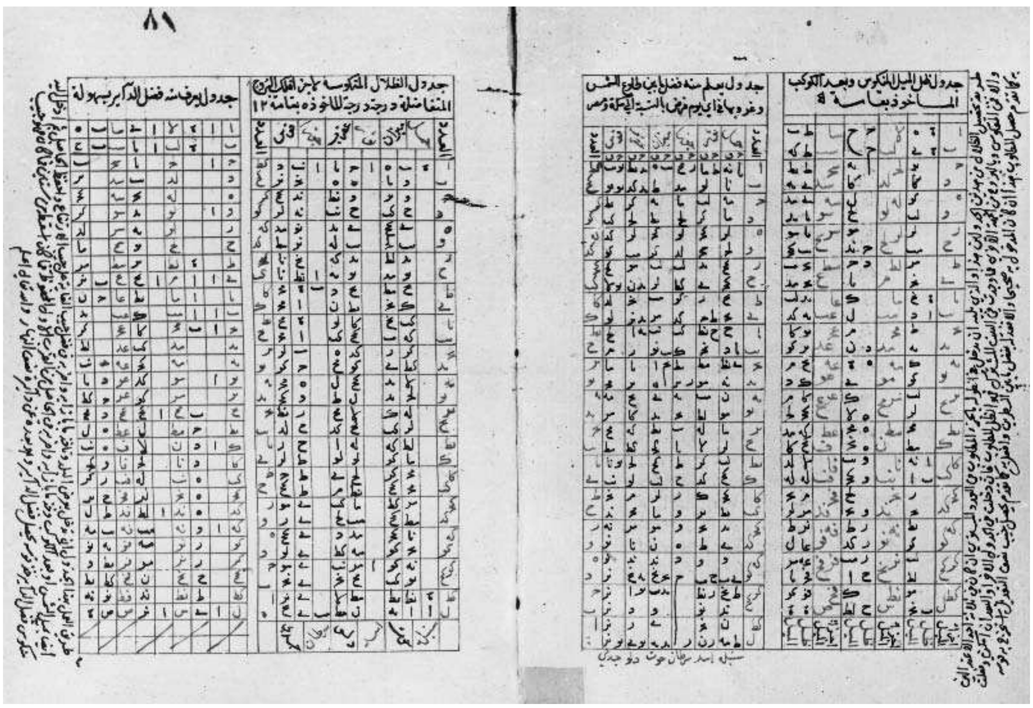


Fig. 6.8.2: The table of the reciprocal of the absolute base in the Tunis corpus, together with a table of the sine of the declination labelled as if it were an auxiliary azimuth function (see **7.1.5**). [From MS Berlin Ahlwardt 5724, fols. 42v-43r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]



6.9 Tables of the secant function for timekeeping

The Muslim astronomers did not generally exploit the standard trigonometric functions other than the sine and (co-)tangent.[15] However, a table of the cosecant function is found in one version of the *Zīj* of the early-9th-century astronomer Ḥabash and others are found in the sets of tables used by the Muslim astronomers for marking curves on astrolabes and quadrants (see further **9.2**). Also, a few simple tables of the secant function have recently come to light but they are not referred to by the standard, if uncommon, Arabic term *quṭr al-ẓill*, "hypotenuse of the (horizontal) shadow" (*i.e.*, the length of the hypotenuse of the triangle formed by a vertical gnomon and its shadow on a horizontal plane). Rather, they are considered as auxiliary tables for timekeeping. See also **8.4** for tables of the secant function intended for azimuth calculations.

I denote the secant function as found in the Arabic sources by G_1, where:

$$G_1(\theta) = R / \text{Cos } \theta .$$

It can be used to find the auxiliary function G (**6.7** and **6.8**) using the relation:

$$G(\Delta,\phi) = G_1(\Delta) \cdot G_1(\phi) .$$

Thus, for example, in the text of the *Zīj* of Ibn al-Bannāʾ (**6.9.4**) the formula outlined for finding t from h, Δ and φ is equivalent to (*cf.* **F9**):

$$\text{Vers } t(h,\Delta,\phi) = (\text{Sin } H - \text{Sin } h) \cdot G_1(\phi) \cdot G_1(\Delta) .$$

Note that each of the four tables of G_1 listed below contains the same significant error in the entry for θ = 60°. (Compare the Cosecant table of Ḥabash discussed in **9.1**.) All of these tables merit a new investigation.

6.9.1 Ibn al-Zarqālluh (Toledo)

The Andalusī astronomer Ibn al-Zarqālluh (*fl.* Toledo, *ca.* 1060) compiled a set of auxiliary tables for generating planetary ephemerides known as the *Almanach of Azarquiel*. These have been published by José-Maria Millás Vallicrosa, using the unique Arabic copy MS Munich ar. 853 and a Catalan version of the *Almanach*.[16] One of the spherical astronomical tables in the set (p. 226) is entitled *jadwal fuḍūl khaṭṭ al-madār ʿalā 'l-irtifāʿ*, literally, "table of the excesses of the line of the day-circle over the altitude". (This makes no sense either in Arabic or English and the Arabic is doubtless corrupt.) The function is in fact $G_1(\theta) = R / \text{Cos } (\theta)$, and values are given to three digits for each 3° of θ and also 88° and 89°. In view of the importance of this table and since Millás' version can hardly be considered an accurate representation at least of the Arabic version, I reproduce the entire table from MS Munich ar. 853, fol. 44v,

←
Fig. 6.9.3: A table of the reciprocal of the absolute base (far left) amidst some anonymous tables for Cairo. Two other tables display the tangent of the declination and the tangent of the solar declination (both omitted from **7.1**!). Another displays for each degree of solar longitude the difference between the lengths of daylight in Cairo and Mecca, which should be investigated in the light of the acompanying treatise. [From MS Cairo MM 204,7, fols. 80v-81r, courtesy of the Egyptian National Library.]

[15] On Islamic trigonometric tables in general see n. 1:23.
[16] Millás, *Azarquiel*, pp. 72-237. More recent studies on Ibn al-Zarqālluh include Toomer, "Solar Theory of al-Zarqāl", p. 331; and Julio Samsó and Eduardo Millás, "Ibn al-Bannāʾ, Ibn Isḥāq and Ibn al-Zarqālluh's Solar Theory", in Samsó, *Studies*, X. See also the article "al-Zarqālī" by Juan Vernet in *DSB*.

The main part of the table is as follows. (The errors in the third digit are indicated in square brackets.)

3°	1;0, 5[a]	[0]	27	1; 7,21	[+1]	51	1;35,27	[+7]
6	1;0,20	[0]	30	1; 9;17	[0]	54	1;42, 2	[-3]
9	1;0,45	[0]	33	1;11,32	[0]	57	1;50,10	[0]
12	1;1,21	[+1]	36	1;14,10	[0]	60	2; 1, 2	[!!!][b]
15	1;2, 5	[-2]	39	1;17,11	[-1]	63	2;12[c],10[d]	[0]
18	1;3, 6	[+1]	42	1;20,44	[0]	66	2;27,31	[0]
21	1;4,16	[0]	45	1;24,51	[0]	69	2;47,27	[+1]
24	1;5;40	[-1]	48	1;29,40	[0]	72	3;14,10	[0]

[a] Text: 30, Millás: 5. [b] See below. [c] Text: 17, Millás: 12. [d] Text and Millás: 2

The remaining entries in the table in the Munich manuscript, compared with Millás' entries and the accurate values of Sec θ, are as follows:

Text		Millás		Accurate	
75°	3;51,50	75°	3;51;50	75°	3;51;49
78	4;48,21	78	4;48,21	78	4;48,35
81	5;23,33(!)	81	6;23,33	81	6;23,33
84	6;14, 1(!)	84	9;34, 1	84	9;34, 0
87	7; 6,30(!)	87	19; 6,30	87	19; 6,25
88	8;18,42(!)	90(!)	29,18,42	88	28;39,11
89	9;17,42(!)	- (!)	56;17,40	89	57;17,56
90	-; 0,47(!!)			90	infinite

The last few entries in the Munich manuscript are thus badly garbled. Of considerable interest is the entry for θ = 60°, namely 2;1,2 rather than the accurate 2;0,0. It is difficult to account for this error in view of the fact that most of the other entries are fairly carefully computed. The tables of $G_1(\theta)$ discussed in **6.9.3-5** have the entry 2;1 for θ = 60°.

6.9.1* Ibn Isḥāq (Tunis)

MS Hyderabad Āṣafiyya 298 of the *Zīj* of Ibn Isḥāq, located only in 1978, is an extremely important new source for the history of Islamic astronomy.1 It contains a table of the function $R^2/\mathrm{Cos}(\theta)$ labelled *nisbat al-madārāt* (table no. 90 in the manuscript). Values are given to two significant sexagesimal digits, and the entry for argument 60 is 1;1, in error for [2];1. See also **7.1.5*** and **II-13.1**.

6.9.2 Anonymous (Cairo)

The table in MS Paris BNF ar. 2513 of the *Muṣṭalaḥ Zīj* which purports to be of the function $G_1(\theta)$ has already been discussed in **6.7.1**.

---

[17] In the 1980s all one that was known about Ibn Isḥāq was what was recorded in Suter, *MAA*, no. 356, and King, *Astronomy in Yemen*, p. 83. His *zīj* is not listed in Kennedy, "*Zīj* Survey". The Zabid manuscrip of the *zīj* of the Yemenī astronomer al-Daylamī (**4.3.4**) contains some planetary tables due to Ibn Isḥāq. MS Milan Ambrosiana C82 of the perpetual almanac of Zacuto (**4.3.7**) contains some spherical astronomical tables due to him. On the Hyderabad manuscript see King, "Astronomy in the Maghrib", p. 32; Samsó, "Maghribī *Zīj*es", p. 93; and, more especially, Mestres, "Hyderabad MS of the *Zīj* of Ibn Isḥāq", and *idem*, Zīj *of Ibn Isḥāq*. In the new edition of Mestres (*ibid*., pp. 275 and 279-281), a value 2;0 occurs at argument 61°, also in the recomputation! The values in the table for 59°-61° are 1;57, 1;1 (*sic*) and 2;5.

6.9.3 Anonymous (Cairo)

In the anonymous set of spherical astronomical tables for Cairo contained in MS Cairo MM 204,7, copied in 1052 H [= 1642/43] and described in **II-4.2**, there is one entitled *jadwal yuʿraf minhu faḍl al-dāʾir bi-suhūla*, "table for finding the hour-angle easily" (fol. 81r): see **Fig. 6.9.3**. The function tabulated is simply $G_1(\theta)$, computed to two digits for each degree of θ, and the entry for θ = 60° is 2;1. The tables are appended to an anonymous treatise on finding the times of sunrise and sunset in localities other than one's own (fols. 77v-80r).

6.9.4 Ibn al-Bannāʾ (Marrakesh)

In the *Zīj* of Ibn al-Bannāʾ,[18] compiled in Marrakesh about 1300, there is a small table entitled *nisbat al-madārāt*, literally, "the ratio of the day-circles". I have consulted MSS Dublin CB 4087, fol. 46r, Escorial ár. 909,1, fol. 36v, and fol. 34v of an unnumbered manuscript in the Museo Naval, Madrid. The function tabulated is $G_1(\theta)$, computed to two digits for each 3° of θ, and the entry for θ = 60° is 2;1. See also **6.9.5** below.

6.9.5 Ibn al-Raqqām (Tunis)

In MS Istanbul Kandilli 249, copied *ca*. 1800, of the *Shāmil Zīj* as well as in the unnumbered manuscript in the Museo Naval, Madrid (fol. 41r), of the *Qawīm Zīj*, both compiled by the early-14th-century Tunisian astronomer Ibn al-Raqqām,[19] there is a table entitled *nisbat al-madārāt* (**6.9.4**) displaying $G_1(\theta)$ to two digits for each degree of θ. The entry for θ = 60° is again 2;1.

6.10 Tables of the altitude of the solar day-circle centre and related functions

Several sources listed below contain tables of:

$$C(\lambda) = \text{Sin } \delta(\lambda) \text{ Sin } \phi \ / \ R$$

for particular latitudes. This function is called *buʿd al-quṭr* in Arabic, literally, "the distance of the diameter", and is easily derived from Sin δ(λ) (**6.1**).

6.10.1 Anonymous: Cairo

In MS Berlin Ahlwardt 5710, fol. 24r, of the prayer-tables of Riḍwān Efendī (**6.1.3**), there is a table of C(λ) to three digits for each degree of λ. The underlying parameters are φ = 30;0° (Cairo) and ε = 23;35°. This table, which is very carefully computed, was doubtless lifted from some earlier source, like most of Riḍwān's tables. His table of the function B(λ) is the same

[18] On Ibn al-Bannāʾ see Suter, *MAA*, no. 399; and *Cairo ENL Survey*, no. F23; as well as Renaud, "Ibn al-Bannāʾ". His *zīj*, not listed in Kennedy, "*Zīj* Survey", is entitled *Minhāj al-ṭālib* and is extant in several manuscripts. The text of the introduction to the *zīj* and a Spanish translation were published in Vernet, *Ibn al-Bannāʾ*, and the planetary tables have recently been studied in Samsó & Millás, "Ibn al-Bannāʾ on Planetary Longitudes".

[19] On Ibn al-Raqqām see Suter, *MAA*, nos. 388 and 417; and *Cairo ENL Survey*, no. F22. His two *zīj*es are not listed in Kennedy, "*Zīj* Survey", but see now *idem*, "Ibn al-Raqqām's Tables". See also Carandell, *Ibn al-Raqqām sobre los cuadrantes solares*.

as that of al-Khaṭāʾī (**6.4.2**), but there is no corresponding table of C(λ) in MS Vatican Borg. ar. 217,2 of al-Khaṭāʾī's set. Note that for $\phi = 30°$:

$$C(\lambda) = {}^1/_2 \text{ Sin } \delta(\lambda) \text{ .}$$

6.10.2 Anonymous: Cairo

MS Istanbul Nuruosmaniye 2925 of al-Bakhāniqī's edition of the Cairo corpus (**2.1.1** and **6.2.4**) contains a table of C(λ) for Cairo. The function is tabulated to two digits for the same parameters as the table in **6.10.1**. MS Alexandria 4441J of the prayer-tables for Cairo of Ibn Abī Rāya (**6.2.4**) contains a corrupt version of this same table.

6.10.3 Anonymous: Cairo

In MS Paris BNF ar. 2513, fol. 53v, of the anonymous recension of the Egyptian *Muṣṭalaḥ Zīj* (**6.7.1** and **II-6.6**), there is a table of the function:

$$\text{arc Sin } \{C(\lambda)\}$$

entitled *irtifāʿ madār al-shams*, "altitude of the (centre of the) solar day-circle". The entries are given to two digits and are based on the parameters $\phi = 30;0°$ (Cairo) and $\varepsilon = 23;35°$. The function measures the altitude in degrees of a point on the celestial sphere at the same distance above the horizon as the centre of the solar day-circle. This corresponds to the altitude of the sun when the hour-angle is 90° (for $\delta > 0$ only). I can think of no good reason for tabulating this function except to facilitate marking C(λ) on a Sine quadrant – see **6.5** above.

6.10.4 Anonymous: Anatolia

MS Istanbul Süleymaniye 1037/32 (**6.4.6**) contains a table of C(λ) to two digits for each degree of λ, based on the parameters $\phi = 38;30°$ (Anatolia ?) and $\varepsilon = 23;35°$.

6.10.5 Anonymous: Tunis

In MS Princeton Yahuda 147c, p. 113, of the Tunisian *Quṣʿī Zīj* of Ḥusayn Quṣʿa (**6.4.7**) there is a table of C(λ) for each degree of λ computed to two digits for the parameters $\phi = 36;40°$ (Tunis) and $\varepsilon = 23;35°$.

6.10.6 Taqi 'l-Dīn: Istanbul

In MS Istanbul Esat Efendi 1976, fol. 25r, of the *Zīj* of Taqi 'l-Dīn (**6.4.8**), C(λ) is tabulated to four significant *decimal* digits of λ and the entries are apparently based on the parameters $\phi = 40;58°$ (Istanbul) and $\varepsilon = 23;28,54°$.

6.10.7 Anonymous: Istanbul

In MS Istanbul UL T1824,1 (**6.4.9**) C(λ) is tabulated to two digits for each degree of λ and parameters $\phi = 41;15°$ (Istanbul) and $\varepsilon = 23;30°$.

6.10.8 al-Dīsṭī: Lattakia

MSS Aleppo Awqāf 911 and Leiden Or. 2808(2) of al-Dīsṭī's prayer-tables for Lattakia (**6.4.10**) contains a table of C(λ) with values to two digits for each degree of λ for the parameters $\phi = 34;30°$ (Lattakia) and $\varepsilon = 23;30°$.

6.10.9 al-Jannād: various localities in the Western Maghrib
MS Cairo TR 338,2 of al-Jannād's prayer-tables for the Western Maghrib (**6.4.11**) contain tables of C(λ) for latitudes φ = 30°, 31° and 34°.

6.10.10 al-Maḥallī: Damietta
Quṭb al-Dīn al-Maḥallī (**6.4.12**) tabulated C(λ) to two digits for each degree of λ and parameters φ = 31;25° (Damietta) and ε = 23;35°.

6.10.11 Anonymous: Alexandria
The anonymous set of prayer-tables preserved in MS Cairo DM 1207 (**6.4.13**) contains a table of C(λ) computed to two digits for each degree of λ and based on the parameter φ = 31;0° (Alexandria).

6.10.12 Yūsuf Kilārjī: Crete
Yūsuf Kilārjī (**6.4.14**) tabulated C(λ) to three digits for each degree of λ for the parameters φ = 35;30° (Crete) and ε = 23;35°.

6.10.13 Anonymous: Sfax
MS Cairo K 7584,1 (**6.4.16**) contains a table of C(λ) on the parameters φ = 34;48° (Sfax) and ε = 23;30,17°.

6.10.14 Ṣāliḥ Efendī: Istanbul
Ṣāliḥ Efendī (**6.4.17**) also tabulated C(λ), giving values to three sexagesimal digits for the parameters φ = 41;0° (Istanbul) and ε = 23;28,54°.

6.10.15 Anonymous: Istanbul
MS Istanbul Kandilli 441 (**6.4.18**) contains a table of C(λ) apparently based on parameters φ = 41;0° (Istanbul) and ε = 23;28°.

6.10.16 ʿAbdallāh al-Ḥalabī (?): Aleppo
MS Aleppo Awqāf 943 of the prayer-tables attributed to ʿAbdallāh al-Ḥalabī (**6.4.19**) contains a table of C(λ) for parameters φ = 35;50° (Aleppo) and ε = 23;30°.

6.10.17 al-ʿAlamī: Fez
MS Cairo TR 132,2 of the treatise by ʿAbd al-Salām al-Ḥusnī al-ʿAlamī (**6.4.20**) contains a table of C(λ) computed to two digits for each degree of λ and based on parameters φ = 34;10° (Fez) and ε = 23;29°.

6.11 Tables of the height of the centre of the arc of visibility for timekeeping by the stars

No tables of C(Δ) for particular latitudes have been located in the manuscript sources. (See, however, **6.16.4** below where C is tabulated for specific stars.) Note that both al-Ṣūfī's first

auxiliary function (**9.10**) and al-Māridīnī's second auxiliary function M_2 (**9.6**) define $C(\phi,\Delta)$ for each degree of both arguments.

6.12 Tables of the modified base

The function:

$$b(h,\lambda) = \text{Sin } h - C(\lambda) ,$$

called *al-aṣl al-muʿaddal*, "the modified base", is useful in timekeeping because the hour-angle $t(h,\lambda)$ is defined in terms of b by the simple relation (**6.0**):

$$\text{Cos } t(h,\lambda) = R \cdot b(h,\lambda) / B(\lambda) = b(h,\lambda) \cdot G(\lambda) .$$

In only one set of hour-angle tables has the compiler left us the set of tables of $b(h,\lambda)$ which he used to compute the hour-angle.

6.12.1 Ṣāliḥ Efendī: Istanbul

In the tables of Ṣāliḥ Efendī for Istanbul (**2.2.3** and **Fig. 2.2.3**), the function $b(h,\lambda)$ is displayed to three digits for each degree of λ and each value of h up to $[H(\lambda)]$. Ṣāliḥ Efendī also tabulated $B(\lambda)$ and $C(\lambda)$ to three digits for each degree of λ (**6.4.17** and **6.10.14**), but not $G(\lambda)$.

6.13 Tables of the difference between the sines of the meridian altitude and the instantaneous altitude

The function:

$$H'(H,h) = \text{Sin } H - \text{Sin } h$$

is useful in timekeeping because the hour-angle $t(H,h)$ is defined in terms of H' by the simple relation (**6.0** and **6.8**):

$$\text{Vers } t(H,h) = H'(H,h) \cdot G(H) .$$

Only one table of this function has come to light.

6.13.1 Ibn al-Mushrif (Cairo)

The early-15th-century Egyptian astronomer Ibn al-Mushrif (**9.8**) tabulated $H'(H,h)$ to two digits for the domains:

$$H = 1°, 2°, \ldots , 90° \quad \text{and} \quad h = 1°, 2°, \ldots , (H\text{-}1) .$$

These tables are contained in MS Cairo MM 241 of his auxiliary tables.

6.14 Tables of the inverse versed sine function

Tables of inverse trigonometric functions are comparatively rare in the Islamic sources. al-Marrākushī (**4.2.4**) tabulated the inverse Sine and Cotangent (base 12) (I.121-124 and 168-169), and a later Egyptian copy of al-Khalīlī's *Universal Table*, MS Cairo MM 43 (**9.5**), contains tables of the inverse Sine and Tangent (base 12). Again, a 14th-century copy of the main Cairo corpus, MS Dublin CB 3673, fol. 8r (**2.1.1**), and also MS London BL Or. 3624,

fol. 169v, of the *Mukhtār Zīj* of Abu 'l-ʿUqūl (**2.1.2**) contain tables of the inverse Sine function, the latter probably taken from an earlier source. Tables of inverse spherical astronomical functions are likewise rare. MS Cairo MM 43, mentioned above, contains a table of the function λ(δ) with values to two digits for each degree of δ, and MS Cairo MM 58, fol. 7r, contains a similar table with argument increment 0;15°.

In timekeeping the most useful tables of inverse trigonometric functions would be for the Cosine and Versed Sine. No tables of the former have come to my attention, but notice that al-Khalīlī's auxiliary function K and al-Ṣūfī's function S (**9.5** and **9.10**) perform the final operation for finding t(h,D). Several tables of the inversed Versed Sine have been located, and the auxiliary functions V and V′ tabulated by al-Khaṭāʾī, al-Khalīlī and Ibn al-Mushrif (**6.15.1**, **9.4** and **9.8**) likewise perform the final operation for finding t(h,H).

6.14.1 al-Wafāʾī (Cairo)

In his auxiliary tables preserved in MSS Istanbul Nuruosmaniye 2921,2 (fols. 22r-26v) and Vatican Borg. ar. 217,1 (fols. 1v-5v), the 15th-century Egyptian astronomer al-Wafāʾī (**9.9**) tabulated arc Vers (x) to two digits for arguments:

x = 0;5, 0;10, ... , 0;30, 0;40, ... , 2;0, 2;15, ... , 3;0, 3;20, ... ,
4;0, 4;30, ... , 16, 17, ... , 120.

The table is entitled *jadwal faḍl al-dāʾir*, "hour-angle table".

6.14.2 Anonymous (Cairo)

Several other Egyptian sources contain tables of arc Vers (x) to two digits for x = 1, 2, ... , 120. Such tables are contained in, for example, MS Oxford Seld. Supp. 99, fol. 63v, of al-Fawānīsī's prayer-tables for Cairo (**6.8.1**), where the function is called *al-dāʾir min al-sahm*, "time from the Versed Sine", and MS Cairo MM 203 of an anonymous set of auxiliary tables (**9.12**), where it is called *qaws al-sahm*, "the arc of the Versed Sine". I have not compared the entries in these two tables.

6.14.3 Anonymous (Damascus)

MS Istanbul Hamidiye 1453 (fols. 232v-266v) of a 15th-century Turkish copy of al-Khalīlī's *Universal Table* (**9.5**) contains (fol. 266r) tables of arc Sin (x) and arc Vers (x), the latter being referred to as *faḍl al-dāʾir min qibal al-sahm*, "the hour-angle as a function of the Versed Sine". See also **7.0**.

6.14.4 Anonymous (Aleppo)

MS Aleppo Awqāf 943, copied in Aleppo *ca*. 1850, contains an anonymous set of tables of the function:

$$V'(x,y) = \text{arc Vers} \{ R \cdot y / x \}$$

for arguments x = 35, 36, ... , 60 and y = 1, 2, ... , 60. The function is referred to as *faḍl al-dāʾir*, "the hour-angle", and the two arguments are labelled *al-aṣl*, "the base", and *mā bayn al-jaybayn*, "the difference between the two Sines". See **9.8** and also **6.2.7** on other tables in this Aleppo manuscript.

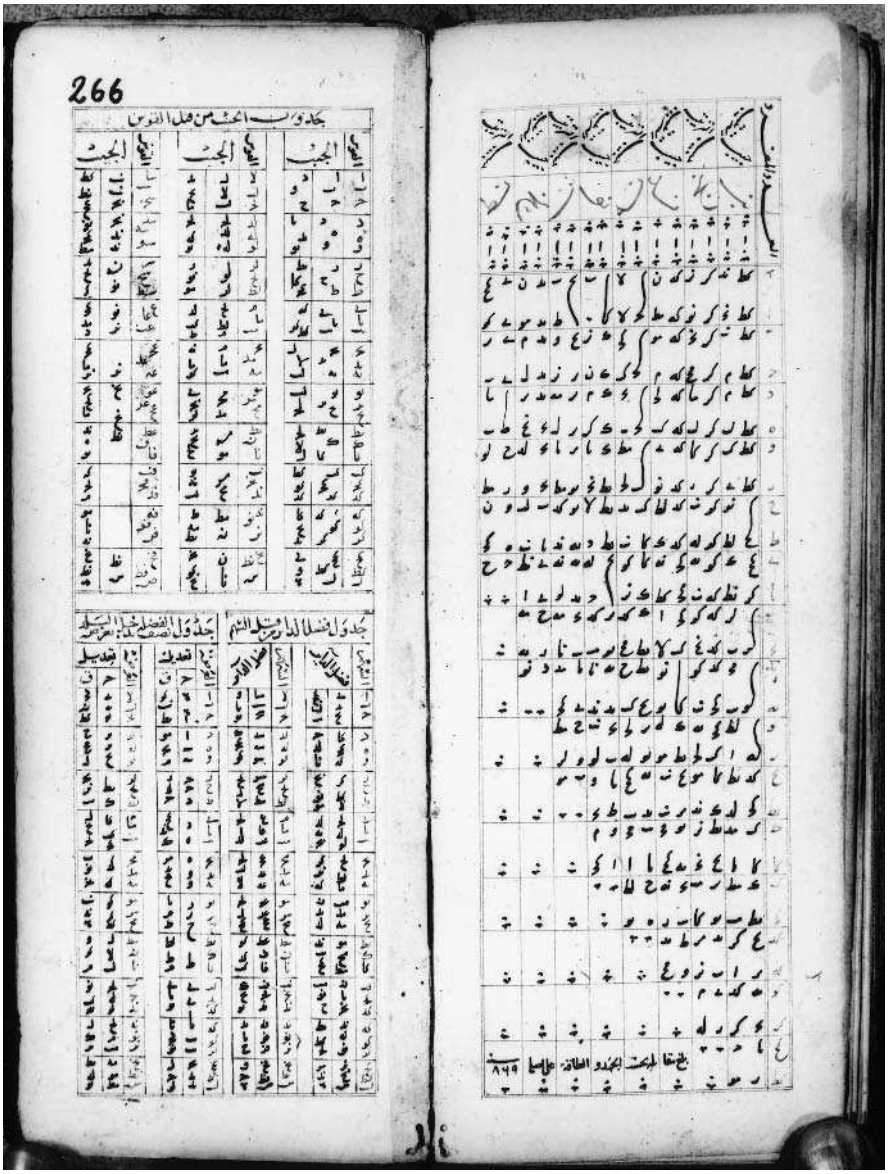


Fig. 6.14.3: Tables of the inverse Sine and Versed Sine functions and a table of the maximum half daylight for all latitudes (based on obliquity 23;35°) appended at the end of a copy of al-Khalīlī's universal auxiliary tables (see **Fig. I-9.5a-b**), of which the last few sub-tables for his third function are shown on the right. From MS Istanbul Hamidiye 1453, fols. 265v-266r, courtesy of the Süleymaniye Library, Istanbul.]

6.15 Tables of the hour-angle as a function of the absolute base

I now describe a table for finding the hour-angle which has the absolute base B as one argument. This is devised for a particular latitude. Note that the auxiliary tables of al-Khalīlī for timekeeping by the sun (**9.4** and also **9.7** and **9.11**) have B′ = $^1/_2$ B as one argument but serve

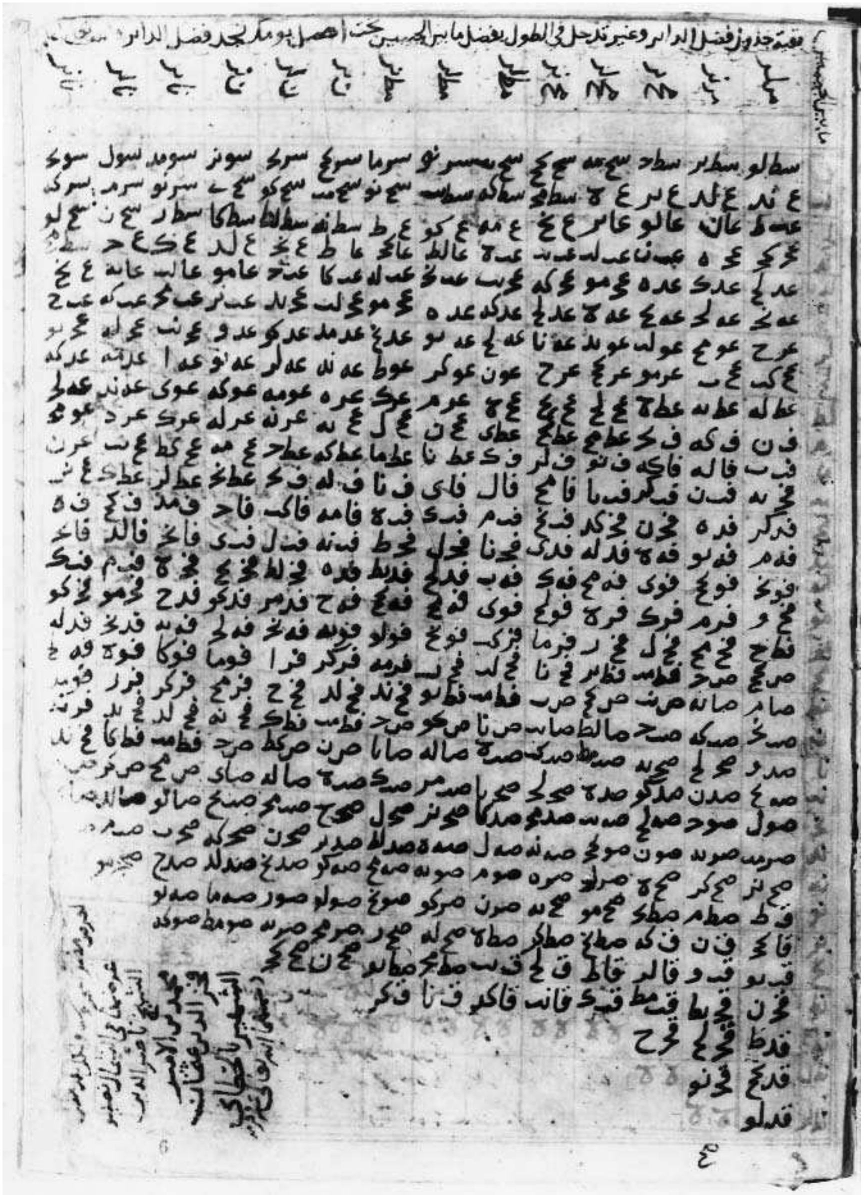


Fig. 6.15.1: An extract from the auxiliary tables of al-Khaṭāʾī, showing the surviving part of the table of V′. [From MS Vatican Borg. ar. 217,2, courtesy of the Biblioteca Apostolica Vaticana.]

all latitudes. Likewise the later auxiliary tables of Ibn al-Mushrif and al-Ṣūfī (**9.8** and **9.10**) have B as one argument and serve all latitudes.

6.15.1 al-Khaṭāʾī: Cairo

MS Vatican Borg. ar. 217,2 (fols. 6r-7r), copied *ca.* 1500, contains part of a set of auxiliary tables intended for use in Cairo, attributed to Muḥammad ibn al-Amīr Fakhr al-Dīn ʿUthmān

al-Khaṭāʾī, an individual whose name is new to the history of Islamic science. His father was a Mamluk prince who flourished around 1450.[20] (The other tables in this manuscript are by al-Wafāʾī: see **9.9**.) The main function tabulated by al-Khaṭāʾī is discussed below. We first consider the smaller tables of the standard functions which accompany the main set. Those having solar longitude as argument have the same format as the tables for timekeeping in the main Cairo corpus (**2.1.1** and **5.1.1**) and the underlying parameters are the same, namely, $\phi = 30;0°$ and $\varepsilon = 23;35°$.

(a) A table of Sin θ to three digits for each degree of θ.

(b) A table of Sin $H(\lambda)$ to three digits (**6.3.1**).

(c) A table of $B(\lambda)$ to three digits (**6.4.2**).

(d) A table of the solar altitude at the beginning of the afternoon prayer $h_a(\lambda)$. The entries are not the same as those in the main corpus of tables for timekeeping compiled for Cairo (**II-4.9**).

(e) A table of the solar azimuth as a function of the altitude, computed for the equinoxes. Values of $a(h)$ are given to two digits for each degree of h from 1° to 60° (= $\bar{\phi}$) and but for minor variants the entries are the same as those of Ibn Yūnus (**5.1.1**).

(f) A table of a function $f(\lambda)$ which I am unable to identify. The table bears no title and is not referred to in the extant fragment of the instructions. The equinoctial value of f is noted in the margin. The following sample entries from the table may eventually facilitate the identification of the function tabulated:

λ	0°	30	60	90	180	210	240	270
$f(\lambda)$	54,45	58;24	59;49	59;59*	54;45	48;23	42;28	39;57

* read 60;0 (?)

At first sight the entries for $\delta > 0$ resemble the values of Cos $\delta(\bar{\lambda})$, but closer investigation reveals that this is not the function tabulated.

The entries for Sin $H(\lambda)$ and $B(\lambda)$ are generally in error by several digits in the third place. Notes at the side of the tables indicate how to derive $B(\lambda)$ from Sin $H(\lambda)$, and indeed the errors in these tables can be explained by the fact that the values of Sin H were found by linear interpolation in the Sine table. al-Khaṭāʾī also points out that the table of a(h) displays $d(\Delta)$ (**5.1.1**), adding that Ibn Yūnus had compiled azimuth tables for the latitude of Cairo. Otherwise there are no references to any of his predecessors, but unfortunately – as already noted – the instructions to his tables in the Vatican manuscript are incomplete.

The main set of tables is likewise incomplete. The function tabulated is:

$$V'(x,y) = \text{arc Vers} \{ R \cdot y / x \},$$

and values are given to two digits for the domains:

$$x = 47;37,\ 47;57,\ \dots,\ 51;57 \quad \text{and} \quad y = 1, 2, \dots, Y(x),$$

where Y(x) is a certain maximum defined below. The instructions for finding $t(h,\lambda)$ indicate that one should first find $B(\lambda)$ and:

$$H'(h,\lambda) = \text{Sin } H(\lambda) - \text{Sin } h$$

[20] de Zambaur, *Manuel*, p. 105.

using tables (a), (b) and (c) above. Then these arguments are to be entered in the main table, which is called *jadwal faḍl al-dāʾir*, "hour-angle table": see **Fig. 6.15.1**. The result is indeed the hour-angle, since:

$$t(h,\lambda) = V \{ B(\lambda), H'(h,\lambda) \} .$$

The argument x runs in intervals of 0;20 between the limits of B(λ) for Cairo and the argument y runs in unit intervals up to Y(x), which is the greatest integer less than the value of Sin H corresponding to the value of δ underlying the argument x. The table originally contained about 825 entries, but in the Vatican manuscript only the page for y ≥ 31 is to be found now. The entries are rather accurately computed.

Being devised for latitude 30° al-Khaṭāʾī's tables are less useful than al-Khalīlī's auxiliary tables for timekeeping by the sun (**9.4**) which serve all latitudes. The possibility that al-Khalīlī knew of al-Khaṭāʾī's tables can be ruled out because the Damascene astronomer preceded the Cairene astronomer by close to a century. Nevertheless, the existence of al-Khaṭāʾī's tables implies that there were no tables of t(h,λ) and/or T(h,λ) currently available for Cairo (see, however, **2.1.1**). Ibn al-Mushrif's auxiliary tables (**9.8**) are an extension of the kind of tables compiled by al-Khaṭāʾī, for timekeeping by both the sun and the stars and intended to serve all latitudes.

6.15.2 Anonymous: Tlemcen

MS London BL Or. 411,2 is the only known copy of an anonymous commentary on the astronomical poem of the late-14th-century Maghribi scholar al-Jādarī.[21] The commentary was written in Tlemcen at the end of the 14th century and it contains several spherical astronomical tables computed for parameters:

$$\phi = 35;0° \text{ (Tlemcen) and } \varepsilon = 23;35° .$$

Details are given in **II-13.6**. In passing, we note that the commentary contains interesting historical accounts of trepidation,[22] twilight and the obliquity of the ecliptic.

Amongst the spherical astronomical tables are three displaying the functions:

$$\text{Sin H}, \ {}^{1}/_{2}\text{ B} \ \text{ and } \ k$$

(**6.3.4**, **6.4.15** and **8.1.4**), as well as another displaying the function:

$$t \{ {}^{1}/_{2}\text{ B}, [\text{Sin H} - \text{Sin h}] \}$$

for the same latitude. I have not been able to compare these tables with the corresponding ones in the Tunisian corpus of auxiliary tables (**9.7**), from which they were probably lifted.

6.16 Tables of auxiliary functions for timekeeping by the stars

Most *zīj*es contain a catalogue displaying the coordinates of prominent stars for a particular epoch, either in the ecliptic or equatorial system or both.[23] A smaller number of *zīj*es and tables

---

[21] On al-Jādarī see Suter, *MAA*, no. 424a, Renaud, "Additions à Suter", no. 424a, and *Cairo ENL Survey*, no. F26. The anonymous commentary has been previously attributed to Ibn al-Ḥabbāk (Suter, *MAA*, no. 435 and *Cairo ENL Survey*, no. F28), which cannot be correct because he is mentioned in the text.

[22] For a survey of trepidation in the Islamic sources see King & Samsó, "Islamic Astronomical Handbooks and Tables", Section 3.7. See also the text to n. **II**-12:20.

[23] See n. 1:44.

for timekeeping give additional numerical information about particular stars.[24] Thus, for example, al-Battānī in his *Zīj*, after his main catalogue which gives the coordinates of 533 stars, tabulates the meridian altitude, half-arc of visibility of five stars, as well as the longitudes of the points of the ecliptic which rise, culminate and set with each star, all computed for the latitude of Raqqa (36°).[25] The two Yemeni astronomers Abu 'l-ʿUqūl (*ca*. 1300) and Ibn *al-M-s-r-b* (?) (*ca*. 1325)[26] tabulated the same functions for 30 stars, computed respectively for the latitude of Taiz (13;40°) and Zabid (14;40°) (in both the Berlin and Sanaa manuscripts). Our present concern is with a category of star catalogue which displays information particularly useful for reckoning time by the stars, of which the following have come to my attention.

6.16.1 Anonymous: Qandahar (?)

Amongst the various tables copied at the end of MS Berlin Ahlwardt 5751 (Mq. 101,1), copied *ca*. 1300, of the *Zīj* of Kūshyār ibn Labbān (**3.2.1**), there is an anonymous set of auxiliary tables (pp. 186-187) for finding the ascendant at night (*al-ṭāliʿ bi-'l-layl*) from stellar altitudes – see **Fig. 6.16.1**. These tables appear not to be related to the tables of the ascendant at night that follow immediately in the Berlin manuscript. For each of 34 stars five quantities are tabulated to two sexagesimal digits, labelled as follows:

1 *taḍrub jayb irtifāʿ al-waqt fīhi abad^{an}*
multiply the Sine of the instantaneous altitude by this;

2 *taqsim mā yakhruj mina 'l-ḍarb ʿalayhi abad^{an}*
divide the product of the multiplication by this;

3 *tanquṣ al-maqsūm minhu abad^{an} wa-tajʿal al-bāqī qaws^{an} maʿkūsa^{tan}*
subtract the quotient from this and take the inverse Versed Sine;

4 *tanquṣ al-qaws al-maʿkūsa minhu aw tuzād ʿalayhi*
subtract or add the arc from or to this; and

5 *yuzād mā baqiya aw balugha mina 'l-qaws ʿalayhi abad^{an}*,
add the result to this.

The quantities mentioned relate to the following functions:

1 and 3: Vers D, 2: Sin H, 4: D, and 5: $\alpha_\phi(\rho)$

and the instructions conform to the two formulae (*cf.* **F9** and **F21**):

$$T(h) = D - \text{arc Vers} \{ \text{Vers } D - \text{Sin } h \text{ Vers } D / \text{Sin } H \}$$

$$\text{and } \alpha_\phi(\lambda_H) = \alpha_\phi(\rho) + T(h).$$

The underlying latitude in these tables is about 30° (this needs to be investigated further) and they were probably intended for use in some city in the South of Greater Iran, possibly Qandahar (see **3.2.1**). We shall return to these tables in **IX**.

---

[24] See Girke, "Die frühesten islamischen Sternkataloge" (cited in n. 1:44). A prime example of such tables are those presented by Ḥabash al-Ḥāsib for his instrument for reckoning time by night for any latitude (**XIIb-12**), on which the various quantities are registered graphically: see Charette & Schmidl, "Ḥabash's Universal Plate", pp. 138-146.

[25] *Cf.* Kennedy, "*Zīj* Survey", p. 155.

[26] King, *Astronomy in Yemen*, no. 12. Perhaps this is Ibn al-Mushrif, who cannot, however, be identical with the 15th-century Cairene astronomer with the same name (see **I-9.8** and **II-6.15**).

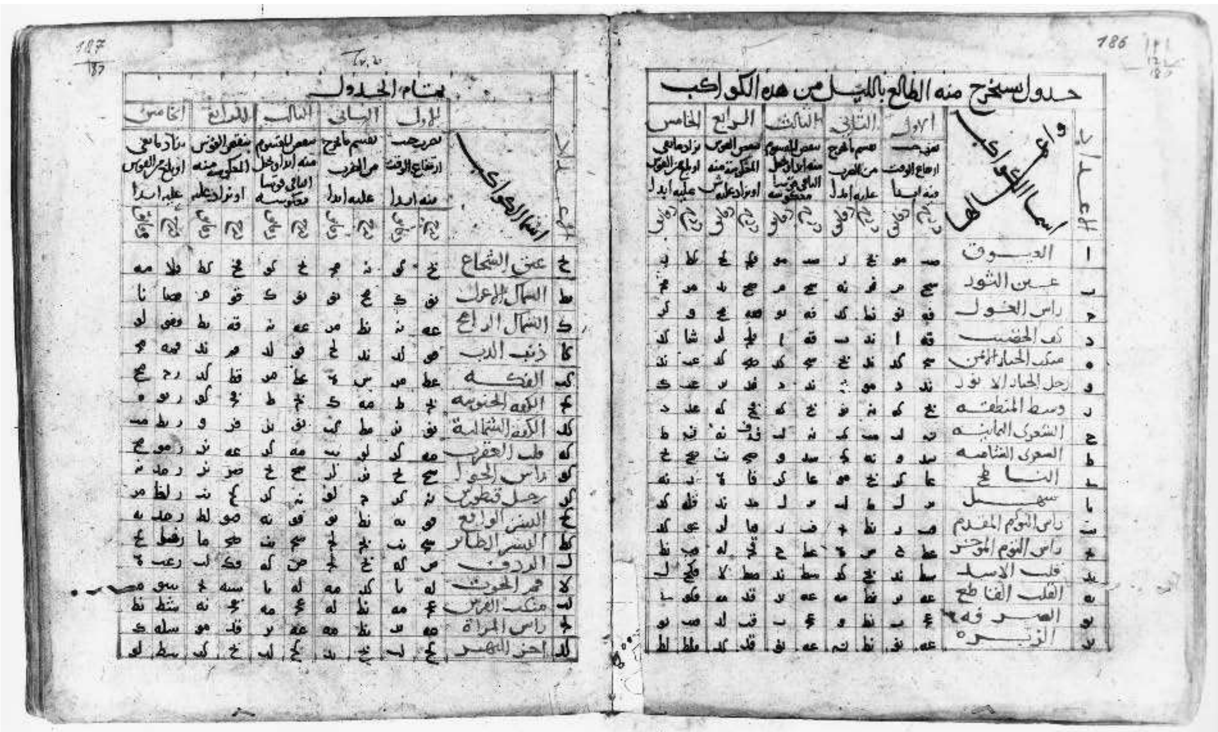


Fig. 6.16.1: Some auxiliary tables for timekeeping by the stars, apparently for the latitude of Qandahar. [From MS Berlin Ahlwardt 5751, pp. 186-187, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

6.16.2 Anonymous: unspecified locality in Iran or al-ʿIrāq

In MS Berlin Ahlwardt 5750, fols. 62v-63r, of the anonymous recension of the *Zīj* of Ḥabash (**9.1** and **9.2**), there is a table displaying the following functions for thirty stars:

λ	*darajat al-kawkab fī 'l-ṭūl*
β	*al-ʿurūḍ*
Δ	*buʿduhā ʿan muʿaddil al-nahār*
α'	*mamarr al-kawkab fī wasaṭ al-samāʾ*
H	*irtifāʿ niṣf nahārihā*
$\alpha_\phi(\rho)$	*maṭāliʿ al-daraja al-ṭāliʿa maʿahu*
D	*niṣf qaws nahārihā*
ρ	*al-daraja al-ṭāliʿa maʿahu*
Cos Δ	*anṣāf al-aqṭār (i.e. radii)*

The last of tabulated functions is of interest to the present study, being the radius of the day-circle, that is, the "day-radius" of Indian astronomy. The hour-angle t corresponding to the instantaneous altitude h of a particular star can be found using (*cf.* **F9**):

$$t = \text{arc Vers } \{ [\text{ Sin H - Sin h }] \cdot R^2 / [\text{ Cos } \Delta \text{ Cos } \phi] \} .$$

Note that it is H rather than Sin H which is tabulated, and also that it would have been more useful (**6.0**) to tabulate the product:

$$B(\Delta,\phi) = \text{Cos } \Delta \text{ Cos } \phi ,$$

rather than simply Cos Δ.

The underlying latitude is 33;25° (Baghdad), which is used elsewhere in the Berlin manuscript, as in the table of oblique ascensions. By virtue of the epoch 304 – whether Hijra (?) [= 916/17] or Yazdigird (?) [= 936] – the star catalogue cannot be due to Ḥabash himself. The star catalogue in MS Istanbul Yeni Cami 784,2, of Ḥabash's *Zīj* (see fol. 192r) is simply that of the *Mumtaḥan Zīj*, compiled for the year 214 H [= 829/30] (**6.16.3**).[27]

6.16.3 Anonymous: Baghdad

MS Escorial ár. 927, copied *ca*. 1300, contains an anonymous recension of the early-9th-century *Mumtaḥan Zīj* (**4.1.1**). It contains a star catalogue dated 380 Yazdigird [= 1012] (fols. 96v-97r), derived by addition of a correction for precession from the *Mumtaḥan* star catalogue dated 214 H [= 829/30], which immediately precedes it in the manuscript (fol. 96r): see **Fig. 6.16.3**. In this later catalogue the following information is given for each of 18 stars:

λ	*mawḍiʿ min falak al-burūj*
β	*al-ʿarḍ*
Δ	*al-abʿād ʿan muʿaddil al-nahār*
μ	*darajat al-mamarr fī wasaṭ al-samāʾ*
D	*niṣf qaws al-nahār*
Vers D	*jayb al-nahār*
H	*irtifāʿ niṣf al-nahār*
Sin H	*jayb irtifāʿ niṣf al-nahār*
$\alpha_\phi(\rho)$	*maṭāliʿ al-daraja al-ṭāliʿa maʿ al-kawkab*
ρ	*al-daraja al-ṭāliʿa maʿ al-kawkab*

The underlying latitude is 33;21° (Baghdad), a parameter used in some other spherical astronomical tables in MS Escorial ár. 927 (**4.1.1**).[28]

6.16.4 Anonymous: Fez and/or Meknes

In 1891 Gäetan Delphin published photographs of an unsigned Moroccan calculating disc (*ṣafīḥa*) dated 1782.[29] This instrument incorporates two small tables displaying the values of the functions:

$$C, B \text{ and } \alpha$$

for 16 prominent stars. Values are given to one sexagesimal digit and the underlying latitude is either 33° as stated on the side of the instrument bearing the tables or 34° as stated on the other side of the instrument. Between the two tables there is engraved a trigonometric grid of the kind known in medieval Arabic as *al-rubʿ al-mujayyab*, complete with semi-circles for finding sines and cosines (**6.5**). With such a grid one could determine the hour-angle from an observation of the altitude of any of the stars using the formula (**6.12**):

---

[27] Debarnot, "*Zīj* of Ḥabash al-Ḥāsib", p. 57. These tables are edited in an unpublished paper by Dorothea Girke (see n. 1:44). See now Charette & Schmidl. "Ḥabash's Universal Plate", p. 139, n. 113.

[28] There are no geographical tables in the Escorial manuscript. According to Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. 55-56, only the *zīj*es of al-Maghribī (Kennedy, "*Zīj* Survey", nos. 41 and 108 – see note 5:8) out of all the numerous Islamic sources investigated have this value for Baghdad. See also King, "Earliest Muslim Geodetic Measurements" (cited in n. 2:26), pp. 221 and 225-227.

[29] See Delphin, "L'astronomie au Maroc".

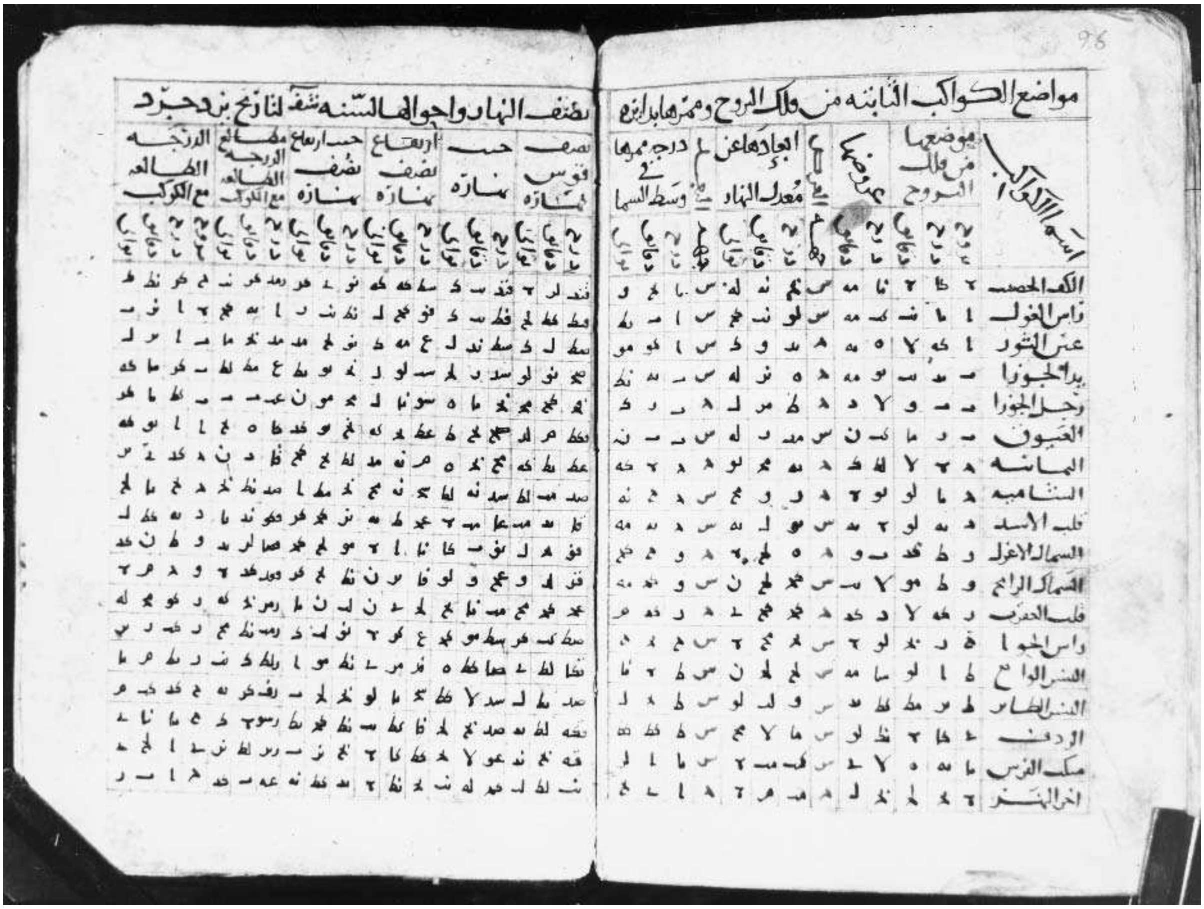


Fig. 6.16.3: The star-table in a recension of the *Mumtaḥan Zīj*, showing various functions useful for timekeeping in addition to the basic ecliptic coordinates of the stars and their declinations. [From MS Escorial ár. 927, fols. 96v-97r, courtesy of the Biblioteca de El Escorial, taken from the 1986 Frankfurt facsimile.]

$$\mathrm{Cos}\ t = R\ [\ \mathrm{Sin}\ h - C\ ]\ /\ B\ .$$

On the other side of the instrument there is an alidade and altitude scale for making such measurements of stellar altitude, as well as a table purporting to display the rising times of the signs of the zodiac for each degree of latitude from 1° to 30° (the entries do not make much sense to me).

CHAPTER 7

TABLES OF AUXILIARY FUNCTIONS FOR CALCULATING THE LENGTH OF DAYLIGHT AND RIGHT AND OBLIQUE ASCENSIONS

7.0 Introductory remarks

As noted in **1.3**, most *zīj*es contain tables of:

$$\delta(\lambda) \text{ and } \alpha(\lambda) \text{ or } \alpha'(\lambda)$$

and of:

$$d(\lambda) \text{ or } D(\lambda) \text{ and } \alpha_\phi(\lambda)$$

for some particular latitude. These functions are related by simple formulae such as (*cf.* **F5**, **F7** and **F18**):

$$\alpha(\lambda) = \text{arc Sin } \{ \text{ R Tan } \delta(\lambda) \text{ / Tan } \varepsilon \}$$

$$d(\lambda,\phi) = D(\lambda,\phi) - 90° = \text{arc Sin } \{ \text{ Tan } \delta(\lambda) \text{ Tan } \phi \text{ / R } \}$$

$$\alpha_\phi(\lambda) = \alpha(\lambda) - d(\lambda,\phi) \ .$$

It is immediately apparent that the function Tan $\delta(\lambda)$ is important in the determination of $\alpha(\lambda)$ and $d(\lambda)$. In **7.1** I list all known Islamic tables of this function. In **7.2** I note the existence of a table of proportional parts of Tan ϕ for various latitudes. Another Islamic formula for $d(\lambda,\phi)$ (which follows from **F5** and **F7**) is:

$$d(\lambda,\phi) = \text{arc Sin } \{ \text{ Sin [max } d(\phi)] \cdot \text{Sin } \alpha(\lambda) \text{ / R } \} \ .$$

Various Islamic sources contain tables of the functions:

$$\text{Sin [max } d(\phi)] \quad \text{and} \quad \text{Sin } \alpha(\lambda) \text{ / R} \ ,$$

which I discuss in **7.3** below. Some of the Cairo and Damascus astronomers who specialized in timekeeping derived approximate formulae for the standard functions of spherical astronomy (see further **II-1.4**): in **7.4** I note the only example known to me of a table based on such an approximation.

The purpose of these auxiliary tables was to facilitate the preparation of tables of d, D or α_ϕ for various latitudes. A table of max $d(\phi)$ to two digits for each degree of latitude is appended to one copy of al-Khalīlī's universal auxiliary tables but being based on $\varepsilon = 23;35°$ is clearly taken from an earlier source: see **Fig. 6.14.3**. His predecessor al-Marrākushī (**4.2.4**, etc.) had tabulated max $d(\phi)$ to three digits for each degree of ϕ from 1° to 66°.[1] In the extensive tables of Najm al-Dīn al-Miṣrī (**2.6.1**) there are tables of $D(\phi,\lambda)$ and $d(\phi,\Delta)$ for each degree of λ and Δ and each degree of ϕ; curiously, alongside these two tables with together over 25,000 entries, there are no associated tables of tan $\delta(\lambda)$ or tan Δ.[2] In the *Ḥākimī Zīj* of Ibn Yūnus there are tables of $\alpha_\phi(\lambda)$ for each degree of both arguments (max ϕ = 48°),[3] and such extensive tables

---

[1] al-Marrākushī, *A-Z of Astronomical Timekeeping*, I, pp. 115, and Sédillot-*père*, *Traité*, I, p. 246.
[2] See now Charette, "Najm al-Dīn's Monumental Table", pp. 22-23.
[3] On these see King, *Ibn Yūnus*, III.14.5.

of oblique ascensions are also contained in the later *zīj*es of al-Ṭūsī, al-Kāshī and Ulugh Beg.[4] Ibn Yūnus' entries are computed to degrees and minutes, al-Ṭūsī's and al-Kāshī's to seconds, and Ulugh Beg's to thirds.

7.1 Tables of the Tangent of the declination for calculating the equation of half-daylight for any latitude and right ascensions

As noted in **7.0**, the function Tan $\delta(\lambda)$ is of use in computing either $d(\lambda)$ or $\alpha(\lambda)$. Most of the tables of Tan $\delta(\lambda)$ listed below have been mentioned in previous studies. Note that tables of the auxiliary functions Sin $d(\lambda)$ and Sin $\alpha(\lambda)$, which are easily derived from tables of Tan $\delta(\lambda)$, also occur in the Islamic sources (see **7.1.2** and **7.1.5**; and **7.2**, **9.2** and **9.3**, respectively). See also **Fig. I-6.9.3** for some other tables of Tan Δ and Tan $\delta(\lambda)$, which by an oversight have not been included in this section.

7.1.1 al-Khwārizmī (Baghdad)

The *Zīj* of the early-9th-century Baghdad astronomer al-Khwārizmī (**4.1.1**) is no longer extant in its original form.[5] However, we do possess a Latin translation by Adelard of Bath based on a greatly modified recension of the *Zīj* by the Andalusī astronomer al-Majrīṭī (*ca*. 1000).[6] There is no table of Tan $\delta(\lambda)$ in this version, but we know from statements by Ibn al-Muthannā (10th century ?) and al-Bīrūnī (*ca*. 1025) that al-Khwārizmī's *Zīj* contained a table entitled *fuḍūl al-maṭāliʿ li-'l-arḍ kullihā*, "ascensional differences for the whole earth".[7] Now in the *Toledan Tables*, a corpus of tables derived mainly from the *Zīj*es of al-Khwārizmī and al-Battānī,[8] as well as in a 15th-century English manuscript,[9] there is a table of the function:

$$e(\lambda) \;=\; 150 \tan \delta(\lambda) \;/\; 12 \;.$$

Values are given to three digits and are based on al-Khwārizmī's parameter $\varepsilon = 23;51°$. See further **10.1**. With this function $d(\lambda,\phi)$ can be found using:

$$d(\lambda,\phi) = \text{arc Sin}_{150} \; \{\; e(\lambda) \cdot \text{Tan}_{12}\; \phi \;\} \;.$$

It is well established that al-Khwārizmī's Sine table[10] was based on the Indian parameter 150

---

[4] See Kennedy, "*Zīj* Survey", pp. 161, 164, and 166, respectively.

[5] On al-Khwārizmī see Gerald Toomer's article in *DSB*, and on various Abbasid works attributed to him, but not all by him, King, "al-Khwārizmī".

[6] See Suter, *al-Khwārizmī*, and Neugebauer, *al-Khwārizmī*, for text, tables, translation, and analysis, and also Goldstein, *Ibn al-Muthannā on al-Khwārizmī*.

[7] See Goldstein, *op*. *cit*., pp. 78-81 and 204-206, and Lesley, "Bīrūnī on Rising Times and Daylight Lengths", pp. 125-127, on these sources.

[8] See Toomer, "Toledan Tables", p. 33, and now Pedersen, *The Toledan Tables*, III, pp. 986-991.

[9] See Neugebauer & Schmidt, "Hindu Astronomy at Newminster", p. 226/430.

[10] al-Khwarizmī's original Sine table has been skilfully reconstructed in Hogendijk, "al-Khwārizmī's Sine of the Hours" (see **4.2.1**).

Another Sine table based on R = 150 survives in the *Toledan Tables* and the same 15th-century English manuscript (*cf.* Toomer, "Toledan Tables", pp. 27-28; Neugebauer & Schmidt, "Hindu Astronomy at Newminster", p. 226/430; North, *Richard of Wallingford*, II, pp. 12-14, and *idem*, *Horoscopes and History*, p. 14). A similar table is found in MS London BL Or. 3624, fol. 170r, of the *Mukhtār Zīj* of the 13th-century Yemeni astronomer Abu 'l-ʿUqūl (see n. 2:11); it is entitled *jadwal jayb al-Hind*, "table of Indian sines", and contains numerous variants from the table published by Neugebauer and Schmidt. Again, in MS Istanbul Ayasofya 4830, copied in Damascus in 626 H [= 1228/29], fol. 188v, amidst various treatises dating from 9th-century al-ʿIrāq (see further **4.1.1**), there is another Sine table based on R = 150. The table from the Latin translation of the

and that he used the value for ε from the *Handy Tables*, namely 23;51° and it seems fairly certain that the above-mentioned tables are ultimately due to al-Khwārizmī.[11] The factor 150/12 essentially converts the product of two tangents (the second to base 12) into a sine (to base 150). See also **7.1.9** on what is probably another such table from 9th-century Baghdad.

7.1.2 Anonymous (Baghdad)

MS Alexandria 5577J of the *Zīj* of the Yemeni astronomer al-Kawāshī (**5.6.1**) contains tables of:

$$\delta(\lambda),\ \delta_2(\lambda) \text{ and } \mathrm{Tan}_{60}\ \delta(\lambda)\ ,$$

each computed to three digits and based on the parameter ε = 23;33°. The same parameter underlies al-Kawāshī's tables of H(λ) and $\alpha_\phi(\lambda)$ for the latitudes of Aden (11°) and Taiz (14;40°), but I suspect that the three tables mentioned above were taken from an earlier ʿIrāqī source. The entry for λ = 90° in the table of Tan δ(λ) is 26;9,53, whereas the accurate value of Tan 23;33° is 26;9,4. See also **7.1.7** on other ʿIrāqī tables of Tan δ(λ) in another Yemeni source and **7.1.9** on a table in a Mamluk Egyptian source also based on obliquity 23;33° and with entries to three digits.

7.1.3 Ibn Yūnus (Cairo)

In MS Leiden Or. 143, p. 271 of the *Ḥākimī Zīj* of Ibn Yūnus (**2.1.1**) the function:

$$e(\lambda) = \mathrm{Tan}_{60}\ \delta(\lambda)$$

is tabulated to four sexagesimal digits for ε = 23;35°.[12] The values are rather accurate: the errors in the fourth digit are less than ±4 in one third of the entries and less than ±30 in the remainder. The entry for λ = 90° is 26;11,33,23 whereas Tan 23;35° is accurately 26;11,33,16 to three sexagesimal fractions. Ibn Yūnus states that he compiled this table by dividing R Sin δ(λ) by Cos δ(λ) for each degree of λ. He refers to the function e(λ) by the same name that al-Khwārizmī used for it and does not explicitly state that it is the Tangent of the declination. His rules for finding α(λ) and δ(λ,ϕ) using the table are equivalent to the formulae:

$$\alpha(\lambda) = \text{arc Sin } \{\ R \cdot e(\lambda)\ /\ e(90°)\ \}$$
$$d(\lambda,\phi) = \text{arc Sin } \{\ e(\lambda) \cdot \text{Sin } \phi\ /\ \text{Cos } \phi\ \}\ .$$

In MS Leiden Or. 143, pp. 270-271 and 347 of the *Zīj*, Ibn Yūnus tabulates the related auxiliary functions:

$$\text{Sin } \alpha(\lambda) \text{ and Sin } d(\lambda)\ (\phi = 30°)\ ,$$

Zīj of al-Majritī published in Suter, *al-Khwārizmī*, pp. 169-170, which is to base 60, is not original to al-Khwārizmī's *Zīj*. On these tables see also King, "al-Khwārizmī", pp. 2 and 34, n. 12.

[11] In MS Berlin Ahlwardt 5793 (Landberg 56), fols. 93v-95v, appended to a unique copy of the treatise on the construction and use of the astrolabe by al-Khwārizmī (see King, "al-Khwārizmī", p. 23), there are some tables which may also be by him. Of particular interest is a table of normed right ascensions α′(λ) with values to 3 digits for each 3° of λ from 273° to 360°. This may be al-Khwarizmī's original table, for as shown in Neugebauer & Schmidt, *op. cit*., p. 224, the values in the table of α(λ) in al-Majrīṭī's *Zīj* do not correspond to values of α(λ) which underlie certain calculations in a text relating to the use of al-Khwārizmī's table of e(λ). Neugebauer and Schmidt noted three values of α(λ) in this text as follows:

0;54,53,40 (λ = 1°), 1;49,33,20 (λ = 2°), and 27;53 (λ = 30°).

The corresponding values which I derive (using linear interpolation for λ = 1° and 2°) from the table of α′(λ) are 0;54,54, 1;49,48, and 27;53,26, so that this table of α′(λ) is probably related to al-Khwārizmī.

[12] See further King, *Ibn Yūnus*, III.13.3d, on this table.

as well as the standard functions:

$$\alpha(\lambda) \text{ and } d(\lambda) \ (\phi = 30°) .$$

His tables of:

$$\text{Tan } \delta(\lambda), \text{ Sin } \alpha(\lambda) \text{ and } \alpha(\lambda)$$

are also found in MS London BL Or. 3624, fol. 173v-176r, of the *Mukhtār Zīj* of Abu 'l-ʿUqūl (**2.1.2** and also **7.2.1**).

7.1.4 Kūshyār ibn Labbān (Iran, locality uncertain)

On fol. 304r of MS Istanbul Yeni Cami 784, fols. 230r-376v, of the *Zīj* of Kūshyār ibn Labbān (**3.2.1**), there is a table[13] entitled *ẓill al-mayl al-awwal*, "tangent of the first declination", displaying the function:

$$e(\lambda) = \text{Tan}_{60}\ \delta(\lambda) .$$

Entries are given to three digits for each degree of λ and are based on ε = 23;35°. The table is considerably less accurate than that of Ibn Yūnus (**7.1.2**). In particular the value of e(90°) is given as 26;11,41 (accurately 26;11,33) and this was probably derived by linear interpolation in a table of Tangents. In the Tangent tables in the Istanbul manuscript (fol. 262v) we find the accurate entries 25;28,7 and 26;42,39 for arguments 23° and 24°, whence by linear interpolation:

$$\text{Tan } 23;35° = 26;11,41,30 = 26;11,41 \text{ (by truncation)} .$$

Kūshyār's table also occurs in MS Paris BNF ar. 5968, fol. 41v, of the anonymous *Dustūr al-munajjimīn*, a *zīj* compiled probably in N. Syria (rather than Alamut in N. W. Iran as has been maintained) in the 12th century and based mainly on the *zīj*es of al-Battānī, Kūshyār, ʿAbū Jaʿfar al-Khāzin and al-Bīrūnī.[14]

In MS Gotha A1402 of the main corpus of tables for timekeeping used in medieval Cairo (**2.1.1**), but in no other such source known to me, there is a table of $\text{Tan}_{60}\ \delta(\lambda)$ with entries based on ε = 23;35° and having 26;11,41 for λ = 90°. I have not been able to ascertain whether this table is related to that of Kūshyār. It is perhaps worth noting that the 13th-century Egyptian astronomer al-Maqsī also used 26;41 for Tan 23;35° in his treatise on sundials (**4.1.3**) rather than the 26;11,40 used by his contemporary al-Marrākushī (**7.1.7**).

7.1.5 Ibn al-Zarqālluh (Toledo)

Amongst the spherical astronomical tables in the *Almanac* of Azarquiel (**6.9.1**) there is one entitled *jadwal fuḍūl al-maṭāliʿ*, "table of the excesses of the ascensions" (p. 225) and the function tabulated is:

$$e(\lambda) = {}^{1}/_{12}\ \text{Tan}_{60}\ \delta(\lambda) .$$

Values are given to three digits and it is stated that the underlying value of ε is 23;33°. This value of ε is the entry for argument λ = 90° in the accompanying table of δ(λ) in the *Almanac*

---

[13] The existence of this table was noted in Kennedy, "*Zīj* Survey", p. 156. There is more work to be done on the two *zīj*es of Kūshyār (see n. 3:4); in particular, an investigation of the tables relating to spherical astronomy in the various available manuscripts might establish the location of his activities. In the *DSB* article on him it is stated, after a medieval source, that he was active in Baghdad, but somehow I doubt this.

[14] On this work, not listed in Kennedy, "*Zīj* Survey", see Zimmermann, "Dustūr al-Munajjimīn".

(p. 174). The entry for $\lambda = 90°$ in the table of e(λ) is 2;10,46 (accurately, 2;10,45). The same table with minor variants occurs in MS Istanbul Kandilli 249 of the *Zīj* of the early-14th-century Tunisian astronomer Ibn al-Raqqām (**6.9.4**), whose declination table, however, is based on the distinctive value 23;32,40° for ε. Here the function is called *nisab juyūb al-faḍlāt li-ʿamal al-maṭāliʿ*, "the ratios of the Sines (*i.e.*, Sines and Cosines) of the excesses (?) for finding ascensions".

A related table in the *Almanac* of Ibn al-Zarqālluh (p. 226) is entitled *jadwal anṣāf fuḍūl ẓuhūrāt al-kawākib*, "table of the half excesses of visibility of stars", and displays values of the function:

$$e(\Delta) = {}^{1}/_{12}\ \mathrm{Tan}_{60}\ \Delta$$

to three digits for each 3° of Δ (and also $\Delta = 88°$ and 89°). It is of interest that the entry for 45° is 4;59,59 rather than 5;0,0: this suggests that the table was not compiled directly from a tangent or cotangent table. (See also **6.9.1**.)

Another table of the same function e(Δ) displaying values to three digits for each degree of argument from 1° to 60°, and also having the entry 4;59,59 for argument 45°, is contained in MS Berlin Ahlwardt 5724, fol. 43r, of an anonymous corpus of tables for Tunis (**2.3.5**). In the Berlin copy the table is entitled *jadwal al-ikhtilāf al-ufuqī*, "table of variation due to the horizon" and the function is labelled *nisbat jayb al-buʿd*, "the ratio of the Sine of the declination" – see **Fig. 6.8.2**.

The advantage to be gained from dividing $\mathrm{Tan}_{60}\ \delta(\lambda)$ or $\mathrm{Tan}_{60}\ \Delta$ by 12 is that using only tables of Cotangents to base 12 and Sines to base 60 one can compute d(λ) or d(Δ) for any latitude with facility. The formulae are:

$$d(\lambda,\phi) = \mathrm{arc\ Sin}_{60}\ \{\ \mathrm{Tan}_{60}\ \delta(\lambda)\ /\ 12 \ \cdot\ \mathrm{Cot}_{12}\ \bar{\phi}\ \}$$
$$d(\Delta,\phi) = \mathrm{arc\ Sin}_{60}\ \{\ \mathrm{Tan}_{60}\ \Delta\ /\ 12 \ \cdot\ \mathrm{Cot}_{12}\ \bar{\phi}\ \}\ .$$

7.1.5* Ibn Isḥāq (Tunis)

MS Hyderabad Āṣafiyya 298 of the *Zīj* of the 13th-century Tunisian astronomer Ibn Isḥāq (**6.9.1***) contains the same table of the function labelled *al-ikhtilāf al-ufuqī* and *jayb nisbat al-buʿd*, "the sine of the ratio of the distance" (table no. 65 in this manuscript), as well as of the function ${}^{1}/_{12}\ \mathrm{Tan}_{60}\ \delta(\lambda)$ labelled *al-fuḍūl* (with maximum value 2;10,46) (table no. 91).[15]

7.1.6 al-Baghdādī (Baghdad)

In MS Paris BNF ar. 2486, fol. 235r, of the late-13th-century *Zīj* of al-Baghdādī (**2.3.1**), there is a table of:

$$e(\lambda) = \mathrm{Tan}_{60}\ \delta(\lambda)$$

with entries to four digits for each degree of λ.[16] The values are based on $\varepsilon = 23;35°$ and are slightly less accurate than those of Ibn Yūnus (**7.1.3**). The entry for $\lambda = 90°$, however, is 26;11,33,16, which is more accurate than Ibn Yūnus' value. I do not discount the possibility that this table was computed by ʿAlī ibn Amājūr in the 10th century (**2.3.1** and **6.4.1***). In MS

[15] See Mestres, Zīj *of Ibn Isḥāq*, pp. 278 and 282.
[16] See Lesley, "Bīrūnī on Rising Times and Daylight Lengths", p. 127.

Paris BNF ar. 2486, fol. 235v, there is a table of the function Sin d(λ), perhaps based on the above-mentioned table of e(λ) and computed for ϕ = 33;25° (Baghdad).

7.1.7 al-Fahhād (?) / al-Marrākushī / al-Fārisī (Cairo / Yemen)

In al-Marrākushī's treatise on spherical astronomy and instrumentation (**4.2.4**), there are tables (I.183-185) of the functions:

$$\delta(\lambda),\ \delta_2(\lambda) \text{ and } \mathrm{Tan}_{60}\ \delta(\lambda)\ (\varepsilon = 23;35°)\ .$$

The first two functions are given to two digits and the third to three digits, for each integral value of λ. Now in the roughly contemporaneous *Muẓaffarī Zīj* of the Yemeni astronomer Muḥammad ibn Abī Bakr al-Fārisī,[17] *e.g.*, MS Cambridge Gg. 3.27, fols. 104v-105r, copied *ca*. 1400, there are tables of the same three functions, each to three digits. al-Marrākushī's values of δ(λ) and $\delta_2(\lambda)$, which contain several errors, can be derived from the corresponding *Muẓaffarī* values by truncation, and the values of Tan δ(λ) in the two sources are identical, but for copyists' errors. The *Muẓaffarī Zīj* is based mainly on the *ʿAlāʿī Zīj* of the mid-12th-century astronomer al-Fahhād.[18] A 15th-century Byzantine astronomical manuscript containing material derived mainly from the *ʿAlāʾī Zīj* likewise contains a table of Tan δ(λ).[19] I have not been able to ascertain whether this Byzantine table is the same as that of al-Marrākushī and al-Fārisī (see also **7.2.2**), but it seems probable that the Islamic tables are due to al-Fahhād or one of his sources. They are characterized by the entry 26;11,40 for argument 90° rather than the 26;11,41 found in the table of Kūshyār (**7.1.4**).

al-Marrākushī also tabulated (I.209-210) the two functions:

$$\mathrm{Tan}_{60}\ \delta(\lambda)\ /\ 12 \quad \text{and } \mathrm{Tan}_{60}\ \Delta\ /\ 12$$

to three and two digits, respectively, for each degree of arguments λ and Δ. He calls the functions *faḍla*, literally "excess". The first table is based on obliquity 23;35° and thus differs from the earlier table of Ibn al-Zarqālluh (**7.1.5**). In MS Cairo MM 43, fol. 42r, of an Egyptian copy of al-Khalīlī's *Universal Table* (**9.5**), two tables of the same functions occur, with numerous variants from al-Marrākushī's tables as published by Sédillot. (See further **7.1.8**.)

7.1.8 Anonymous (Cairo)

In MS Paris BNF ar. 2513, fol. 81r, of a recension of the 13th-century Egyptian *Muṣṭalaḥ Zīj* (**6.7.1**) there is a table of:

$$e(\lambda) \ = \ \mathrm{Tan}_{60}\ \delta(\lambda)\ /\ 12$$

based on ε = 23;35° and with entries computed to three digits. The entries differ from those of al-Marrākushī (**7.1.7**) but the same table occurs in MS Cairo MM 43, fol. 42r, in a set of anonymous spherical astronomical tables for Cairo which also contains al-Khalīlī's universal auxiliary tables (**9.5**); in MS Damascus Ẓāhiriyya 3116, fol. 61r, where it is appended to a

[17] On al-Fārisī and the *Muẓaffarī Zīj* see Suter, *MAA*, nos. 349 and 349N; Kennedy, "*Zīj* Survey", no. 54, and King, *Astronomy in Yemen*, no. 6. An 1822 study Lee, "Astronomical Tables of Al Farsi", is still most useful. See also n. 7:26 below and **II-2.2** and **12.0**.

[18] On al-Fahhād see Kennedy, "*Zīj* Survey", nos. 23 and 84, *inter alia*, and, most recently, Pingree, *Astronomical Works of Gregory Chioniades*.

[19] See Neugebauer, "Studies in Byzantine Astronomy", p. 36b (entries for fols. 43lr-433r).

set of al-Khalīlī's hour-angle tables for Damascus (**2.1.4**); and in MS Cairo TR 275, p. 73, copied in 858 H [= 1454], of the *zīj* entitled *al-Lumʿa* by the early-15th-century Egyptian astronomer al-Kawm al-Rīshī[20] (but not in other copies of this *zīj*); and in the manuscript in a private collection in Sanaa of the astronomical miscellany of the 14th-century Yemeni ruler al-Sulṭān al-Afḍal (**2.1.2**). A significant entry in all of these sources is 1;50,41 for $\lambda = 60°$: al-Marrākushī has the accurate value 1;50,48.

7.1.9 Anonymous (Cairo)

In MS Paris BNF ar. 2520, fol. 69v, copied *ca*. 1400, of another recension of the *Muṣṭalaḥ Zīj* there is a table of:

$$e(\lambda) = \mathrm{Tan}_{60}\ \delta(\lambda)$$

with entries computed to three digits – see **Fig. 9.1a**. The underlying value of ε is 23;33° and I suspect that the table was taken from earlier source, not least because it precedes a copy of the auxiliary tables of Ḥabash (**9.1**). The entry for argument 90° is 26;9,3, whereas the accurate value of Tan 23;33° is 26;29,4. See also **7.1.2** above.

7.1.10 Anonymous (Yemen)

In the anonymous late-14th-century Yemeni *zīj* MS Paris BNF ar. 2523 (**2.5.3** and **4.3.3**) there is a table (fol. 85v) entitled *jadwal ẓill al-mayl al-awwal al-mankūs*, "table of the Tangent of the first declination", displaying the function:

$$e(\lambda) = \mathrm{Tan}_{60}\ \delta(\lambda)$$

to three digits for each degree of λ and apparently based on ε = 23;35° – see **Fig. 4.3.3**. The entries are rather inaccurate: for example, the table gives e(90°) = Tan ε = 26;12,0°, whereas the accurate value of Tan 23;35° is 26;11,33.

7.1.11 Riḍwān Efendī (Cairo)

In MS Istanbul S. Esad Efendi Medresesi 119, fol. 59v, of the prayer-tables of Riḍwān Efendī for Cairo (**6.1.3**), the function:

$$e(\lambda) = \mathrm{Tan}_{60}\ \delta(\lambda)$$

is tabulated alongside:

$$\delta(\lambda),\ \mathrm{Sin}\ \delta(\lambda) \text{ and } \mathrm{Cos}\ \delta(\lambda)\ .$$

Values are given to three digits for each degree of λ and are based on Ulugh Beg's parameter ε = 23;30,17°. I see no reason to doubt that these tables are due to Riḍwān.

7.1.12 Taqi 'l-Dīn (Istanbul)

In MS Istanbul Nuruosmaniye 2930, fol. 23r, of the *zīj* entitled *Sidrat muntaha 'l-afkār* of Taqi 'l-Dīn (**6.1.4**), there is a table of the same function with entries to four *sexagesimal* digits for each degree of λ from 1° to 89°. The underlying parameter is ε = 23;28,54°.

Again, in MS Istanbul Esat Efendi 1976, fol. 25v, of Taqi 'l-Dīn's later *zīj* called *Jarīdat al-durar* (**6.4.8**), the function:

$$e(\lambda) = \mathrm{Tan}_{100}\ \delta(\lambda)$$

[20] On al-Kawm al-Rīshī (Suter, *MAA*, no. 428) see *Cairo ENL Survey*, no. D41.

is tabulated to four significant *decimal* digits for each degree of λ. This table and accompanying one of $\delta(\lambda)$ correspond closely to recomputation with the parameter ε = 23;28,54°. Taqi 'l-Dīn's Sine and Tangent tables in MS Istanbul Esat Efendi 1976 are also given decimally, but to base 10 rather than base 1.[21]

7.2 Tables of auxiliary functions for calculating the equation of half daylight for any latitude

As noted in **7.0**, a widely-used Islamic formula for $d(\lambda,\phi)$ is the following:

$$d(\lambda,\phi) = \text{arc Sin } \{ \text{ Sin } [\max d(\phi)] \cdot \text{ Sin } \alpha(\lambda) \ / \ R \ \} \ ,$$

and tables of the two auxiliary functions:

$$\text{Sin } [\max d(\phi)] \quad \text{and Sin } \alpha(\lambda) \ / \ R$$

are found in various *zīj*es. They are generally labelled *jayb taʿdīl al-nahār al-kullī*, "Sine of the maximum equation of (half-)daylight", and *daqāʾiq al-nisab*, which means "minutes of the interpolation factor". Note that Abū Naṣr's auxiliary functions f_2 and f_3 are equivalent to these (**9.3**). Other combinations of functions with less limited practical application are found in our sources, such as the tables of $\alpha(\lambda)$ and Sin $\alpha(\lambda)$ (for each 10° of λ) and of max $d(\phi)$ (for each 0;30° of ϕ) in the *Zīj* of al-Battānī.[22] Tables of max $d(\phi)$ are attested already in the *Handy Tables*.[23] In **7.4** I remark on the similarity between a Byzantine table for finding $d(\lambda,\phi)$ approximately and the Islamic tables of Sin $\alpha(\lambda)$ / R. Note that the tables in **7.2.0***, which came to my attention only in December, 2001, have Δ as the argument and are for a fixed ϕ.

7.2.0* Early anonymous (N. Iran)
MS Istanbul U.L. A 314 (**6.4.1****) contains tables of

$$\text{Sin } d(\Delta) \qquad \text{and} \qquad d(\Delta)$$

with values to three digits for each degree of argument up to 55°. The tables appear to be computed for a latitude of about 38°, but this copy is extremely corrupt. It seems that we are dealing with an early production for a locality in Northern Iran.

7.2.1 Abu 'l-ʿUqūl (?) (Taiz)
Tables of the three functions:

$$\max d(\phi) \ , \text{ Sin } [\max d(\phi)] \quad \text{and Sin } \alpha(\lambda) \ /R$$

occur in MS London BL Or. 3624, fol. 185v, of the *Mukhtār Zīj* of the Yemeni astronomer Abu 'l-ʿUqūl (**2.1.2**). Values are given to two digits and are based on ε = 23;35°. The argument domains are:

$$\phi = 1°, 2°, \ldots , 60° \text{ and } \lambda = 1°, 2°, \ldots , 90°.$$

A marginal note states that to find the third function one should take the Sine of $\alpha(\lambda)$ and divide it by 150 or 60, according to which base the Sines are computed. A table of "Indian Sines" to base 150 as well as another to base 60 are found elsewhere in the *Zīj* (*cf*. n. 7:10 to **7.1.1**).

---

[21] See n. 1:22.
[22] Nallino, *al-Battānī*, II, pp. 58-59; also Neugebauer, *HAMA*, II, pp. 980-982.
[23] See, for example, Stahlman, *Handy Tables*, p. 264.

The entries in the tables are rather accurately computed, but, unlike many of the other standard spherical astronomical tables in the *Zīj*, are not taken from the *Ḥākimī Zīj* of Ibn Yūnus (**7.1.3**). Indeed it would appear that Abu 'l-ʿUqūl took these auxiliary tables from the same source from which he took the table of "Indian Sines", the identity of which is not yet established. The attribution of these auxiliary tables to Ibn Yūnus is put in question by the fact that the values of max d(ϕ) do not generally correspond to his values for λ = 90° in his tables of oblique ascensions, and also, for example, by the fact that the auxiliary tables have Sin [max d(45°)] (= Tan ε) = 23;11, whereas Ibn Yūnus' value for Tan ε is 23;11,33,23 (≈ 23;12). The formula for d(λ,ϕ) noted in **7.2** was used by Ibn Yūnus, but it was also used by certain of his predecessors such as al-Battānī and anyway is equivalent to the method proposed by Ptolemy in the *Almagest*.[24]

7.2.2 al-Khāzinī (Merw) / al-Fārisī (Aden)

In MS Vatican ar. 761, fol. 152r, of the *Sanjarī Zīj* of al-Khāzinī,[25] compiled in Marw *ca*. 1120, there is a set of tables of the same three functions also based on ε = 23;35° but less accurately computed. Certain significant errors in these tables are also found in the set in MS Cambridge Gg. 3.27, fol. 108r, of the *Muẓaffarī Zīj* of al-Fārisī, compiled in the Yemen about 1260 (**7.1.6**). Since al-Fārisī did not know of the *Sanjarī Zīj*,[26] both sets of tables must have a common source. Auxiliary tables of this kind are found in an early-15th-century Byzantine astronomical manuscript that has been investigated successively by Otto Neugebauer and Alexander Jones: this manuscript includes considerable material from the *Sanjarī Zīj*.[27] Tables of max d(ϕ) and Sin [max d(ϕ)] for ϕ = 16°, 17°, ... , 45° based on ε = 23;35° are also contained in MS Paris BNF ar. 5968, fol. 171r, of the anonymous *Dustūr al-munajjimīn* (**7.1.4**).

In MS Paris BNF supp. pers. 1488, fol. 109v, of the *Ashrafī Zīj* (**2.3.3**) there is a table entitled *taʿādīl al-nahār al-kulliyya al-muʿtabara*, "universal checked equations of daylight", also computed for ε = 23;35°. Three functions are displayed for each degree of ϕ from 1° to 60°, namely:

max d(ϕ), Sin [max d(ϕ)], and f(ϕ) ,

where f(ϕ) is a function labelled *taʿdīl ajzāʾ al-sāʿāt*, "equation of the degrees of the hours", whose nature and purpose escapes me. I can find no reference to the function in the text immediately preceding these tables. Many years ago, Jan Hogendijk explained its purpose to me but both of us have forgotten what it was.

A table of the function Sin α(λ) /R accompanies these three, and the set is followed by tables of oblique ascensions for each degree of λ and each degree of ϕ from 1° to 42°, with separate tables for 0° and 29;30° (Shiraz).

---

[24] *Cf.* King, *Ibn Yūnus*, III.14.1d; Nallino, *al-Battānī*, I, p. 188; Neugebauer, *al-Khwārizmī*, p. 51; and *Almagest*, II.7 (pp. 92-94).

[25] On al-Khāzinī and the *Sanjarī Zīj* see Kennedy, "*Zīj* Survey", no. 27, and the article by Robert E. Hall in *DSB*. The existence of these tables is noted by Kennedy on p. 159. More recent studies include King, *Mecca-Centred World-Maps*, pp. 71-75 and 564-585, and Pingree, "Editing the *Zīj al-Sanjarī*", pp. 105-113.

[26] al-Fārisī lists some 28 *zīj*es known to him (see Lee, "Notice" (cited in n. 7:17), and also Kennedy, "*Zīj* Survey", no. 54), and the *Sanjarī* is not one of them.

[27] Neugebauer, "Studies in Byzantine Astronomy", p. 30 (Appendix 12).

7.2.3 Anonymous (locality uncertain)

In MS Baghdad Awqāf 2966/6294, fol. 9v, of an anonymous *Zīj* containing tables mainly by Ibn al-Shāṭir, al-Ṣāliḥī and Ulugh Beg (**4.3.5**), there is a table of $\delta(\lambda)$ based on Ibn al-Shāṭir's parameter, ε = 23;31°, and a set of the same three auxiliary functions (for ϕ = 1°, 2°, ... , 66°) based on ε = 23;35°. The entries are significantly different from those in the tables noted in **7.2.1** and **7.2.2**. Likewise, in MS Istanbul Hafid Efendi 181, fol. 93v, of the redaction of the *Īlkhānī Zīj* for Damascus prepared by Shihāb al-Dīn al-Ḥalabī (**5.1.2**) the auxiliary functions appear again, also based on ε = 23;35°. In MS Damascus Ẓāhiriyya 7387 of a recension of the *Zīj* of Ibn al-Shāṭir by al-Qazwīnī the same set based on ε = 23;35° occurs once more. However, there are no such tables in MS Oxford Seld. A30 of the original *Zīj* of Ibn al-Shāṭir or in MS Cairo TR 275 of the Egyptian recension by al-Kawm al-Rīshī (**7.1.8**). The same set of tables occurs in the late Tunisian source MS Cairo K 7584,1 (**6.4.16**), in which other spherical astronomical tables are based on Ulugh Beg's parameter ε = 23;30,17°.

7.2.4 Ḥusayn Quṣʿa (Tunis)

MS Princeton Yahuda 147c of the *Quṣʿī Zīj* of the Tunisian astronomer Ḥusayn Quṣʿa (**4.3.6**) contains a page ruled and titled for the three auxiliary functions but there are no entries. It is stated in the title that ε = 23;30°.

7.3 Tables of proportional parts of the tangent of the local latitude, for calculating the equation of half-daylight

7.3.1 Ibn al-Mushrif (Cairo)

In MS Cairo MM 241 of the auxiliary tables of the Egyptian astronomer Ibn al-Mushrif (**9.8**) there is one set of tables of a function f(x,y). The arguments x and y are labelled *ẓill ʿarḍ* and *ẓill mayl*, that is, tangents of the latitude and declination. The horizontal arguments are:

21°, 24°, 30°, 31°, 32° and 33;24° ,

and the vertical arguments are 1°, 2°, ... , 60. The function tabulated is:

$$f(\phi,n) = n/R \cdot \mathrm{Tan}_R\ \phi \qquad (R = 60)$$

where ϕ and n are the horizontal and vertical arguments, so that the table can be used to find the quantity:

$$\mathrm{Tan}\ \Delta / R \cdot \mathrm{Tan}\ \phi$$

which is Sin d(Δ,ϕ). The latitudes correspond to Mecca, Medina, Cairo, Alexandria, Jerusalem and Damascus. The value for the latitude of Damascus, 33;24°, is attested in only one other known Islamic source, namely, the fragment of a *zīj* preserved in MS Utrecht Or. 1442.[28]

The values in the table are rather carelessly copied, which suggests that they were taken from an earlier source. The underlying values of Tan ϕ are respectively:

23;1,55, 26;42,49, 34;38,23, 36;3,6, 37;29,32 and 39;33,46 ,

all of which are accurate. Ibn al-Mushrif notes that this table can also be used to find the auxiliary azimuth function, k(h): see further **8.2.1**.

---

[28] Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 473 (*sub* UTT). See also n. **II**-10:34.

7.4 Tables of auxiliary functions for calculating the arc of daylight approximately

In this section I describe a table for computing the arc of daylight for any latitude, underlying which is the assumption that $d(\Delta,\phi)$ varies linearly with each of Δ and ϕ. It is worth mentioning that in various Byzantine manuscripts of the *Handy Tables* there is a table of a function $\xi(\lambda)$, tabulated to one digit for each degree of λ, with which the equation of daylight is given approximately using:

$$d(\lambda,\phi) \approx \xi(\lambda) \cdot \max d(\phi) .$$

William Stahlman has suggested that this table is a Byzantine addition to the *Handy Tables*.[29] By virtue of the fact that:

$$\xi(\lambda) \approx \mathrm{Sin}\,\alpha(\lambda) / R$$

and that tables of $\mathrm{Sin}\,\alpha(\lambda) / R$ (to two digits) were used by certain Muslim astronomers to determine $d(\lambda,\phi)$ (**7.2**), it may be that this particular table is ultimately of Islamic provenance. On the other hand, if the table is due to Theon then it may have provided the inspiration for the more elaborate Islamic tables.

7.4.1 Anonymous (Cairo)

In MS Paris BNF ar. 2513, fol. 61r, of the recension of the 13th-century Egyptian *Muṣṭalaḥ Zīj* (**6.7.1**) there is a table entitled *jadwal li-maʿrifat qaws al-nahār fī kull balad*, "table for finding the arc of daylight in any locality". The arguments are labelled *al-mayl*, "declination" and the entries *al-faḍla*, "equation of daylight"; the tabulated function is simply:

$$f(\delta) = 0;11\ \delta$$

for $\delta = 1°, 2°, \ldots, 24°$. There are no instructions accompanying the table, but what is intended is that the equation of daylight in equinoctial hours d' is very approximately given by:

$$d'(\delta,\phi) = 2\ d(\delta,\phi) / 15 \approx f(\delta) \cdot \phi / R .$$

This same approximation is mentioned in MS Istanbul Hamidiye 1453, fol. 222v, of an anonymous Egyptian treatise on timekeeping, which I suspect is by Najm al-Dīn al-Miṣrī (**2.6.1** and **II-6.5**). Another such approximation, namely:

$$\max d(\phi) \approx {}^{1}/_{2}\ \phi ,$$

is recorded on the title folio of MS Cairo DM 188, a copy of al-Asyūṭī's prayer-tables for Assiut (**II-6.16**).

---

[29] See Stahlman, *Handy Tables*, pp. 265 and 117-122, for table and commentary; also Neugebauer, *HAMA*, II, pp. 980-982.

CHAPTER 8

TABLES OF AUXILIARY FUNCTIONS FOR AZIMUTH CALCULATIONS

8.0 Introductory remarks

As noted above (**5.1**), the standard Islamic method for finding the solar azimuth from the solar altitude involves the use of two auxiliary functions (*cf.* **F15**). The first of these is called in Arabic *ikhtilāf al-ufq*, literally "difference of the horizon" or *ḥiṣṣat al-samt*, "azimuth component", and is defined by:

$$k(h,\phi) = \mathrm{Sin}\ h\ \mathrm{Sin}\ \phi\ /\ \mathrm{Cos}\ \phi\ .$$

For lack of simple, meaningful English equivalent, I shall refer to this function as the "auxiliary azimuth function". The second auxiliary function is called in Arabic *jayb saʿat al-mashriq* or simply *jayb al-saʿa*, which means "Sine of the rising amplitude". This is determined by:

$$L(\delta,\phi) = \mathrm{Sin}\ \psi(\delta,\phi) = R\ \mathrm{Sin}\ \delta\ /\ \mathrm{Cos}\ \phi\ .$$

With these two functions the azimuth is defined by:

$$a(h,\delta,\phi) = \mathrm{arc\ Cos}\ \{\ R\ [\ k(h,\phi) - L(\delta,\phi)\ ]\ /\ \mathrm{Cos}\ h\ \}\ .$$

In **8.1** and **8.3** I describe the tables of k(h) and L(δ,φ) for specific latitudes which have been located in the Islamic sources, and in **8.5** al-Khalīlī's tables of both functions for each latitude. The quantity:

$$m(h,\delta,\phi) = k(h,\phi) - L(\delta,\phi)$$

is usually called *taʿdīl al-samt*, "equation of the azimuth", in late medieval Arabic. At least one Muslim astronomer compiled a table of the secant function $G_1(h)$ (see already **6.9**) specifically to facilitate azimuth calculations. With this the determination of the azimuth reduces to:

$$a(h,\delta,\phi) = \mathrm{arc\ Sin}\ \{\ m(h,\delta,\phi) \cdot G_1(h)\ \}.$$

I discuss this secant table in **8.4**. al-Khalīlī further tabulated a double-argument auxiliary function with the equation of the azimuth as one argument and the solar altitude as the other (**8.5.1***): the function gives the azimuth without further ado.

8.1 Tables of the auxiliary azimuth function, for a specific latitude

Tables of k(h) for particular latitudes are found in the following sources. Note that al-Khalīlī's auxiliary function g_ϕ (**9.5**) defines k for each degree of both h and φ, thus:

$$g_\phi(h) = \mathrm{Sin}\ h\ \mathrm{Sin}\ \phi\ /\ \mathrm{Cos}\ \phi = k(h,\phi)\ .$$

8.1.1 Ibn Yūnus (Cairo and Baghdad)

Ibn Yūnus (**2.1.1**) compiled tables of k(h) for latitudes 30;0° (Cairo-Fustat) and 33;25° (Baghdad).[1] These tables occur in the *Ḥākimī Zīj*, MS Leiden Or. 143, pp. 381 and 357,

[1] On these tables and those in **8.3.1** see further King, *Ibn Yūnus*, III.18.2 and 19.2. The table for Baghdad is published already in Schoy, *Gnomonik der Araber*, p. 81.

respectively. Values are given to three digits for each degree of argument and are extremely accurately computed. Ibn Yūnus doubtless used his table for Cairo to compile his extensive azimuth tables (**5.1.1**).

8.1.2 Sanjar al-Kamālī: Shiraz, Rayy, and two other cities in Iran

In MS Paris BNF supp. pers. 1488, fol. 208r, of the *Ashrafī Zīj* (**2.3.1**) there are tables of k(h) for the latitudes of four cities in Iran – see **Fig. 6.2.1**. They bear the title *jadwal al-sumūt*, "table of azimuths", but the tabulated function is referred to as *aṣl*, "base". Values are given to two digits for each degree of h and the underlying latitudes are stated to be:

$$29;30°, \quad 30;22°, \quad 33;18° \quad \text{and} \quad 36;0° \; .$$

The first latitude is that of Shiraz and is used in other tables in the *Zīj* (see, for example, **6.4.5**), but the second and third values do not occur in the *Ashrafī* geographical tables, or in any other known medieval geographical tables,[2] and so the tables of k(h) may have been taken from an earlier source. The fourth value is standard for Rayy and the 4th climate.[3]

8.1.3 Anonymous: Tunis

In MS Berlin Ahlwardt 5724, fols. 30v-31r, of the anonymous corpus of tables for Tunis (**2.3.4**) there is a table of k(h) with entries to two digits for ϕ = 37;0° (Tunis). See also **8.3.3** and **8.4.1** on related tables in the same source.

8.1.4 Anonymous: Tlemcen

One of the anonymous tables for Tlemcen in MS London BL Or. 411,2 (**6.4.15**) displays the function k(h) with entries to two digits for ϕ = 35;0° (Tlemcen). There is no accompanying table of Sin $\psi(\lambda)$ or even $\psi(\lambda)$.

8.2 Tables of proportional parts of the tangent of the local latitude, for calculating the auxiliary azimuth function

8.2.1 Ibn al-Mushrif (Cairo)

Ibn al-Mushrif, in his instructions to his auxiliary tables in MS Cairo MM 241 (**7.3.1**), notes that his tables of the function:

$$f(\phi,n) \;=\; n/R \;\cdot\; \mathrm{Tan}_R\,\phi \qquad (R = 60)$$

for various latitudes can also be used to find the auxiliary azimuth function (*ḥiṣṣat al-samt*). One simply feeds in Sin h as the vertical argument, since:

$$k(h,\phi) \;=\; f(\phi,\mathrm{Sin}\ h) \; .$$

---

2 In other words, they are not found in Kennedy & Kennedy, *Islamic Geographical Coordinates*. For the geographical table in the *Ashrafī Zīj* see now King, *Mecca-Centred World-Maps*, pp. 74-75 and 564-585.

3 See Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 284, and King, "Geography of Astrolabes", pp. 6-8.

8.3 Tables of the Sine of the solar rising amplitude, for a specific latitude

The function $L(\delta,\phi) = \text{Sin}\ \psi(\lambda)$ is related to $\text{Sin}\ \delta(\lambda)$ (**6.1**) by the relations:

$$\text{Sin}\ \psi(\lambda) = \text{R Sin}\ \delta(\lambda)\ /\ \text{Cos}\ \phi = \text{Sin}\ \varepsilon\ \text{Sin}\ \lambda\ /\ \text{Cos}\ \phi = \text{Sin}\ [\max\ \psi(\phi)]\ \text{Sin}\ \lambda\ /\ \text{R}\ .$$

Thus $\text{Sin}\ \psi(\lambda)$ can be computed from $\text{Sin}\ \delta(\lambda)$ (**6.1**) or $\text{Sin}\ [\max\ \psi(\phi)]$. Note that Abū Naṣr's auxiliary function $f_1(\phi)$ (**9.3**) defines $\text{Sin}\ [\max\ \psi(\phi)]$, and that al-Khalīlī's auxiliary function f_ϕ (**9.5**) defines the Sine of the rising amplitude for each degree of declination and latitude, thus:

$$f_\phi(\Delta) = \text{R Sin}\ \Delta\ /\ \text{Cos}\ \phi = \text{Sin}\ \psi(\Delta,\phi)\ .$$

Tables of $L(\lambda) = \text{Sin}\ \psi(\lambda)$ for particular latitudes are located in the sources indicated below. Related tables of $\psi(\lambda)$ and $\max\ \psi(\lambda)$ occur in various Islamic sources (**5.4** and **5.8**).

8.3.1 Ibn Yūnus: Cairo and Baghdad

In MS Leiden Or. 143, pp. 356-357, of the *Ḥākimī Zīj*, Ibn Yūnus tabulates $\text{Sin}\ \psi(\lambda)$ for the latitudes of Cairo and Baghdad (**8.1.1**).[4] He also tabulates $\psi(\lambda)$ for Cairo (**5.6.2**). The underlying value of ε is 23;35° and entries are given to three digits for each degree of λ.

8.3.2 al-Khalīlī: Damascus

MS Cairo DM 184 of al-Manāshīrī's prayer-tables for Damascus (**6.1.2**), which he attributes to al-Khalīlī, contains a table of $\text{Sin}\ \psi(\lambda)$ with entries computed to two digits for the parameters:

$$\phi = 33;30° \text{ and } \varepsilon = 23;31°.$$

The same table is contained in MS Damascus Ẓāhiriyya 9233, p. 156, of the main Damascus corpus, where it is introduced in the name of al-Ṭanṭāwī (see also **6.1.2**). al-Khalīlī also tabulated $\psi(\lambda)$ for these parameters (**5.6.6**).

8.3.3 Anonymous: Tunis

In MS Berlin Ahlwardt 5724, fol. 30r (**8.1.3**), there is a table of $\text{Sin}\ \psi(\lambda)$ with entries to two digits for the parameters $\phi = 37;0°$ (Tunis) and $\varepsilon = 23;35°$. There is also a table of $\psi(\lambda)$ for these parameters (**5.6.7**).

8.4 Tables of the secant function for azimuth calculations

On tables of the secant function intended to facilitate the computation of the hour-angle see **6.9** above.

8.4.1 Anonymous (Tunis)

In MS Berlin Ahlwardt 5724, fols. 30v-31r, alongside the table of k(h) for Tunis (**8.1.3**), there is a table of the function:

$$G_1(h) = \text{R}\ /\ \text{Cos h}$$

with entries to two digits for each degree of argument. The two tables are labelled jointly *jadwal*

[4] The table for Baghdad is published already in Schoy, *Gnomonik der Araber*, p. 81.

Fig. 8.4.2: The very special auxiliary function, actually just the secant, for calculating the rising amplitude, found in the anonymous Sfax treatise. That such a table just "pops up" in such a work points to the fact that a large proportion of our potential sources are lost for all time. [From MS Cairo K 7584,1, fol. 55r, courtesy of the Egyptian National Library.]

al-samt li-'l-shams wa-'l-kawākib, "table for solar and stellar azimuth" and there is no reference to the use of the function G_1 in timekeeping calculations.

8.4.2 Anonymous (Sfax)

MS Cairo K 7584,1, fol. 55r, of the anonymous astronomical compilation for Sfax (**6.4.16**) contains a table of the function:

$$G(\phi) = R / \text{Cos } \phi$$

labelled *ḥiṣaṣ*, literally, "shares" or "portions",[5] with values to three digits for each degree of ϕ. The table, shown in **Fig. 8.4.2**, occurs in a section dealing with the computation of ψ and the text prescribes the formula:

$$\text{Sin } \psi(\delta,\phi) = \text{Sin } \delta \cdot G(\phi) \;.$$

[5] See n. 1:48.

8.5 Auxiliary tables for finding the solar azimuth for any latitude

Only in January, 2001, did the following tables come to my attention. I mention them here only in passing: a more detailed account is in **9.4*** and **II-10.3***.

8.5.1* al-Khalīlī

MS Bursa Haraççioğlu 1177,4 (fols. 72r-90r), copied *ca*. 1450, is a unique copy of a set of tables by Shams al-Dīn al-Khalīlī, not identified as the compiler, which contain sub-tables for the following functions:

$$L(\lambda) = \mathrm{Sin}\ \psi(\lambda) \text{ and } k(h)$$

for each 1° of ϕ from 1° to 49°, and each 1° of the arguments λ and h. The value of ε underlying the first table is about 23;30°, possibly 23;31°. With these the instructions describe how to determine the *taʿdīl al-samt*. The third sub-table displays the function:

$$K(x,y) = \mathrm{arc\ Cos}\ \{\ x \bullet R\ /\ \mathrm{Cos}\ h\ \}$$

for each unit value of x and each 1° of h. These universal auxiliary tables represent the ultimate solution to the problem of determining the solar azimuth a(h,λ,ϕ). See further **9.4***, where they are considered in the light of al-Khalīlī's other tables.

CHAPTER 9

TABLES OF AUXILIARY FUNCTIONS FOR SOLVING THE PROBLEMS OF SPHERICAL ASTRONOMY FOR ALL LATITUDES

9.0 Introductory remarks

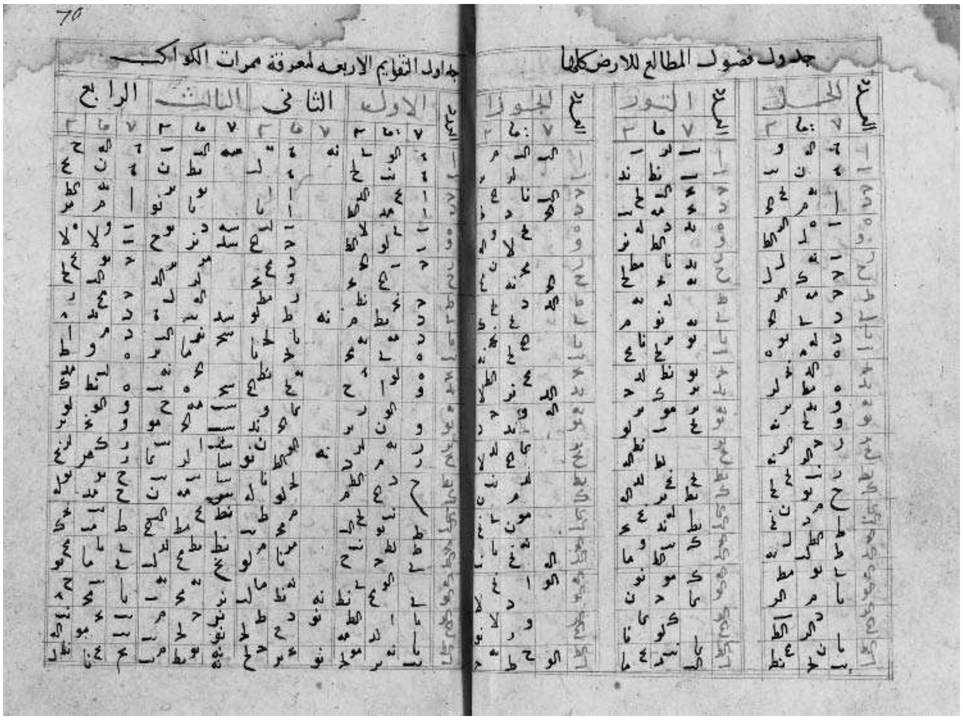
In the 9th century Ḥabash compiled a set of tables of functions which had no specific astronomical significance, but which were so conceived that combinations of them would lead to the solution of certain problems in spherical astronomy. In the 10th century al-Nayrīzī and Abū Naṣr compiled other sets of such functions. These three sets of tables contain a few hundred entries and they are not ideally suited for use in the solution of problems in timekeeping. However, in the early 14th century, the Cairo astronomer compiled a monumental table for timekeeping, with close to half a million entries, and proposed its use as a universal auxiliary table for solving all of the standard problems of spherical astronomy, including the determination of the qibla and the coversion of ecliptic and equatorial coordinates. Such a table, before the age of printing, was doomed from the outset, and it is small wonder that it survives only in a single copy in the hand of its author. In the mid 14th century, the Damascus astronomer al-Khalīlī compiled his more subtle, and eventually more useful, universal auxiliary tables. These contain over 13,000 entries and were likewise devised to solve the standard problems of timekeeping for all latitudes, with no calculation beyond addition, subtraction and interpolation.

In the sequel I briefly describe the auxiliary tables of Ḥabash, al-Nayrīzī, Abū Naṣr and al-Khalīlī (**9.1**, **9.2**, **9.3** and **9.5**), all of which are known from previous studies, and I also present a description of several other sets of auxiliary tables which have come to light during recent research. Most of these, like al-Khalīlī's tables, were intended to be used for problems in timekeeping. Most of the functions displayed in these tables are extensions of the auxiliary functions discussed in **Chs. 6** and **8** to serve more than one latitude. Only the auxiliary tables of al-Khalīlī, Ibn al-Mushrif and al-Wafāʾī (**9.5**, **9.8** and **9.9**) take advantage of the fact that the problems of determining the hour-angle and azimuth from the altitude, declination and latitude, are mathematically equivalent.

9.1 The "Rectification Table" of Ḥabash al-Ḥāsib

Ḥabash, called al-Ḥāsib, meaning "the astronomer" rather than "the calculator", lived for over a hundred years and was most active in Baghdad, Samarra and Damascus in the first half of the 9th century.[1] He was the author of several *zīj*es, one of which survives in a later recension

[1] On Ḥabash (Suter, *MAA*, no. 22) see Kennedy, "*Zīj* Survey", nos. 15 and 16; Sezgin, *GAS*, V, pp. 275-276, and VI, pp. 173-175; King, *Mecca-Centred World-Maps*, pp. 40-41 and 345-349; and also nn. 2-4 below.



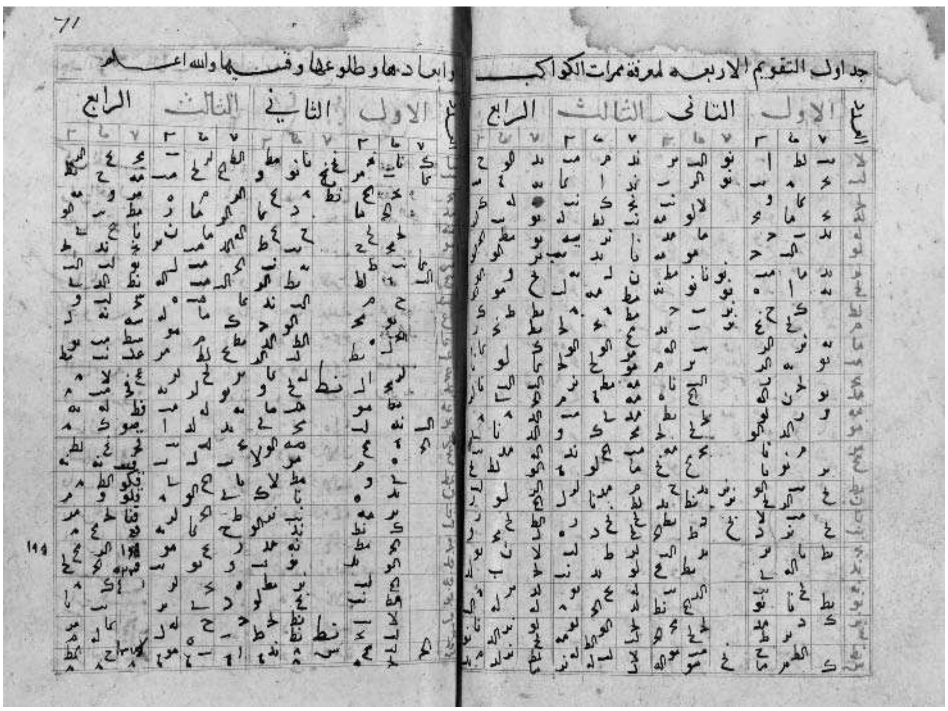


Fig. 9.1a-b: The auxiliary tables of Ḥabash as they occur in a Mamluk *zīj*. On the right of the first double page is a table of the tangent of the solar declination (**7.1.9**). [From MS Paris BNF ar. 2520, fols. 69v-71r, courtesy of the Bibliothèque Nationale de France.]

in MS Istanbul Yeni Cami 784, fols. 69v-229r, from the 13th century. Another anonymous recension of a different *zīj* by Ḥabash is contained in MS Berlin Ahlwardt 5750 (Wetzstein 90), copied *ca*. 1300. The first source has been studied in detail by Marie-Thérèse Debarnot.[2]

The Istanbul manuscript contains two related sets of auxiliary tables, both apparently compiled by Ḥabash himself and both entitled *Jadwal al-taqwīm*, "Rectification Table".[3] The two sets of tables in the Istanbul manuscript have been analyzed in detail in a valuable study by Rida A. K. Irani, alas unpublished.[4] The main difference between the two sets is that one is based on $\varepsilon = 23;35°$ and the other on $\varepsilon = 23;33°$. Ibn Yūnus (**2.1.1**) noted with disapproval that in his "*Mumtaḥan Zīj* called *al-Qānūn*" Ḥabash used $\varepsilon = 23;35°$ in the solar declination tables and $\varepsilon = 23;33°$ in the auxiliary tables.[5]

A set of Ḥabash's tables for $\varepsilon = 23;35°$ is also contained in a treatise preserved in MS Bankipore 2468,8 (fols. 50v-66r), copied 631 H [= 1233/34], which consists of a discussion by Abū Naṣr (**9.3**) of Ḥabash's tables and procedures.[6] Yet other sources exist for the study of Ḥabash's tables.[7] For example, there are various related auxiliary tables in MS Berlin Ahlwardt 5750 of the recension of Ḥabash's *Zīj* (**9.2**), and another copy of the auxiliary tables has been located in MS Paris BNF ar. 2520 of a recension of the Egyptian *Muṣṭalaḥ Zīj* – see **6.7.1** and **Fig. 9.1a-b**. Again, a table of one of Ḥabash's functions occurs in MS Paris BNF supp. pers. 1488 of the Persian *Ashrafī Zīj* – see **6.2.1** and **Fig. 6.2.1**.

Ḥabash's auxiliary functions are the following (R = 60):[8]

$$F_1(\lambda) = \delta_2(\lambda) = \text{arc Tan } \{ \text{ Tan } \varepsilon \text{ Sin } \lambda \ / \ R \ \}$$
$$F_2(\lambda) = \text{Cos } \delta(\bar{\lambda}) = \text{Cos } \{ \text{ arc Sin } [\text{ Cos } \lambda \text{ Sin } \varepsilon \ / \ R \ ] \ \}$$
$$F_3(\lambda) = R \text{ Cos } \lambda \ / \ F_2(\lambda) = R \text{ Cos } \lambda \ / \ \text{Cos } \delta(\bar{\lambda})$$
$$F_{4a}(\theta) = \text{Tan } \theta \text{ Sin } \varepsilon \ / \ R$$
$$F_{4b}(\theta) = \text{Tan } \theta \ .$$

They are of use in solving particular problems of spherical astronomy, though not those directly relating to timekeeping. Values are given to three digits for each degree of argument and I have not investigated their accuracy. Maybe before I retire I shall organize a seminar to prepare a publishable version of Ḥabash's work.

MS Leiden Or. 468 (280 fols., copied *ca*. 1400), contains the introduction and first of five *maqāla*s of a work entitled *Kanz al-yawāqīt fi 'stīʿāb al-mawāqīt* and compiled by an anony-

---

[2] See Debarnot, "*Zīj* of Ḥabash".

[3] The use of the word *taqwīm* here is curious: in medieval Arabic scientific terminology it generally means "the operation of finding the longitudes of the sun, moon, and planets", or "an ephemeris (in which the planetary longitudes are displayed for intervals of a few days)". See n. 1:42.

[4] Irani, "*Jadwal al-Taqwīm* of Ḥabash al-Ḥāsib". I have a photocopy of a carbon copy.

[5] MS Leiden UB Or. 143, p. 223, lines 10-13, translated in Schoy, "Bestimmung der Ortsbreite nach Ibn Yūnus", p. 11.

[6] See Sezgin, *GAS*, VI, p. 243.

[7] Note that: (i) The tables of F3 in MSS Istanbul YC 784 (fols. 226v-227r) and Paris ar. 2520 need further investigation since if they are indeed intended to display the function associated with F_3 the errors are larger than in the rest of the table. See Irani, *op. cit*., p. 75. (ii) The table of F_2 in MS Berlin 5750 (fols. 85r-87v) contains significant variants from those in MSS Istanbul YC 784 (fols. 147r-148v) and Bankipore 2468,8 and may not be due to Ḥabash.

[8] As noted in Irani, *op. cit*., p. 51, the functions tabulated in MS Istanbul YC 784 (fols. 226v-227r) (see previous note) are to base 1. Those in MS Paris ar. 2520, however, are to base 60. Abū Naṣr confirms that Ḥabash used bases 60 and 1 in his two sets of auxiliary tables (*ibid*., p. 128).

mous Egyptian author *ca*. 1350.[9] The work consists mainly of material culled from the *Muṣṭalaḥ Zīj* and the *Kitāb al-Mabādiʾ wa-ʾl-ghāyāt* of al-Marrākushī (**II:6.6-7**), both compiled in Cairo in the late 13th century. Besides various spherical astronomical tables also found in al-Marrākushī's treatise, the *Kanz al-yawāqīt* contains a set of auxiliary tables not found in any other known sources. The tables (fol. 147v-150r) are entitled *jadwal al-nisab*, "table of ratios" and display values of four functions for each half-degree of argument θ from 0;30° to 90° to three sexagesimal digits. The entries in the tables are garbled but the tabulated functions are recognizable as sexagesimal multiples of:

$$\cos\delta(\bar{\theta})\ ,\quad \cos\theta\ /\ \cos\delta(\bar{\theta})\ ,\quad \tan\varepsilon\tan\theta\quad \text{and}\quad \tan\theta\ .$$

I have not compared the entries in this Egyptian source with those found in MS Paris ar. 2520 of the Egyptian *Muṣṭalaḥ Zīj* mentioned above.

9.2 The auxiliary tables of al-Nayrīzī and an anonymous set

In MS Berlin Ahlwardt 5750 of the anonymous recension of one of the *zīj*es of Ḥabash (**9.1**) there are two sets of auxiliary tables, the first (fols. 82r-84v) entitled *al-jadāwil al-jāmiʿa*, "the universal tables", and the second (fols. 85r-87v) entitled *jadwal al-nisab*, "table of ratios". Various operations with the first set are outlined in the text of the *Zīj*. These tables have not been discussed before in any detail,[10] and they deserve a more thorough investigation than is possible here. Indeed our knowledge of early Islamic mathematical astronomy in general will be greatly improved when each of MSS Istanbul Yeni Cami 784 (fols. 69v-229r) and Berlin Ahlwardt 5750 of Ḥabash's *Zīj*, as well as MS Escorial ár. 927 of the anonymous recension of the *Mumtaḥan Zīj*, have been subjected to a critical comparative analysis.[11]

The first set is clearly related to *al-jadwal al-jāmiʿ*, "the universal table", of the late-9th-century astronomer al-Nayrīzī.[12] Abū Naṣr (**9.3**) mentions that al-Nayrīzī incorporated Ḥabash's *jadwal al-taqwīm* into his *Zīj* and added some more functions of his own.[13] This *zīj* is alas no longer extant. The first set of auxiliary tables in the Berlin manuscript displays ten functions, with entries computed to three digits for each degree of argument. Five of these are simply as follows (R = 60):

$$\mathrm{Sin}\,\theta \text{ and } \mathrm{Vers}\,\theta\ ,$$
$$\delta(\lambda),\ \mathrm{Sin}\,\delta(\lambda) \text{ and } \mathrm{Cos}\,\delta(\lambda)\ (\varepsilon = 23;35°)\ .$$

The remainder are labelled "Ḥabash's second, third and fourth (functions), and al-Nayrīzī's first and second (functions)". The first three are indeed Ḥabash's functions F_2 (computed for $\varepsilon = 23;33°$), F_3 and F_{4a} (computed for $\varepsilon = 23;35°$), and the remaining two are simply F_2 and F_3 (also for $\varepsilon = 23;35°$), to base 150 rather than 60.

---

[9] King, *Mecca-Centred World-Maps*, pp. 80-83 and 114, esp. n. 82 on p. 82.

[10] The existence of some of these tables was first noted in Schoy, "Beiträge zur arabischen Trigonometrie", p. 392. See also Irani, *op. cit*., pp. 32-33.

[11] On these three sources see Kennedy, "*Zīj* Survey", nos. 15, 16, and 51, and the abstracts of contents on pp. 145-147 and 151-154. On the *Mumtaḥan Zīj* see also n. 4:7.

[12] On al-Nayrīzī (Suter, *MAA*, no. 88) see Kennedy, "*Zīj* Survey", nos. 46 and 75, and Sezgin, *GAS*, V, pp. 283-285, and VI, pp. 177-182.

[13] See Irani, *op. cit*., p. 19.

This curious assortment of tables is followed immediately by the second set displaying another ten functions. The entries are also computed to three digits for each degree of argument, and the functions tabulated are the following (R = 60):

$g_1(\lambda) = \text{Cos } \delta(\bar{\lambda})$ [$= F_2(\lambda)$]
$g_2(\lambda) = \text{Sin } \alpha(\bar{\lambda})$ [$= g_{10}(\bar{\lambda})$]
$g_3(\lambda) = \alpha(\bar{\lambda})$
$g_4(\phi) = \text{Tan } \phi \text{ Tan } \varepsilon / R$
$g_5(\theta) = \text{Tan } \theta$
$g_6(\lambda) = \lambda(\alpha)$
$g_7(\lambda) = \delta(\lambda)$
$g_8(\theta) = R / \text{Sin } \theta$
$g_9(\theta) = \text{Tan } \theta / R$
$g_{10}(\lambda) = \text{Sin } \alpha(\lambda)$ [$= g_2(\bar{\lambda})$]

Note the following points. First, the tables of g_1, g_2, g_3, g_4, g_6, g_7 and g_{10} are based on ε = 23;35° but the entries in the table of g_{10} do not correspond precisely to those in the table of g_2. Second, the table of g_1 differs from both of Ḥabash's tables of F_2. Third, the value of Tan ε underlying the table of g_4 is 26;11,34 (the accurate value is 26;11,33). (See **7.1** on other values of this parameter.) Fourth, $g_4(\phi)$ is tabulated only up to ϕ = 60°. For the remaining arguments up to 90° a different function, which I am unable to interpret, is tabulated. Sample entries are:

61°	0;47,15	.		85	4;49,22
62	0;49,13	.		86	6;14,34
63	0;51,53	75	1;37,45	87	8;19,20
64	0;53,41	.		88	12;31,30
65	0;56,11	.		89	26;30, 0
.		80	2;33,33	90	50; 0, 0
.		.			
70	1;11,18	.			

Fifth, the function f_8 is simply the modern cosecant: tables of this function were also used in Islamic instrument construction.[14]

Here again we have at first sight a rather motley set of auxiliary tables. But we do well not to forget that we are dealing with tables compiled almost over a thousand years ago. The anonymous compiler has included a function equivalent to Ḥabash's F_2 and Abū Naṣr's f_5 as well as others equivalent to Abū Naṣr's f_2 and f_3. I have not investigated the accuracy of either set of auxiliary tables in the Berlin manuscript.

[14] The cosecant table is published in Schoy, "Beiträge zur arabischen Trigonometrie", p. 15, from defective photographs (Schoy was able to publish only 60 entries rather than the 90 that are found in the manuscript).

For an example of a cosecant table for use in marking curves on astrolabes and quadrants see the Ottoman source MS Princeton UL Yahuda 373, fol. 74r. Here the cosecant and cotangent are tabulated side by side to three sexagesimal digits for each degree of arc: on one possible application of such a pair of tables see King, "al-Māridīnī's Universal Quadrant", App. A.

9.3 Abū Naṣr's "Table of Minutes"

Abū Naṣr ibn ʿIrāq[15] worked in Khwārizm *ca*. 1000 and was a teacher of the illustrious al-Bīrūnī.[16] He was the author of numerous works of considerable merit, two of which concern the present study. The first of these is a commentary on the auxiliary tables of Ḥabash and al-Nayrīzī (**9.1-2**), and the second is a treatise written to al-Bīrūnī in which he introduces a set of auxiliary tables of his own. Both of these texts were published in Hyderabad in 1948 using MS Bankipore 2468,14 (fols. 86v-93r), copied 631 H [= 1233/34], and Abū Naṣr's auxiliary tables have been analyzed in a valuable study by Claus Jensen.[17] MS Oxford Bodleian Thurston 3, fols. 111r-114r, copied 675 H [= 1276/77] is another copy of Abū Naṣr's treatise on his own auxiliary tables – see **Fig. 9.3**.

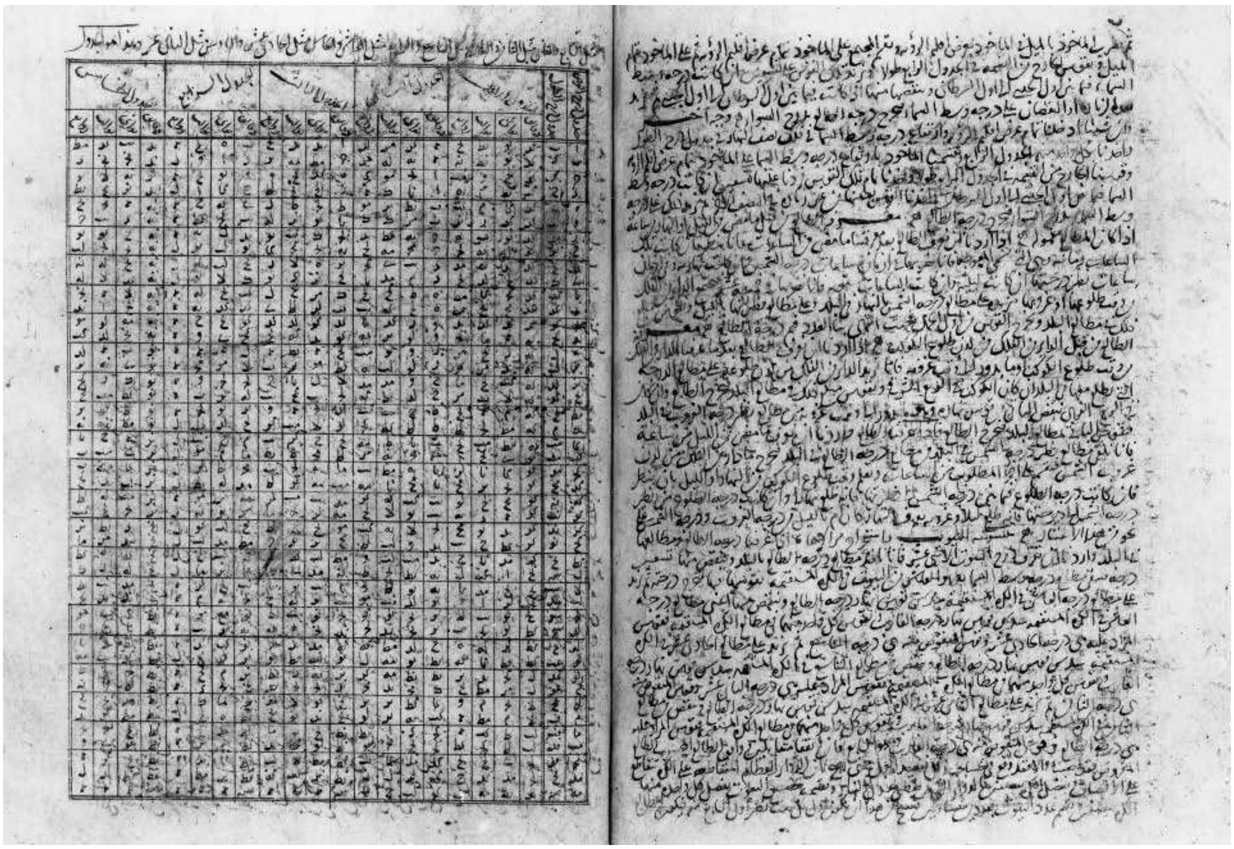


Fig. 9.3: The end of Abū Naṣr's treatise on his auxiliary tables and the first tables. [From MS Oxford Bodleian Thurston 3, fols. 113v-114r, courtesy of the Bodleian Library.]

[15] On Abū Naṣr (Suter, *MAA*, no. 186) see the article "Manṣūr ibn ʿAlī ibn ʿIrāq" by Julio Samsó in *DSB*, and Sezgin, *GAS*, V, pp. 338-341, and VI, pp. 242-245, and more especially Samsó, *Estudios sobre Abū Naṣr*.

[16] On al-Bīrūnī see the splendid article by Ted Kennedy in *DSB*, and also **II-2.2**.

[17] See Jensen, "Abū Naṣr's *Table of Minutes*".

The underlying trigonometric functions used by Abū Naṣr are to base 1 rather than 60, but values are, of course, given sexagesimally. This explains the title *Jadwal al-daqāʾiq*, "Table of Minutes (*i.e.*, sexagesimal fractions)", which he gave to his auxiliary tables. The functions tabulated are the following:

$$f_1(\phi) = \sin\varepsilon / \cos\phi$$
$$f_2(\phi) = \tan\varepsilon \tan\phi$$
$$f_3(\lambda) = \tan\delta(\lambda) / \tan\varepsilon$$
$$f_4(\theta) = \sin\theta$$
$$f_5(\lambda) = \cos\delta(\bar{\lambda}) \ .$$

Abū Naṣr used Ptolemy's value 23;51,20° for ε rather than the value 23;35° which was generally accepted in his time. His tables contain 225 entries, rather carelessly computed to four sexagesimal digits.[18]

Abū Naṣr's $f_4(\theta)$ is simply the sine function, and the other functions $f_1(\phi)$, $f_2(\phi)$ and $f_3(\lambda)$ are respectively:

$$\sin[\max\psi(\phi)] \ , \quad \sin[\max d(\phi)] \quad \text{and} \quad \sin\alpha(\lambda) \ .$$

Tables of max $\psi(\phi)$, max $d(\phi)$ and its Sine, and Sin $\alpha(\lambda)$ occur in several other Islamic sources (**7.2** and **9.2**). Note that Abū Naṣr's f_5 is equivalent to Ḥabash's F_2.

Abū Naṣr's functions are not ideally suited to solving the problems of timekeeping. For example, to find $T(h,\lambda,\phi)$ he suggests a method which involves first finding $\delta(\lambda)$ and $\sin d(\lambda,\phi)$. Thus we begin by using the relations:

$$\delta(\lambda) = f_4^{-1}\ \{\ f_4(\lambda) \cdot f_4(\varepsilon)\ \} \quad \text{and} \quad \sin d(\lambda,\phi) = f_2(\phi) \cdot f_3(\lambda)$$

and then apply:

$$T(h,\lambda,\phi) = f_4^{-1}\ (\sin d) + f_4^{-1}\ \{\ [\ f_4(h) \cdot \sin d + f_4(h)\ ]\ /\ f_4(\ \bar{\phi}+\delta\ ) - \sin d\ \} \ .$$

It is not difficult to show that this last formula is equivalent to **F11**.

Abū Naṣr does not describe the computation of the corresponding azimuth using his tables. The procedure would be clumsy, although the value of $\sin\psi(\lambda)$ is conveniently given by:

$$\sin\psi(\lambda) = f_1(\phi) \cdot f_4(\lambda) \ .$$

9.3* Najm al-Dīn al-Miṣrī's universal auxiliary tables

Najm al-Dīn al-Miṣrī (**I-2.6.1**) intended his monumental triple argument table of a function $F(x,y,z)$ to be used as a universal auxiliary table. A commentary on the use of this table for this purpose is in MS Dublin 102,1, copied *ca*. 1325, and has been studied in detail by François Charette.[19] I refrain here from a detailed commentary, not least in order to encourage the reader to consult Charette's study, which is to be regarded as a supplement to the present work. Suffice it to say that Najm al-Dīn describes not only the use of the table to find the time since rising and the azimuth, and special cases thereof such as the determination of the length of daylight and rising amplitude, but also to find the qibla using the accurate formula and to covert between ecliptic and equatorial coordinates. Also, that Najm al-Dīn shows as much originality in his

---

[18] On the accuracy of these tables see King, "Al-Khalīlī's Auxiliary Tables", p. 107, n. 7.

[19] Charette, "Najm al-Dīn's Monumental Table".

Figs. 9.4a-b: Two extracts from al-Khalīlī's universal auxiliary tables for timekeeping by the sun. The first (a) shows a sub-table for the (half) *aṣl* function for latitudes 31°, 32°, 33°, 33;30° (Damascus), and 34°, with the solar longitude entered vertically. The second (b) shows a sub-table for the hour-angle as a function of the same *aṣl* function entered horizontally, here with arguments 29 and 30, and the difference of the Sines of the meridian and instantaneous altitudes entered vertically. [From MS Dublin CB 4091, fols. 163r and 166r, courtesy of the Chester Beatty Library.]

instructions as in his tirelessness at compiling his enormous table. However, all this seems to have taken its toll on his mind.

9.4 al-Khalīlī's auxiliary tables for timekeeping by the sun

In the unique source MS Dublin CB 4091, copied 833 H [= 1429], there is a set of auxiliary tables by the Damascus astronomer al-Khalīlī (**2.1.4**) for finding t(h,λ,φ), which is quite different from his *Universal Table* (**9.5**). The principal functions tabulated are the following:

$$B'(\phi,\lambda) = \mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi\ /\ 2R = {}^1/_2\ B(\phi,\lambda) \qquad (\varepsilon = 23;31°)$$

for the domains:

$$\phi = 1°, 2°, \ldots, 49° \text{ as well as } 33;30° \text{ (Damascus) and } \lambda = 1°, 2°, \ldots, 90°,$$

and:

$$V'(x,y) = \mathrm{arc\ Vers}\ \{\ R \cdot y\ /\ 2x\ \}$$

for the domains:

$$x = 30, 29, \ldots, 19 \text{ and } y = 0;10, 0;20, \ldots, 60;0 .$$

These two sets of tables – see **Figs. 9.4a-b** – contain respectively 4,500 and 4,680 entries, rather accurately computed to two digits. Horizontal differences are shown in the second set. The first function is called *al-aṣl*, "the base" (*cf.* **6.0**) but a marginal note at the beginning of the table points out that it is in fact *niṣf al-aṣl*, "half the base". The second function is called *faḍl al-dāʾir āfāqī min al-aṣl wa-min faḍl jayb al-ghāya ʿan jayb al-irtifāʿ*, "the hour-angle for all latitudes as a function of the base and the excess of the Sine of the meridian altitude over the Sine of the (instantaneous) altitude".

al-Khalīlī's tables are complete with instructions and are preceded by two small tables of the function:

$$\delta'(\lambda) = 90° + \delta(\lambda)\ (\varepsilon = 23;31°)$$

and the Sine function, with entries to two digits for each degree of argument. To find t(h,λ,φ) al-Khalīlī proposes first using the table of δ′(λ) to find H(λ,φ), thus:

$$H(\lambda,\phi) = \delta'(\lambda) - \phi ,$$

and then using the Sine table to find Sin H and Sin h in order to establish their difference:

$$H'(h,\lambda,\phi) = \mathrm{Sin}\ H(\lambda,\phi) - \mathrm{Sin}\ h\ (> 0) .$$

Then, using the two main sets of auxiliary tables one should find B′(φ,λ), with which:

$$t(h,\lambda,\phi) = V'\ \{\ B'(\phi,\lambda)\ ,\ H'(h,\lambda,\phi)\ \} .$$

al-Khalīlī's tables are clearly an extension of those of al-Khaṭāʾī (**6.15.1**), which were devised for a specific latitude, but the two sets may have been compiled independently. Although only one copy of these original auxiliary tables of al-Khalīlī has come to light, as opposed to the several extant copies of his *Universal Table*, it appears that they became known in both Tunis and Cairo: see further **9.7** and **9.11** and also **II-10.11.**

9.4* al-Khalīlī's universal auxiliary tables for finding the solar azimuth

The following tables are mentioned by al-Khalīlī is the introduction to his main universal auxiliary tables (**9.5**) but the unique surviving copy came to my attention only in January, 2001.

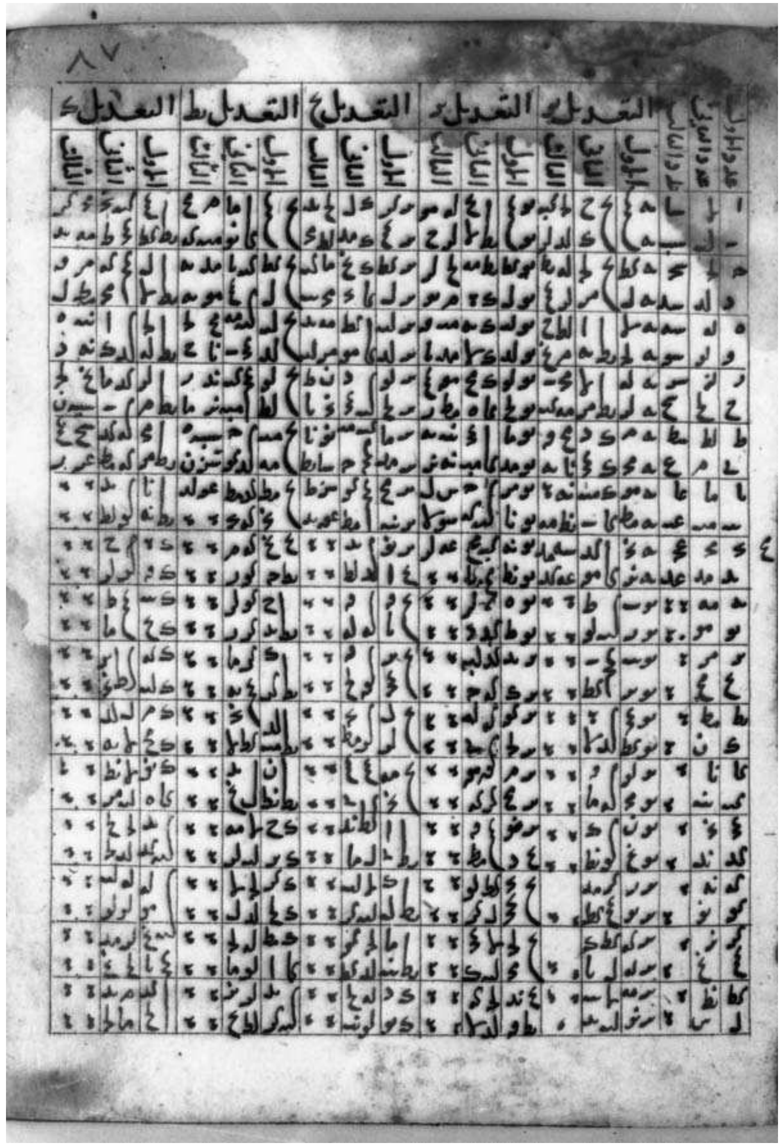


Fig. 9.4*a-b: Extracts from al-Khalīlī's tables of f'_{ϕ} and g_{ϕ} for latitudes 33° and 33;30° (a) and his table of K for arguments 16, 17, ... , 20 (b). See also **Fig. II-10.3*a-b**. [From MS Bursa Haraççioğlu 1177,4, fols. 80v-81r and 86v-87r, courtesy of the Genel Kütüphanesi.]

I have mentioned them briefly in **8.5.1***, and they will be treated again in the context of al-Khalīlī's other tables in **II-10.3***.

MS Bursa Haraççioğlu 1177,4 (fols. 72r-90r), copied *ca*. 1450, is a unique copy of a set of tables entitled *Jadwal al-samt li-kulli 'rtifāʿ fī ʿurūḍ al-aqālīm al-sabʿa*, "table of the azimuth for all altitudes in the latitudes of the seven climates (*i.e*., for all latitudes).[20] No compiler is mentioned, but it can only be Shams al-Dīn al-Khalīlī (**9.4** and **9.5**). The set contain sub-tables for the following functions:

$$f'_{\phi}(\lambda) = R \text{ Sin } \delta(\lambda) \ / \text{ Cos } \phi \quad \text{and} \quad g_{\phi}(h) = \text{Sin } h \text{ Tan } \phi \ / \ R$$

for the domains:

$$\lambda \text{ and } h = 1°, 2°, \ldots, 90° ,$$
$$\phi = 1°, 2°, \ldots, 48°, \text{ as well as } 33;30° \text{ (Damascus)} ,$$

and:

$$K(x,h) = \text{arc Sin } \{ R \cdot x \ / \text{ Cos } h \}$$

for the domains:

$$x = 1, 2, \ldots, 59 \text{ and } h = 0°, 1°, \ldots, n(x) ,$$

where n(x) is the largest integer such that $x \leq \text{Cos } n(x)$. The functions f'_{ϕ} and g_{ϕ} are labelled *jayb saʿat al-mashriq* and *ḥiṣṣat al-samt* by al-Khalīlī, and it is clear that they represent Sin $\psi(\lambda)$ and k(h) – see **5.0** and **8.5**. The value of ε underlying the first table is about 23;30°, possibly 23;31°.

The instructions describe how to determine the *taʿdīl al-samt* using the first two tables, and then how to find the azimuth from the third table. These universal auxiliary tables represent the ultimate solution to the problem of determining the solar azimuth $a(h,\lambda,\phi)$. It is clear that all al-Khalīlī had to do to compile his splendid tables described in the next section was to replace the table of $f'_{\phi}(\lambda)$ with a new one of $f_{\phi}(\theta)$ and rewrite the instructions!

9.5 al-Khalīlī's "Universal Table"

After compiling his two sets of auxiliary tables for timekeeping by the sun (**9.4**) and for finding the solar azimuth (**9.4***), al-Khalīlī prepared another set which can be used for solving all of the standard problems of spherical astronomy for any latitude.[21] These tables are appropriately called *al-Jadwal al-āfāqī*, "The Universal Table". They are preserved in several sources, including MSS Paris BNF ar. 2558, Berlin Ahlwardt 5754/5/6 (Wetzstein 1138) and 5739 (Wetzstein 1144), Escorial ár. 931,8, fols. 171r-211v, Istanbul Ayasofya 2590, Istanbul Hamidiye 1453 (fols. 232v-266v), Istanbul Serez 1914, Oxford Marsh 95 (Uri 961), Cairo MM 43, Cairo MM 98, Cairo DM 758, and Princeton Yahuda 861,2. See also **II-10.7**.

The functions tabulated by al-Khalīlī are:

$$f_{\phi}(\theta) = R \text{ Sin } \theta \ / \text{ Cos } \phi \quad \text{and} \quad g_{\phi}(\theta) = \text{Sin } \theta \text{ Tan } \phi \ / \ R$$

for the domains:

$$\theta = 1°, 2°, \ldots, 90° ,$$
$$\phi = 1°, 2°, \ldots, 55° \text{ as well as } 21;30° \text{ (Mecca) and } 33;30° \text{ (Damascus)} ,$$

[20] I owe the reference to this manuscript to İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 805. My thanks are due to Dr. Sonja Brentjes of Berlin for showing me a microfilm thereof.

[21] See King, "al-Khalīlī's Auxiliary Tables", for the first analysis.

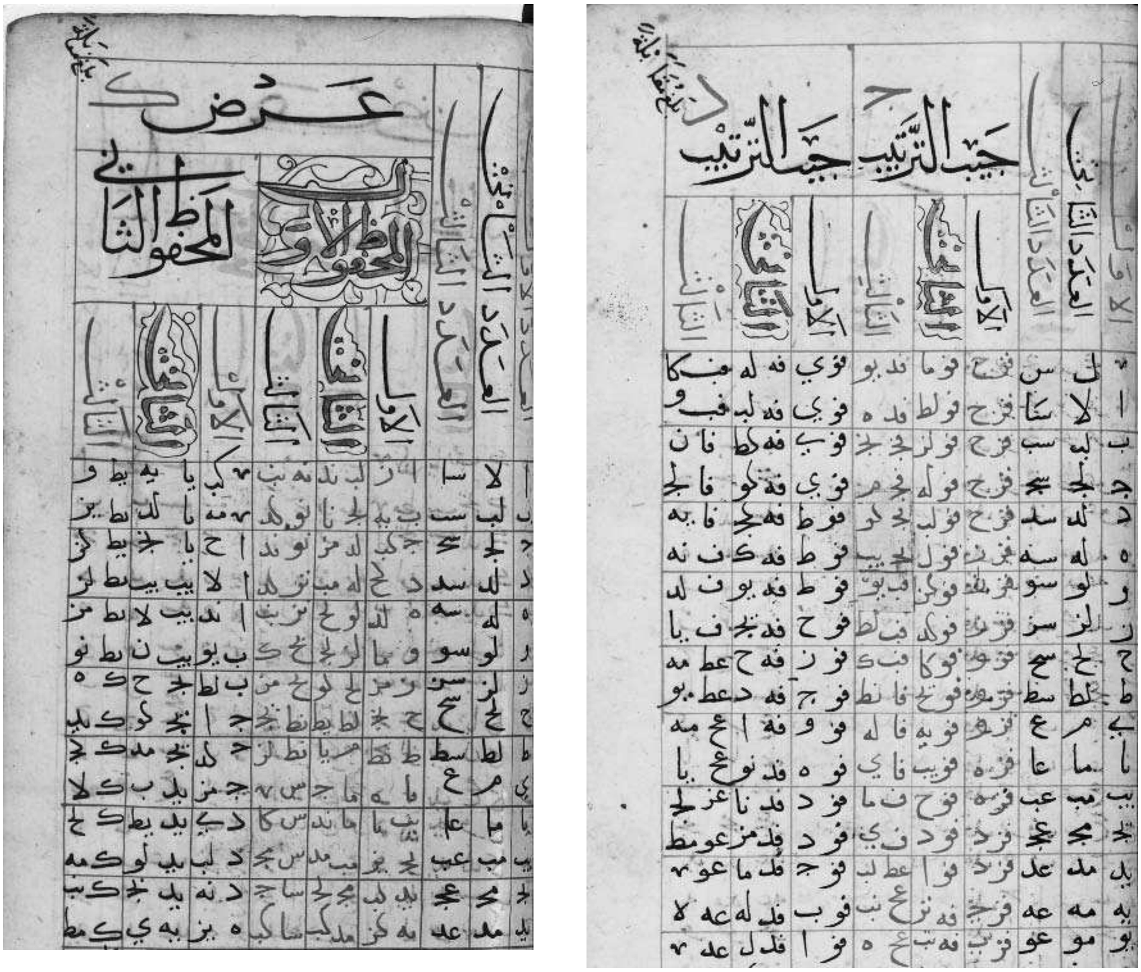


Fig. 9.5a-b: The sub-tables of al-Khalīlī's first and second functions for latitude 20° (a) and his third function for arguments 3-4 (b). See also **Fig. I-6.14.3**. [From MS Paris BNF ar. 2558, courtesy of the Bibliothèque Nationale de France.]

and:

$$K(x,y) = \text{arc Cos } \{ R \cdot x / \text{Cos } y \}$$

for the domains:

$$x = 1, 2, \ldots , 59 \text{ and } y = 0°, 1°, \ldots , n(x) ,$$

where n(x) is the largest integer such that $x \leq \text{Cos } n(x)$. al-Khalīlī calls f_ϕ and g_ϕ *al-maḥfūz al-awwal* and *al-thānī*, "the first and second functions", and he refers to K by the same expression used for the argument x, namely *jayb al-tartīb,* "the auxiliary Sine". His tables contain over 13,000 entries, rather accurately computed – see **Fig. 9.5a-b**. The values for the second and third function were, of course, simply taken over from his universal auxiliary tables for finding the solar azimuth (**9.4***).

To find $D(\Delta,\phi)$, $t(h,\Delta,\phi)$ and $a(h,\Delta,\phi)$ using his tables al-Khalīlī outlines the following rules:

$$D(\Delta,\phi) = 180° - K \{ g_\phi(\Delta) , \Delta \} \ (\Delta>0) \quad \text{or} \quad K \{ g_\phi(\Delta) , \Delta \} \ (\Delta<0)$$

$$t(h,\Delta,\phi) = K\ \{\ [f_\phi(h) - g_\phi(\Delta)],\ \Delta\ \} \quad \text{and} \quad a(h,\Delta,\phi) = K\ \{\ [g_\phi(h) - f_\phi(\Delta)],\ h\ \}\ .$$

The functions are devised to take advantage of the equivalence of the problems of determining the hour-angle and azimuth. Note that al-Khalīlī's f_ϕ and g_ϕ are simply extensions of the auxiliary functions k and Sin ψ used in Islamic azimuth calculations (**8.0**), since:

$$f_\phi(\Delta) = \text{Sin}\ \psi(\Delta,\phi) \quad \text{and} \quad g_\phi(h) = k(h,\phi)\ .$$

al-Khalīlī's *Universal Table* was used for several centuries in Syria and also in Cairo, the Maghrib, and Turkey. MSS Cairo MM 43, Cairo MM 98, Cairo DM 758 and Princeton Yahuda 861,2 are Egyptian copies of these tables, and it is reasonable to assume that both al-Wafāʾī and al-Ṣūfī (**9.9** and **9.10**) had seen the *Universal Table*. MS London BL Add. 9599,31 is a late Maghribi copy of al-Khalīlī's table. MS Istanbul Hamidiye 1453 (fols. 232v-266v) is a copy of the tables prepared *ca*. 1475 in Edirne, and MS Istanbul Ayasofya 2590 is another Turkish copy of the tables preceded by a Turkish translation of al-Khalīlī's instructions, prepared in 896 H [= 1491] by the Ottoman astronomer Muḥammad ibn Kātib Sinān (**2.7.2**). In an unnumbered manuscript formerly (*ca*. 1970) in the private collection of the late Professor Buhairi of the American University of Beirut, there are tables of f_ϕ and g_ϕ for $\phi = 41°$ (Istanbul) and of G for x = 41.[22] The hapless Ottoman copyist who put these three tables together was unaware that the argument x is in no way related to the latitude (**II-10.11** and **14.6**).

9.6 al-Māridīnī's auxiliary tables

The late-14th-century astronomer Jamāl al-Dīn al-Māridīnī, who appears to have worked in both Damascus and Cairo,[23] compiled an auxiliary table which is extant in MS Paris BNF ar. 2525,1 (fols. 1v-16v), copied *ca*. 1450, and also in the much later copy MS Cairo K 4026. al-Māridīnī called his table *al-shabaka*, which means "net" or "grid" and I have analyzed it in detail in a previous publication.[24] The *shabaka* displays values of three trivial functions, arranged in a single 90 × 90 table containing 8,100 entries. No indication is given in the instructions that more than one function is tabulated and in the sequel I use M(x,y) to denote all three functions, which are in fact:

$$M_1(x,y) = R \cdot y\ /\ x \qquad \text{for } 0 \le x \le 59,\ y < x$$
$$M_2(x,y) = \text{Sin } x° \cdot \text{Sin } y°\ /\ R \qquad \text{for } 0° \le x \le 90°,\ x < y \le 90°$$
$$M_3(x,y) = 24\ y\ /\ x \qquad \text{for } 60 \le x \le 90,\ y < x$$

The function M_3 is intended to be used for calculating conjunctions and does not concern the present study.

To derive the hour-angle using his table al-Māridīnī suggests finding the three quantities:

$$B(\phi,\Delta) = M(\bar{\phi},\bar{\Delta})\ (\phi > \Delta) \quad \text{or} \quad M(\bar{\Delta},\bar{\phi})\ (\phi < \Delta)$$
$$C(\phi,\Delta) = M(\phi,\Delta)\ (\phi < \Delta) \quad \text{or} \quad M(\Delta,\phi)\ (\phi > \Delta)$$
$$b(h,\Delta,\phi) = \text{Sin } h \mp C(\phi,\Delta)\ (\text{as } \Delta \gtrless 0)\ ,$$

[22] I am indebted to the late Prof. M. Buhairi of the American University of Beirut for showing me this manuscript from his personal collection.

[23] On al-Māridīnī (Suter, *MAA*, no. 421) see *Cairo ENL Survey*, no. C47; and King, "al-Māridīnī's Universal Quadrant", p. 219, n. 2.

[24] *Ibid*., pp. 231-240. The term *shabaka* is not to be confused with the same term used for a table displaying the solar longitude over a four-year period: see n. 9:26.

which he calls respectively *al-aṣl al-muṭlaq*, *jayb irtifāʿ quṭr al-madār and al-aṣl al-muʿaddal* (see further **6.0**). With these t is defined by:

$$\text{Cos } t(h,\Delta,\phi) = M\{\ B(\phi,\Delta)\ ,\ b(h,\Delta,\phi)\ \}\ .$$

To find the azimuth a(h,Δ,φ) one of al-Māridīnī's methods involves first finding the arc:

$$\xi = \text{arc Sin } (\text{Tan } \phi)$$

and then the three quantities:

$$k(h,\phi) = M(\xi,h)\ (\xi < h) \quad \text{or} \quad M(h,\xi)\ (\xi > h)$$
$$\text{Sin } \psi(\Delta,\phi) = M(\text{Cos } \phi, \text{Sin } \Delta)$$
$$m(h,\Delta,\phi) = k(h,\phi) \mp \text{Sin } \psi(\Delta,\phi)\ (\text{as } \Delta \gtreqless 0)$$

which he calls respectively *ḥiṣṣat al-samt*; *jayb al-saʿa* and *taʿdīl al-samt.* al-Māridīnī also suggests the following alternative method. First form the three quantities:

$$p(h,\phi) = M(h,\phi)\ (\phi > h) \quad \text{or} \quad M(\phi,h)\ (\phi < h)$$
$$q(h,\Delta,\phi) = p(h,\phi) - \text{Sin } \Delta$$
$$r(h,\phi) = M(\bar{\phi},\bar{h})\ (\phi > h) \quad \text{or} \quad M(\bar{h},\bar{\phi})\ (\phi < h)$$

and then:

$$\text{Sin } a(h,\Delta,\phi) = M\{\ r(h,\phi),\ q(h,\Delta,\phi)\ \}\ .$$

9.7 Anonymous Tunisian tables for timekeeping by the sun

MS Cairo DM 689, copied *ca*. 1600, contains an extensive set of auxiliary tables copied in an elegant Maghribi hand.[25] The tables conclude with a star catalogue dated 801 H [= 1398] and they appear to have been compiled in Tunis. The title folio, instructions and first few tables are missing from the manuscript, which begins with the last page of a set of tables of the *shabaka* variety[26] displaying the solar longitude for each day of a period of four Syrian years (see **Fig. 1.2d**). The tables for timekeeping follow this – see the extracts in **Figs. 9.7a-b**.

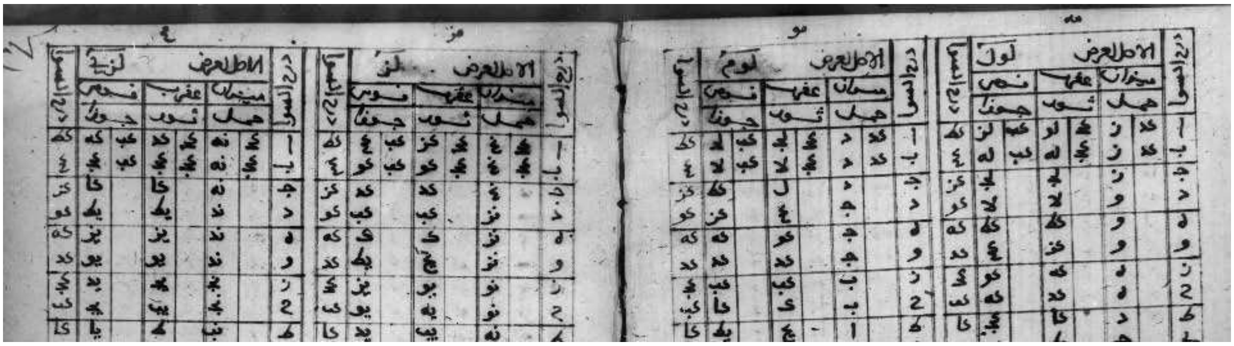
The main functions tabulated are the same as those in al-Khalīlī's auxiliary tables for timekeeping by the sun (**9.4**), namely B′(φ,λ) and V′(x,y). In fact these Tunisian tables are merely an extension of al-Khalīlī's tables and the corresponding entries in both sets are the same. The Tunisian tables of B′ are computed for each degree of φ from 1° to 48° and also 21;40° (Mecca) and various non-integral latitudes between 30° and 40° some of which were intended to serve specific localities in Ifriqiyya and the Maghrib and perhaps also Sicily. al-Khalīlī had a separate table for latitude 33;30° (Damascus) and the Tunisian set has separate tables for latitudes:

30;30°, 31;30°, 32;30°, 33;30°, 33;40°, 34;30°, 35;30°,
36;30°, 36;40°, 37;10°, 37;30°, 38;30°, 39;30° .

The Tunisian tables of V′(x,y) are simply those of al-Khalīlī rearranged so that the horizontal argument is increasing.

In MS Cairo DM 689 the tables of B′ are preceded by a set of tables of the function Sin H(λ), called *jayb al-ghāya*, computed to two digits for each degree of λ and the latitudes of

[25] *Cairo ENL Survey*, no. F30.
[26] See n. 1:43.



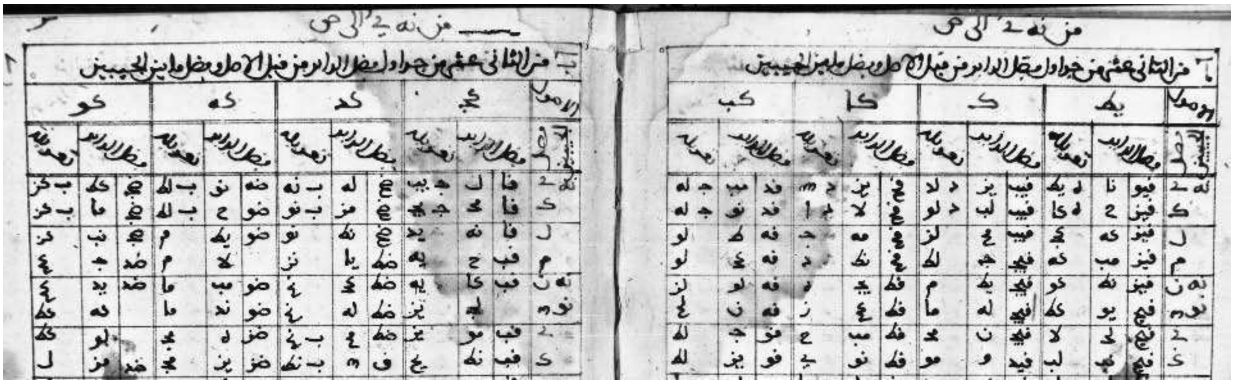


Fig. 9.7a-b: (a) The sub-tables for the *aṣl* B(λ) in the Tunisian corpus for latitudes 36;30°, 36;40°, 37° and 37;10°. (b) This extract shows the table of the hour-angle V′(x,y) for arguments x = B from 19 to 26, and y = H′ = Sin H - Sin h from 55;10 to 60. [From MS Cairo DM 689, fols. 26v-27r and 47v-48r, courtesy of the Egyptian National Library.]

Mecca (21;40°) and Medina (25°), as well as each of the latitudes between 30° and 38° for which B′ is tabulated. These tables were probably used by the anonymous Tunisian astronomer to compile his tables of B′, since:

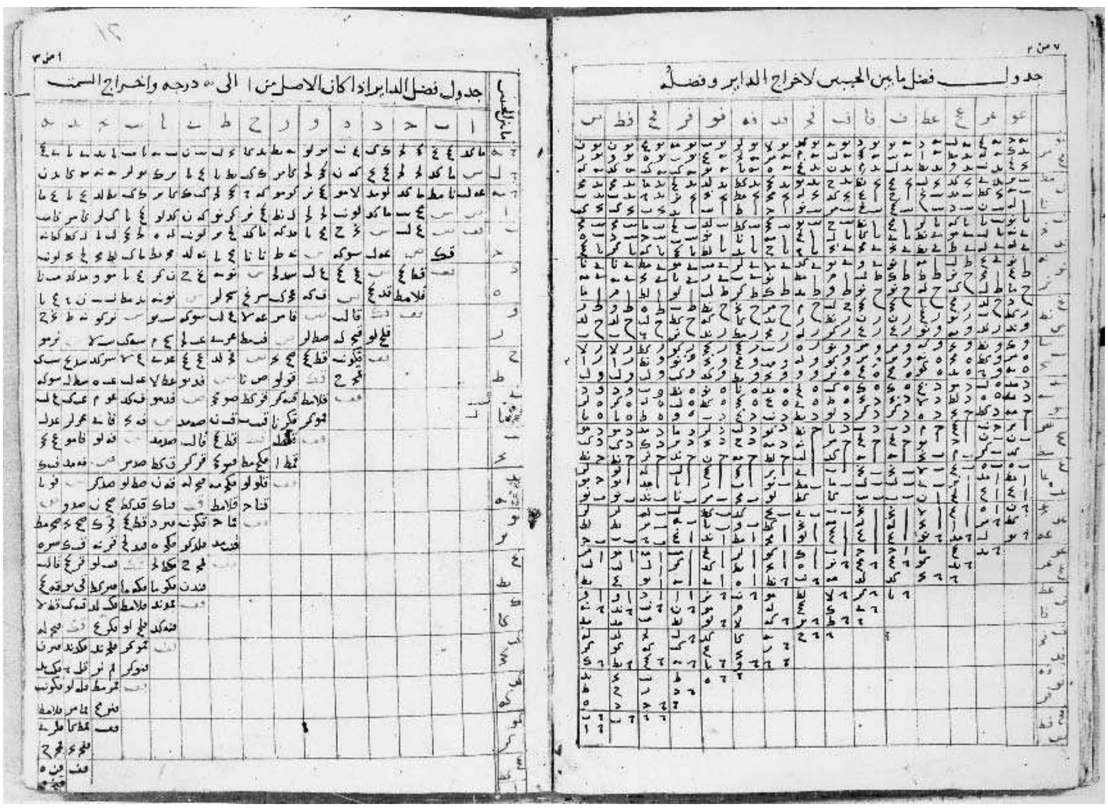
$$B'(\phi,\lambda) = \tfrac{1}{2} \ \{ \text{ Sin } H(\phi,\lambda) + \text{Sin } H(\phi,\lambda^*) \ \} \ .$$

The underlying value of ε is 23;35°, whereas al-Khalīlī used 23;31°. However, the change in ε hardly affects the values of B′(λ) given to two digits. The Tunisian tables also contain a table of Sin H(λ) for φ = 0°, which is simply Cos δ(λ). See further **6.2.5** and **6.3.3**, and **II-13.4**.

9.8 Ibn al-Mushrif's auxiliary tables

MS Cairo MM 241 is an apparently unique copy from *ca*. 1450 of a set of auxiliary tables by Abū Bakr ibn Ismāʿīl, known as Ibn al-Mushrif (see already **6.13.1**, **7.3.1** and **8.2.1**).[27] The work is entitled *Nūr al-aḥdhāq li-maʿrifat aʿmāl al-falak fī sāʾir al-āfāq*, "The Light of the

[27] On Ibn al-Mushrif see *Cairo ENL Survey*, no. C43. He was still active in 848 H [= 1444/45]: see **II-5.7**, so he cannot be identical with the individual with the same name mentioned in King, *Astronomy in Yemen*, no. 12, and **6.16**.


جدول فضل الداير اذا كان الاصل من ا الى ۹۰ درجه واخراج السمت
جدول فضل ما بين الجيبين لاخراج الداير وفضله

Intelligent for Performing Astronomical Calculations for all Latitudes", and appears to have been compiled in Cairo in 1435. The title-folio of this manuscript is shown in **Fig. 9.8a**. The three main functions tabulated are:

$$B(\phi,\Delta) = \text{Cos } \phi \text{ Cos } \Delta \ / \ R$$

for the domains:

$$\phi = 0°, 1°, \ldots , 89° \text{ and } \Delta = 0°, 1°, \ldots , 90° \ ;$$

$$H'(H,h) = \text{Sin } H - \text{Sin } h$$

for the domains:

$$H = 1°, 2°, \ldots , 90° \quad \text{and} \quad h = 1, 2°, \ldots , H - 1° \ ;$$

and:

$$V'(x,y) = \text{arc Vers } \{ \ R \cdot y \ / \ x \ \}$$

for the domains:

$$x = 1, 2, \ldots , 60 \quad \text{and} \quad y = 0;15, 0;30, 0;45, 1, 2, \ldots , 60 \ .$$

These three functions are called *al-aṣl*, "the base", *faḍl mā bayn al-jaybayn*, "the difference between the two Sines", and *faḍl al-dāʾir idhā kān al-aṣl kadhā daraja wa-ikhrāj al-samt*, which means "hour-angle or azimuth for base so-and-so". An extract is shown in **Fig. 9.8b**. The tables are complete with instructions and to find the hour-angle $t(h,\Delta,\phi)$ one simply finds $B(\phi,\Delta)$ and $H'(H,h)$ and uses these as arguments in the third table, thus:

$$t(h,\Delta,\phi) = V' \ \{ \ B(\phi,\Delta) \ , \ H'(H,h) \ \} \ .$$

The method for finding the azimuth $a(h,\Delta,\phi)$ is to find $B(\phi,h)$ and:

$$H' \ \{ \ (\ \bar{\phi} + h \ ), \ \Delta \ \} = \text{Sin } (\ \bar{\phi} + h \ ) - \text{Sin } \Delta$$

using the first two tables, and then the azimuth (measured from the north point) is given by:

$$a(h,\Delta,\phi) = V'\{ \ B(\phi,h), \ H'\{ \ (\ \bar{\phi} + h \ ), \ \Delta \ \} \ \} \ .$$

Various other minor functions are tabulated by Ibn al-Mushrif including $\delta(\lambda)$ for ε = 23;35° and $B(\lambda)$ for ϕ = 30° (**6.4.3**) and the inverse Versed Sine function (**6.14**), as well as a table for finding the equation of half daylight or the auxiliary azimuth function (**7.3.1** and **8.2.1**). One small table, which is not referred to in the instructions and whose purpose eludes me, is entitled *jadwal taḥwīl juyūb al-tartīb*, "Table for converting the auxiliary Sines". The argument is *al-ʿarḍ*, "latitude", and the values of the function $f(\phi)$ are labelled *al-taʿdīl*, "equation" or "interpolation factor". The tabulated function displays discontinuities between ϕ = 30° and 31° and between ϕ = 31° and 32°. Sample entries are:

←

Figs. 9.8a-b: (a) The title-folio of the unique copy of Ibn al-Mushrif's tables. The owner was the copyist Abu 'l-Yumn Muḥammad ibn Muḥammad ibn Muḥammad ibn ʿArab (*Cairo ENL Catalogue*, I, pp. 714 and 761). There is a note in a different hand listing the planetary apogees for epoch 740 H [= 1339/40] from the *Muṣṭalaḥ Zīj*, the most popular *zīj* in medieval Egypt. A further note in the lower left corner is in the hand of Ibn Abi 'l-Fatḥ al-Ṣūfī, the leading astronomer in Egypt *ca*. 1500: it deals with what one might call "advanced timekeeping". (b) The last part of the table of the second function and the beginning of the table of the third function in Ibn al-Mushrif's set. By the nature of the functions both tables are of the *ṭaylasān* variety. [From MS Cairo MM 241, fols. 1r and 11v-12r, courtesy of the Egyptian National Library.]

φ	f(φ)	.		30	; 0,35,24	.	
1°	; 0, 0, 0	.		31	1; 0,37,11	.	
2	; 0, 1, 27	20	; 0,20,36	32	1; 1,16,48	40	1; 7,49,50
3	; 0, 2, 21	.		33	1; 1,24,24	.	
.		.		.		.	
.		29	; 0,33,37	35	1; 3,25,19	45	1;13,29, 4
10	; 0, 9, 16						

The tabulated function is certainly neither Sec φ nor arc Vers (x) / x nor Vers φ.

MS Cairo DM 512, copied *ca*. 1080 H [≈ 1670], contains an extract from Ibn al-Mushrif's tables by Sūdūn al-Bashtakī.[28] The author presents only the tables of V′(x,y), with his own introduction. MS Cairo MM 209,1 (fols. 1r-37r) contains tables of normed right ascension to three digits for each minute of ecliptic longitude attributed to "Abū Bakr ibn al-Mushrif", copied in 873 H [= 1468/69] from a copy by the author dated 848 H [= 1444/45].

9.9 al-Wafāʾī's auxiliary tables

al-Wafāʾī was an Egyptian astronomer who died *ca*. 1470.[29] MSS Vatican Borg. ar. 217,1 (fols. 1v-5v), copied *ca*. 1500, and Istanbul Nuruosmaniye 2921,2 (fols. 22r-26v) contain a small set of auxiliary tables attributed to him entitled *Kifāyat al-waqt li-maʿrifat al-dāʾir wa-faḍlihi wa-'l-samt*, which might be rendered "All you need to find the time, hour-angle and azimuth".

The first function tabulated is:

$$G(\phi,\theta) = R^3 / [\ \text{Cos}\ \phi\ \text{Cos}\ \theta\ ]$$

with values to three digits for horizontal arguments (latitudes):

φ = 12° (Hadramawt, Aden), 15° (Zabid, Sanaa), 18° (Ethiopia), 21° (Aswan, Jidda), 24° (Qift, Yathrib (= Medina)), 27° (Akhmim, Yenbo), 30° (Fayyum, Aqaba), 32° (Jerusalem, Hebron), 33;30° (Damascus, Tripoli), 36° (Mardin, Tunis), 39° (Malatya), 42° (Edirne) and 45° (Constantinople)

and vertical argument:

$$\theta = 0°, 1°, \dots , 89° ,$$

and the second function is simply:

$$V(x) = \text{arc Vers}(x) ,$$

with values to degrees and minutes for:

x = 0;5, 0;10, ... , 0;30, 0;40, ... , 2;0, 2;15, ... , 3;0, 3;20, ... , 4;0, 4;30, ... , 16, 17, 18, ... , 120 .

The two tables of these functions are labelled *jadwal al-ḥiṣaṣ* and *jadwal faḍl al-dāʾir*, that is "argument table" and "hour-angle table". These titles generally refer to tables giving the times of prayer and tables in which the hour-angle is given as a function of solar altitude, respectively. An extract is shown in **Fig. 9.9a-b**.

---

28 On Sūdūn see *Cairo ENL Survey*, no. C81.

29 On al-Wafāʾī see Suter, *MAA*, no. 437, and *Cairo ENL Survey*, no. C98.

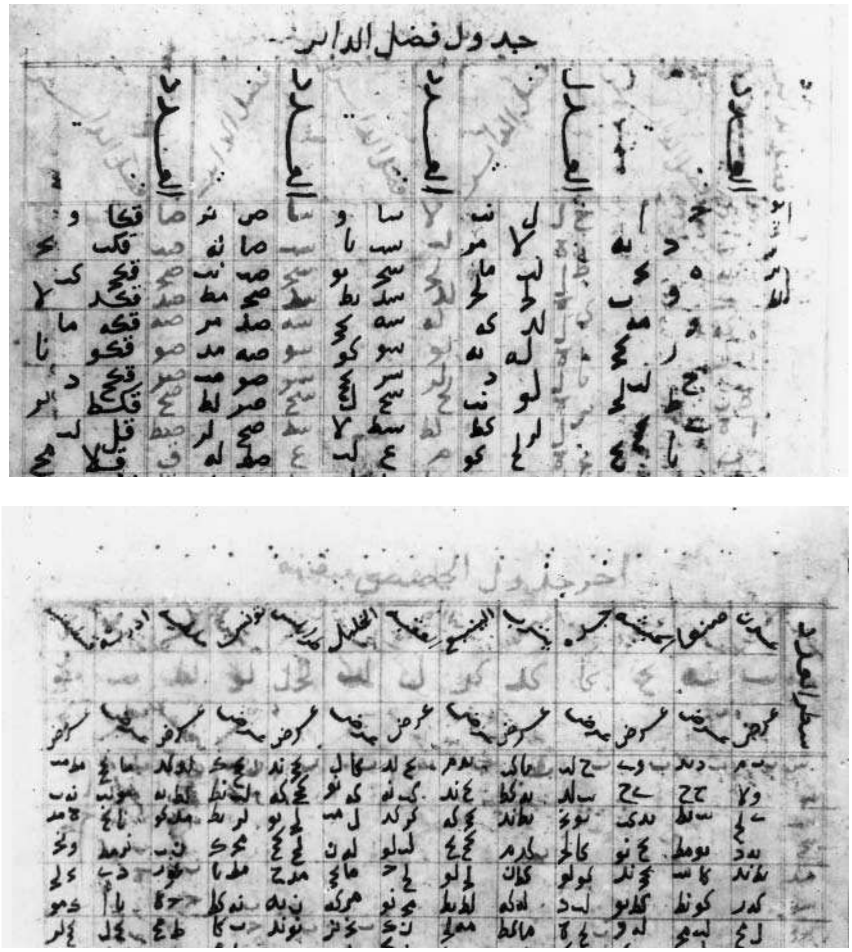


Fig. 9.9a-b: Extracts from the auxiliary tables of al-Wafāʾī. They are followed by part of the incomplete auxiliary tables of al-Khaṭāʾī (**6.15.1**). [From MS Vatican Borg. ar. 217,1, courtesy of the Biblioteca Apostolica Vaticana.]

al-Wafāʾī's tables, which contain 1,320 entries, are rather accurately computed and are complete with instructions. To find t from h, Δ and ϕ one first computes:

$$H'(h,\Delta,\phi) = \text{Sin } H(\Delta,\phi) - \text{Sin } h = \text{Sin } (\bar{\phi} + \Delta) - \text{Sin } h$$

and then uses:

$$t(h,\Delta,\phi) = V \{ H'(h,\Delta,\phi) \cdot G(\phi,\Delta) \} .$$

Similarly, to find $a(h,\Delta,\phi)$ one computes:

$$H'(\Delta,h,\phi) = \text{Sin } (\bar{\phi} + h) - \text{Sin } \Delta$$

and then uses:

$$a(h,\Delta,\phi) = V \{ H'(\Delta,h,\phi) \cdot G(\phi,h) \} ,$$

where a is measured from the north point. al-Wafāʾī's use of the functions G and H to determine t or a with equal facility is ingenious and it fully exploits the equivalence of the underlying mathematical problems. He also demonstrates how to compute the duration of twilight and the direction of Mecca with his tables.

9.10 Ibn Abi ’l-Fatḥ al-Ṣūfī’s auxiliary tables

Ibn Abi ’l-Fatḥ al-Ṣūfī was an Egyptian astronomer who died *ca*. 1495.[30] He prepared a recension of the early-15th-century *Zīj* of Ulugh Beg of Samarqand, adapting the planetary tables for the longitudes of Cairo and he was also the author of several short treatises on quadrants and sundials. His auxiliary tables are contained in the apparently unique source MS Oxford Seld. Supp. 101 (Uri 1040), copied in 918 H [= 1513], and they are entitled *Kitāb al-Jawāhir fī maʿrifat al-samt wa-faḍl al-dāʾir*, “Book of Jewels for Finding the Azimuth and the Hour-angle”.

The first and second functions tabulated by al-Ṣūfī are the following:

$$C(\xi,\eta) = \mathrm{Sin}\ \xi\ \mathrm{Sin}\ \eta\ /\ R \quad \text{and} \quad B(\xi,\eta) = \mathrm{Cos}\ \xi\ \mathrm{Cos}\ \eta\ /\ R\ ,$$

computed to two digits for the domains:

$$\xi = 0°, 1°, \ldots , 89°,\ \eta = 0°, 1°, \ldots , 90° \text{ and also } \eta = 33;30° \text{ and } 23;35° \ (= \varepsilon)\ .$$

The third is:

$$S(x,y) = \mathrm{arc\ Cos}\ \{\ R \cdot x\ /\ y\ \}\ ,$$

computed likewise to two digits for the domains:

$$x = 0;30,\ 1;0,\ \ldots ,\ 59;30 \quad \text{and} \quad y = [x] + 1,\ [x] + 2,\ \ldots ,\ 60\ .$$

The first two functions are called *ufq* and *aṣl* respectively, that is, “horizon” and “base” (see further **6.0**). The third function is referred to by the term applied to the argument x, namely, *al-muqanṭara*, “almucantar”, for what reason is not clear.

al-Ṣūfī’s tables, which contain about 15,000 entries, are more sophisticated than those of al-Māridīnī (**9.6**) but are hardly an improvement over the *Universal Table* of al-Khalīlī (**9.5**). In fact, it seems that they owe their inspiration to both al-Māridīnī’s set and al-Khalīlī’s first set of auxiliary tables (**9.4**): compare al-Ṣūfī’s function C with al-Māridīnī’s M_1 and his functions B and S with al-Khalīlī’s B′ and V′. The link with al-Khalīlī is also apparent from the fact that al-Ṣūfī, working in Cairo, tabulates B and C for $\phi = 33;30°$, which serves Damascus. Likewise it may be that al-Ṣūfī noticed that al-Māridīnī’s function M_1 would be more useful if one could feed in the non-integral value of ε as one argument in order to facilitate the computation of δ(λ): his tabulating C for η = 23;35° satisfies this need. al-Ṣūfī clearly had not learned much from the investigations of the Damascus and Samarqand astronomers on the secular change of ε: he preferred to use the standard value, 23;35°. He begins his work with a table for finding the solar longitude for the period 877-1004 H [= 1472-1596], based on Ibn Yūnus’ solar parameters derived five centuries previously. al-Ṣūfī’s auxiliary tables can be used for computations involving the stars as well as the sun, whereas al-Khalīlī’s first set of auxiliary tables are for timekeeping by the sun only. al-Ṣūfī notes that his two functions B and C are related by:

$$B(\xi,\eta) = C(\bar{\xi},\bar{\eta})$$

but states that he preferred to tabulate both functions rather than only one. To find the hour-angle t(h,Δ,φ) his instructions prescribe the simple rule:

[30] On Ibn Abi ’l-Fatḥ al-Ṣūfī see Suter, *MAA*, no. 447; Kennedy, “*Zīj* Survey”, no. 37; *Cairo ENL Survey*, no. C98; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 116-126, no. 58 (where this work is not mentioned).

$$t(h,\Delta,\phi) = S \{ [\mathrm{Sin}\ h - C(\phi,\Delta)] \cdot B(\phi,\Delta) \} ,$$

but to find the corresponding azimuth $a(h,\Delta,\phi)$, as in the case of al-Māridīnī's tables, the procedure is more complicated. First form:

$$m(h,\Delta,\phi) = \mathrm{Cos} [S \{ C(\phi,h), \mathrm{Cos}\ \phi \}] + \mathrm{Sin}\ \psi(\Delta,\phi)$$

and then:

$$a(h,\Delta,\phi) = 90° - S \{ m(h,\Delta,\phi) , \mathrm{Cos}\ h \} .$$

Note that Cos $\{S(x,y)\}$ is simply $R \cdot x / y$ and that al-Ṣūfī's tables are not suited to determine Sin $\psi(\Delta,\phi)$ easily.

9.11 Two sets of anonymous Egyptian auxiliary tables for timekeeping by the sun

MS Istanbul S. Esad Efendi Medresesi 119,2 (fols. 82v-97v), copied *ca*. 1700, contains a set of auxiliary tables bound at the end of a set of prayer-tables for Cairo by Riḍwān Efendī (**6.1.3**). The same auxiliary tables occur in MS Cairo DM 644,1 from *ca*. 1700, where they are followed by a star catalogue dated 1061 H [= 1651]. In both sources the work is anonymous. The title is *Fatḥ al-Karīm al-Bāqī fī maʿrifat al-dāʾir wa-faḍlihi āfāqī*, "The Victory of God, the Noble and Eternal, for Finding the Time since Sunrise and the Hour-angle for all Latitudes".[31]

The main functions tabulated are:

$$B'(\phi,\lambda) = {}^1/_2\ B(\phi,\lambda) = {}^1/_2\ \mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi\ /\ R$$

for the domains:

$\lambda = 1°, 2°, \ldots , 90°$ and $\phi = 1°, 2°, \ldots , 27°, 28°, 30°, 32°, 33;30°, 36°, 38°, \ldots , 48°$,

and:

$$V'(x,y) = \mathrm{arc\ Vers} \{ R \cdot y / 2x \}$$

for the domains:

$$x = 19, 20, \ldots , 30 \quad \text{and} \quad y = 0;5, 0;10, \ldots , 60;0 .$$

The first auxiliary function is labelled *al-aṣl*, "(half) the base", and the arguments in the table of the second function are called *al-aṣl al-muʿaddal*, "the modified base", and *faḍl al-jaybayn*, "the difference between the two Sines". The entries in the tables, which are given to two digits, were lifted from al-Khalīlī's auxiliary tables for timekeeping by the sun (**9.4**). Note that in al-Khalīlī's set B′ is tabulated for some 50 values of ϕ and the increment in the argument y for the tables of V′ is 0;10 rather than 0;5. Also, in the later Egyptian set no horizontal differences are given in the tables of V′ but the simplified format makes them slightly easier to use.

MS Cairo MM 203,3 (fols. 153r-160v), copied *ca*. 1700, contains an anonymous set of tables, probably of Egyptian provenance, entitled *Daqāʾiq al-raqāʾiq fī maʿrifat faḍl al-dāʾir li-sāʾir al-āfāq*, which means something like "Nice Numbers for Finding the Hour-angle for any Latitude". The set consists of simple tables of the inverse Versed Sine (**6.14.2**) and Sine functions and solar declination (based on $\varepsilon = 23;35°$), followed by a table of a function:

$$G_3(\phi,\delta)$$

tabulated to *one* significant digit for the domains:

$$\phi = 1°, 2°, \ldots , 60° \quad \text{and} \quad \delta = 1°, 2°, \ldots , 24° .$$

---

[31] A third copy in Diyarbakır, also anonymous, is mentioned in İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, p. 743.

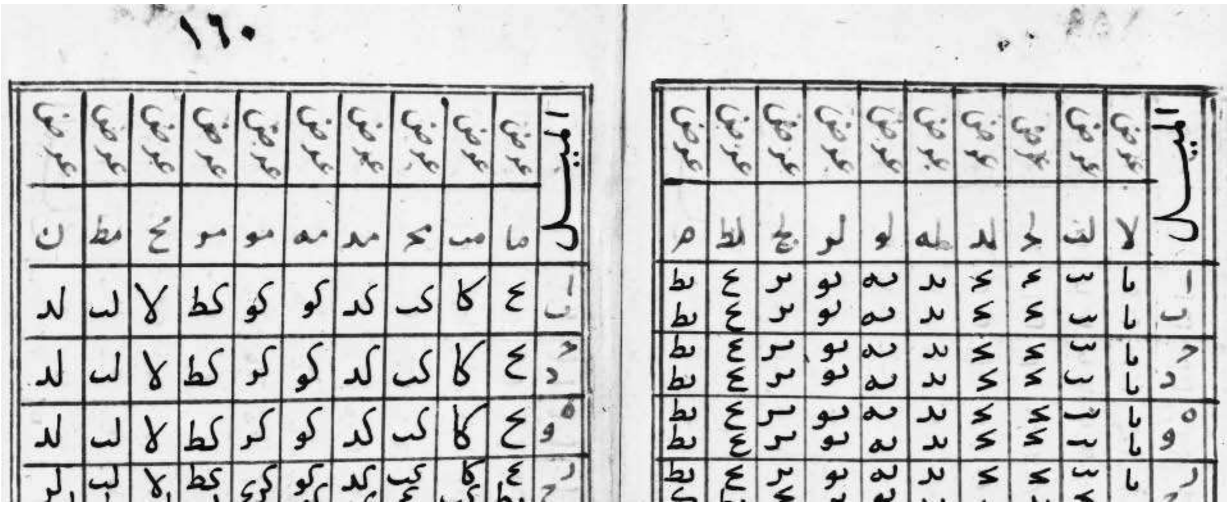


Fig. 9.11: An extract from some anonymous Egyptian auxiliary tables, serving latitudes 31°-50°. [From MS Cairo MM 203, fols. 159v-160r, courtesy of the Egyptian National Library.]

See the extract in **Fig. 9.11**. The methods outlined in the instructions for finding D and t by means of this function are equivalent to the formulae:

$$\text{Vers } D(H,\phi) = \text{Sin } H \;+\; \text{Sin } H \cdot G_3(\phi,\delta) \;/\; R \;,$$

$$\text{Vers } t(h,H,\phi) = [\;\text{Sin } H - \text{Sin } h\;] \;+\; [\;\text{Sin } H - \text{Sin } h\;] \cdot G_3(\phi,\delta) \;.$$

It is clear that for these formulae to be valid we must have:

$$G_3 = R^3 \;/\; [\;\text{Cos } \delta \text{ Cos } \phi\;] \;-\; R \;,$$

which is confirmed by inspection of the table. The inspiration for tabulating this function was probably the little table of G_3 for the latitude of Cairo due to al-Marrākushī (**6.7.2**).

9.12 Saʿīd Beg Zāde's auxiliary tables for timekeeping

It is not inappropriate that I should conclude this study with a brief description of the late Ottoman set of auxiliary tables for timekeeping which shows the influence of Western mathematics on traditional Islamic astronomy. MS Istanbul Kandilli 226 is a copy of the auxiliary tables of Saʿīd (or Sayyid ?) Beg Zāde for computing the prayer-times in Turkish time for any latitude.[32] The main tables are based on the principle that to find the hour-angle in equinoctial hours and minutes corresponding to a particular prayer time t_i one uses three functions:

$$H_i''(H),\; G''(\lambda,\phi) \text{ and } V''(x) \;,$$

thus:

$$t_i = V''\{\; H_i''(H) + G''(\lambda,\phi) \;\}.$$

Likewise, to find the hour-angle corresponding to a particular solar altitude one uses other tables to find a function $H''(H,h)$, and then uses:

---

32 Apparently not mentioned *ibid*.

$$t_i = V''\{ H_i''(H,h) + G''(\lambda,\phi) \} .$$

These functions are the following (compare the auxiliary tables of al-Wafāʾī discussed in **9.9** above !):

$$H''(H,h) = \log [\mathrm{Sin}\ H - \mathrm{Sin}\ h] = \log H'(H,h) ,$$
$$G''(\lambda,\phi) = \log \{ R^2 / [\mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi] \} = \log G(\lambda,\phi),$$
$$V''(x) = {}^1/_{15}\ \mathrm{arc\ Vers}\ (\mathrm{alog}\ x).$$

Both the trigonometric functions and the logarithms are to base 60 and are written sexagesimally! Further details are given in **II-14.16**.

MS Istanbul UL T4203 contains another set of auxiliary tables for computing the hour-angle, also using logarithms but not the sexagesimal system. The tables were computed by an individual identified simply as Jamāl,[33] and I have not investigated their underlying structure.

---

33 *Ditto.*

CHAPTER 10

EUROPEAN TABLES FOR TIMEKEEPING

10.1 Some medieval European tables for timekeeping

"Über das Wie und Warum der Benutzung dieser Tabellen können wir nur spekulieren."
D. A. King, in *Schweinfurt 1993 Exhibition Catalogue*, p. 352.

I have not looked systematically for tables for timekeeping in medieval European manuscripts or early printed works; however, I am confident that these are far more widespread than has hitherto been thought. I mention here just a few examples from the published literature and from European manuscripts which have come to my attention by chance. One feature of these tables is clear: medieval European astronomers liked to tabulate the solar altitude as a function of time. The reason is not completely clear, although it had to do with the regulation of clocks.[1] Muslim astronomers, on the other hand, preferred tabulating something that one wants to know, namely, the time, as a function of something that one can measure, namely, solar or stellar altitude.

We begin with the *Toledan Tables*, a motley corpus of tables derived mainly from the *Zīj*es of al-Khwārizmī and al-Battānī, which has been surveyed by Gerald Toomer, with a detailed new study by Fritz Pedersen.[2] In this corpus, as well as in a 15th-century English manuscript, there is a table of the function $e(\lambda) = {}^{1}/_{12}\ 150 \tan \delta(\lambda)$ (**7.1.1**), labelled *tabula differencie ascensionum universe terre*.[3] A table of h(T,H) for all latitudes with entries tabulated to two digits for each 0;15° of T (up to 6^{sdh}) and each 0;30° of h from 10° to 90°, with a total of over 3,500 entries (**4.3**), in a 14th-century English astronomical manuscript of both the *Toledan* and *Alphonsine Tables*, has also been noted by Toomer.[4] Universal tables of T(H,h) and h(H,T) based on the standard approximate formula are also found in two Byzantine manuscripts.[5]

The *Alphonsine Tables* generally replaced the *Toledan Tables* and were used in different recensions all over Europe until the 16th century. An isolated table displaying the function $\lambda_H(T,\lambda)$ for a particular latitude (**3.0**) is found in an early-15th-century manuscript of the *Alphonsine Tables* written in Germany: it is entitled *Tabel der ofsteygenden Czeychn* and the entries are given in zodiacal signs and degrees for each hour and each degree of solar longitude.[6]

---

[1] Geoffrey Chaucer referred to them as "tables ... for the governaunce of a clokke": see North, *Chaucer's Universe*, p. 87, and also Eisner, ed., *Kalendarium of Nicholas of Lynn*, pp. 29ff.

[2] See Toomer, "Toledan Tables", and now Pedersen, *The Toledan Tables*.

[3] *Ibid*., p. 33, and Neugebauer & Schmidt, "Hindu Astronomy at Newminster", p. 226. See also North, *Richard of Wallingford*, II, pp. 12-14, and *idem*, *Horoscopes and History*, p. 14. See now Pedersen, *Toledan Tables*, III, pp. 986-991.

[4] Toomer, "Toledan Tables", p. 155. See now Pedersen, *op. cit.*, III, pp. 1134-1138.

[5] Jones, *Byzantine Astronomical Manual*, p. 174.

[6] I have examined MS Gotha Forschungsbibliothek Membr. I. 109, apparently dating from 1428, in which the tables occur on fols. 22r-24v.

ENERO

días	horas	X	XIIII	XVII	XXII	XXV	XXVII	XXX
	VII	grados						
	VIII	VII	VII	VIII	VIII	IX	X	XI
	IX	XVI	XVI	XVII	XVII	XVIII	XVIII	
	X	XXIIII	XXIIII	XXV	XXV	XXVI	XXVII	XXVIII
	XI	XXIX	XXX	XXXI	XXXII	XXXIII	XXXIIII	XXXV
	XII	XXXI	XXXII	XXXIII	XXXIIII	XXXV	XXXVI	XXXVII
	horas	grados	gra.	gra.	gra.	gra.	gra.	gra.

JUNIO

días		X	XXII
horas	h. V	g. VII	VI
grados	VI	g. XIX	XVIII
	VII	XXX	XXIX
	VIII	XXXXI	XXXX
	IX	LII	LI
	X	LXIIII	LXIII
	XI	LXXII	LXXI
	XII	LXXVI	LXXV

Fig. 10.1a: Extracts from the table of solar altitude at the hours in the treatise on astrology attributed to Enrique de Villena. The format of the table is dictated by the changes in meridian altitude of 1°. [From Cátedra & Samsó, *Astrología de Enrique de Villena*, pp. 171 and 173.]

A table of $h_i(H)$ for the equatorial hours and each degree of H was computed for the latitude of Baeza (38°) by pseudo-Enrique de Villena *ca*. 1430: see **Fig. 10.1a**.[7] The entries display error patterns which could be taken as implying that they were based on the approximate formula; however, the table may have been compiled using an instrument such as an astrolabe, for even the values at the equinoxes contain errors of as much as 2°. A less extensive table of this kind, with entries for each 5° of h, is found in a medieval Hebrew manuscript.[8]

Tables of $h_i(\lambda)$ ($\Delta\lambda$ = 30°) and/or the associated vertical or horiontal shadows abound in the medieval European sources, but few have been published, and even fewer investigated.[9] There is, for example, a table of solar altitudes h_i for latitude 48° in the treatise on the astrolabe attributed to Hermannus Contractus (1013-1054).[10] The author states that he derived the entries, which are given to degrees, using an astrolabe and they do indeed correspond quite well to computation with the exact formula. The available version of the table of vertical shadows $z'_i(\lambda)$

[7] See Cátedra & Samsó, *Astrología de Enrique de Villena*, pp. 171-176, and the commentary on pp. 56-57.
[8] MS Munich heb. 343, which was brought to my attention by Professor Bernard Goldstein in the 1970s.
[9] Zinner, *Astronomische Instrumente*, pp. 159 and 50-51, also lists tables for Oxford, London, Rome, Venice, Nuremberg, Augsburg and Regensburg, which I have not investigated. See North, *Richard of Wallingford*, I, p. 18, on a table for Oxford by John Maudith (14th century), and *idem*, *Chaucer*, p. 114, on various other tables of this kind.
[10] On Hermannus see the article by Claudia Kren in *DSB*. For the table see Gunther, *Early Science in Oxford*, II, p. 419 (but it is not contained in the treatise translated in Joseph Drecker, "Hermannus Contractus über das Astrolab", *Isis* 16 (1931), pp. 200-219).

= $\mathrm{Tan}_{12}\ h_i(\lambda)$ ($\Delta\lambda = 30°$) for latitude 47;46° (Vienna) compiled by John of Gmunden (*fl. ca.* 1430) for marking the hour-curves on a cylindrical sundial, is full of errors.[11]

In the *Kalendarium* of Nicholas of Lynn (1386) there are tables of the solar altitude h_i and shadows z (base 6!) for the seasonal hours on each day of the year for the latitude of Oxford, taken as 51;50°.[12] Petrus de Dacia, canon in Roskilde, Denmark, *ca.* 1300, presented tables of meridian altitudes H and lengths of day and night 2D and 2N for each day of the year, computed for latitude 48;50°, serving Paris, and similar to others by William of St Cloud in his *Kalendarium* edited in 1296.[13] In one manuscript of the treatise on astrolabe construction by Jean Fusoris (Paris *ca.* 1400) there is a table, also for Paris, of the solar altitude $h_i(\lambda)$ at each equinoctial hour for each degree of λ and a separate table for each 15 days of the year specifically for constructing a cylindrical sundial.[14]

This activity was clearly continued by Renaissance astronomers, but there is precious little documentation as yet. A manuscript in the Stadtsarchiv in Schweinfurt contains a mixture of tables in the hands of two 16th-century German astronomers, Johannes Hommel and Johannes Praetorius.[15] The former contributed two tables for a latitude of 51° (location unspecified), displaying of $h(T,\lambda)$ and $a(T,\lambda)$ for each equinoctial hour and each 1° of λ. The latter contributed tables of $h(T,\lambda)$ for latitudes 50° (locality unspecified) and 49;26° (Nuremberg is specifically mentioned), for $\Delta\lambda = 3°$ and 5°, respectively, as well as a set of tables for coordinate conversion for latitude 50° (copied in Cracow) after the model of the tables appended to Ptolemy's *Analemma*.[16] Also in the hand of Hommel we find a table of $h(\phi,\lambda)$ for each 1° of φ from 39° to 63° and each 1° of λ. No other such universal table is known from the medieval and Renaissance European sources.

In Europe, between the late 18th and the early 20th century, there appeared a series of extensive tables for timekeeping for purposes of navigation, of which I present scant details.[17] Already in 1770 the French astronomer César-François Cassini (= Cassini III) prepared in his *Almanach perpetuel pour trouver l'heure par tous les degrés de hauteur du soleil* a "skeleton set" of tables for latitudes between 34° and 51°. Joseph-Jérôme Lalande indicated the usefulness of such tables in the second edition of his *Traité de l'astronomie* (1771), and referred to the tables computed by Cassini. We can take a closer look at the tables published in Leipzig in 1791 by one Friedrich Christoph Müller. These display the two times of day in the morning and afternoon for each day of the year when the sun has a given altitude. Entries are given

---

[11] On John of Gmunden see the article by Kurt Vogel in *DSB*. The table is published in Claudia Kren, "The Traveller's Dial", p. 431.

[12] Eisner, *Kalendarium of Nicholas of Lynn*, pp. 68-69 (Jan.), 74-75 (Feb.), *etc*., and the commentary on p. 26. On the latitude see *ibid*., p. 17. See also North, *Chaucer*, pp. 115-116, and on the problems of the edition see *ibid*., p. 115, n. 54. These tables include values for twilight: see also Pedersen, *Toledan Tables*, III, pp. 1132-1133.

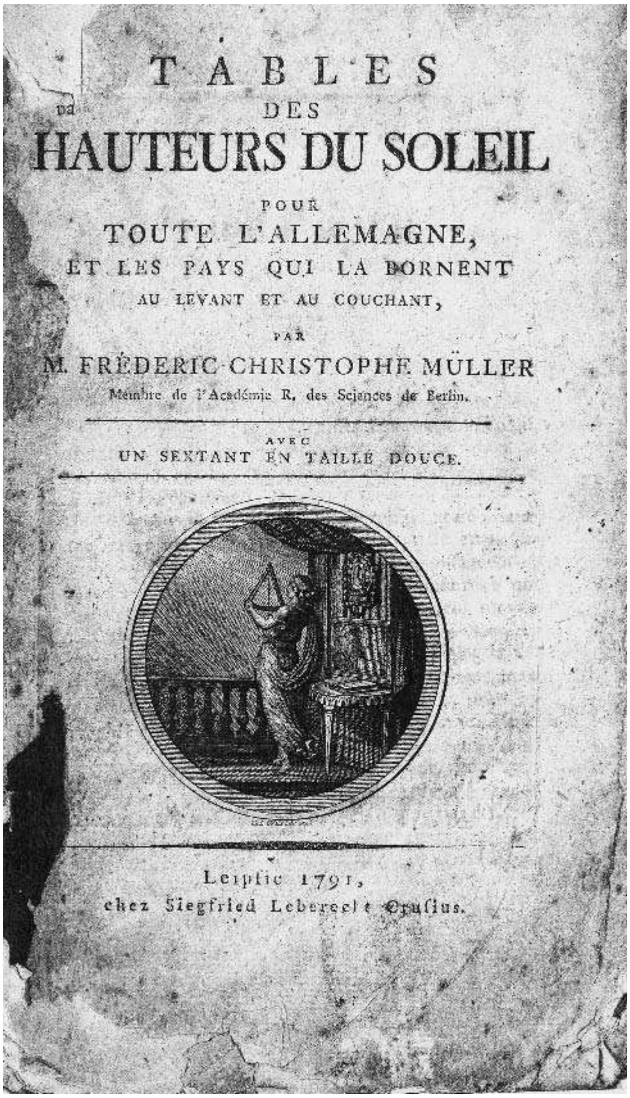
[13] Pedersen, F. S., ed., *Astronomical Works of Petrus de Dacia and Petrus de S. Audomoro*, I, pp. 39 and 320-322.

[14] Poulle, *Fusoris*, pp. 184-185 and 186. On this manuscript see also King, *Ciphers of the Monks*, p. 155, and for more on Fusoris see *ibid*., pp. 397-398.

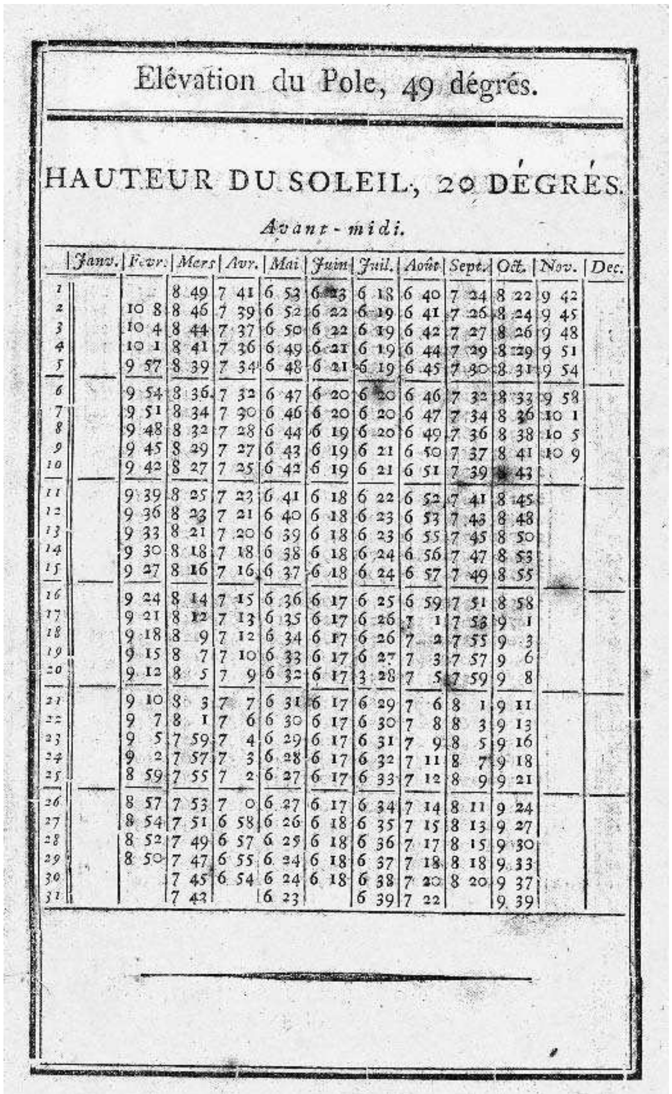
[15] On the manuscript (AvS Ha 21) see *Schweinfurt 1993 Exhibition Catalogue*, pp. 351-353, no. 169.

[16] See n. 1:40.

[17] The information in this paragraph is taken from Cotter, "Nautical Tables". Alas the details of the individual tables are not standardized. The subject could use a new study.



TABLES
DES
HAUTEURS DU SOLEIL
POUR
TOUTE L'ALLEMAGNE,
ET LES PAYS QUI LA BORNENT
AU LEVANT ET AU COUCHANT,
PAR
M. FRÉDERIC CHRISTOPHE MÜLLER
Membre de l'Académie R. des Sciences de Berlin.
AVEC
UN SEXTANT EN TAILLE DOUCE.
Leipsic 1791,
chez Siegfried Lebereckt Crusius.



Elévation du Pole, 49 dégrés.

HAUTEUR DU SOLEIL, 20 DÉGRÉS.

Avant-midi.

	Janv.	Fevr.	Mars	Avr.	Mai	Juin	Juil.	Août	Sept.	Oct.	Nov.	Dec.
1			8 49	7 41	6 53	6 23	6 18	6 40	7 24	8 22	9 42	
2		10 8	8 46	7 39	6 52	6 22	6 19	6 41	7 25	8 24	9 45	
3		10 4	8 44	7 37	6 50	6 22	6 19	6 42	7 27	8 26	9 48	
4		10 1	8 41	7 36	6 49	6 21	6 19	6 44	7 29	8 29	9 51	
5		9 57	8 39	7 34	6 48	6 21	6 19	6 45	7 30	8 31	9 54	
6		9 54	8 36	7 32	6 47	6 20	6 20	6 46	7 32	8 33	9 58	
7		9 51	8 34	7 30	6 46	6 20	6 20	6 47	7 34	8 36	10 1	
8		9 48	8 32	7 28	6 44	6 19	6 20	6 49	7 36	8 38	10 5	
9		9 45	8 29	7 27	6 43	6 19	6 21	6 50	7 37	8 41	10 9	
10		9 42	8 27	7 25	6 42	6 19	6 21	6 51	7 39	8 43		
11		9 39	8 25	7 23	6 41	6 18	6 22	6 52	7 41	8 45		
12		9 36	8 23	7 21	6 40	6 18	6 23	6 53	7 43	8 48		
13		9 33	8 21	7 20	6 39	6 18	6 23	6 55	7 45	8 50		
14		9 30	8 18	7 18	6 38	6 18	6 24	6 56	7 47	8 53		
15		9 27	8 16	7 16	6 37	6 18	6 24	6 57	7 49	8 55		
16		9 24	8 14	7 15	6 36	6 17	6 25	6 59	7 51	8 58		
17		9 21	8 12	7 13	6 35	6 17	6 26	7 1	7 53	9 1		
18		9 18	8 9	7 12	6 34	6 17	6 26	7 2	7 55	9 3		
19		9 15	8 7	7 10	6 33	6 17	6 27	7 3	7 57	9 6		
20		9 12	8 5	7 9	6 32	6 17	3 28	7 5	7 59	9 8		
21		9 10	8 3	7 7	6 31	6 17	6 29	7 6	8 1	9 11		
22		9 7	8 1	7 6	6 30	6 17	6 30	7 8	8 3	9 13		
23		9 5	7 59	7 4	6 29	6 17	6 31	7 9	8 5	9 16		
24		9 2	7 57	7 3	6 28	6 17	6 32	7 11	8 7	9 18		
25		8 59	7 55	7 2	6 27	6 17	6 33	7 12	8 9	9 21		
26		8 57	7 53	7 0	6 27	6 17	6 34	7 14	8 11	9 24		
27		8 54	7 51	6 58	6 26	6 18	6 35	7 15	8 13	9 27		
28		8 52	7 49	6 57	6 25	6 18	6 36	7 17	8 15	9 30		
29		8 50	7 47	6 55	6 24	6 18	6 37	7 18	8 18	9 33		
30			7 45	6 54	6 24	6 18	6 38	7 20	8 20	9 37		
31			7 43		6 23		6 39	7 22		9 39		

Figs. 10.1b: The title-page and an extract from the tables of F. C. Müller, 1791, showing time as a function of solar altitude throughout the year for a range of latitudes in Germany. The corpus bears the title "tables of solar altitudes", which is mathematically inexact, and curiously corresponds precisely to the Turkish expression *irtifāʿ jadwali* (*irtifā cedveli*) for tables of the same kind. Müller's introduction makes no mention of contemporaneous Turkish tables, and the West was to remain innocent of any knowledge of them for almost another two centuries. [From Müller, *Tables des hauteurs du soleil*.]

to hours and minutes for each degree of altitude, and there are tables for each degree of latitude from 47° to 54°, intended to serve all of Germany and the countries surrounding it. See **Fig. 10.1b** for an extract from the French edition, apparently published simultaneously with the original German version.[18] In his preface the author complained about the difficulties of

[18] It is a pleasure to thank Dr. Živa Vesel of Paris for sending me a copy of the French edition of these tables, listed in the bibliography as Müller, *Tables des hauteurs du soleil*.

compiling the tables and finding a publisher, and mentioned that he was considering compiling similar tables for the solar azimuth.

In his *Abrégé de navigation* (1793) Lalande presented tables of t(h,δ,φ) for each value of h between 2° and 48°, each degree of δ between 0° and 24°, and each degree of φ from 0° to 60°. The tables displayed the first differences and an auxiliary table was provided to facilitate interpolation. The first English tables of this kind were published in 1827 by Thomas Lynn: they were not widely used except by officers of the Honourable East India Company, to whom Lynn was engaged as an examiner in navigation. The *Time-Azimuth-Tables* of the naval officer John Burdwood published from 1852 onwards became most popular on British vessels. Percy Davis of the Royal Naval Office published in 1902 a set of tables of t(h,δ,φ) with values for each degree of each argument. In 1913 a set of tables of stellar altitudes and azimuths, for declinations between -30° and +30°, and specifically for latitude 55°, was published in Berlin for the German Navy; within the next few years companion volumes were published for latitudes 50° and 70°. (It is not clear what the advantage of such tables would have had beyond enabling a vessel to be steered along one of three parallels of latitude.) A set of tables published by the United States military authorities in 1919 displayed altitude and azimuth in parallel columns (no further information available). In fact, various astronomers from the late 19th century to the mid 20th century compiled tables of the hour-angle and the solar azimuth, both as functions of solar altitude and declination, or solar altitude as a function of hour-angle and declination, all for extensive ranges of latitudes.

10.2 European auxiliary tables for spherical trigonometry and astronomy

Although previous studies of Islamic auxiliary tables have failed to draw attention to European tables of the same kind, the tradition of compiling auxiliary tables for solving problems of spherical trigonometry – inevitably relating mainly to spherical astronomy – was continued in Europe.[19]

Regiomontanus' table for solving spherical triangles in his *Tabula primi mobilis* prepared in 1463 displays values of:

$$x(a,b) = \text{arc sin } \{ \text{ sin a} \cdot \text{sin b } \}$$

for each degree of both arguments. Values are given in degrees, minutes and seconds, and are generally accurate in the last digit. First horizontal and vertical differences are also tabulated. As Anton von Braunmühl has shown, linear interpolation applied horizontally and vertically yields results in which the seconds are not at all to be trusted (his example shows an error of about 30″). This table is useful for solving right-angled spherical triangles with any of the rules involving only sines and cosines.

[19] The information in this section is summarized from Cantor, *Geschichte der Mathematik*, II, pp. 273-275; von Braunmühl, *Geschichte der Trigonometrie*, I, pp. 122-124 and 231-236, II, p. 231; Wolf, *Handbuch der Astronomie*, I, p. 438; and Fletcher *et al.*, *Index of Mathematical Tables*, esp. I, pp. 224-227 (altitude tables, azimuth tables and hour-angle tables). The tables of F. C. Müller are not mentioned in any of these works. As far as I know, the Renaissance tables have never been studied in any depth. Glowatzki & Göttsche, *Die Tafeln des Regiomontanus*, deals only with standard trigonometric tables, but not only those of Regiomontanus.

Regiomontanus' tables called *Tabulae directionum* for converting ecliptic to equatorial coordinates were first published in 1475 and again in a new edition by Erasmus Reinhold in 1554. For each degree (?) of ecliptic longitude values of the following functions are tabulated:

$$\alpha' ,\quad \delta_2,\quad \sin\gamma \quad \text{and} \quad \cot\gamma ,$$

where δ_2 is the "second declination" (measured perpendicular to the ecliptic). With these Δ and α are found using:

$$\sin\Delta = \sin\{\beta + \delta_2\}\sin\gamma \quad \text{and} \quad \sin\{\alpha' - \alpha\} = \tan\Delta\cot\gamma .$$

The auxiliary tables of Ḥabash also include tables of δ_2 and Sin γ for the same purpose.

The year 1604 saw the publication in Venice of Magini's *Tabulae primi mobilis, quas directorium vulgo dicunt*. Five years later an extensive commentary was published in Bologna, which was one of the most widely used books among contemporary astronomers. Magini's tables display four main functions:

$$\begin{aligned} x(a,b) &= \text{arc sin}\{\sin a \cdot \sin b\} \\ y(a,b) &= \text{arc sin}\{\sin a / \sin b\} \\ z(a,b) &= \text{arc tan}\{\sin a \cdot \tan b\} \\ w(a,b) &= \text{arc sin}\{\tan a \cdot \tan b\} , \end{aligned}$$

again with values for each degree of both arguments. In the 1609 edition values are given for each 0;10°. The first function is that of Regiomontanus, and all four serve the direct solution of all of the standard formulae of spherical astronomy with the exception of the cosine formula, which requires two applications. In addition, Magini's *Magnus canon trigonometricus* contains tables of the following functions for each minute of argument:

$$\sin x,\ \text{arc sin}\{(\sin x)/10\},\ \text{vers } x,\ \text{arc sin}\{(\text{vers } x)/10\},\ \tan x,$$
$$\text{arc sin}\{(\tan x)/10\},\ \text{arc sin}(\tan x),\ \sec x \text{ and arc sin}\{(\sec x)/10\}.$$

Although Delambre noted that Magini's tables were soon rendered superfluous by the invention of logarithms, von Braunmühl was more considerate, writing that:

> "Magini with his *primum mobile* had given Italy a work the like of which was not attested in the history of trigonometry, neither previously [*sic* !!] nor for decades thereafter."

Of course, neither scholar was aware of any Islamic tables of this kind.

In 1849 Adolf Heegemann prepared new tables displaying the functions:

$$\begin{aligned} x(a,b) &= \text{arc sin}\{\sin a \cdot \sin b\} \\ z(a,b) &= \text{arc tan}\{\sin a \cdot \tan b\} , \end{aligned}$$

apparently without knowing that Regiomontanus and Magini had produced similar tables several centuries before. It was only in the 19^{th} century with Inman's *Nautical Tables* and Raper's *Practice of Navigation* that tables for finding an angle A of a general spherical triangle became available. These displayed sin $^1/_2$ A, computed using:

$$\sin^2(^1/_2 A) = {}^1/_2 \text{ vers } A = \sin(s-b)\sin(s-c) / (\sin b \sin c) ,$$

where $s = {}^1/_2 (a + b + c)$. We should point out that, using this same notation, it may be said that the tables of such Muslim scholars as Najm al-Dīn al-Miṣrī and al-Khalīlī serve to determine the angle A(a,b,c) in terms of the sides of a general spherical triangle for problems of spherical astronomy. When:

$$a = 90° - h ,\quad b = 90° - \delta \quad \text{and} \quad c = 90° - \phi ,$$

A is the hour-angle, and, when a and b are switched, A is the azimuth.

It is not necessary to assume a knowledge of the spherical cosine formula when an equivalent solution occurs in an ancient or medieval text. Thus, for example, it is often asserted that al-Battānī (*ca.* 910) knew the formula and used it to derive the hour-angle from an instantaneous celestial altitude.[20] This is simply not the case. al-Battānī merely used an Indian procedure for deriving the hour-angle by projection methods. His formula can, of course, be shown to be *equivalent* to an application of the spherical cosine rule. As far as we know, no Muslim scholar wrote on the cosine formula *per se*, and we cannot even assume that even Najm al-Dīn al-Miṣrī or al-Khalīlī knew of it in its generality.[21] All that can be asserted is that both of them and many other medieval Muslim astrnomers were fully aware of the mathematical equivalence of the two major problems of spherical astronomy (determination of the hour-angle and the azimuth from an instantaneous celestial altitude) which in modern terms can most readily be solved by an application of the formula. As far as the available sources indicate, most of the astronomers who produced the tables I have described preferred projection methods to spherical trigonometry anyway.

There is no evidence that Regiomontanus or any of the later European astronomers were familiar with the Islamic auxiliary tables. Nor did the orientalists at Oxford in the 16th and 17th centuries, for all their interest in the Islamic astronomical texts and tables preserved in Oxford, encounter any Islamic tables for timekeeping.[22] Nevertheless, the auxiliary tables of the Muslim astronomers, particularly the splendid universal tables of Najm al-Dīn al-Miṣrī and al-Khalīlī, as well as some of the more impressive of tables of other Muslim astronomers from the 9th to the 16th century, would surely have been of interest to any serious European astronomer, navigator or naval officer from the 18th to the 20th century. For during that time, numerous sets of tables of trigonometric functions with two or three arguments were compiled.[23]

Certainly the tables of the Muslim astronomers merit appropriate consideration in any new history of astronomy or history of trigonometry. But most people would think that these histories have already been written, and it is unlikely that newly-discovered historical sources, such as a few hundred medieval tables, will change old attitudes.

---

20 Similar confusion about the use of the cosine formula for deriving the azimuth from the altitude is found in Sezgin, *GAS*, V, pp. 261 and 288, and VI, p. 185 (based on earlier secondary sources).

21 Similar confusion relates to the attribution of the so-called prosthaphaeresis formula of trigonometry to Ibn Yūnus: see the remarks at the end of **2.1.1**.

22 See Mercier, "English Orientalists and Islamic Astronomy". As I have shown in a study listed as "Ibn Yūnus and the Pendulum", the misconception that Ibn Yūnus discovered the principle of the pendulum, an association now widely spread on the Internet, actually stems from Edward Bernard in 17th-century Oxford.

23 Fletcher *et al.*, *Index of Mathematical Tables*, esp. I, pp. 169-195 (tables of trigonometric functions and miscellaneous functions associated with the circle and sphere), 207-229 (trigonometric functions of two or three arguments).

Part II

A survey of tables for regulating the times of prayer

This page intentionally left blank

To Bernard Goldstein

This page intentionally left blank

ACKNOWLEDGEMENTS AND NOTES TO THIS VERSION

This study is dedicated to the best teacher and thesis advisor a graduate student could ever have wished for. Bernard Goldstein was just that, a man whose generosity, humour and breadth of scholarship I have always admired and felt fortunate to have experienced and enjoyed. It was he who taught me how to read Arabic scientific manuscripts and what to do with the numbers one finds in them: in his seminar we read the Arabic version of Ptolemy's *Planetary Hypotheses*, in which he had been the first to discover Ptolemy's inter(n)esting spheres.

One day Bernie told me to get hold of microfilms of the Leiden, Oxford and Paris manuscripts of the *Zīj* of Ibn Yūnus and redo Delambre: this would serve as a doctoral thesis. But I should also take a look at a certain Dublin manuscript: it had been catalogued by A. J. Arberry as another copy of the *Zīj*. The Dublin manuscript turned out to be unrelated to the *Zīj*, rather it was a copy of the Cairo corpus of tables for timekeeping, and the reader will imagine my surprise and delight as I drooled over the microfilm for the first time. When I returned to Yale in 1971 after a year in Beirut with Ted Kennedy, I showed Bernie not only the first draft of my thesis on Ibn Yūnus, but also my analysis of the corpus of tables for timekeeping attributed to him in the Dublin manuscript. Bernie corrected the first draft of my thesis over a weekend. "Should I include the timekeeping stuff in the thesis?", I asked. "No", he replied, fully aware of the syndrome of the graduate student who cannot finish a thesis. He was right, of course, and as soon as the thesis was submitted, I turned back to the time-keeping. Thus Bernie is directly responsible for the genesis of this study, albeit unintentionally. (Now this study is published, and the thesis not.)

When my wife and I went off to Cairo in 1972, Bernie gave me a compass and a book entitled *How to Survive in the Outdoors*, both of which I have always cherished. I came out of Egypt after "seven fat years" rather than the formidable "forty years" which others are said to have taken. I used Bernie's compass off and on, not to get back to the U.S. but to measure orientations of various medieval mosques; thus he also unintentionally inspired another major area of my research (see **VIIa**). In 2001 he further gave me a very useful little Eskimo man, carved out of bone, and with a flat head: this one should turn upside down and address when one has problems with colleagues.

At the special session of the Annual Meeting of the (U.S.) History of Science Society held in Bernie's honour in Pittsburg in November, 1999, four colleagues presented state-of-the-art reports on their joint research with him, in ancient Greek, medieval European and Renaissance astronomy, and, last but not least, things Keplerian. There were no presentations on Islamic or Hebrew astronomy, which was a pity because Bernie's contributions to both have been of the same order of magnitude. However, the *Festschrift* dedicated to him in 2003 contains contributions relating to each of these six widely-different various areas of the history of astronomy and is appropriately entitled *Astronomy and Astrology from the Babylonians to Kepler*. In each of these areas Bernie Goldstein has made ground-breaking discoveries, and not only because he too had the best teacher.

TABLE OF CONTENTS

CHAPTER 1

ON ISLAMIC PRAYER-TABLES

"But now Islam has appeared in the eastern and western parts of the world and has spread between Andalusia in the West and parts of China and central India in the East, and between Abyssinia and Nubia in the south and the Turks and Slavs in the North. It has united all the different nations in one bond of love, a handiwork which can be made by God only."
al-Bīrūnī (Afghanistan, *ca*. 1025), *Taḥdīd*, p. 225, transl. by J. Ali, p. 190-191.

"The historian of the Islamic exact sciences is frequently confronted with an *embarras de richesse* – hundreds of manuscript sources which have never been studied in modern times."
E. S. Kennedy, "Islamic Mathematical Geography" (1996), p. 185.

1.0 Introduction

The times of the five daily prayers of Islam are astronomically defined and for a given locality vary throughout the year. Given a clear sky and unrestricted view of the local horizon, it is possible to regulate the daytime prayers using a gnomon and the nighttime prayers by observation of twilight phenomena. The legal scholars of medieval Islam, who spoke with authority, advocated the methods of folk astronomy, devoid of any mathematics beyond simple arithmetic, in their discussions of this subject (see **III**). However, the astronomers of medieval Islam developed a new branch of science out of the religious obligation to pray at specific times, which they called *ʿilm al-mīqāt*, the science of astronomical timekeeping. In the early centuries of Islamic astronomy, this topic was treated almost as in passing in *zīj*es and treatises on instruments. But in Egypt in the 13th century a new professional class emerged, astronomers who specialized in this discipline and who were associated with a particular religious institution. These men were called *muwaqqit*s, literally, "timekeepers"; astronomers who also worked in this discipline independently bore the epithet *mīqātī* (see **V**). The writings of these men consist mainly of treatises on the determination of the prayer-times either by direct calculation or with the aid of analogue computers such as the astrolabe and astrolabic quadrant or calculating devices such as the trigonometric quadrant. But they also produced an enormous amount of tables of one sort or another as aids to their task. Muslim astronomers were also concerned with the determination of the qibla, that is the direction of Mecca, which the Muslim should face in prayer, and also the determination of the visibility of the lunar crescent, and hence the regulation of the lunar Muslim calendar with its holy months of fasting and religious festivals.

We have already seen the practical aids to their work which the astronomers and *muwaqqit*s had at their disposal in the form of tables for timekeeping by the sun and stars (**I**). The most tangible and colourful evidence of the activity of the *muwaqqit*s and their role in Islamic societies is to be found in the mathematical tables which they prepared displaying the prayer-times throughout the year for a particular locality. It is clearly convenient to have the prayer-

times tabulated, either for each degree of solar longitude or for each day of the year, particularly in localities where clouds are frequent or where the local horizon is obscured by mountains or tall buildings. However, such tables must be used in conjunction with some kind of instrument for measuring the passage of time, be it an astrolabe, a quadrant or a sundial, or, from the $17^{th}/18^{th}/19^{th}$ century onwards (depending on the location), a clock. The times of the muezzin's call to prayer in the modern Islamic world are still regulated by tables, compiled by modern methods, and displayed in almanacs, calendars and pocket-diaries (see **V-13**).

The purpose of this study is to present a survey of the medieval prayer-tables which have been located in recent years in various libraries around the world.[1] The vast majority have never been studied previously, indeed, they were not known to exist. These tables are of interest not only by virtue of their mathematical sophistication and accuracy and for the light they cast on medieval computational techniques, but also for the limited but significant information they reveal on one aspect of devotional life in medieval Islam. I discuss mainly tables that were used in Egypt and Syria, since these were the centres of *ʿilm al-mīqāt* in medieval times. However, I have also included descriptions of all medieval prayer-tables from other areas known to me in 1975 (and in 1999 I know of no others). I make no claim to have exhausted the available material even for the study of Egyptian and Syrian prayer-tables, although I suspect that any fresh tables from these two areas that may come to light will not differ greatly from those described in the sequel. However, new Egyptian and Syrian manuscripts may clarify the picture which I present of the transmission of prayer-tables in the medieval period and help to establish the identity of those who compiled the tables, where this is still in doubt. The reader will observe that many of the tables that I discuss exist in what appear to be unique manuscripts. However, the main corpuses of tables for Cairo and Damascus and also Istanbul, exist in dozen manuscripts.

In an early publication I analyzed in some detail the corpus of tables for timekeeping that was used in medieval Cairo.[2] In each of the seven manuscripts that formed the basis of that analysis the tables are attributed to the 10^{th}-century astronomer Ibn Yūnus. Fresh manuscript material enables me now to present new information on the main Cairo corpus (**Chs. 4** and **5**), showing that some of the tables were in fact not computed by Ibn Yūnus but by later Egyptian astronomers. There are still many questions surrounding our knowledge of astronomy in Egypt before about 1400, particularly with regard to the activities of such individuals as Najm al-Dīn al-Miṣrī and al-Marrākushī (**Ch. 6**). Other later Egyptian tables for timekeeping (**Chs. 7-8**) can generally be shown to be closely related to the main Cairo corpus. The

[1] I use the terms "medieval" to cover the entire period from the 8th to the 19th century: see n. **I**-1:1. I use the expression "prayer-table" to denote any set of spherical astronomical tables displaying functions relating to twilight or the afternoon prayer. Many medieval prayer-tables also contain tables of standard spherical astronomical functions: these I have discussed. Many *zīj*es (see n. **I**-1:3) contain similar tables of spherical astronomical functions: these I have generally not discussed. In this study I also treat tables for finding the local qibla by means of the sun, but not tables displaying the qibla for a whole range of latitudes and longitudes, except for that of al-Khalīlī (**10.8**), the only such table in a corpus of tables for timekeeeping.

[2] See n. 4:1 below.

impressive developments in astronomical timekeeping in 14th-century Syria and their later influence (**Chs. 9-11**) seem to owe at least part of their inspiration to the main Cairo corpus. Likewise, the tables compiled in the Yemen, in Tunis and in Istanbul (**Chs. 12-14**) are related to the Egyptian and Syrian traditions. Various treatises on timekeeping and on the use of instruments (**Ch. 2**) cast light on the mathematical techniques that were used by the astronomers of medieval Islam for this purpose.

I have grouped together various sets of prayer-tables for timekeeping compiled in al-ʿIrāq and Iran that have been located in the manuscript sources (**Ch. 3**), but these are inadequate to convey any clear picture of the development of tables for timekeeping in these two areas. Notice that even al-Bīrūnī, the leading scientist of Islam, when writing on timekeeping at the beginning of the 11th century (**2.2**), does not mention the existence of any such tables. Likewise, I have found no tables of Andalusī origin. I suspect that new material located during future research might demand a reappraisal of the development of *mīqāt* particularly in al-ʿIrāq and Iran, as well as al-Andalus. Prayer-tables of the modern kind (**V-13**) are, together with the determination of lunar crescent visibility at the beginning and end of Ramaḍān, and the orientation of the *miḥrābs* in mosques towards Mecca, the sole vestiges of a tradition of mathematical astronomy which knew no rival for 750 years.

As was the case with the tables surveyed in **I**, the tables discussed in this study had, with very few exeptions, never been investigated previously before *ca*. 1970. I know no reference in the Arabic literary sources to tables for regulating the times of prayer, and there is not a trace of such tables in the standard bibliographical works of Carl Brockelmann, Fuat Sezgin and Charles Storey.[3] Furthermore, tables for regulating the prayer-times are seldom contained in the Islamic astronomical handbooks known as *zīj*es.[4] Most of the manuscripts on which this study is based were either miscatalogued or not catalogued at all, and in general the tables were discovered only by foraging through large numbers of manuscripts. Catalogues such as that of Wilhelm Ahlwardt for the Berlin manuscripts give sufficiently detailed descriptions of each manuscript, and even individual tables are mentioned.

The information that I present on Islamic prayer-tables represent the results of only a preliminary survey of the available material. Certainly many of the sources that I describe in the sequel deserve further investigation.

1.1 On the definition of the times of Muslim prayer

The definition of the intervals during the five daily prayers should be performed have been traditionally traced to certain verses of the *Qurʾān* and various *ḥadīth* attributed to the Prophet Muḥammad. The *ḥadīth* dealing with the prayer-times have been subjected to a preliminary analysis by Arent J. Wensinck,[5] and some of the various definitions found in the later astronomical sources have been discussed in a valuable study by Eilhard Wiedemann and

[3] See nn. **I**-1:13 and 19.
[4] See n. **I**-1:3.
[5] See the article "Mīḳāt, i. Legal Aspects" by Arent J. Wensinck in *EI*$_2$.

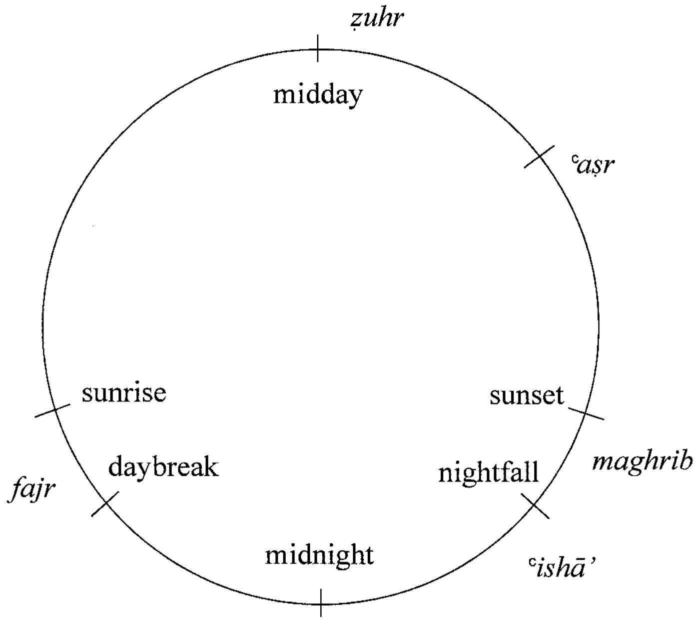


Fig. 1.1a: The Islamic day, showing the times for the beginnings of the five prayers at sunset, nightfall, daybreak, midday and mid-afternoon.

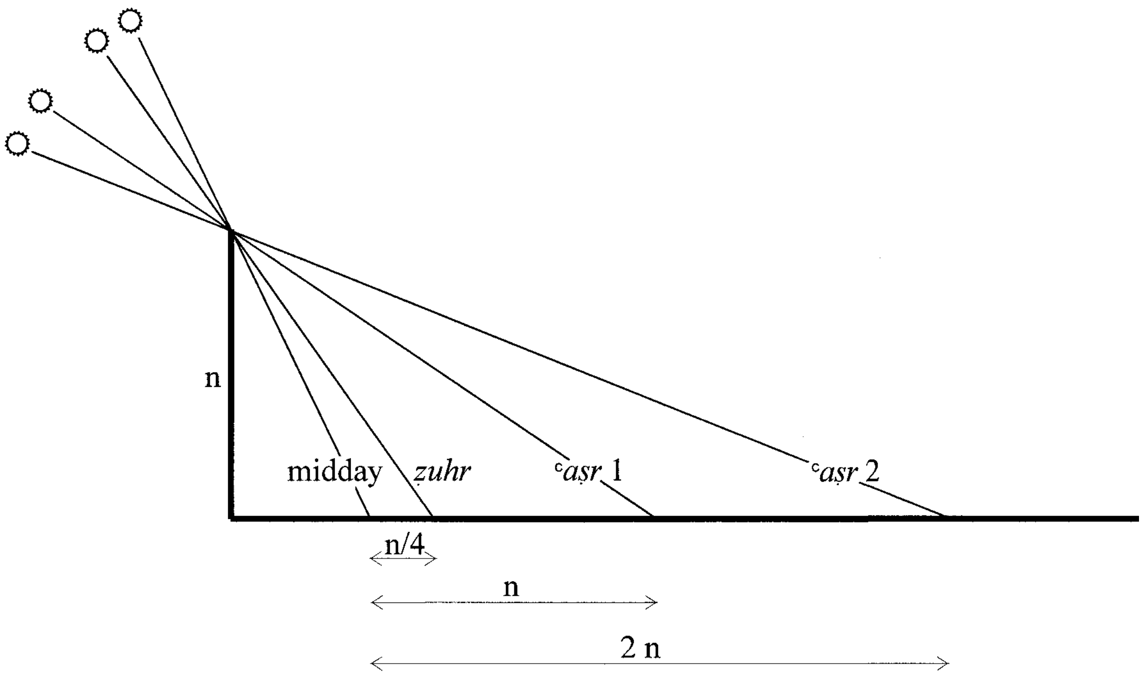


Fig. 1.1b: The standard definition of the beginning of the interval for the *ʿaṣr* is the time when the shadow increase equals the gnomon-length. In some circles, notably amongst the Ḥanafīs, the end of the permitted interval is defined when the shadow increase is double that amount. In Andalusī and Maghribī practice, the interval for the *ẓuhr* prayer begins when the shadow increase is one-quarter of the gnomon-length.

Joseph Frank.[6] In the 1970s Ted Kennedy published a translation and commentary of al-Bīrūnī's book *On Shadows*, probably the most important single work on timekeeping compiled in the Middle Ages, albeit without tables.[7]

The definitions which were used by the Muslim astronomers, and which underlie most of the tables discussed in this study, are as follows: The Muslim day begins at sunset, and the interval during which the first prayer (*al-maghrib*) is to be performed lasts from sunset to nightfall. The interval for the second prayer (*al-ʿishāʾ*) begins at nightfall and lasts until daybreak. The third prayer (*al-fajr*) is to be performed during the interval between daybreak and sunrise. The permitted time for the fourth prayer (*al-ẓuhr*) begins when the sun has crossed the meridian and ends when the interval for the fifth prayer (*al-ʿaṣr*) begins; namely, when the shadow of an object equals its meridian shadow increased by the length of the object. The interval for the fifth prayer may last until the shadow increases again by the length of the object or until sunset. In the medieval period there were occasional variations to this practice, such as, for example, the definition of the *ẓuhr* in al-Andalus and the Maghrib, whereby the prayer began not at midday but when the shadow increase over its midday minimum was one-quarter of the length of the gnomon. Also in some communities there was a prayer at mid-morning called the *ḍuḥā*. Indeed there is sufficient material available to explain the origin of the curious definitions for the *ẓuhr* and *ʿaṣr*, which are not specifically mentioned either in the *Qurʾān* or the *ḥadīth*. I have investigated the origin of these definitions in **IV**.

The determination of the prayer-times either by observation or by computation was a relatively simple problem for a competent medieval astronomer. The midday and afternoon prayer-times can be determined with a facility using a gnomon. The corresponding solar altitude can be readily computed so that one could also use an astrolabe or a quadrant to know when the time for prayer had arrived. To determine the times from sunrise to midday, and from midday to the beginning of the afternoon prayer (when the sun has a predetermined altitude), are standard problems of medieval spherical astronomy. The times of the prayers at nightfall and daybreak can be likewise be determined either by observation or by calculation. To compute the duration of twilight one must assume that the transitions between twilight and total darkness, that is, daybreak and nightfall, occur when the sun has a certain angle of depression below the horizon. The determination of the duration of twilight is then a simple extension of the problem of finding the time to sunrise or since sunset when the sun has a particular altitude, which is again a standard problem of medieval spherical astronomy. Thus from an astronomical or mathematical point of view the determination of the prayer-times is a straightforward application of certain standard procedures in spherical astronomy.

Underlying several of the tables amongst those I shall be describing is a value for the qibla or local direction of Mecca. the reader is referred to **VIIa-c** for an introduction to this topic.

---

6 See Wiedemann & Frank, "Gebetszeiten".

7 Listed under al-Bīrūnī, *Shadows*. For a summary of the parts relevant to this study see Kennedy, "al-Bīrūnī on Prayer-Times".

1.2 On the development of astronomical timekeeping in Islam

In early Islam the times of prayer were regulated by observation. In the case of the daytime prayers, the *ẓuhr* and *ʿaṣr*, a gnomon erected in the mosque could be used to determine the times with facility. Such a gnomon exists to this day in the beautiful Mosque of Janad in the Yemen (see **Fig III-1.2b**); the mosque dates from the 7th century, but I do know when this gnomon, a (concrete?) block of rectangular cross-section about 2 metres high and about 15 cm thick, was erected. In the case of the nighttime prayers, the moment of sunset can generally be ascertained from the vantage of a minaret, and the twilight phenomena are likewise readily observable.

In early Islam, time in general was reckoned either in seasonal hours of the day or night or with respect to the times of prayers. The time of day could be calculated from shadow length by simple formulae ultimately of Indian origin, as recorded in the treatises of al-Bīrūnī and Ibn Raḥīq (**2.2-3**). The Bedouin of the Arabian peninsula have possessed, since pre-Islamic times, a detailed astronomical folklore relating the changing night sky to meteorological and agricultural patterns; in particular, timekeeping by night was effected by observing the lunar mansions.[8] The time of night in early Islam could also be measured, for example, by a simple candle-clocks: one such clock described in the medieval sources consists of a set of twelve oil lamps which are filled in such a way that one goes out at each hour of the night.[9]

From the 9th century onwards the times of the *ẓuhr* and *ʿaṣr* were sometimes regulated by sundials.[10] Also, individual prayer-times could be determined using special curves marked on astrolabes and quadrants.[11] However, the passion of the early Muslim astronomers for the compilation of tables found a ready outlet in astronomical timekeeping. Already in the mid 10th century ʿAlī ibn Amājūr in Baghdad compiled two extensive tables displaying the time since sunrise as a function of instantaneous solar altitude and meridian altitude. One of these was based on an accurate formula and computed for the latitude of Baghdad, and the other was based on an approximate formula and was intended to serve all latitudes (**3.2**). Ibn Amājur and various other Muslim astronomers, even from the 9th century, likewise tabulated certain functions relating to the times of prayer, such as the solar altitude at the beginning of the *ʿaṣr* as a function of its meridian altitude, and the duration of twilight assuming that daybreak and

[8] See, for example, the articles "Anwāʾ" and "Layl and Nahār" by Charles Pellat and "Manāzil" by Paul Kunitzsch in EI_2, and now Varisco, "Islamic Folk Astronomy".

[9] This clock is described in Kennedy & Ukashah, "The Chandelier Clock of Ibn Yūnis". The author is the ʿIrāqī craftsman Yūnus al-Asṭurlābī, not the celebrated Egyptian astronomer: see King, *Ibn Yūnus*, I.2.10 and III.15.1c, and *idem*, "Review of Hill, *al-Jazarī*", n. 2 on p. 286.

[10] On Islamic gnomonics see Schoy, *Gnomonik der Araber*, and, more recently, the survey in the article "Mizwala" in EI_2, repr. in King, *Studies*, C-VIII, and also **X-7**. Many new insights are to be gained from Charette, *Mamluk Instrumentation*.

[11] On the astrolabe in Islam see the articles "Asṭurlāb" by Willy Hartner in EI_2, also listed under Hartner, "Astrolabe". On the quadrant see the ground-breaking study Schmalzl, *Geschichte des Quadranten*, and my article "Rubʿ" in EI_2. Many new insights are to be gained from Charette, *op. cit.* Several Islamic instruments displaying the curves for the various times of prayer are illustrated in Gunther, *Astrolabes*, I, and Michel, *Traité de l'astrolabe*. See further **X-4**.

nightfall occur when the sun is at a fixed arc of depression below the horizon (**3.1** and **3.3**). Later some astronomers in Iran followed the tradition of Ibn Amājūr in compiling tables of the time since sunrise for the latitudes of Maragha and Shiraz, *inter alia* (**3.8**, **3.11** and **3.12**).

The corpus of tables for Cairo associated with Ibn Yūnus (**Ch. 4**) marks a new trend in astronomical timekeeping in Islam. The major part of the corpus, which contains some 200 tables and over 30,000 entries, consists of tables for reckoning time and solar azimuth from solar altitude. Other tables relate to the beginning of the afternoon prayer, the duration of morning and evening twilight, and the time when the sun is in the azimuth of Mecca. Numerous simple spherical astronomical functions are also tabulated, such as the solar meridian altitude and the rising amplitude. All of the functions are displayed for each integral degree of solar longitude, and the times are usually given in equatorial degrees and minutes. Not all the tables in this corpus were computed by Ibn Yūnus (**Ch. 5**), but the corpus set the pattern for most subsequent prayer-tables in Egypt and Syria. In certain tables prepared by later Egyptian *muwaqqit*s (**Chs. 7-8**) we find the values of the functions relating to the prayer-times in the Cairo corpus tabulated for each minute of solar longitude, with values derived from these by interpolation. The extensive tables for regulating time by the sun and stars for all latitudes compiled by Najm al-Dīn al-Miṣrī in the early 14th century (**6.2**) are rather different in character from those of the Cairo corpus, but they were not widely used. In another set of tables, apparently dating from the 15th century, we find prayer-tables like those in the Cairo corpus computed for each degree of terrestrial latitude (**8.1**). The Cairo corpus itself was used in Egypt as late as the 19th century alongside some of its later manifestations.

In Syria, tables for timekeeping were computed independently by three 14th-century astronomers of exceptional competence, namely, al-Mizzī, Ibn al-Shāṭir and al-Khalīlī (**Chs. 9-10**). al-Mizzī, who studied astronomy in Cairo and then moved to Damascus, computed a set of hour-angle tables and prayer-tables for his city which were virtually identical in their conception to those of the Cairo corpus. Ibn al-Shāṭir computed some prayer-tables for an unspecified locality near Damascus, but this was his colleague al-Khalīlī who was responsible for the extensive corpus of tables for Damascus that was used in Syria until the beginning of the 20th century. The main corpuses of tables that were used in the Yemen and in Tunis also owe their inspiration to the Egyptian and Syrian traditions. The achievements of the *muwaqqit*s in Egypt and Syria after about 1500 are unimpressive, but this is partly explained by the fact that the tables which were their major concern had already been computed by competent astronomers in earlier centuries.

In the earliest Islamic prayer-tables, the functions relating to the times of prayer are tabulated with the solar longitude as argument. In later tables of this kind, the argument was the date in a calendar based on the solar year, such as the Syrian or Coptic calendars. Additional calendrical tables could enable the user to find the corresponding date in a solar-based calendar to use in such prayer-tables. Tables displaying the prayer-times for a given Muslim year were also compiled.

In medieval prayer-tables it was usually the permitted intervals for the prayers that were tabulated, these being expressed in equatorial degrees and minutes. In Ottoman times it generally became the practise to tabulate the times of the prayers in equinoctical hours and

minutes, according to the Ottoman convention that sunset is 12 o'clock.[12] The rationale for this system derives from the fact that the Muslim day begins at sunset and also that the Muslim astronomers generally used equinoctial hours rather than seasonal hours. However, this system is hardly convenient when used with mechanical clocks, and when Western clocks were introduced in the Ottoman Empire they were at first generally used with the Turkish time system, the hands of the clock being adjusted to 12 o'clock every evening at sunset. This custom still prevails in the Arabian peninsula, although Yemeni friends of mine have claimed, and not during a *qāt* session, that their watches register 12 o'clock at sunset throughout the year. Modern prayer-tables are in a sense an extension of the Ottoman-type tables, now displaying the times in "Western time": see some examples in **Fig. V-13.1**. Also, since they are intended to be used with clocks, the time given for midday usually incorporates the equation of time.[13]

1.3 Notation used in the analysis

In the text notation in the form "(**1.2.3**)" stands for "(see Section 1.2.3)". Occasionally in **Part II** there are also cross-references to **Part I** in the form "(**I-1.2.3**)", and *vice versa*.

In the mathematical analysis of tables the notation f(x,y) is used for a function f tabulated so that x is the horizontal argument and y is the vertical argument. I use the following mathematical notation:

Latin

a	azimuth, measured from the prime vertical
a′	earliest permitted time for the afternoon prayer (Ottoman convention)
-_a	relates to the earliest time for the afternoon prayer
b′	time for the end of the afternoon prayer (Ottoman convention)
-_b	relates to the end of the interval for the afternoon prayer
$b_{1,2}$	auxiliary functions related to B
B	the "absolute base", an auxiliary function of Islamic timekeeping (see **I-6.0**)
C	the "altitude of the day-circle centre", an auxiliary function of Islamic timekeeping (see **I-6.0**)
d	half-excess of daylight, or equation of half daylight (d = D-90°)
∂	("uncial d") time of the *ḍuḥā*
-_∂	relates to the time of the *ḍuḥā* (only in Maghribi and Ottoman sources)
D	semi diurnal arc
e	the auxiliary function tan δ tan φ
G	an auxiliary function for timekeeping related to sec δ sec φ
h	altitude
h_0	solar altitude in the prime vertical
h_a	solar altitude at the beginning of the interval for the afternoon prayer
h_b	solar altitude at the end of the interval for the afternoon prayer
h_q	solar altitude in the azimuth of the qibla

[12] On this see Würschmidt, "Osmanische Zeitrechnung", and the text to n. **I**-1:36.
[13] On this notion in medieval astronomy see the article "Taʿdīl al-zamān" in EI_2.

h_{q*}	solar altitude in the direction perpendicular to the qibla
h_r	solar depression at daybreak
h_s	solar depression at nightfall
$h_{\tilde{v}}$	solar altitude in the azimuth of the ventilator (*i.e.,* the azimuth of the rising sun at the winter solstice)
h_z	solar altitude at the beginning of the noon prayer (usually equals H)
$-^h$	equinoctial hours (also used in the representation of functions, thus: $2D^h(\lambda) = D(\lambda) / 15$)
$\tilde{h}$	number of equatorial degrees corresponding to one seasonal day-hour
$1/\tilde{h}$	factor for converting equatorial degrees to seasonal day-hours
$\tilde{h}'$	number of equatorial degrees corresponding to one seasonal night-hour
H	solar meridian altitude
H′	the difference between the Sines of the meridian and instantaneous altitudes (H′ = Sin h - Sin h)
$-_H$	relates to the ascendant (horoscopus)
i′	time of the *imsāk* (Ottoman convention)
j′	time of the call to prayer in Ramaḍān (Ottoman convention)
k	an auxiliary function for azimuth calculations (see **I-8.0**)
L	terrestrial longitude
m′	time of midday (Ottoman convention)
$-_M$	relates to upper midheaven
$-^m$	equinoctial minutes
n	length of darkness or length of gnomon
2N	nocturnal arc
q	azimuth of the qibla, measured from the meridian
$-_q$	relates to the qibla
q*	the direction perpendicular to the qibla
$-_{q*}$	relates to the direction perpendicular to the qibla
r	duration of morning twilight
r′	time of daybreak (Ottoman convention)
$-_r$	relates to daybreak
R	sexagesimal base (60)
R′	time of sunrise (Ottoman convention)
s	duration of twilight
s′	time of nightfall (Ottoman convention)
$-_s$	relates to nightfall
S	midday shadow
$-^{sdh}$	seasonal day-hours
$-^{snh}$	seasonal night-hours
t	hour-angle (actually an arc measured in equatorial degrees rather than an angle; measured from the meridian)
t_a	time from midday to the beginning of the interval for the afternoon prayer (see also a′ for the time of the *ʿaṣr* according to the Ottoman convention)

$\underline{t}_{a/b}$	refers to a time between the beginning and end of the ʿ*aṣr* (see **14.10**)
t_b	time from midday to the end of the interval for the afternoon prayer (see also b′)
t_q	hour-angle when the sun is in the azimuth of the qibla
t_{q*}	hour-angle when the sun is in the direction perpendicular to the qibla
t_z	hour-angle at the beginning of the midday prayer (see h_z)
-_t	relates to the *taʾhīb* (Maghribi sources only)
T	time since sunrise or remaining until sunset (measured from horizon)
T_a	time from the beginning of the interval for the afternoon prayer to sunset
T_q	time from sunrise to the moment when the sun is in the azimuth of the qibla
z	horizonal shadow of a vertical gnomon (base given in parentheses as subscript)
z_a	shadow at the beginning of the afternoon prayer
z′	vertical shadow
-_z	relates to the *ẓuhr*
Z	horizontal shadow at midday
Greek	
α	right ascension
α'	normed right ascension ($\alpha' = \alpha - 90°$)
α_a	oblique ascension of the ascendant at the ʿ*aṣr*
α_r	oblique ascension of the ascendant at daybreak
α_s	oblique ascension of the ascendant at nightfall
α_σ	ascension of the ascendant at the time of the *salām*
α_ϕ	oblique ascension for latitude ϕ
δ	solar declination
δ^*	solar declination augmented by 90° (useful since $H = \delta^* - \phi$)
δ_2	second declination (see **F6** in **I-1.3**)
ΔD	correction to D for horizontal refraction
Δz	increase of the shadow over its midday minimum
$\Delta\alpha$	rising times of the zodiacal signs
ε	obliquity of the ecliptic
ι'	time of the prayer at the ʿ*īd* (Ottoman convention)
θ	independent variable
λ	solar longitude or ecliptic longitude
λ'	elongation of sun from nearer equinox
λ^*	longitude of point of ecliptic opposite point with longitude λ ($\lambda^* = \lambda + 180°$)
λ_H	longitude of the ascendant (horoscopus)
λ_M	longitude of upper midheaven
σ	time from sunset to the *salām*
-_σ	relates to the time of the *salām*
τ	time from sunset to the time of the *ṭafy* in Ramaḍān
τ_a	time before the beginning of the afternoon prayer
τ_z	time from the beginning of the midday prayer (see h_z) to the beginning of the afternoon prayer

$-_\tau$	relates to the time of the *ṭafy*
ϕ	terrestrial latitude
ψ	rising amplitude

Miscellaneous

$\approx$	approximately equals
-*	opposite point of ecliptic
$\bar{\theta}$	complement of θ to 90°
-′	for times reckoned according to the Ottoman convention
; ,	sexagesimal semi-colon and comma

In all of the tables consulted in the manuscripts, the entries are, unless otherwise stated, given to two sexagesimal digits and are expressed in standard medieval Arabic alphabetical notation.[14] Most intervals of time are expressed in equatorial degrees and minutes, where:

$$360° = 24^h \quad \text{and} \quad 1° = 4^m .$$

In the few tables in which times are given in seasonal day or night-hours the reader should bear in mind that:

$$12^{\,sdh} = (2D)° \quad \text{and} \quad 12^{snh} = (2N)° .$$

In various Ottoman tables other numerical notation and units are used, and the times:

$$m, \; r', \; R', \; s' \; \text{and} \; a'$$

are given according to the convention whereby sunset is reckoned as 12 o'clock.[15]

The capital notation for medieval trigonometric functions, now standard in modern literature on the history of the exact sciences,[16] indicates that the function is computed to a base R, usually 60 for the Sine and Cosine of a general arc θ, thus, for example:

$$\mathrm{Sin}_R\, \theta = R \sin \theta .$$

The Cotangent or Tangent is usually defined with an argument that is usually a celestial altitude, since these functions measure, respectively, the horizontal shadow of a vertical gnomon length n or the vertical shadow of a horizontal gnmon. Thus for any celestial altitude h:

$$\mathrm{Cot}\, h = n \cot h .$$

The most common bases for these functions are 12 (digits) and 7 (feet). However, other bases were used, such as 1, 6, $6^1/_2$, $6^2/_3$, 10, 20 and 60,[17] and one "artificial" base of 5 (= 60/12).[18] I use Z for Cot H and z for Cot h and occasionally z′ for Tan h, with the base indicated in parentheses in the subscript, thus, for example, $Z_{(6;40)}$ stands for $\mathrm{Cot}_{6;40}$ H. I also use z_a, *etc*., for the shadow at the ʿ*aṣr* prayer. The 'z', for Arabic *ẓill*, shadow", is intended as a reminder that the medieval Cotangent function is of a different nature from the medieval Sine function.

I assume that the reader has some familiarity with spherical astronomy and is familiar with **I**.[19] Note that the hour-angle t(h,λ,ϕ) corresponding to an instantaneous solar altitude h, solar

---

[14] On this notation see the text to n. **I**-1:27.
[15] See the text to n. **I**-1:12.
[16] See the text to n. **I**-1:28.
[17] See the text to n. **I**-1:31-35. On the reasoning behind the use of these bases see the text to n. **III**-0:17.
[18] See, for example, the various tables discussed in **I-7.1.5** and **7-8**.
[19] A brief account of the standard medieval formulae and references to the published literature on spherical astronomy in medieval Islam is given in **I-1.2**. A discussion of the determination of the prayer-times by modern methods is contained in Arvanitaki, "L'heure arabe" (1915), and, more recently, Ilyas, *Astronomy of Islamic Times*, and *Islamic Calendar, Times & Qibla*.

longitude λ and local latitude ϕ, is symmetrical with respect to the solstices and so the equinoctial value and 180 values suffice to define it for each integral degree of solar longitude. This also holds for functions defining the permitted intervals for the prayers. In virtually all the tables consulted, full advantage is taken of the symmetry of the functions tabulated. I denote the longitude argument for such functions by λ' ($\delta \gtreqless 0°$), where λ' measures the solar elongation from nearer equinox. Generally early tables begin with entries for Aries and Virgo and later ones begin with entries for Capricorn and Sagittarius.

1.4 On some approximate formulae for timekeeping

Some of the Muslim astrononomers whose writings and tables we shall be considering used approximate formulae, as a result of which computation is much facilitated. I mention just two of these here, since they recur frequently.

First, an Indian "arithmetical" formula for finding the time in seasonal hours ($T < 6$) from the increase in the shadow length Δz for a gnomon length n is equivalent to the following:

$$T \approx 6 \cdot n / (\Delta z + n) .$$

Note that when $\Delta z = 0$ then $T = 6$, and when $\Delta z \to \infty$ then $T = 0$. For intermediate hours the results are respectable to a first approximation. As we shall see, this formula underlies the standard definitions for the *ʿaṣr* as well as the Andalusī and Maghribī definitions of the *ẓuhr* (see further **IV-2.4**). See **2.3**, **2.4**, **2.5**, **2.9**, and **9.3** (Ibn al-Shāṭir!) for the formula in the present study, and also **III** and **IV**.

Second, an approximation based on trigonometric rather than simple arithmetical considerations. The time in seasonal day-hours T (< 6) is given in terms of the instantaneous solar altitude h and the meridian altitude H thus:

$$T \approx {}^{1}/_{15} \cdot \text{arc Sin} \{ \text{R Sin h} / \text{Sin H} \} .$$

Here $T = 0$ when $h = 0°$ and $T = 6$ when $h = H$. The formula is in fact accurate when the sun is at the equinoxes. For intermediate solar longitudes and latitudes between 15° and 40° this formula too gives results that are reasonable – actually more reasonable than one would expect – to a first approximation. We find this formula in numerous of the sources studied here, from the 9th century onwards: see **I-2.5** and **4.3**, **II-2.4**, **2.5**, **2.9**, **3.2** and **3.3**, and further **XI**. It underlies the universal horary quadrant that was a standard feature of astrolabes, both Islamic and European, although those who used it in Europe were apparently aware that the formula is no longer reliable at European latitudes.

In addition, various approximate rules for quick calculation of various spherical astronomical functions are proposed by certain Muslim astronomers: see **2.4**, **2.4a** (Andalusī source), **2.5**, **2,9**, **3.1** (early ʿAbbāsid source!), **3.11**, **6.1**, **6.3**, **6.12**, and **9.3** (Ibn al-Shāṭir!). It seems that those who proposed such rules looked at tables of a function $f(\lambda)$, noted the changes between the values at the equinoxes f_0 and solstices, and then determined the factors m and n (+ or -) in simple fractions which would best fit in a simple relation of the kind:

$$f(\lambda) = f_0 + m \cdot \delta(\lambda) \text{ for } \delta > 0 \quad \text{and} \quad f_0 + n \cdot \delta(\lambda) \text{ for } \delta < 0 .$$

Sometimes other variables, such as d or $\bar{H}$ were used instead of δ. I am not aware of any tables based on such approximations.

1.5 On the recomputation of medieval prayer-tables

Given the algebraic expression defining a particular function and the underlying parameters it is possible to compile extensive tables of the function with an electronic computer (see already **I-1.5**). The use of the computer in the analysis of medieval Islamic tables was pioneered by Ted Kennedy, and is now being pursued by Benno van Dalen, Glen Van Brummelen and François Charette.[20] Once the underlying latitude is established, it is often useful to consult the lists of coordinates taken from Islamic sources prepared by Ted and Mary Helen Kennedy.[21]

In 1970-71 I recomputed the main Cairo corpus in the Computer Centre at the American University of Beirut and in 1971-72 recomputed similar sets for other latitudes at the Yale University Computer Center. At the Computer Centre of the American University in Cairo, programmes were prepared in 1972-73 to recompute prayer-tables of each of the various types described in the sequel for any latitude. A given set for a particular latitude can be recomputed in a matter of seconds once the underlying parameters have been established.[22]

The tables discussed in the present study contain a total of several million entries. Some of them merit publication *in toto*, others are of little scientific or historical interest. In this study I present sample entries only from tables that I consider to be of singular interest for one reason or another. The values agree with recomputation except where the error in the second digit is shown in square brackets, computed according to the convention:

error = (value in text) - (recomputed value) .

Rather than give sample entries, I have generally considered it more important to identify the structure and underlying parameters of all the tables, to comment on their accuracy, as well as to present illustrations of as many tables as possible.[23]

---

[20] See n. **I-**1:24.

[21] See n. **I-**1:4.

[22] Computer time was made available by the Department of Mathematics, The American University of Beirut; the Department of Near Eastern Languages and Literatures, Yale University; and the Smithsonian Institution, respectively.

[23] Ideally, sample entries should be given for tables forming part of a clearly defined corpus of prayer-tables for a given locality. (A model for such an endeavour for standard astronomical tables is Toomer, "Toledan Tables".) In most cases such entries (say for each 30°) are adequate to assess the accuracy of the tables and to identify related material. In King, "Astronomical Timekeeping in Medieval Cairo", pp. 380-384, I present sample entries from the tables in the Cairo corpus, and in an unpublished study *Shams al-Dīn al-Khalīlī and the Culmination of the Islamic Science of Astronomical Timekeeping*, yet more extensive samples from the Damascus corpus. The latter work is unpublished not least because the prospective publisher, the Third World Academy of Sciences, some years ago understandably baulked at the prospect of printing masses of tables as well as photos of the most important manuscripts (including MSS Paris ar. 2558 and Dublin CB 4091). The text, without the tables, is now incorporated in **Ch. 10** of the present study.

CHAPTER 2

SELECTED MEDIEVAL ARABIC TREATISES ON TIMEKEEPING

2.0 Introductory remarks

An enormous amount of Arabic scientific literature deals with astronomical timekeeping. Also, the topic of spherical astronomy is dealt with in the majority of Islamic *zīj*es. In order to display the importance of this literature for the documentation of the history of the development of mathematical methods, and the history of astronomical inquiry into such phenomena as twilight and refraction at the horizon, I have selected a few of the more historically interesting of the available treatises and have attempted to survey their contents. I have deliberately included one treatise in the folk astronomical tradition (**2.3**). There are others, also of widely varying levels of competence on the part of their original authors, to which the interested reader may have recourse: Kūshyār ibn Labbān on spherical trigonometry;[1] Abū Naṣr's treatises on spherical astronomy;[2] al-Bīrūnī's treatise on spherical trigonometry applied to problems of spherical astronomy;[3] al-Kāshī's treatment of spherical astronomy in his *Khāqānī Zīj*;[4] and Abū Miqraʿ's treatment of timekeeping by folk astronomical methods.[5] Also the simple techniques for reckoning time of night by the lunar mansions have been studied recently by Miquel Forcada.[6] In addition there are several accounts in the published literature of the solution of such problems using astronomical instruments, such as al-Khwārizmī on the astrolabe;[7] Ḥabash on the celestial globe;[8] Ibn al-Zarqālluh on the universal plate (*ṣafīḥa shakkāziyya*);[9] Ibn al-Shāṭir on an astronomical "compendium" (multi-purpose instrument) of his own design;[10] and Jamāl al-Māridīnī on the universal quadrant.[11] See further **X**. As this book goes to press, I am gratified that Julio Samsó has published an account of a medieval Maghribī treatise preserved in MSS London B.L. 411 and Cairo K 4311 which deals critically with parameters for twilight (and also trepidation) as discussed by various earlier authors.[12]

---

[1] See Berggren, "Spherical Trigonometry in the *Jāmiʿ Zīj*".
[2] See Samsó, *Estudios sobre Abū Naṣr*.
[3] See the study of Marie-Thérèse Debarnot listed under al-Bīrūnī, *Maqālīd*.
[4] See Kennedy, "Spherical Astronomy in al-Kāshī's *Khāqānī Zīj*".
[5] See Colin & Renaud, "Abū Miqraʿ".
[6] See Forcada, "*Mīqāt* en los calendarios andalusíes".
[7] See already Frank, "al-Khwārizmī über das Astrolab". A new edition, translation and commentary have been prepared by François Charette and Petra Schmidl.
[8] Lorch & Kunitzsch, "Ḥabash on the Sphere".
[9] See n. **I**-1:45.
[10] See Janin & King, "Ibn al-Shāṭir's Compendium".
[11] See King, "al-Māridīnī's Universal Quadrant" (n. **I**-9:23).
[12] Samsó, "Astronomical Observations in the Maghrib", pp. 174-175. On the unidentified author and his work see *Cairo ENL Survey*, no. F26, and on some of his tables for timekeeping see **13.6**.

2.1 A poem on the times of prayer attributed to the Imām al-Shāfiʿī and Ibn Yūnus

In MS Cairo DM 181, fols. 46v-48r, copied *ca*. 1800, there is a short poem on the prayer-times in *ṭawīl* metre attributed to Ibn Yūnus (**4.1**): see Fig. **III-9.7b**. The Egyptian astronomer was renowned as a poet and several of his poems are preserved in various anthologies and biographical lexicons.[13] However, in MSS Berlin Ahlwardt 5820 (Wetzstein 175), fol. 65r, and 5700 (Landberg 953), fol. 11r, copied in 944 H [= 1537/38] and 1231 H [= 1816], respectively, a similar poem is attributed to the famous legal scholar al-Shāfiʿī (d. 819/20). This poem is also attributed to al-Shāfiʿī in various later Egyptian works on timekeeping, such as those of ʿAbd al-Raḥmān al-Tājūrī and Yūsuf Kilārjī (**8.8**). al-Shāfiʿī was renowned for his poetry, but this particular poem is not included in a modern anthology of his poetical works.[14] The poem in both Berlin copies is extremely corrupt and is clearly an improvisation of the original poem by someone who was not very gifted in either astronomy or poetry. I present a prose translation of the poem from MS Cairo DM 181, incorporating a few corrections necessary to the sense of the poem, some of which are confirmed by the readings in the second Berlin manuscript (Ahlwardt 5700):[15]

> "Knowing the prayer-times is a prescribed duty for discerning Muslims. This is summarized in the *Qurʾān*, my friend, and was explained by Aḥmad [*i.e.*, the Prophet Muḥammad, referred to as Aḥmad in the *Qurʾān*], the most outstanding of men. Perform the midday prayer whenever you observe the shadow starting to increase. Add a length (of the gnomon) to the shadow: this gives you (the shadow) at the time of the afternoon prayer. At sunset get up and perform the evening prayer: this is the only permissible time. Perform the night prayer when, looking at the sky, you see the upper part of the evening twilight fade away and disappear. As for the end of this prayer time, if you wish you can wait until one-third of the night; indeed, it is better that you wait. But do not wait until whiteness appears (on the eastern horizon): it will last for a time in the sky. Bear in mind that there are two stages of daybreak according to our doctrine: distinguish between them carefully – you are the one who decides this. The first daybreak looks like a wolf's tail rising in the sky: this is the false dawn. The later one is the true dawn: you see it illuminate the sky like a fire. The end of this prayer-time is sunrise: at that moment the best time for prayer is over. There is no virtue in a person who is neglectful of the prayer-times and he has no knowledge of Him who is to be worshipped."

Ibn Yūnus is the more likely author because in the time of al-Shāfiʿī the standard definitions mentioned here were not yet definitively formulated (**IV-3**).

---

[13] On Ibn Yūnus' poetry see King, *Ibn Yūnus*, I.1, and the references there cited.

[14] On al-Shāfiʿī's poetry see Sezgin, *GAS*, III, p. 647.

[15] The text of the poem is published in my contribution to the *Aleppo 1976 Symposium Proceedings*, I (Arabic section), pp. 393-394, but is not included in the English version "Astronomical Timekeeping in Medieval Syria", repr. in King, *Studies*, A-X.

2.2 al-Bīrūnī on tables for timekeeping

al-Bīrūnī was the most outstanding scientist of the Islamic Middle Ages.[16] He was a prolific writer and did original work in fields as varied as ethnography, geology and astronomy. Our present concern is with his *Ifrād al-maqāl fī amr al-ẓilāl*, "Unique Treatise on Matters Relating to Shadows", in which he devotes a whole chapter to the opinions of the religious leaders about the times of prayer and the obligation to determine them properly. The Arabic text of this work was published in Hyderabad in 1948 and E. S. Kennedy published a translation and commentary in 1976.[17] al-Bīrūnī's treatise is a mine of information concerning the prayer-times and time-keeping in general. However, al-Bīrūnī does not make a single reference to tables for regulating the prayer-times, although in his treatise on the construction of the astrolabe entitled *al-Istīʿāb* he does mention *ṭaylasān* tables for time-keeping.[18] Thus this study is in a sense a supplement to Kennedy's study of al-Bīrūnī's *Shadows*.

2.3 Ibn Raḥīq's treatise on timekeeping

MS Berlin Ahlwardt 5664 (Landberg 108), achieved *ca*. 1350, is an apparently unique copy of a treatise on folk astronomy by an individual named Abū ʿAbdallāh Muḥammad ibn Raḥīq ibn ʿAbd al-Karīm, otherwise unknown to the literature.[19] The treatise was compiled in Mecca in the early 11th century and it contains information of a very simple kind on the calendar, the lunar mansions and timekeeping. Some of the material on the latter topic is said to be taken from "the large book" of Abū ʿAlī ʿAraqa, the muezzin of the mosque of ʿAmr (*al-jāmiʿ al-ʿatīq*) in Fusṭāṭ. The treatise is an important source for our knowledge of the practices of the muezzins of early Islam, before sophisticated mathematical methods were applied to time-keeping. A detailed study of this and two other Rasulid Yemeni treatises on the same subjects – by Ibrāhīm ibn ʿAlī al-Aṣbaḥī[20] and Muḥammad ibn Abī Bakr al-Fārisī[21] – is currently being conducted by Petra Schmidl.[22]

Ibn Raḥīq begins his treatise by asserting that the times of prayer should be determined by observation with one's own eyes (*al-ʿiyān wa-'l-raṣad*), not by using an astrolabe or "any of this astronomy nonsense" (Arabic, *tanjīm*: the word usually means "astrology"). He states that he will follow the *ḥadīth* about the Archangel Gabriel recorded in the canonical collections of Muslim and al-Bukhārī (**IV-1.2**), rather than the writings of those take their learning from "the infidels and the *Sindhind*". (By these he means the Greek and Indian traditions in Islamic astronomy, respectively.)

With regard to the *ẓuhr* prayer Ibn Raḥīq recommends the use of an "Indian sundial" (*al-rukhāma al-mabniya* (?) *al-hindiyya*) with which one can watch for the moment when the

---

[16] On al-Bīrūnī see the splendid article by E. S. Kennedy in *DSB*. His work on timekeeping is listed as al-Bīrūnī, *Shadows*.
[17] See n. 1:7.
[18] See n. **I**-1:53.
[19] On Ibn Raḥīq and his works see King, *Astronomy in Yemen*, no. 2, and also **III-3.2** and **IV-1.5**.
[20] On al-Aṣbaḥī see King, *Astronomy in Yemen*, no. 5; *Cairo ENL Survey*, no. E5; and **IV-1.5**, *etc*.
[21] On al-Fārisī see n. **I**-7:17, and also **III-4.2** and **IV-1.5**.
[22] See Schmidl, *Islamische Volksastronomische Abhandlungen*, and also *eadem*, "Qibla und Winde".

shadow cast by the sun begins to increase.[23] Ibn Raḥīq records that ʿAraqa had reported that Aḥmad al-Rāzī, the muezzin at the Sacred Mosque in Mecca, had observed the sun over a period of seven years and had found that the shadow length for the Coptic months, expressed in terms of the height of a man (*qāma*), were as follows (beginning with the 10th month):

X: $^1/_3 \cdot {}^1/_5$	XI: 0 [a]	XII: $^1/_6$
I: $^1/_3 + {}^1/_2 \cdot {}^1/_6$ [b]	II: $^1/_2$	III: $^2/_3 + {}^1/_4$
IV: $1 + {}^1/_6 \cdot {}^1/_8$	V: $1 - {}^1/_2 \cdot {}^1/_8$	VI: $^1/_3 + {}^1/_4$
VII: $^1/_4 + {}^1/_8$	VIII: $^1/_6$	IX: 0

[a] Arabic: *lā shay*ʾ [b] text has: $1/33 + {}^1/_2 = {}^1/_6$

These fractions, written in words in the text, can be simplied thus:

$^1/_{18}$ 0 $^1/_6$ $^5/_{12}$ $^1/_2$ $^{11}/_{12}$ $1^1/_{48}$ $^{15}/_{16}$ $^7/_{12}$ $^3/_8$ $^1/_6$ 0

Ibn Raḥīq states that the beginning of the ʿ*aṣr* prayer should be when the shadow is $6^1/_2$ feet (*qadam*, pl. *aqdām*) more than the midday shadow, which indicates that he favored $6^1/_2$ units for the length of the gnomon. He records the following maximum shadow lengths reported by Abū ʿAlī ʿAraqa:

Mecca	$7 - {}^1/_5$	(= $6^4/_5$)
Medina	$7 + {}^1/_2$	
Sirrayn (in Yemen)	$7 - {}^1/_3$	(= $6^2/_3$)
Alexandria, Baghdad, Crete	10	
Jerusalem	9 + 2/3	
Cairo-Fusṭāṭ (*Miṣr*)	9	
Ifrīqiyya (*i.e.*, Tunis), Rayy	11	
Qum	13	
Kufa	$8 + {}^1/_2$	
Armenia	6	(read 16?)

ʿAraqa also measured the shadow at Sanaa on Kayhak 25 (year unspecified), in the company of the *ḥadīth* scholar Aḥmad ibn Ṣāliḥ, and found it to be $4^1/_2$ feet.

In a section on measuring time of day approximately from shadow lengths Ibn Raḥīq presents a method for finding the seasonal hours from the excess of the instantaneous shadow over the midday shadow, which he asserts is valid for the latitudes of al-ʿIrāq, Syria, the Maghrib and Egypt. When these excesses are:

$$39, \quad 19^1/_2, \quad 6^1/_2, \quad 3^1/_4, \quad 1+$$

according to Ibn Raḥīq, the time is:

$$1, 2, 3, 4, 5 \quad \text{or} \quad 11, 10, 9, 8, 7$$

seasonal hours after sunrise. The figure for 5 and 7 seasonal hours is expressed as *qadam rājiḥ*, which means "just more than one foot". No explanation is given for these figures. Consider, however, the following approximation for the time T in seasonal day-hours as a function of the excess Δz of the instantaneous shadow of a gnomon length n over the midday shadow, namely:

$$T \approx 6 \cdot n / [\, n + \Delta z \,] .$$

[23] It is not completely clear what is meant here. The name "Indian circle" (*al-dāʾira al-hindiyya*) is usually given in the Arabic sources to a simple device and procedure – a gnomon and a a geometric construction based on observations with it – for determining the local meridian. On this see Kennedy, *al-Bīrūnī's* Shadows, II, pp. 80-90, and, most recently, King, *Mecca-Centred World-Maps*, pp. 93 and 94, and also n. 24 on p. 134.

This formula, which is found in several Islamic sources (**1.5**), is ultimately of Indian origin (**III-1.3** and **IV-2.4**). If we substitute T = 3, 4 and 5^{sdh}, we obtain respectively:

$$x = 6^1/_2 ,\ 3^1/_4 \quad \text{and} \quad 1^3/_{10} ,$$

as in the text. Further, if x is large compared with n and we use the cruder approximation:

$$T \approx 6 \cdot n / x ,$$

we obtain for T = 1^{sdh} and 2^{sdh} Ibn Raḥīq's values:

$$x = 39 \quad \text{and} \quad 19^1/_2 .$$

Thus each of the five values is explained. Ibn Raḥīq adds a note that the 9th hour marks the beginning of the *ʿaṣr* prayer according to the fourth caliph ʿAlī ibn Abī Ṭālib (**III-2.2** and **IV-2.2**).

Here we have, perhaps, the explanation of the origin of the definition of the *ʿaṣr* which was not stated in the *ḥadīth* but which was adopted already in the 8th century and has persisted to this day. If we postulate that the *ʿaṣr* divides the period from midday to sunset into two equal intervals, then, according to the approximate Indian formula quoted above, the length of the shadow at the *ʿaṣr* is equal to the midday shadow increased by the length of the gnomon. If this is indeed the origin of the definition of the *ʿaṣr*, Ibn Raḥīq has underestimated the debt of Islam to at least the Indian tradition in early Islamic astronomy. Note also that the shadow length at the beginning of the 10th hour, according to the Indian formula though not according to Ibn Raḥīq, is in excess of the midday shadow by twice the length of the gnomon. This is Abū Ḥanīfa's definition of the *ʿaṣr* (**1.1** and **IV-3**).

Ibn Raḥīq states that the evening prayer (*al-maghrib*) should begin when redness appears in the western sky after sunset. He distinguishes between the true and false dawn for the beginning of the morning prayer (*al-fajr*).

Our author quotes from a work of Muhammad ibn Surāqa al-ʿĀmirī (d. 1019) information about the determination of the qibla in various parts of the Muslim world. The following extract is typical of the information contained in an extensive corpus of material dealing with what I call sacred geography and which I have surveyed elsewhere.[24] It is traditional knowledge, independent of the scientific tradition of mathematical geography and cartography in medieval Islam.

> "The people of Alexandria and Egypt up to Kairouan and Sus (*al-Sūs al-aqṣā*) and the Black Sea and all places in the same direction (with regard to Mecca) should face the same direction as (when one is standing in front of the Kaʿba) between its western corner to the draindripe (at the middle of the north-western wall). In any of these places, if one should stand with the three main stars of the Great Bear (*banāt naʿsh*) at one's right shoulder when they are setting and at one's left shoulder when they are rising, with the celestial pole behind one's back and the west wind (*al-dabūr*) on one's right, then one is facing the direction of the Kaʿba."

A large part of Ibn Raḥīq's treatise consists of simple information about each of the lunar mansions throughout the year, such as is found in several later Yemeni works (**12**). The following is an extract:

---

24 This kind of qibla determination is surveyed in my unpublished study *The Sacred Geography of Islam*, summarized in the article "Makka. iv. As centre of the world" in EI_2, repr. in King, *Studies*, C-X. See also Schmidl, *Islamische Volksastronomische Abhandlungen* (n. 2:22).

"The lunar mansion *al-Iklīl* rises at daybreak on Hātūr 17, which is Tishrīn II 13, November 13 and Ādharmāh 10. Morning twilight (?: *al-ẓill*) is when the next mansion *al-Qalb* (is rising over the horizon) and the sun is in (the next mansion) *al-Shawla*. The beginning of the night is when al-*Haqʿa* is rising. The mansions culminating at dawn and nightfall are *al-Jabha* and *al-Muʾakhkhar*. This mansion is (completely) visible on the 12th day of its rising and contains three bright stars which (form) the crown of the Scorpion. The midday shadow at Mecca – may God exalt her – is six and one third feet."

2.4 Ibn al-Ḥammāmī's treatise on timekeeping

MS Istanbul Haci Mahmud Efendi 5713, penned in the 12th century, is a unique copy of three Egyptian astronomical treatises of considerable historical interest. These are: (a) a treatise on timekeeping by an individual called Ibn al-Ḥammāmī (fols. 1r-10v), (b) a treatise on the astrolabic quadrant by the same author (fols. 10v-25v), and (c) a treatise on the trigonometric quadrant by Fatḥ al-Dīn al-Qaysī (fols. 25v-34v). The treatise on the astrolabic quadrant predates the earliest previously-known Arabic treatises on quadrants (by al-Mizzī) by at least a century.[25] I have no information on either of the two authors.[26]

Our present concern is with the first of these treatises, which contains various approximate rules for the standard functions of timekeeping, such as those given by the later Egyptian astronomers Najm al-Dīn al-Miṣrī, al-Bakhāniqī and al-Fawānīsī (**3.3**, **3.11** and **6.12**).

Ibn al-Ḥammāmī first records the following approximate changes in δ and α_ϕ for each 30° of λ measured from the equinox:

$$\delta\text{: } 12° \quad 8° \quad 4° \quad \text{and} \quad \alpha_\phi\text{: } 21° \quad 24° \quad 30° \quad 34° \quad 36° \quad 35° \ ,$$

and the following midday shadows for the zodiacal signs:

7 4 2 1;21 2 4 7 10 14 16;16 14 10

To find the time T in seasonal hours since sunrise from the instantaneous and midday shadows z and Z he prescribes the rule:

$$T = 72 / [(z + n) - Z] \qquad (n = 12) \ .$$

To find the shadow z_1 and z_2 at the (second optimum?) time for the midday prayer (*ẓill ikhtiyār al-ẓuhr*) and the end of the midday prayer (*ẓill ākhirihi*), the author prescribes:

$$z_1 = Z + {}^1/_4\, n \quad \text{and} \quad z_2 = Z + n \qquad (n = 12) \ .$$

Likewise the shadows z_3 and z_4 at the beginning and end of the afternoon prayer (*ẓill awwal waqt al-ʿaṣr* and *ẓill ākhirihi*) are given as:

$$z_3 = z_2 + \varepsilon \quad \text{and} \quad z_4 = Z + 2n \ ,$$

where ε is referred to as *adnā ziyāda*, "the last (perceptible) increase". I know of no other Egyptian treatise in which the times of the midday and afternoon prayers are defined in this way (**IV-4**).

---

[25] On quadrants see my EI_2 article "Rubʿ" and **X-6**.

[26] Their full names are Abu 'l-Ḥasan ʿAlī Ibn Muḥammad ibn al-Ḥammāmī and Fatḥ al-Dīn Abū ʿUthmān ibn Hibatallāh ibn Abi 'l-Ḥawāfir. The copyist of the manuscript was ʿAbd al-Raḥmān ibn Muḥammad ibn ʿAbd al-Muhayman al-Shāfiʿī, and the date of copying is not clearly legible. The day was Wednesday, Ramaḍān 1, and the year ??1 Hijra. The Süleymaniye card index has 504 H, which is untenable.

In the course of his discussion of the determination of the standard functions of timekeeping Ibn al-Ḥammāmī presents several approximate rules. As an example I quote the rule for finding t_a and T_a:

> "To find the time between the midday and afternoon prayers and between the afternoon prayer and sunset, add one-half of one-sixth of the solar declination to 52° for the northern signs and subtract one-quarter of the declination and one-sixth of the declination for the southern signs. The result will be the time between the midday and afternoon prayers. Subtract this from half the diurnal arc and the remainder will be the time between the afternoon prayer and sunset. God knows best."

In all there are five such approximate rules:

(a)	$h_a \approx 32;30° + {}^2/_5\ \delta$	$(\delta \gtrapprox 0)$
(b)	$h_q \approx 47° + \delta + {}^2/_5\ \delta$	$(\delta \gtrapprox 0)$
(c)	$t_a \approx 52° + {}^1/_2 \cdot {}^1/_6\ \delta$	$(\delta > 0)$
	$t_a \approx 52° - {}^1/_4\ \delta - {}^1/_6\ \delta$	$(\delta < 0)$
(d)	$s \approx 18;30° + {}^1/_8\ \delta$	$(\delta > 0)$
	$s \approx 18;30° + {}^1/_2 \cdot {}^1/_8\ \lvert\delta\rvert$	$(\delta < 0)$
(e)	$r \approx 23;30° + {}^1/_5\ \delta$	$(\delta > 0)$
	$r \approx 23;30° + {}^1/_2 \cdot {}^1/_6\ \delta$	$(\delta < 0)$

Rule (b) corresponds slightly more closely to the paramater q = 52° than to 53° (though see below) and rules (d) and (e) imply that Ibn al-Ḥammāmī accepted the parameters 20° and 16° for morning and evening twilight, which is confirmed on fols. 22r and 22v of this Istanbul copy of his treatise on the astrolabic quadrant. I consider it likely that Ibn al-Ḥammāmī derived these simple rules by inspecting some tables of these five functions. Those in the main Cairo corpus give the following values for the equinoxes and the solstices:

	h_a	h_q q=52°	h_q q=53°	t_a	s h_s=-16°	r h_r=-20°
EQ (λ=1°)	32;33	47;23	46;43	51;54	18;35	23;18
SS	41;58	80; 2	79;48	53;48	22;13	28;28
WS	23; 0	13;39	12;32	41;53	20; 4	24;54

This having been said, it should be pointed out that Ibn al-Ḥammāmī predates al-Maqsī and so was more likely to have access to earlier tables that may not have been preserved for us. See, for example, **5.3**.

Our author concludes his treatise by describing some simple operations with an astrolabe, including how to use the alidade to indicate the direction of the qibla, stated as 37° south of east (q = 53°).

2.4a Ibn Bāṣo's rules for the prayer-times

The Andalusī astronomer Ibn Bāṣo (d. 1316) is best known for his ingenious universal plate, which became a feature of many later Maghribi and even a few Eastern Islamic astrolabes.[27]

[27] See Calvo, *Ibn Bāṣo and his Universal Plate*, where the definitions are discussed on pp. 78-80.

In his treatise on this plate he advocates approximate rules for the times of the *ẓuhr* and *ʿaṣr*, as follows:

$$h_z \approx (H - 10°) - {}^1/_{10}(H - 30°) \text{ for } H > 30° \quad \text{and} \quad h_z \approx H - 10° \text{ for } H < 30°$$

$$h_a \approx {}^1/_2 H + {}^1/_{10}(80° - H) \text{ for } H < 80° \quad \text{and} \quad h_a \approx {}^1/_2 H \text{ for } H > 80°$$

The second rule is curious because for the latitude of, say, Cordova, the solar altitude does not reach 80°. I have not encountered these elsewhere, although certain Egyptian treatises (**2.9** and **6.1**) have a similar rule for h_a with underlying latitude 30°.

2.5 Najm al-Dīn al-Miṣrī's treatise on timekeeping

MS Istanbul Hamidiye 1453, fols. 219r-228v and 228v-230, copied in Damascus in 869 H [= 1464/65], contains two related treatises on timekeeping, the second being attributed to Najm al-Dīn al-Miṣrī, who worked in Cairo in the early 14th century (see **I-2.6.1** and **I-9.3***, and also **II-6.5** on his extensive tables for timekeeping). The first treatise is entitled *Taḥrīr al-maqāla fī maʿrifat al-awqāt bi-ghayr āla*, and the second *Ikhtiṣār al-maqāla ...* , meaning respectively "Redaction of" and "Abridgement of" of a "Treatise on Determining the Times of Prayer without Instruments". A third treatise on timekeeping by Najm al-Dīn al-Miṣrī is preserved in MS Milan Ambrosiana 227a (C49), fols. 85v-97r, and represents a much more sophisticated level of spherical astronomy. Neither of the first two treatises is related to the third, since they both deal only with approximate methods, and not always the same ones at that. However, I see no problem in accepting Najm al-Dīn as the author of all three.

In the first treatise Najm al-Dīn first records the following approximate changes in δ, α and α_ϕ for each 30° of λ measured from the equinox:

$$\delta: \ 12° - 8° - 4°$$

$$\alpha: \ 28° - 30° - 32°$$

$$\alpha_\phi: \ 21° - 24° - 30° - 35° - 35° - 35°$$

To find the shadow lengths (base 12) corresponding to a particular altitude he considers the four cases:

$$\text{(a)} \quad h < 27°, \quad \text{(b)} \quad 27° < h < 45°, \quad \text{(c)} \quad 45° < h < 63° \quad \text{and} \quad \text{(d)} \quad h > 63°$$

stating that the corresponding shadow lengths are respectively:

$$\text{(a)} \quad h / 4^1/_2, \quad \text{(b)} \quad (h-27)/3 + 6, \quad \text{(c)} \quad (63-h)/3 + 6, \quad \text{and} \quad \text{(d)} \quad (90-h) / 4^1/_2,$$

where the first two are vertical (*mankūs*) and the last two horizontal (*mabsūṭ*). He also notes that the horizontal and vertical shadows z and z′ are related by:

$$z \cdot z' = 144 \quad (= 12^2).$$

Najm al-Dīn then discusses the altitudes corresponding to (vertical) shadow lengths in feet (*aqdām*) and mentions the values:

z	1	2	3	4	5	6	6;40
h	9°	17	24	31	37	42	45

For timekeeping by the sun he notes that the length of half-daylight in equinoctial hours D′ is given approximately by:

$$D' \approx {}^{11}/_{60} \cdot \delta \cdot \phi,$$

a formula which underlies a simple table in MS Paris BNF ar. 2513 of the contemporary

Muṣṭalaḥ Zīj (**6.6** and **I-7.4.1**), and then prescribes the following rules for finding the time in seasonal hours since sunrise, T, from the instantaneous and midday shadows, z and Z:

$$T \approx 40 / [(6;40 + z) - Z] \qquad \text{(shadows in feet)}$$
$$T \approx 40 / [(12 + z) - Z] \qquad \text{(shadows in digits)}$$

The author also presents the following approximate rules for finding the duration of morning and evening twilight r and s in seasonal hours when the solar longitude is λ:

$$r \approx 72 / [(33+12) - Z^*] \quad \text{and} \quad s \approx 72 / [(42+12) - Z^*] ,$$

where Z^* is the horizontal midday shadow ($n = 12$) cast by the sun when it has longitude $\lambda^* = \lambda + 180°$. Note that $\mathrm{Cot}_{12}\,20° \approx 33$ and $\mathrm{Cot}_{12}\,16° \approx 42$, so that these rules are simply extensions of the above rule for reckoning time by day.

In the second treatise Najm al-Din records the following approximate changes in δ, α and α_ϕ for each 30° of λ measured from the equinox:

δ: 12° - 8° - 4° α: 28° - 30° - 32° and α_ϕ: 21° - 24° - 30° - 34° - 36° - 35°

and the following midday shadows ($n = 12$) for the zodiacal signs beginning with Aries:

7 4 2 1;20 2 4 7 10 14 16;15 14 10 .

Note that the solar meridian altitude in Cairo at the equinox and solstices are 60°, 83;35°, 36;25° and the corresponding shadows are 6;56, 1;21, and 16;16.

The remainder of the treatise deals with the determination of the standard functions of timekeeping. Najm al-Dīn mentions the following approximate formulae:

(a)	$h_a \approx 32;32° \pm {}^2/_5\,\lvert\delta\rvert$	$(\delta \gtrless 0)$
(b)	$d \approx {}^1/_2\,\delta + {}^1/_4\,\delta$	
(c)	$t_a \approx 52° + {}^1/_3 \cdot {}^1/_5\,d$	$(\delta > 0)$
	$t_a \approx 52° - {}^1/_3\,\lvert d\rvert$	$(\delta < 0)$
(d)	$\psi \approx \delta + {}^1/_6\,\delta$	
(e)	$h_q \approx 47° + \delta + {}^1/_6\,\delta + {}^1/_4\,\delta$	$(\delta \gtrless 0)$
(f)	$h_0 \approx 2\delta$	$(2\delta < 15°)$
	$h_0 \approx 30° + (4\delta - 30°) + {}^1/_4\,(4\delta - 30°)$	$(2\delta > 15°)$
(g)	$\alpha_s \approx \alpha_\phi(\lambda^*) + 20°$	
(h)	$\alpha_r \approx \alpha_\phi(\lambda) - 25°$	

Note that in (a) the value 32;32° was probably taken from a table of $h_a(\lambda)$ in which it was the first entry (for Aries 1°): the tables of this function in the main Cairo corpus (**4.9**) have 32;33° for the first entry but the corresponding table in the *Muṣṭalaḥ Zīj* (**6.6**) and in the auxiliary tables of al-Khaṭāʾī (**6.8**) have 32;32°. Rules (b) and (c) are rather poor approximations, but may be garbled. The values 20° and 25° in (g) and (h) are approximations for the duration of twilight at the equinoxes and the summer and winter solstices, which are approximately:

$18^1/_2°$, $22^1/_4°$, 20 and $23^1/_3°$, $28^1/_2°$, 25

respectively. Rules (d) and (e) are also used by al-Bakhāniqī and al-Fawānīsī (**2.9** and **7.5**).

The treatise concludes with an approximate rule for finding the solar azimuth a_a at the time of the afternoon prayer, namely:

$$a_a \approx 21° + \delta + {}^1/_8\,\delta \qquad (\delta \gtrless 0)$$

(measured south from the east point). The azimuths at the equinoxes and solstices are in fact close to:

EQ: 21°S SS: 6°N WS: 48°S ,

so the rule is a fair approximation. (No tables of $a_a(\lambda)$ are known to have been prepared for Cairo, but the function is discussed in several later treatises on sundial theory.)

In passing I note the existence of a related anonymous treatise preserved in MS Escorial ár. 961,3 (fols. 13v-16v), copied in 863 H [= 1459]. This consists of a preface and 21 very short chapters, each dealing with one of the standard functions of timekeeping. The approximations for d, ψ, α_s and α_r given in this source are the same as those of Najm al-Dīn, and the other approximations are clearly related to his:

(a) $h_a \approx 32° + {}^2/_5\ \delta$

(b) $t_a \approx 52° + {}^1/_3 \cdot {}^1/_5\ d$

(c) $h_q \approx 47° + {}^1/_6\ \delta + {}^1/_4\ \delta$

(d) $h_0 \approx 2\delta$ $(\delta < 15°)$

$h_0 \approx 30° + {}^1/_4\ (2\delta - 30°)$ $(\delta > 15°)$

(e) $a_a \approx 21° + {}^1/_8\ \delta$.

The work attributed to Najm al-Dīn al-Miṣrī in MS Milan Ambrosiana 227a (C49), fols. 85v-97r, is entitled *al-Risāla al-ḥisābiyya fī 'l-aʿmāl al-āfāqiyya*, which means "Treatise on Computations in Spherical Astronomy for all Latitudes". This short treatise of 31 chapters contains material of considerably greater sophistication than his two other treatises. All of the theoretical procedures advocated are exact, and are expressed in the terse technical Arabic standard in later Egyptian and Syrian works on astronomy. I shall refer to the section on reckoning time from celestial altitude elsewhere (**5.2**), and here restrict attention to the only historical reference in the work, which occurs in Ch. 11 on the determination of twilight. Najm al-Dīn advocates the parameter 18° for both morning and evening phenomena but states that if one is calculating "for Cairo in particular according to the opinions of the celebrated Ibn Yūnus", one should use 20° and 16°. This reference confirms that Ibn Yūnus wrote more on twilight than has been preserved in MS Leiden Or. 143 of his *Ḥākimī Zīj* (**4.5** and **5.1**) and indicates that the Egyptian tradition of using the parameters 20° and 16° which we shall encounter frequently was much older than the 13th century.

2.6 al-Dīrīnī's poem on timekeeping

ʿAbd al-ʿAzīz ibn Aḥmad al-Dīrīnī was an Egyptian dervish who lived from about 1215 to 1297. He was the author of numerous works, one of which deals with astronomical timekeeping.[28] This particular compilation is a poem with intermittent prose passages and simple tables and diagrams (see **Fig. 2.6a**). I have examined MS Istanbul Hamidiye 1453, fols. 85v-102v, copied in Edirne in 869 H [=1464/65], and MS Istanbul Ayasofya 2711 of this poem, which is entitled *al-Yawāqīt fī ʿilm al-mawāqīt*, "Gems for Timekeeping".

al-Dīrīnī's discussion of calendars, lunar mansions and the motions of the sun and moon do not directly concern the present study. He notes the following midday shadows for the twelve signs beginning with Cancer, which appear to be based on n = 6;40:

$^2/_3$ 1 2 $3^1/_2$ $5\ ^1/_4 + {}^1/_8$ $7^1/_4$ $8^1/_8$ $7^1/_4$ ...

[28] On al-Dīrīnī see Brockelmann, *GAL*, I, pp. 588-589, and SI, pp. 810-811; and *Cairo ENL Survey*, no. C14.

Figs. 2.6a-b: Illustrations of the bright stars (*ʿayyūqāt*) in each of the 28 lunar mansions (a), and a curious table purporting to show the solar altitude for each integral shadow length (to base 6;40!) (b). [From MS Istanbul Hamidiye 1453, fols. 87v-88r and 97v-98r, courtesy of the Süleymaniye Library, Istanbul.]

He also notes the daily change in shadow length for each sign. In one of the two manuscripts there is a table displaying the midday shadows for each of the Coptic year. al-Dīrīnī's values for the solar meridian altitude and for the lengths of daylight in equatorial degrees are:

84°	80	72	60	48	40	36	40	...
210°	205	195	180	165	155	150	155	...

His values for the change in oblique ascensions for each 30° of ecliptic longitude are:

21°	24	30	34	36	35	..

A table of solar altitude corresponding to particular horizontal shadow is also presented (see **Fig. 2.6b**). The first six entries:

z	1	2	3	4	5	6
h	81°	73	66	59	53	48

correspond to those in the first treatise of Najm al-Dīn al-Miṣrī (**2.5**); the remaining entries are somewhat confused. The reader should bear in mind that they are written in *abjad* notation in the manuscripts:

z (Hamidiye)	6;47	7;50	8;48	11;15	14; 6	22;30	44;*60*
z (Ayasofya)	6;*80*	7;50	8;48	11;15	14;*100*	22;30	44;*60*
h	45°	42°	37°	31°	24°	17°	9°

It is difficult to explain these numbers. However, it is clear that z(45°) was intended to be 6;40 and the *mīm* representing 40 has been miscopied as *mīm-zāy* 47 and (by a further stretch of the imagination) *fāʾ* 80. The values of h in the second part of the table are the complements of those in the first part, and for these complementary values we should have shadows z and z′ such that:

$$z \cdot z' = (6;40)^2 = (6\,{}^2/_3)^2 = 400 / 9 .$$

In order to complete the table one must divide this product by 1, 2, 3, 4, 5 and 6 to obtain the shadow lengths corresponding to 9°, 17°, 24°, 31° and 42°. The values that one obtains this way are:

z	7;24	8;53	11; 7	14;49	22;13	44;27
h	42°	37	31	24	7	9

I suggest that whoever compiled the table in al-Dīrīnī's treatise was incapable of expressing the quotient 400/9 as a sexagesimal and wrote z(9°) = 44;60. This explains the values z(17°) = 22;30 and z(31°) = 11;15. To find z(24°) our author divided 44;60 by 3 and somehow derived 14;100 (the *qāf* for 100 was confused for *wāw*, 6 in the Ayasofya copy) and hence 7;50 for z(42°). To find z(37°) he divided 44;60 by 5 and was content to a settle for $8\,{}^4/_5$ = 8;48 as the quotient. The above explanation is somewhat far-fetched, but how else can one explain the absurd entries in the table?

2.7 The spherical astronomy in al-Marrākushī's A-Z

Abū ʿAli al-Marrākushī was the author of a substantial *summa* of spherical astronomy and instruments entitled *Jāmiʿ al-mabādiʾ wa-'l-ghāyāt*. Part of this work was translated by Jean-Jacques Sédillot (*père*) in 1834-35 as *Traité des instruments astronomiques des Arabes*, although in fact the part that he translated deals only with spherical astronomy and sundials.

A summary of part of the remainder of al-Marrākushī's treatise was published by Louis-Amélie Sédillot (*fils*) in 1844 as *Mémoire sur les instruments des Arabes*, but considerable material of interest was omitted.[29] It has not been adequately stressed in previous studies that al-Marrākushī's treatise was compiled in Cairo; al-Marrākushī is invariable listed amongst Moroccan astronomers since, as his name indicates, his family originated in Marrakesh. In his treatise he mentions that he travelled extensively in al-Andalus, North Africa and the Near East, but in all of his tables numerical examples are computed for Cairo. Sédillot-*père* misdated al-Marrākushī to about 1230 in spite of the fact that his solar longitude tables and star catalogue are computed for 1275/76 and 1282, respectively.

al-Marrākushī's treatise was very popular amongst later Muslim astronomers, if only in the central lands of Islam: Egypt, Syria and Turkey.[30] It exists in several manuscript copies and numerous manuscripts that I have examined in Cairo contain spherical astronomical tables extracted from his treatise. It is rather curious that al-Marrākushī makes no reference to Ibn Yūnus, in spite of the fact that he recomputed several tables already computed by Ibn Yūnus three centuries previously, and for the same parameters, namely:

$$\phi = 30;0^\circ \text{ and } \varepsilon = 23;35^\circ\ .$$

Likewise al-Marrākushī does not refer to al-Maqsī (**3.5**), who must have been working in Cairo at the same time, and who compiled tables for sundial construction similar to, but more extensive than his own. al-Maqsī in the introduction to his tables does not mention al-Marrākushī either. On the other hand, al-Khalīlī (**10.8**) mentions that he favoured al-Marrākushī's method for determining the qibla, and his colleague Ibn al-Shāṭir (**7.3**) surely found his inspiration for his list of 184 standard spherical astronomical formulae in al-Marrākushī's list of 62 of the same (see below).

Whilst considerable uncertainty clouds our knowledge of al-Marrākushī's sources, his work in Cairo, and his later influence, we are at least fortunate in having the studies of the Sédillots on his major work. In the following brief survey of his spherical astronomy and related tables I have used the sudies of *père* and *fils*, hereafter labelled I and II respectively and referred to by page numbers, and also the 14th-century manuscript used by the Sédillots, namely MS Paris BNF ar. 2507-2508, as well as MS Istanbul Selim Ağa 866.[31] We should bear in mind that the work is a monumental encyclopaedia on the subject, to which the following extracts can hardly do justice. See also **6.7** on the extensive tables in the work.

Simple problems in spherical astronomy

al-Marrākushī uses the approximation Sin $\varepsilon \approx 24$ (I.180) and the value Tan $\varepsilon = 26;11,40$ (I.185). al-Maqsī, for comparison, used 24 and 26;11,41. al-Marrākushī gives the following approximation for finding $\delta(\lambda)$:

[29] On al-Marrākushī (**I-4.2.4**, *etc.*) see Suter, *MAA*, no. 363; *Cairo ENL Survey*, no. C17; my article in *EI*$_2$; and now Charette, *Mamluk Instrumentation*, pp. 9-13. Sédillot-*père*, *Traité*, contains a translation of the first half of al-Marrākushī's treatise on spherical astronomy and sundials. Sédillot-*fils*, *Mémoire*, contains a summary of the second half dealing with instruments.

[30] That is, as far as I know, it was not known in the either in the Maghrib or in Iran and points further east. In *London Khalili Catalogue*, pp. 191-192, no. 118, a manuscript copied by a Tabrīzī is presented supposedly from "Iran or India", and this thesis of mine countered. However the manuscript is in a typical Ottoman hand.

[31] The latter was published in facsimile, after a fashion, by the Institut für Geschichte der Arabisch-Islamischen Wissenschaften, Frankfurt, in 1984.

$$\text{Sin } \delta(\lambda) = \text{Sin } \varepsilon \text{ Sin } \lambda \ / \ R \approx 24 \text{ Sin } \lambda \ / \ 60 = {}^{2}/_{5} \text{ Sin } \lambda \ .$$

Later in his treatise he gives an example of the computation of the solar rising amplitude, choosing the summer solstice (I.303). He derives 27;31° and also suggests the approximation:

$$\text{Sin } \psi = R \text{ Sin } \delta \ / \ \text{Cos } \phi = 1;9,16 \text{ Sin } \delta \approx 1^{3}/_{20} \cdot \text{Sin } \delta$$

for $\phi = 30°$. In a similar example for the solar altitude in the prime vertical at the summer solstice (I.306), he computes Sin h_0 = 48, from which I derive 53;8° for h_0 rather than the more accurate value 53;9°. He also notes that for ϕ =30°:

$$\text{Sin } h_0 \ = \ R \text{ Sin } \delta \ / \ \text{Sin } \phi \ = \ 2 \text{ Sin } \delta \ .$$

Timekeeping

I have discussed elsewhere (**I-6.7.2**) al-Marrākushī's small auxiliary table for timekeeping operations (I.264). It is rather curious that he does not tabulate the functions:

$$B(\lambda) = \text{Cos } \delta(\lambda) \text{ Cos } \phi \ / \ R \quad \text{or} \quad B(\Delta) = \text{Cos } \Delta \text{ Cos } \phi \ / \ R \ ,$$

which he calls the *aṣl*, "base", and uses freely throughout his discussion of the determination of time from solar and stellar altitudes (I.258ff.). I have also discussed (**I-7.1.7**) al-Marrākushī's auxiliary tables for finding the half-excess of daylight (I.209). The method he expounds is repeated by al-Bakhāniqī (see **5.6**, note (k) to the translation).

al-Marrākushī advocates the parameters 20° and 16° for morning and evening twilight (I.295-296). He states that observations which he had made in various latitudes between 20° and 45° led him to the conclusion that these parameters, or "what differs from them by one degree", by which he surely means 19° and 17°, were the most suitable.

Standard formulae of spherical astronomy

al-Marrākushī presents a list of some 62 proportions, which represent the standard formulae of spherical astronomy (pp. 352-359). As an example I quote no. 55 in his list, which reads (in words):

1	2	3	4
Cos h	Cos δ	Sin t	Cos a

and indicates that:

$$\text{Cos } h \ / \ \text{Cos } \delta \ = \ \text{Sin } t \ / \ \text{Cos } a \ .$$

Less than a century later Ibn al-Shāṭir compiled a list of 184 such proportions in his *Zīj* (MS Oxford Seld. A30) and his colleague al-Khalīlī 24 proportions in his minor auxiliary tables in MS Dublin CB 4091 (**10.3b**).

My investigations thus far show that there is no material in the *Jāmiʿ al-mabādiʾ wa-'l-ghāyāt* which was taken directly from Ibn Yūnus' more sophisticated treatment of spherical astronomy in the *Ḥākimī Zīj*. al-Marrākushī makes no reference to Ibn Yūnus or any other Egyptian astronomers, and does not mention the tables for timekeeping which were available in Cairo in his time. Some of his individual tables are found in several later collections of tables, such as those in MSS Cairo MM 43, copied *ca*. 1450, and the manuscript in a private collection in Sanaa, copied *ca*. 1375, which were used in Cairo and the Yemen respectively (see **II-6.14** and **12.4**), but further research is necessary before we can ascertain the extent of his influence in later Islamic astronomy.

2.8 al-Abharī's treatise on instruments

MS Escorial ár. 965, copied in Syria in 719 H [= 1319/20], contains a treatise on instruments entitled *Lawāmiʿ al-wasāʾil fī maṭāliʿ al-rasāʾil* by Amīn al-Din ʿAbd al-Raḥmān ibn ʿUmar al-Abharī.[32] The author pays tribute in his introduction to the scholar-prince Abu 'l-Fidāʾ Ismāʿīl (*reg.* 1310-1332)[33] and (his father?) Nūr al-Dīn Abu 'l- Ḥasan ʿAlī ibn Maḥmūd, and from this and other internal evidence it is clear that the Escorial manuscript was copied not long after the treatise was finished. MS Gotha A1414 (date?), which I have not examined, is a later copy of the same treatise.[34]

al-Abharī discusses at great length (the Escorial manuscript contains 170 folios) the problem that can be solved using four instruments: (1) the universal plate known as *shakkāziyya*;[35] (2) the celestial globe (*al-kura dhāt al-kursī*); (3) a trigonometric grid (*al-dustūr wa-rubʿ al-mujayyab*); and (4) what appears to be the standard astrolabe (here called *al-āfāqiyya al-asṭurlābiyya*). He gives no information on any previous writings on these instruments, but does present a set of planetary parameters that are not attested in any known *zīj*. Of relevance to the present discussion are the parameters 18° and 16° advocated for morning and evening twilight (fols. 49v-50r and 171r-171v of the Escorial manuscript), although 18° is given for both phenomena elsewhere (fols. 83v-84r). Also of interest is the statement (fol. 62v) that the ʿ*aṣr* prayer begins when $z_a = Z + n$ and ends when $z_b = Z + 2n$, where the gnomon length n is either 12 or 7 (fol. 134v).

2.9 al-Bakhāniqī's treatise on timekeeping

The mid-14th-century Egyptian astronomer al-Bakhāniqī is known to us as the editor of a recension of the main Cairo corpus (**4.1.5** and **5.6**). He also wrote a treatise on spherical astronomy and the use of the quadrant, extant in MS Berlin Ahlwardt 5860 (Sprenger 1835), copied *ca*. 1400. This treatise contains some approximate formulae for determining certain of the standard functions of *mīqāt* for the latitude of Cairo. al-Bakhāniqī does not say how he derived these formulae and does not refer to any of his predecessors, such as Najm al-Dīn al-Miṣrī (**2.5**), who had presented similar formulae. The principal formulae proposed by al-Bakhāniqī, outlined in words in the text, are the following:

(a) $d \approx {}^1/_2\,(\delta + {}^1/_4\,\delta)$

(b) $h_a \approx {}^1/_2\,H + {}^1/_{10}\,[\max H - H]$

(c) $t_a \approx 52° + 2\,\delta/\varepsilon$ $(\delta > 0)$

$t_a \approx 52° - 10\,|\delta|/\varepsilon$ $(\delta < 0)$

(d) $\psi \approx \delta + {}^1/_6\,\delta$

---

[32] On Amīn al-Dīn al-Abharī (not to be confused with the well-known Athīr al-Dīn al-Abharī) see Suter, *MAA*, nos. 369 and 393 (confused).

[33] On Abu 'l-Fidāʾ see Suter, *MAA*, no. 392, and the articles in *DSB* by Juan Vernet and in EI_2 by H. A. R. Gibb.

[34] *Gotha Catalogue*, pp. 64-65.

[35] See the article "Shakkāziyya" in EI_2, and **X-5.2**.

(e) $h_0 \approx 2\delta$ $(\delta < 15°)$
$h_0 \approx 2\delta + {}^1/_4\,(2\,\delta - 30°)$ $(\delta > 15°)$
(f) $h_q \approx 47° + \delta + {}^1/_4\,\delta + {}^1/_6\,\delta$
(g) $s \approx 18;30° + {}^1/_6\,\delta$ $(\delta > 0)$
$s \approx 18;30° + {}^1/_2 \cdot {}^1/_6\,|\delta|$ $(\delta < 0)$
(h) $r \approx 23;30° + {}^1/_6\,\delta$ $(\delta > 0)$
$r \approx 23;30° + {}^1/_2 \cdot {}^1/_6\,|\delta|$ $(\delta < 0)$

Other rules are given for determining ascensions, the times of rising and setting of the moon, and the rising of the lunar mansions.

The formulae for twilight (*cf.* Ibn al-Ḥammāmī's formulae in **2.4**) are based on parameters 20° and 16° for morning and evening. However in the Berlin copy (fol. 25v) of his treatise on the quadrant al-Bakhāniqī prescribes the parameters 19° and 17°, which also underlie the twilight tables in his edition of the Cairo corpus **4.10** and **5.6**.

Another approximate formula propounded by al-Bakhāniqī is for finding the time of day in seasonal hours, T, from the length of the shadow, z, cast by a gnomon of length n. The formula appears to be garbled in the text (fols. 9v-10r of the Berlin copy), and may be represented algebraically thus:

$$T \approx \{\, 36 \,/\, [\, (z+n) - Z \,] \,\}^{\text{sdh}} ,$$

where the gnomon length n is not specified. Elsewhere in this same treatise al-Bakhāniqī uses base 12 for the Cotangent function, for which the corresponding formula would be:

$$T \approx \{\, 6 \cdot n \,/\, [\, (z+n) - Z \,] \,\}^{\text{sdh}} \qquad (n = 12) ,$$

It is difficult to explain the figure 36 in the numerator rather than 72. However, in al-Bakhāniqī's introduction to his edition of the Cairo corpus (**5.5**) in MS Cairo DM 108 he mentions Cotangents to base 6. The use of this base is uncommon in the Islamic sources,[36] but in light of this evidence it seems that al-Bakhāniqī's approximate formula for the time of day was intended to be:

$$T \approx \{\, 6 \cdot n \,/\, [\, (z+n) - z' \,] \,\}^{\text{sdh}} \qquad (n = 6) .$$

2.10 Sibṭ al-Māridīnī on the differences of opinion of the religious scholars and the *muwaqqit*s

Sibṭ al-Māridīnī was one of the leading astronomers in Cairo towards the end of the 15th century; he was a prolific author, mainly of treatises on spherical astronomy and instrumentation.[37] One of his works (see also **5.8** on another) exists in an apparently unique copy, MS Princeton Garret Hitti 531 (1960). This is a discussion of the validity of the times of prayer determined by the *muwaqqit*s, and it is disappointing for the paucity of information which it presents on our subject. Nevertheless I present a free translation of the entire treatise.

> "Questions and answers by the scholar Sibṭ al-Māridīnī concerning the determination of the prayer-times, and whether or not the opinion of the astronomers agrees with the opinion of the legal scholars, and other similar problems.

---

[36] See n. **I**-1:32.
[37] On Sibṭ al-Māridīnī see Suter, *MAA*, no. 445; *Cairo ENL Survey*, no. C97; and İhsanoğlu, *et al.*, *Ottoman Astronomical Literature*, II, p. 1014 (index).

In the Name of God, the Merciful and Compassionate. The lawyers, who are the *imām*s of religion and the scholars of the Muslims – may God grant them all success for their obedience – have written in their books, and commented thereon with their legal opinions, concerning those daily phenomena which can actually be seen, such as: sunset, when someone who is fasting can break the fast; the disappearance of the red twilight glow, when the time for the evening prayer begins; and the first visibility of the lunar crescent, when the fast of Ramaḍān begins.
They also dealt with similar problems like the determination of midday, on which the determination of all the prayer-times depends, noting that when the sun is in the east the shadow it casts will be in the west, and that as the sun's altitude increases the length of this shadow decreases, and stops decreasing when the sun culminates; and that as soon as the sun starts to sink from the meridian, the midday shadow starts to increase, and this phenomenon marks the beginning of the time of the midday prayer. If the sanctity of this prayer-time is neglected, and this increase has not taken place, the prayer is not correct. Now it has become apparent that most muezzins for a long time have not been taking heed of this. They say: "We take heed of what the professional time-keepers have done, and of what they have prescribed in their treatises on quadrants and astrolabes; this is what we go by." What they mean is that they consider the actual time of midday as the beginning of the prayer-time and do not wait to observe the increase in shadow length prescribed by the legal scholars. It is clear that the opinion of the muezzins is less correct than that of the legal scholars, and it is the latter opinion which should be used as a basis for the determination of the prayer-time."

2.11 A late Syrian treatise on fifteen times of day with religious significance

MS Aleppo Awqāf 970, copied *ca*. 1850, contains a short treatise of a few folios compiled in Aleppo and deals with 15 times of day with religious significance. This treatise is of considerable interest for late Ottoman practice (**IV-5.4** and **6.1**). I have come across no other treatise in which these 15 times are astronomically defined. The times, which include an indication of when the performance of the prayer is no longer approved (*karāha*), are as follows:

(1)	the *ẓuhr*;
(2)-(5)	the first and second *ʿaṣr* and the times of *karāha* for both *ʿaṣrs*;
(6)-(7)	sunset and the time of *karāha* for the *maghrib* prayer;
(8)-(9)	the first and second *ʿishāʾ*s;
(10)	the *imsāk*;
(11)-(12)	the false dawn and the true dawn;
(13)	sunrise;
(14)	the small *ḍaḥwa*; and
(15)	the large *ḍaḥwa*.

The author states that the time of *karāha* for the *ʿaṣr* is when the solar altitude is $4^1/_2$°. The time of the *karāha* for the *maghrib* is when the stars become visible (*waqt iḥtibāk al-nujūm*),

namely, when sun is 10° below the horizon (h = -10°). The first and second *ʿishāʾ* occur at the disappearance of the red and the white twilight glow, respectively, that is, when h = -17° and -19°. The false dawn occurs when h = -20°, altered in the text to -21°, and the true dawn when h = -19°. The *imsāk* is 20 minutes before the true dawn. The small *ḍaḥwa*, the time before which prayer is forbidden and "the time for the prayer at the *ʿīd* and the *ḍuḥā*", is when h = +4$^1/_2$°. The large *ḍaḥwa*, which is "the time after which the statement of intention to fast is not valid and after which the *ḍuḥā* prayer should not (be performed)", is halfway between the true dawn and sunset, when the sun is at the mid-point of the "legal arc of daylight" (*qaws al-nahār al-sharʿī*).

The treatise includes some remarks about the effect of refraction at the horizon and concludes with some tables for facilitating the determination of the hour-angle at these times: see further **11.15**.

CHAPTER 3

ʿIRĀQĪ AND IRANIAN TABLES FOR TIMEKEEPING

3.0 Introductory remarks

Islamic astronomical timekeeping and indeed Islamic mathematical astronomy in general started in 8th-century Baghdad. However, the number of Abbasid astronomical sources which survive is small indeed compared with the original output of the early Muslim astronomers. Ted Kennedy's survey of the Islamic astronomical handbooks known as *zīj*es indicates that at least two dozen *zīj*es, that is a substantial fraction of all significant known Islamic *zīj*es, were compiled in Abbasid al-ʿIrāq, Syria and Iran.[1] Very little material on timekeeping survives from these regions in Abbasid times:[2] the few tables I have found are discussed below. These, although of considerable historical interest, hardly constitute a corpus of tables that could have been used by muezzins. The Abbasid tables occur mainly in *zīj*es, in contrast to later tables for timekeeping from other parts of the Muslim world. We can, however, detect a tradition, which apparently started in Baghdad with the astronomer ʿAlī ibn Amājūr (**3.2**), of compiling tables of the time of day as a function of solar altitude (and solar meridian altitude) and of time of night as a function of stellar altitudes, and which influenced later astronomers at least in Iran and the Yemen (*cf.* **3.8**, **3.10-11** and **12.3**). The writings of al-Bīrūnī[3] illustrate the development of the mathematical methods of astronomical timekeeping, from early Abbasid sources when they relied on Indian techniques down to his own time when he and his predecessors had developed spherical trigonometry. Certain of the tables which have survived from al-ʿIrāq and Iran are based on the approximate methods adopted from Indian astronomy (**3.2-4**). There is every reason to suppose that further research on Arabic scientific manuscripts, particularly those preserved in Iran and Turkey, will bring to light other examples of tables for timekeeping from the early period of Islamic astronomy.

After the fall of Baghdad in 1258, no scientific works of any consequence are known to have been compiled in al-ʿIrāq. The few astronomical manuscripts preserved in modern Iraqi libraries are virtually untouched by modern scholarship, but the manuscript catalogues seem to confirm this lack of scientific endeavour.[4] I have found no complete sets of ʿIrāqī tables for timekeeping postdating the 13th century, although prayer-tables of the Ottoman kind were doubtless prepared for such important centres as Mosul, Baghdad and Basra.

---

[1] King & Samsó, "Islamic Astronomical Handbooks and Tables", pp. 31-44.

[2] The standard bio-bibliographical source for early Islamic astronomy is Sezgin, *GAS*, VI. Sezgin does not mention astronomical timekeeping at all.

[3] I think here especially of his works *Ifrād al-maqāl fī amr al-ẓilāl* and *al-Maqālīd fī ʿilm al-hayʾa*, listed in the bibliography as al-Bīrūnī, *Shadows*, and *Maqālīd*.

[4] Apart from Suter, *MAA*, and Brockelmann, *GAL*, the Arabic work Azzawi, *History of Astronomy in Iraq*, is the only bio-bibliographical source for this area and period. Unfortunately the author was unaware of any other modern scholarship in the history of Islamic astronomy, but occasionally he mentions treatises and manuscripts not listed in other modern sources.

On the other hand, creative interest in astronomy continued in Iran and Central Asia, both under the Timurids and later under the Safavids.[5] But the tables for timekeeping that we find are also few and far between. Nevertheless, there is every hope that such materials will be uncovered when the rich manuscript collections in Iran are exploited for their scientific contents.

3.1 al-Khwārizmī's tables for Baghdad

The anonymous tables in MS Berlin Ahlwardt 5793 (Landberg 56), fols. 93v-95v, copied *ca.* 1450, are appended to al-Farghānī's treatise on the construction of the astrolabe (fols. 1r-77r)[6] and two treatises on the construction and use of the astrolabe by al-Khwārizmī (fols. 77v-93r).[7] In view of the nature of these tables, I deem likely that they were compiled in the time of al-Farghānī and al-Khwārizmī, namely the early 9th century, and the most probable author is al-Khwārizmī himself.

Firstly, there is a table (fol. 93v) of normed right ascensions, $\alpha'(\lambda)$, with entries to three digits for $\lambda = 273°, 276°, \ldots, 360°$, accurately computed but for a few scribal errors for $\varepsilon = 23;33°$, the first *Mumtaḥan* value of this parameter.[8] The second table (fol. 94r – see **Fig. 3.1**) displays the shadows to base 12 at the beginning of the midday prayer, and at the beginning and end of the afternoon prayer, that is:

$$z_z, \quad z_a \quad \text{and} \quad z_b \; .$$

Entries are given to one digit for each 6° of λ' ($\delta \gtrless 0°$). The latitude is stated as 13°, which is a scribal error for 33°. The values in the table correspond *roughly but not precisely* to the standard definitions of later Islamic practice:

$$z_{z(12)} = Z_{(12)} \; , \quad z_{a(12)} = Z_{(12)} + 12 \quad \text{and} \quad z_{b(12)} = Z_{(12)} + 24 \; .$$

In particular, it seems that the values of z_a and z_b were computed using functions of H different from these. The table merits detailed investigation.

In al-Khwārizmī's treatise on the astrolabe the following definitions are given for the solar altitude at the *ẓuhr* and *ʿaṣr* (fol. 89v of the Berlin manuscript):

$$h_z \approx H - 7° \quad \text{and} \quad h_a \approx {}^1/_2\, H + {}^1/_{10}\, \bar{H} \; .$$

[5] See the survey articles Kennedy, "Seljuq and Mongol Science", and "Timurid Science", and Winter, "Safavid Science", and on Safavid astronomy see more recently King, *Mecca-Centred World-Maps*, pp. 128-138. The standard Western bio-bibliographical reference work on Persian manuscripts is Storey, *PL*, II:1, although much can now be added from Iranian reference works, such as those of Āghā Buzurg, *al-Dharīʿa*, and Monzawi, *Persian Manuscripts*.

[6] On al-Farghānī see the article in *DSB* by A. I. Sabra, and Sezgin, *GAS*, VI, pp. 149-151. On his astrolabe tables see King & Samsó, "Islamic Astronomical Handbooks and Tables", Section 4.11. His astrolabe treatise is currently being edited by Richard Lorch (Munich).

[7] On al-Khwārizmī (**I-7.1.1**) see the article in *DSB* by G. Toomer; Sezgin, *GAS*, VI, pp. 140-143; and on various Abbasid works attributed to him, but not all by him, King, "al-Khwārizmī". One of his treatises, on the use of the astrolabe, is translated in Frank, "al-Khwārizmī über das Astrolab". On the other, on the construction of the astrolabe, see King, "al-Khwārizmī", pp. 23-27. An edition of both treatises with English translation and commentary has been prepared by François Charette and Petra Schmidl.

[8] See **I-5.6.1** and **9.1**.

Fig. 3.1: al-Khwārizmī's prayer-tables for Baghdad. Note the sporadic use of Hindu-Arabic numerals in the table: this is most unusual in tables copied in the Mamluk period. It may indicate that al-Khwārizmī used such notation in his original table. From MS Berlin Ahlwardt 5793 fol. 94r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

It remains to explain these formula, the first of which is not attested in any other source known to me, and the second of which is similar to that used by later astronomers in both Egypt and Syria (see, for example, **2.9**).

al-Khwārizmī's definition of the *ẓuhr* is intended to correspond to a time one seasonal hour after midday as defined by shadow lengths using the Indian formula:

$$T \approx \{\ 6 \cdot n\ /\ (\Delta z + n)\ \}^{\text{sdh}}\ ,$$

discussed above (**1.4**): see further **IV-4.5**.

The second of al-Khwārizmī's formulae is more a careful approximate solution to the condition $\Delta z = n$. If we add $n = 12$ to the midday shadows for Baghdad noted above we obtain:

30;29	19;48	13;54

and the corresponding altitudes are:

$$21^1/_2° \qquad 31° \qquad 41° .$$

We notice that these altitudes are a little more than one-half the meridian altitudes. The rationale behind al-Khwārizmī's formula in which a fraction of $\bar{H}$ is added to $^1/_2$H to obtain h_a can be understood from the following:

	h_a	$^1/_2$ H	h_a - $^1/_2$ H	$\bar{H}$	$^1/_{10}$ $\bar{H}$
WS	$21^1/_2$°	$16^1/_2$°	5°	57°	6°
EQ	31	$28^1/_2$	$2^1/_2$	33	3
SS	41	$40^1/_2$	$^1/_2$	9	1

The shadow lengths given in the tables for the *ẓuhr* and *ʿaṣr* are not based on solar altitudes derived by these formulae of al-Khwārizmī.

The tables which follow the prayer-tables (fols. 94v-95v of the Berlin manuscript) display the solar altitude as a function of the seasonal hours and the meridian altitude, as well as a curious (and superfluous) auxiliary function for finding the sine of the altitude from the same arguments. See further **I-4.2.1** and **4.3.1** and especially **XI-9.3**.

In the margin by the side of the prayer-table is the following note, with no further explanation:

> "If you want to know the shadow at midday (*ẓill niṣf yawmika*) subtract the arc of half daylight (*qaws niṣf al-nahār*) from the rising time (of the arc of the ecliptic) from the beginning of Cancer to the end of Sagittarius (*maṭāliʿ mā bayn awwal al-sarṭān ilā ākhir al-qaws*) and divide the remainder by 15: the result will be the shadow at midday, if God – may He be exalted – wills."

Now the rising time of the arc between λ = 90° and λ = 270° is either:

$$180° \quad \text{or} \quad 2 \max D(\phi) ,$$

according as the ascensions are measured in *sphaera recta* or in *sphaera obliqua*. Thus this method may be represented by the formula:

$$Z = [\ 2 \max D(\phi) - D(\lambda)\ ] / 15 .$$

This method requires further investigation.

3.2 ʿAlī ibn Amājūr's *ṭaylasān* tables

The most impressive known achievement of any Abbasid astronomer in astronomical timekeeping is the *ṭaylasān* table of Abu 'l-Qāsim ʿAlī ibn Amājūr, who worked in Baghdad in the mid 10th century.[9] This displays the function T(H,h) for each degree of both arguments such that 21° ≤ H ≤ 84° and 1° ≤ h ≤ H. Values are given in equatorial degrees to two digits and are based on the parameter ϕ = 33;25° (Baghdad). They are extremely carefully computed, being either accurate or in error by ±1 in the second digit, rarely more. Since for the sun 33;0° = $\bar{\phi} - \varepsilon \leq H \leq \bar{\phi} + \varepsilon$ = 80;10°, ʿAlī ibn Amājūr clearly intended his table to be used for timekeeping by the stars as well. See further **I-2.3.1**. Ibn Amājūr's table is contained in MS Paris BNF ar.

[9] On Ibn Amājūr (n. **I**-2:29) see Sezgin, *GAS*, VI, pp. 177-178, where the table in MS Paris BNF ar. 2486 is listed.

2486, fols. 239v-255r, of the *Zīj* of al-Baghdādī (copied by the author in 1285), where it is expressly attributed to the earlier scholar and called *al-zīj al-ṭaylasān* (fol. 239r), the word *zīj* here apparently meaning a large table rather than an astronomical handbook.

In MS Paris BNF supp. pers. 1488, copied *ca*. 1500, of the early-14th-century Persian *Ashrafī Zīj*,[10] there is a similar table computed for an unspecified locality, actually Shiraz (**3.10**), preceded by yet another *ṭaylasān* table (fols. 201v-204v), of which it is stated: "this table was compiled by Abu 'l-Qāsim al-Mājūr in his *zīj* called *al-Zīj al-Ṭaylasān*". The title reads *istikhraj al-sāʿāt al-zamāniyya min irtifāʿ al-waqt wa-nisf al-nahār*, "for finding the temporal hours from the instantaneous and meridian altitudes". The function tabulated is T(H,h) in seasonal hours and minutes for degree of both arguments such that $1° \leq h \leq H \leq 90°$. The table is in fact intended to display the time since rising for any latitude and the tabulated function is based on the approximate formula:

$$T(H,h) \approx \text{arc Sin} \{ R \text{ Sin } h \,/\, \text{Sin } H \}^{sdh} .$$

See further **I-2.5.1** on the table and **II-1.4** on the formula, also **XI**.

Thus Ibn Amājūr's work contained at least two extensive tables, one for Baghdad and the other for all latitudes. I suspect that several of the other spherical astronomical tables in the *Zīj* of al-Baghdādī (**3.2**) are also due to Ibn Amājūr. We are most fortunate to have these *ṭaylasān* tables of his, since the other five *zīj*es compiled by him and his sons have vanished almost without trace. The little evidence we have about the Amājūr family ranks them close to the top of the list of Muslim astronomers.

3.3 The prayer-tables for Baghdad in the *Zīj* of al-Baghdādī

The unique MS Paris BNF ar. 2486, copied 684 H [= 1285], of the *Zīj* of al-Baghdādī (**3.2**) contains numerous spherical astronomical tables, but I here restrict comments to those found on fols. 117r-122v. Several functions are tabulated to *four* digits for the parameters $\phi = 33;25°$ (Baghdad) and $\varepsilon = 23;35°$, namely:

$$d,\ 2D^h,\ \tilde{h} \text{ and } H .$$

These are followed by a set of prayer-tables – see **Fig. 3.3** – for the same parameters, with values given to one or two digits for each day of the Syrian year. The corresponding position of the sun (*mawḍiʿ al-shams*) in the zodiacal signs (*al-aḥwāl*) is given (Tishrīn I 1 corresponds to Libra 16°) and the functions tabulated are as follows:

(1) Cot_7 H is given to two digits.

(2) A function is labelled *ṭulūʿ al-fajr*, "daybreak", is actually the number of seasonal night-hours and minutes from sunset to daybreak (see below). To find the duration of evening twilight one should subtract the values from 12 (see below).

(3) Values of functions H, h_a and h_b are given to one digit.

al-Baghdādī's twilight table is based on the parameter 17°. The function tabulated is:

$$r(\lambda) = 12^{snh} - s(\lambda)$$

[10] On the *Ashrafī Zīj* see n. **I**-2:32.

Fig. 3.3: An extract from the prayer-tables in the *Zīj* of al-Baghdādī. This double page serves the month of Tishrīn I (October). [From MS Paris BNF ar. 2486, fols. 120v-121r, courtesy of the Bibliothèque Nationale de France.]

where s(λ) is defined by the approximate formula:

$$s(\lambda) \approx {}^{1}/_{15} \text{ arc Sin } \{ R \cdot \text{Sin } 17° / \text{Sin } H(\lambda^*) \}^{\text{snh}} .$$

This same formula, though with parameter 18° is prescribed in MS Berlin Ahlwardt 5750, fol. 153v, of a recension of the *Zīj* of Ḥabash (**I-9.1**). Likewise, the twilight tables in the *Dustūr al-munajjimīn* (**3.7**) are based on this formula but with parameter 16°. The formula is derived from approximate formula for T(h,H) in seasonal day-hours that underlies, for example, the universal table of T(h,H) prepared by ʿAlī ibn Amājūr (**3.2**). The last of al-Baghdādī's tables displays the functions:

$$h_a(H) \text{ and } h_b(H)$$

to two digits for each degree of H up to 90°.

3.4 Anonymous timekeeping tables for Baghdad (?)

MS Paris BNF ar. 2514 contains an anonymous set of tables for timekeeping, copied or maybe compiled in 612 H [= 1215]. The work consists of two parts, for timekeeping by day and night, respectively. The first set of tables display the time since rising in seasonal hours as a function of meridian altitude and instantaneous altitude. The underlying formula is approximate and the entries can be used for all latitudes; however, for meridian altitudes such that $\bar{\phi} - \varepsilon \leq H \leq \bar{\phi} + \varepsilon$, the time since rising for the sun is given in equinoctial hours for the latitude $\phi \approx 33;30°$, which could serve Baghdad or Damascus. For each value of H the value of h_a is also given. On this table see further **I-2.5.2**.

The remainder of the tables (fols. 28v-48v) display the longitude of the horoscopus and its oblique ascensions as a function of the altitudes of various stars. See further **I-3.2.2**. On some similar tables for latitude 30;25° (Qandahar) see also **I-3.2.1**.

3.5 al-Qāyinī's twilight tables for Qayin

A pair of tables for the duration of morning and evening twilight was computed by an individual named ʿAlī ibn ʿAbdallāh al-Qāyinī who lived in the city of Qayin in Eastern Iran, probably *ca*. 1000.[11] These tables are preserved in the unique MS Bankipore 2468,23 (fols. 114v-115r), copied in 632 H [= 1234/35], from which the text was published in Hyderabad in 1947; they have been studied by Marie-Louise Davidian and Ted Kennedy.[12]

al-Qāyinī states in his introduction that he has tabulated the functions $r(\lambda_H)$ and $s(\lambda_H + 180°)$, where $\lambda_H(\lambda)$ is the longitude of the horoscopus at daybreak in Qayin when the solar longitude is λ. He also states that the latitude of Qayin is 33;55°, a value not known from other sources,[13] and that he is using the parameter 17° for both morning and evening twilight. In the introduction it is noted that:

$$r(\lambda_H) = s(\lambda_H + 180°) = s(\lambda_H^*) ,$$

[11] On al-Qāʾinī see Sezgin, *GAS*, V, p. 337, and VI, p. 242.
[12] Kennedy & Davidian, "Al-Qāyinī on Twilight".
[13] Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 258.

and this relation also holds for the tabulated functions, values of which are given in degrees and minutes for each 5° of argument. However, the tabulated functions do not correspond to the accurate values of these functions for these parameters. Kennedy and Davidian demonstrated that the entries corresponded more closely to the lengths of twilight as a function of solar longitude, but then we are confronted with the fact that if λ is considered constant for a given day then $r(\lambda) = s(\lambda)$.

To compute the function $r(\lambda_H)$ using an accurate formula we first determine the arc of the ecliptic $\Delta\lambda$ which rises during morning twilight. The simplest formula is suggested by al-Qāyinī, namely:

$$\Delta\lambda = \text{arc Sin } \{ r \cdot \text{Sin } 17°/ \text{ Cos } \nu \}$$

where $\nu(\lambda_H)$ is the function known as *ʿarḍ iqlīm al-ruʾya*, "the latitude of visible climate", which measures the angle between the ecliptic and the horizon.[14] With this $r(\lambda_H)$ is given by:

$$r(\lambda_H) = \alpha_\phi(\lambda_H + \Delta\lambda) - \alpha_\phi(\lambda_H) .$$

No tables of $\nu(\lambda)$ or $\alpha_\phi(\lambda)$ specifically for latitude 33;55° are attested in the known manuscript sources.

Like Kennedy and Davidian I am unable to explain al-Qāyinī's table. I have considered the possibility that he used a table for the "latitude of visible climate" which was computed for the latitudes of one of the climates rather than for Qayin, and also that the quantity $\Delta\lambda$ was computed by an approximate formula. But neither of these possibilities seems to account for the peculiarities of al-Qāyinī's functions.

3.6 al-Bazdawī's qibla table for Transoxania

Abu 'l-Yusr al-Bazdawī was a Ḥanafī legal scholar active in Samarqand in the 11th century.[15] He wrote a treatise on the qibla in early Islamic Transoxania which is extant in a unique manuscript preserved at one time in Sohag, from which MS Cairo B 19385 was copied in 1355 H [= 1936].[16] al-Bazdawī promises a table displaying the solar altitude in the azimuth of the qibla h_q which will serve Samarqand, Bukhara and Nasw. The table is not contained in the Cairo manuscript, but al-Bazdawī mentions three values from it which are the following:

EQ: 40° SS: 66° W: 11° .

For $\phi = 40°$, which al-Bazdawī gives for Samarqand, these correspond roughly to $q = 45°$, which is a happy compromise between the directions of due west, used for the qibla by the Ḥanafīs in Samarqand, and due south, used there by the Shāfiʿīs.[17]

[14] On this concept see Kennedy, "*Zīj* Survey", p. 145b. See also nn. 3:29 and 7:15.
[15] On al-Bazdawī see Brockelmann, *GAL*, I, p. 460, and SI, pp. 637-638, and Sezgin, *GAS*, I, pp. 412-428.
[16] Text, translation, and commentary are contained in King, "al-Bazdawī on the Qibla in Tranoxania". That study is based on the Cairo manuscript because the original in Sohag had disappeared when I tried to locate it in the 1970s.
[17] *Op. cit*., and King, *Mecca-Centred World-Maps*, p. 125.

3.7 Anonymous twilight tables for the fourth climate

The anonymous *zīj* preserved in MS Paris BNF ar. 5968, copied *ca*. 1250, and known as the *Dustūr al-munajjimīn* is based mainly on the *zījes* of al-Battānī, Kushyār, Abū Jaʿfar al-Khāzin and al-Bīrūnī.[18] There are two tables for morning and evening twilight in this *zīj* (fols. 187v-188), which are based on the approximate formulae noted in **3.3**. The tables are anonymous, and someone has written a non-commital *li-baʿḍihim*, "by one of the astronomers", by each of the titles – see **Fig. 3.7**. The underlying parameter is found by inspection to be 16°, although the parameters mentioned in the accompanying text are firstly 18° and then 17° (fols. 187r). Corresponding entries in the two tables add up to twelve seasonal night-hours: the table for evening twilight measures the duration of twilight and the other measures the time from sunset

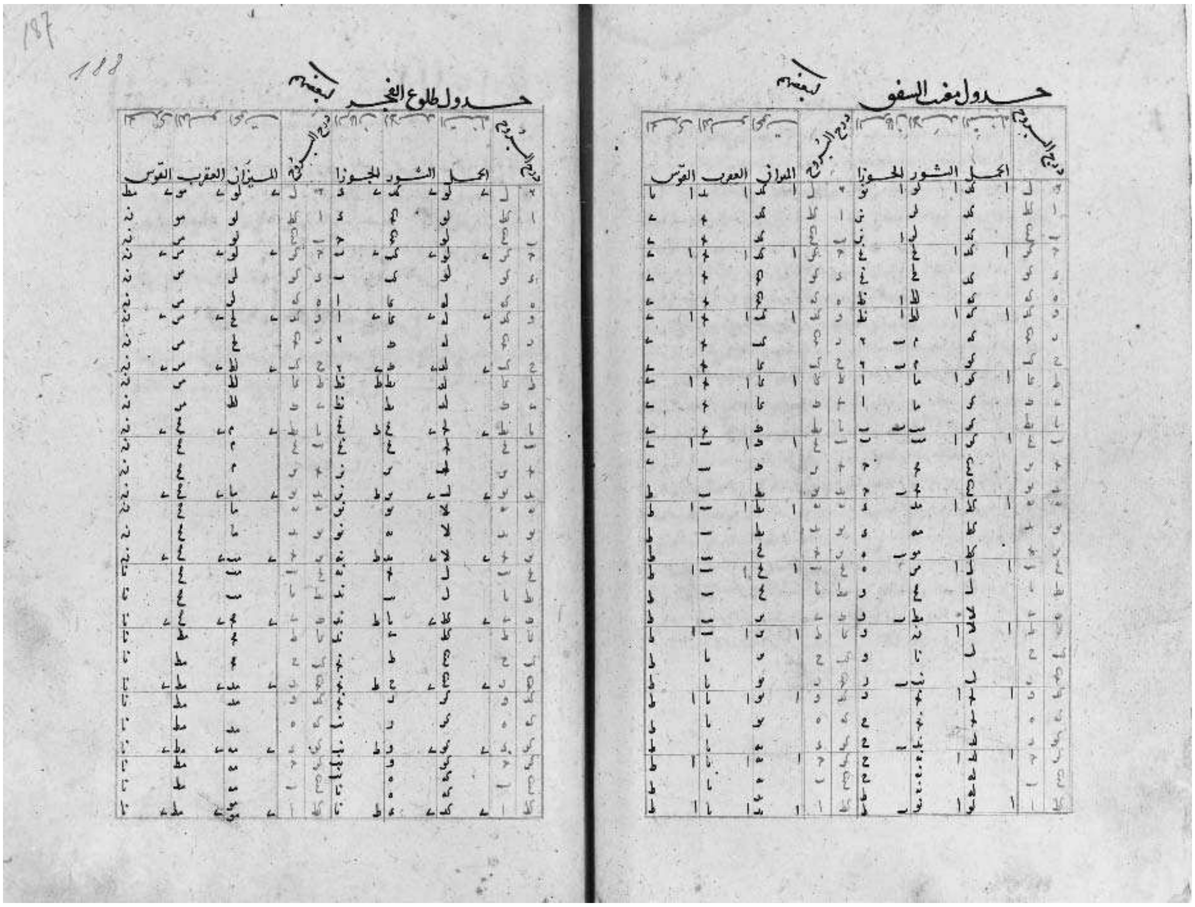


Fig. 3.7: The tables for the duration of twilight in the the *Dustūr al-munajjimīn*. The format is the same as that of the tables in the Cairo corpus, and it may be that there is Egyptian influence behind these Iranian tables. [From MS Paris BNF ar. 5968. fols. 187v-188r, courtesy of the Bibliothèque Nationale de France.]

[18] This *zīj* is not listed in Kennedy, "*Zīj* Survey", but the introduction has been studied in Zimmermann, "Dustūr al-munajjimīn" (cited in n. **I**-7:14). See also Sezgin, *GAS*, VI, pp. 63-64.

until daybreak. The underlying latitude is as stated, namely, $\phi = 36;21°$. This is the value used in several Islamic sources for the Ismāʿīlī stronghold of Alamut (and also Nishapur), although it was probably derived by calculation as the latitude of the 4th climate rather than derived from any observation.[19] Elsewhere in the *Dustūr* (fols. 161r-163r) there are tables of the functions:

$$H, \quad d, \quad D, \quad 2D^h \quad \text{and} \quad \tilde{h} \,,$$

with values to two digits for parameters $\phi = 36;21°$ and $\varepsilon = 23;35°$. See further **XI-5.2**.

3.8 The *Īlkhānī ṭaylasān* table for Maragha

The *Īlkhānī Zīj* of Naṣīr al-Dīn al-Ṭūsī and his collaborators at the Observatory in Maragha *ca*. 1260[20] contains a *ṭaylasān* table displaying the times since sunrise as a function of the solar meridian altitude and instantaneous altitude for $\phi = 37;20°$ (Maragha). The table is not contained in all of the numerous available copies of the *Zīj* since its use is restricted to one latitude, but it is contained, for example, in MS Florence Medici 269, fols. 150r-152v, and Oxford Hunt. 143, fols. 155v-160v, of the *Zīj*. See further **I-2.3.2**.

3.9 al-Maghribī's qibla table for Maragha

Muḥyi 'l-Dīn al-Maghribī was one of the astronomers who assisted Naṣīr al-Dīn al-Ṭūsī at the Observatory in Maragha (**I-4.3.1*** and **5.6.3**).[21] His own *Zīj* purports to be more representative of the work done at Maragha than al-Ṭūsī's *Īlkhānī Zīj*, but neither *Zīj* has been fully studied yet: a brief survey has been made of the *Īlkhānī Zīj*, but the *Zīj* of Muḥyi 'l-Din al-Maghribī has hardly been studied at all. Whilst the *Īlkhānī Zīj* exists in several copies, only three copies are known of al-Maghribī's *Zīj*, namely, MSS Dublin CB 3665, Meshed Shrine Library 332(103), and Medina Aref Hikmet *mīqāt* 1.

Certain features of al-Maghribī's *Zīj* indicate his debt to Ibn Yūnus (**5.1**). First, his Sine tables are based on values lifted from the *Ḥākimī Zīj*. (Likewise, his Tangent table is based on values lifted from al-Bīrūnī's *Qānūn*.) Second, other than Ibn Yūnus he is the only known compiler of a *zīj* who tabulates both the solar rising amplitude and the solar altitude in the azimuth of the qibla (MSS Dublin CB 3665, fols. 96v-97r and Meshed Shrine Library 332(103), fols. 99r-99v).

al-Maghribī tabulated $\psi(\lambda)$ to three digits for each degree of λ, and the latitude of Maragha, taken as 37;20,30° (**I-5.6.3**). Ibn Yūnus had also tabulated the function to three digits for the latitude of Cairo (**5.1**). al-Maghribī's table of $h_q(\lambda)$ gives values to two digits for each degree of λ, as did Ibn Yūnus' table for Cairo (see also **5.1**). At the head of al-Maghribī's table of $h_q(\lambda)$ in both sources it is stated that the underlying parameter is $q = 16;59°$. In al-Maghribī's

---

[19] Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. 15 and 245-246; also King, "Geography of Astrolabes", pp. 6-9.

[20] On al-Ṭūsī and the *Īlkhānī Zīj* see n. **I**-2:31. The existence of the *Īlkhānī ṭaylasān* table was noted in Kennedy, "*Zīj* Survey", p. 161.

[21] On Muḥyi 'l-Dīn al-Maghribī see n. 5:8.

geographical tables (MSS Dublin CB 3665, fols. 57v-59r, and Meshed Shrine Library 332(103), fols. 61r-61v) we find the coordinates:

Mecca	ϕ: 21;40°	L: 77;0°
Maragha	37;20,(30)°	82;0°

Accurately computed for these coordinates q is 16;49°. However, in MS Istanbul Ayasofya 2984, fols. 85-86v, of the early-14th-century Persian *Zīj* of al-Wābiknawī,[22] the calculation is presented on which the value 16;59° is based. The value of q which underlies the table of $h_q(\lambda)$, however, appears to be 16;49° rather than 16;59° and the value used for ε is 23;30°. al-Maghribī has the three values: 51;27°, 75;38°, 27;15° for h_q at the equinoxes and summer and winter solstices. Recomputation with the parameter 16;49° reproduces precisely these values. Recomputation with the parameter 16;59° yields 51;25°, 75;37°, 27;13°. al-Maghribī's table of h_q differs in format from Ibn Yūnus' table in the *Ḥākimī Zīj*. The latter is arranged in 12 columns of 30 entries for each degree of λ, so that no advantage is taken of the symmetry of the function. In the main Cairo corpus $h_q(\lambda)$ is displayed in six columns of 30 entries and no value is given for the equinoxes. In al-Maghribī's table the function is displayed in six columns of 30 entries beginning with Aries, but the vertical argument runs from 0 to 29 instead of Ibn Yūnus' 1 to 30. At the bottom of the third column (for Gemini) and the sixth column (for Sagittarius) an extra entry is added for argument 30. Thus values are given for the equinoxes and both solstices. al-Maghribī's entries are rather accurately computed.

The Medina manuscript, but not the other two copies of the *Zīj,* contains a table displaying the solar altitude at each seasonal hour for each degree of solar meridian altitude. This table is based on an approximate formula – see further **I-4.3.1***.

3.10 The *ṭaylasān* table for Shiraz in the *Ashrafī Zīj*

MS Paris BNF supp. pers. 1488, copied *ca*. 1550, is an unique copy of the *Ashrafī Zīj,* compiled in 702 H [= 1302/03] by the Iranian astronomer Sanjar al-Kamālī also known as Sayf al-Munajjim (**3.2**). It contains two *ṭaylasān* tables, the first being the universal table of Ibn Amājūr (**3.2**) and the second being computed for ϕ = 29;30° (Shiraz). Various other tables in the *Ashrafī Zīj* relate to spherical astronomy and have been discussed in **I-2.3.3**, *etc*. – see **Fig. I-6.2.1** for some of these. However, there are no tables for regulating the times of prayer.

3.11 An anonymous *ṭaylasān* table for Northern Iran

MS Leiden Or. 199, fols. 21v-27v, contains an anonymous *ṭaylasān* table of the function t(H,h) for latitude 36°. This table, which contains many errors difficult to attribute to copyists, has been studied by Bernard R. Goldstein.[23] See further **I-2.3.4** and, on the structure, **XI-4.2** (illustrated).

[22] Kennedy, "*Zīj* Survey", no. 35 (not in Storey, *PL*, II:1); and King, *Mecca-Centred World-Maps*, n. 84 on p. 158.

[23] See Goldstein, "Medieval Table for Reckoning Time", the first modern study of a medieval Islamic table for timekeeping.

3.12 Anonymous timekeeping tables for Baghdad

MS Cambridge Add. 3527 (date?) contains a set of anonymous timekeeping tables for Baghdad, bound in some confusion. The underlying latitude is ϕ = 33;25° (Baghdad). The functions tabulated for each degree of argument λ are α_ϕ (fols. 120v-121r), $2D^h$ (fol. 121v), and α' (fols. 158v-159). In view of the format of the tables, which have the main vertical argument running from 0 to 29° rather than the more sensible 1° to 30°, I estimate that they are rather late, perhaps from the 16th century. There is also a table for converting hours and minutes to equatorial degrees for each minute from 1^m to 15^h59^m (fols. 122v, 157r-158r) and another displaying the astrological houses for latitude 33;25°.[24] Two other tables in the set, each copied in different hands, are spurious, namely:

(1) a table displaying the astrological houses for latitude 32° (fols. 166v-172r); and
(2) a table displaying the time of daybreak for Baghdad according to the Ottoman convention for each day of the Persian year (fol. 165v) – see the next section.

3.13 An anonymous table for twilight at Baghdad

The anomalous table in MS Cambridge Add. 3527 (**3.12**), fol. 165v, displays the duration of twilight (*sāʿāt ṣubḥ wa-shafaq*) for the latitude of Baghdad, stated to be 33;25°. Values are given in equinoctial hours and minutes for each day of the Persian year starting with the month of Farwadīn and ending with the five intercalary days. The maximum value of $1;41^h$ corresponds nicely to a solar depression of 17° and the minimum of $1;17^h$ equally well to 16°, so the table invites further investigation.

3.14 Muḥammad Zamān al-Mashhadī's prayer-tables for Meshed

MS Cairo ṬFF 14, copied *ca*. 1700, is the only copy known to me of an astronomical work entitled *Tuhfa-yi Sulaymani* compiled in 1078 H [= 1667/68] by Muḥammad Zamān ibn Sharaf al-Dīn Ḥusayn al-Mashhadī, well-known as a maker of astronomical instruments.[25] The treatise contains some tables of various spherical astronomical functions (fols. 103r-105v) such as:

$$\alpha(\lambda),\ \alpha'(\lambda),\ \text{and}\ \alpha_\phi(\lambda)$$

based on the parameter ϕ = 37;0° (Meshed), a standard value.[26] There is also a table displaying the time of culmination of the star *al-kaff al-khaḍīb* (= β Cas) as a function of solar longitude (fol. 42v) – see further **I-2.8.2**. Two prayer-tables concern the present study. They are entitled *jadwal-i irtifāʿ-i aftāb dar samt-i qibla-yi Mashad-i muqaddas bi-taqwim-i shams birikrand* (fol. 102v) – see **Fig. 3.14** – and *jadwal-i sāʿāt mā bayn ṭulūʿ-i ṣubḥ wa shams wa ghurūb-i shams wa maghīb-i shafaq* (fol. 111r) and display the functions:

$$h_q(\lambda) \quad \text{and} \quad r^h(\lambda)\ \text{or}\ s^h(\lambda)\ .$$

[24] On the houses see n. **I**-3:2.
[25] On Muḥammad Zamān see n. **I**-2:45.
[26] See Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 221, and King, *Mecca-Centred World-Maps*, pp. 172-175.

Fig. 3.14: Muḥammad Zamān's table of the solar altitude in the azimuth of the qibla at Meshed, followed by the first and second tables in a set for each of the zodiacal signs for the equalisation of the astrological houses. [From MS Cairo ṬFF 14, fols. 102v-103r, courtesy of the Egyptian National Library.]

The format is the same as that of the twilight tables in the Cairo corpus (**4.10**). The underlying latitude here is stated on other tables to be 37°. The qibla used by Muḥammad Zamān for Meshed in the gazetteers engraved on one of his astrolabes is 45;23° (accurately for his coordinates 45;9°); on another he has 55;28°(!), and on another no value at all (!).[27]

3.15 Anonymous prayer-tables for Isfahan

MS Istanbul Topkapı B 411, an encyclopedia compiled *ca*. 1413 for Isakandar Sulṭān ibn ʿUmar Shaykh of Isfahan, contains an extensive section on astronomy with numerous tables and illustrated constellation figures.[28] Amidst the tables is a set of prayer-tables for Isfahan. Values of the following functions are displayed to two digits for each degree of λ' ($\delta \gtreqless 0$):

d, ψ, d, D, $\tilde{h}$, $2D^h$, H, $Z_{(12)}$, $Z_{(7)}$, h_a, $z_{a(12)}$, $z_{a(7)}$, $t_a^{\ h}$, t_a, $12/2D$, h_b, $z_{b(12)}$, $(2N\text{-}r)^h$ and r^h .

The underlying parameters are:

$$\phi = 32;25° \text{ and } \varepsilon = 23;30° .$$

Although no locality is specified, elswhere in this compilation there are tables of the function called *ʿarḍ-i iqlīm-i ruʾya*, "latitude of visible climate"[29] for:

$$\phi = 29;30° \text{ (Shiraz) and } 32;25° \text{ (Isfahan).}$$

The times defined by h_a and h_b are called *awwal al-ʿaṣr* and *ajzāʾ-yi ikhtiyārāt*, the latter suggesting an astrological association but simply an unfortunate error for *ākhir al-ikhtiyār*, "the end of the most favourable time". The names of all the functions tabulated are given in Arabic although the remainder of the astronomical section is in Persian.

---

[27] *Ibid*., p. 504.
[28] See *ibid*., pp. 143-145, on the world-map in this manuscript.
[29] See n. 3:14.

3.16 Prayer-tables for Isfahan on instruments by ʿAbd al-Aʾimma

The celebrated Iranian astrolabist ʿAbd al-Aʾimma, who worked in Isfahan about 1700 and is best known for his elegant astrolabes,[30] engraved a set of shadow tables on a sundial and qibla-indicator which he made and which is now preserved in the Time Museum, Rockford, Illinois. Another unsigned instrument of the same kind is in the Museum of the History of Science, Oxford: see **Fig. 3.16**.[31] The instrument is fitted with a pin-gnomon at the centre a set of

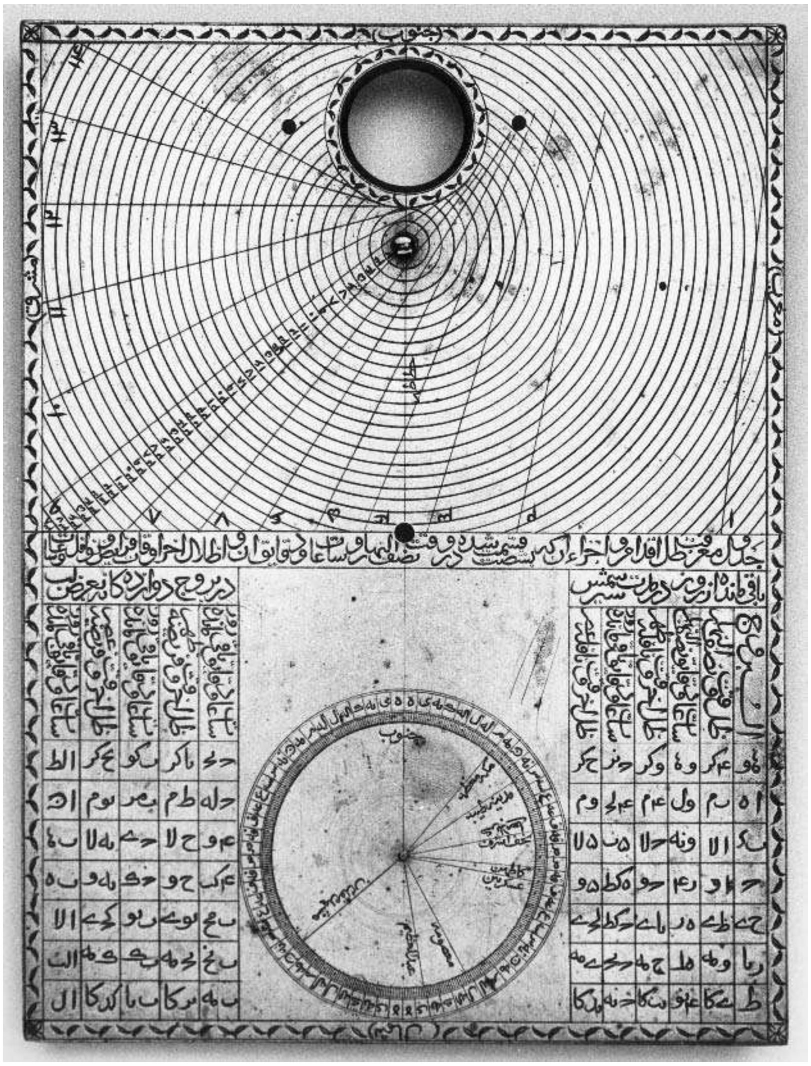


Fig. 3.16: The prayer-tables on the compass *cum* qibla-indicator attributable to ʿAbd al-Aʾimma. [Photo courtesy of Museum of the History of Science, Oxford.]

---

[30] On ʿAbd al-Aʾimma see Mayer, *Islamic Astrolabists*, pp. 23-26, and supplement, p. 293; Gingerich & King & Saliba, "The ʿAbd al-Aʾimma Astrolabe Forgeries"; and also King, *Mecca-Centred World-Maps*, pp. 175-177.

[31] On these two instruments see *ibid*., pp. 118-121. On a fake see *Khalili Collection Catalogue*, II, p. 274 (no. 162).

quadrants of a circle radii 1, 2, ... , 40 where 12 is the length of the gnomon. These curves can be used to regulate the *ẓuhr* and *ʿaṣr* using the tables, which display values of the functions:

$$z_z\ (= Z), \quad z_a \quad \text{and} \quad T_a \text{ (in hours)}$$

to two sexagimals for each zodiacal sign. The underlying latitude is $\phi = 32°$ (Isfahan).[32] Given the paucity of Safavid manuscript sources on timekeeping these tables are of particular historical interest. It is unlikely that ʿAbd al-Aʾimma computed them himself, for, as far as we know, neither he nor any of his colleagues – astronomers and instrument-makers – from 17th-century Isfahan were not into compiling tables.[33]

3.17 Anonymous Ottoman-type prayer-tables for Yarqand

I recall seeing a set of such tables in a manuscript at the Oriental Institute in Tashkent. My notes thereon have disappeared.

---

[32] On the qibla values for various cities also engraved on these instruments see *ibid*., pp. 118-121, 177, 518-519, and 545.

[33] *Ibid*., pp. 128-138.

CHAPTER 4

THE MAIN CAIRO CORPUS OF TABLES FOR TIMEKEEPING

4.0 Introductory remarks

The Cairo corpus exists in a number of a manuscript sources, usually containing tables from three main categories:

(1) tables of solar azimuth as a function of solar altitude and longitude;
(2) tables of time as a function of solar altitude and longitude;
(3) tables of standard spherical astronomical functions of solar longitude and various other functions for regulating the prayer-times.

All of these tables are computed for the latitude of Cairo taken as 30;0° and obliquity 23;35°. No two manuscripts have been found yet containing precisely the same tables.

In the first category the function tabulated is $a(h,\lambda)$, and the tables contain over 10,000 entries. In the second the functions tabulated are either $t(h,\lambda)$ or $T(h,\lambda)$ or both. These are closely related and the computation of one given the other is trivial, since their sum is $D(\lambda)$. Each set contains over 10,000 entries. The third category consists of a total of about 30 tables, each containing 90 or 180 entries. Most of these can be derived very easily from basic tables of $\delta(\lambda)$ (as is the case with, say, H, whence z_a and h_a can be found), or $d(\lambda)$ (as is the case with, say, D, 2N, $\tilde{h}$, $1/\tilde{h}$, $\tilde{h}$), or directly from the tables of $T(h,\lambda)$ (as in the case with r and s). Likewise, given h_a or h_q, it is not difficult to find the corresponding hour-angles and time since sunrise, t_a and T_a, and t_q and T_q, using the tables of $t(h,\lambda)$ and $T(h,\lambda)$ or $D(\lambda)$.

Most of the sources which I used in my first study of the Cairo corpus[1] mention the 10th-century astronomer Ibn Yūnus as the compiler, but several sources identified thereafter include instructions by the the late-13th-century astronomer al-Maqsī and others contain some notes by the mid-14th-century astronomer al-Bakhāniqī. In numerous late manuscripts in which only the tables of standard spherical astronomical functions and the prayer-tables are found, these tables are attributed to the 16th-century astronomers al-Lādhiqī and al-Ikhṣāṣī (**7.8**).

The location in the manuscripts of the various categories of main tables, as well as the authors to whom they are attributed, is shown below (see already **I-2.1.1**). (The tables in MS Cairo TR 354 falsely attributed to Ibn Yūnus are computed for Alexandria – see **8.5**.) The notation T/t means that T and t are tabulated on facing pages, and the notation T/t/a means that T, t and a are tabulated together, with triplets of values for each pair of arguments. Otherwise the functions listed are tabulated separately. An asterisk indicates that tables are preceded by an introduction on their use prepared by al-Maqsī (**5.4**), and a double asterisk indicates that this introduction is accompanied by some notes by al-Bakhāniqī describing the way in which he had rearranged the tables (see further **5.6**).

[1] See King, "Astronomical Timekeeping in Medieval Cairo", cited in n. **I**-2:2. That study was based mainly on MSS Dublin CB 3673 and Cairo DM 108, also Berlin Ahlwardt 5753, Cairo Azhar *falak* 4382, Cairo TR 191, and Escorial ár. 924,7.

Berlin Ahlwardt 5753	T/t and a	Ibn Yūnus
Cairo TR 191	t	Ibn Yūnus
Dublin CB 3673	T/t and a	Ibn Yūnus*
Escorial ár. 924,7	a	Ibn Yūnus
Cairo DM 108	T/t/a	Ibn Yūnus**
Cairo Azhar *falak* 4382	t and a	Ibn Yūnus
Cairo MM 137	a	Ibn Yūnus
Dublin CB 4078	T/t	anon.
Gotha A 1410	a	Ibn Yūnus
Cairo DM 53	T/t/a	al-Maqsī**
Cairo DM 45	T/t/a	anon.
Gotha A1402	T	al-Maqsī
Istanbul Kılıç Ali Paşa 684	t	Ibn al-Kattānī
Leipzig 817	T/t	anon.
Istanbul Nuruosmaniye 2903	T/t/a	anon.*
Istanbul Nuruosmaniye 2925	T/t/a	al-Maqsī**
Cairo TR 354	T/t/a (ϕ=31°)	Ibn Yūnus
Cairo MM 64	a	anon.
Cairo DM 444	T	al-Maqsī*
Cairo DM 690	T/t/a	anon.
Cairo DM 777	T/t	anon.
Cairo DM 616	T/t/a	al-Maqsī**
Cairo DM 739	T/t/a	al-Maqsī**
Cairo DM 776,1	T	anon.
Cairo DM 778	t	anon.
Cairo DM 786	T/t/a	anon.
Cairo DM 1101	a	anon.
Cairo DM 1108,9	a	anon. (**I-5.2.1**)
Cairo DM 1109	T/t/a	Ibn Yūnus
Cairo DM 1224	T/t/a	Ibn Yūnus
Cairo K 4044	t and a	al-Maqsī (t)/ Ibn Yūnus (a)

Having already analyzed the main Cairo corpus from a mathematical point of view in my previous study, I propose to consider each of the tables afresh (**4.3** to **4.9**), paying particular attention to the question of authorship. I begin with a few brief remarks (in **4.1**) about each of the individuals known to have contributed to the corpus. I then (in **4.2**) present brief descriptions of several new manuscripts of the corpus to supplement those in my original analysis.

My main purpose in **4.3-9** is to describe the tables in two manuscripts of the corpus, MS Dublin CB 3673, copied in the 14th century, and MS Cairo DM 108, copied at the beginning of the 19th century, in the light of the several dozen related manuscripts that I have located.

In **4.3** I discuss the tables of standard functions for spherical astronomy. In **4.4** I discuss the tables of a(h,λ) and show that they were indeed compiled by Ibn Yūnus. Certain manuscripts

of the azimuth tables contain some nonsensical values added by an incompetent anonymous and in others these have been corrected by the early-14th-century astronomer Ibn al-Rashīdī. In **4.5** I discuss the tables of t(h,λ) and T(h,λ) and present the evidence which makes it clear that these tables in their present form were not compiled by Ibn Yūnus. Three centuries after his time, al-Maqsī claimed to have prepared the tables of T(h,λ), but I suspect that he had at his disposal some tables for timekeeping by Ibn Yūnus which have not survived in their original form. In the 14th century Ibn al-Kattānī prepared a set of tables of t(h,λ) using al-Maqsī's tables of T(h,λ). In **4.6-11** I attempt to cast some light on the highly complicated problem of the authorship of the minor tables in the corpus. Most of the mathematically-significant tables were either computed by Ibn Yūnus or owe their inspiration to him. The development of the main Cairo corpus from the 10th to the 14th century is then discussed in **Ch. 5**.

4.1 The main contributors

4.1.1 Ibn Yūnus

Abu 'l-Ḥasan ʿAlī ibn 'Abd al-Raḥmān known as Ibn Yūnus worked in Fusṭāṭ in the last quarter of the 10th century. Much of the activity described in this book owes its original inspiration to him. According to his biographers, Ibn Yūnus had a comic appearance: in the only illustration of him, in which he appears with his patron al-Ḥākim, he looks reasonably normal – see **Fig. I-4.1.1**. He is already well known to the history of Islamic science as the author of an extensive *zīj* called the *Ḥākimī Zīj*, which is one of the finest works of its genre.[2] His extensive and highly sophisticated treatment of spherical astronomy in the *Ḥākimi Zīj* includes numerous tables (**5.1**). Indeed, virtually all of the theory underlying the mathematically-significant tables in the main Cairo corpus and also some of the individual tables are contained in the *Ḥākimi Zīj*. Ibn Yūnus compiled more than one *zīj*: for example, the *Mukhtār Zīj* by the Yemeni astronomer Abu 'l-ʿUqūl (**12.1**) is based on a *zīj* of Ibn Yūnus other than the *Ḥākimī*. Ibn Yūnus also appears to have compiled extensive tables of the Sine and Cotangent functions for each minute of argument,[3] as well as a large set of double-argument tables for the lunar equation.[4] In **5.3** I assess the evidence that there is a "missing link" between his known works and the main tables of the corpus in the form of a lost work of his dealing with practical aspects of timekeeping.

4.1.2 al-Maqsī

Shihāb al-Dīn Aḥmad ibn ʿUmar al-Maqsī was one of the leading astronomers in Cairo about 1275.[5] In my previous study of the Cairo corpus, on the basis of my examination of the set of tables in MS Cairo DM 53, introduced in al-Maqsī's name, I suggested that he had merely plagiarized some of Ibn Yūnus' tables. I have since been able to consult other such sets, for example, MSS Gotha A1402, Istanbul Nuruosmaniye 2925 and Cairo DM 616, and al-Maqsī's

---

[2] On Ibn Yūnus see n. **I**-2:3.
[3] King, *Ibn Yūnus*, pp. 85-89 and 109. See also *ibid*., pp. 96-99, on tables of the solar declination for each minute of solar longitude, with differences for each second, attributed to him.
[4] King, "Lunar Equation Table".
[5] On al-Maqsī see n. **I**-2:4.

Fig. 4.1.1: A miniature showing a rather angry-looking Caliph al-Ḥākim apparently giving Ibn Yūnus a hard time. The astronomer is holding his *zīj*, marked with Hindu-Arabic numerals on the spine. Actually the *Ḥākimī Zīj* would have filled several volumes of this size. [From MS Istanbul Topkapı Revan Köşku 1638 (400 folios) of *al-Qānūn fī 'l-dunyā*, an astrological work of the prolific Ibn Zunbul (İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 183-184, no. 84); photo courtesy of the Topkapı Library.]

contribution is reconsidered in **5.4** below. I have also studied the only other known work attributed to al-Maqsī, a treatise on sundial theory extant in MSS Cairo Azhar *falak* 5528, Dublin CB 4090 and Cairo DM 103. This consists mainly of tables, which are to be used for marking shadow traces and curves for the seasonal hours on sundials erected in various planes.[6] There are over 100 tables each containing about 30 entries and this illustrates that al-Maqsī was not afraid of extensive calculation. These tables are related to the more accurate but less extensive ones of his contemporary Abū ʿAlī al-Marrākushī (**6.7**) although neither individual refers to the other. The treatment of the standard problems of spherical astronomy in al-Maqsī's introduction to his sundial tables is not impressive, and it is of interest that in this introduction al-Maqsī used some material from the *Ḥākimī Zīj* without mentioning his illustrious predecessor.

[6] On al-Maqsī's sundial tables see King, "Islamic Astronomical Tables", pp. 51-53, and *idem*, "Astronomy of the Mamluks", pp. 547-548, and also **I-4.1.3** and **X-7.1**.

4.1.3 Ibn al-Rashīdī

Shams al-Dīn Muḥammad ibn Ibrāhīm known as Ibn al-Rashīdī appears to have worked in Cairo about 1300,[7] after al-Maqsī and before al-Bakhāniqī. He was the compiler of a set of prayer-tables for Mecca (**6.11**) and is known to us also by the references of al-Karakī (**9.4**), who was probably his late contemporary, and of al-Bakhāniqī (**5.6**). al-Karakī states that Ibn al-Rashīdī compiled some hour-angle tables, but does not specify the underlying latitude. I am inclined to think that al-Karaki's own tables for Jerusalem were in fact based on a set of tables for latitude 32° computed by Ibn al-Rashīdī. al-Bakhāniqī mentions that Ibn al-Rashīdī had recomputed some of the values in Ibn Yūnus' azimuth tables where they had been inaccurately copied. On Ibn al-Rashīdī's contribution to the Cairo corpus see **4.8** and **5.2**.

4.1.4 Ibn al-Kattānī

Muḥammad ibn Muḥammad ibn Muḥammad ibn ʿAbd al-Qawī al-Qurashī known as Ibn al-Kattānī was an astronomer and instrument-maker and also a fine penman.[8] MSS Cairo MM 72, Istanbul Kılıç Ali Paşa 684, and Dublin CB 3673,1 are copied in his elegant hand and are dated 747 H [= 1346/47], 768 H [= 1361] and 772 H [= 1371], respectively (see **Figs. 4.2b**, **4.7a** and **I-4.1.1**). The first of these manuscripts contains a set of tables of solar altitude for Cairo: the title folio is missing and no compiler is mentioned, although I suspect that these tables are due to Najm al-Dīn al-Miṣrī (**6.5**). The second manuscript contains a set of tables for timekeeping for Cairo which Ibn al-Kattānī says he computed himself. These include a complete set of tables of $t(h,\lambda)$ and various additional tables for regulating the times of prayer. Ibn al-Kattānī's name is not mentioned in any other known source in connection with the authorship of part of the Cairo corpus. The third manuscript is a copy of Ibn Yūnus' tables of $a(h,\lambda)$ (**5.2**) which contains additional garbled entries for solar altitude up to 88°. Ibn al-Kattānī's integrity is put in question by the fact that he claimed to have computed $t(h,\lambda)$ but was incapable of copying tables of $a(h,\lambda)$ properly. Other inconsistencies in his tables in MS Istanbul Kılıç Ali Paşa 684 confirm that Ibn al-Kattānī was a better copyist than astronomer. I discuss his tables in **5.5**.

4.1.5 al-Bakhāniqī

Shams al-Dīn Aḥmad ibn Muḥammad al-Bakhāniqī worked in Cairo and also in the Yemen in the 14th century.[9] He was at some time associated with the Azhar Mosque since he is also called al-Azharī. MS Istanbul Topkapı A3343 of the *summa* of al-Marrākushī (**2.7** and **6.7**) bears his mark of ownership dated 776 H [= 1374/75]. However, he must have been active for rather a long time because one of his works was compiled for a Yemeni *wazīr* and can be dated *ca*. 1325. This particular compilation, extant only in MS Dublin CB 4092, was an extension of the tables of the early 9th-century Baghdad astronomer al-Farghānī for constructing the various markings on astrolabe plates.[10] al-Farghānī's table enabled the user to construct

---

[7] On Ibn al-Rashīdī (**I-2.1.5**), whose full name was Shams al-Dīn Abū ʿAbdallāh Muḥammad ibn Burhān al-Dīn Ibrāhīm al-Rashīdī, see n. **I**-2:18.

[8] On Ibn al-Kattānī see n. **I**-2:6.

[9] On al-Bakhāniqī see n. **I**-2:5.

[10] On al-Farghānī and his astrolabe tables see the article in *DSB* by A. I. Sabra and King, "Islamic

the altitude and azimuth circles on plates for each degree of latitude from 15° to 50° and are easily derived from a single very useful little table of an auxiliary trigonometric function; al-Bakhāniqī simply completed the set for each degree of latitude from 0° to 90°. Another work by al-Bakhāniqī is a treatise on the prayer-times in which he outlines a set of approximations for the standard functions of timekeeping: I have summarized this in **2.9**.

al-Bakhāniqī was responsible for rearranging the tables in the Cairo corpus so that triplets of values (T,t,a) are displayed side by side as functions of (h,λ). I assess his other contributions to the corpus in **5.6**.

4.2 The manuscript sources

The manuscripts in which the main tables of the Cairo corpus are to be found date from the 14th to the 19th century.[11] No two copies contain the same tables. No single manuscript contains reliable information on the identity of the compiler of the individual tables. In my previous analysis I relied on MSS Berlin Ahlwardt 5753, Cairo TR 191, Dublin CB 3673, Escorial ár. 924,7, Cairo DM 108, Cairo Azhar *falak* 4382 and Cairo MM 137; all of these happened to be preserved in collections which had been catalogued and all happened to be attributed to Ibn Yūnus, which explains how I located them first and why I had no problem attributing all of the tables to him. I also noted the existence of MS Cairo DM 53, in which part of the corpus is attributed to al-Maqsī, of al-Bakhāniqī's notes in MS Cairo DM 108, and of various copies of al-Lādhiqī's prayer-tables, but I assumed that each of these authors had merely plagiarized tables of Ibn Yūnus. The new sources listed below help explain more precisely the development of the corpus between the 10th and the 14th centuries and although my doubts about the integrity of al-Maqsī, al-Bakhāniqī and al-Lādhiqī still appears to be correct (**5.4, 5.6** and **7.8**), one should bear in mind that in the world of the Cairo astronomers in the 13th and 14th centuries it must have been difficult to simply "plagiarize" tables.[12] The discovery of other early copies of the corpus might well cast new light on its development and on the identity of the compilers of some of the tables, where this is still in doubt.

MS Berlin Ahlwardt 5753 (Landberg 574):[13]
This is a disordered fragment of the corpus copied *ca*. 1400. The title on fol. 1r reads *Kitāb fīhi 'l-Samt li-'bn Yūnus al-ḥāsib*, "Azimuth Tables by Ibn Yūnus, the astronomer" (see **Fig. 4.2a**). There are several notices of possession on the title folio, one dated 840 H [= 1436]. The tables on fols. 1v-20r are in considerable confusion. Each is headed only by the degree of solar altitude. For altitudes 1° through 19° (fols. 1v-10v) the function tabulated is a(h,λ);

Astronomical Tables", pp. 53-55; *idem* & Samsó, "Islamic Astronomical Handbooks and Tables", pp. 91-92; and **X-4.2**.

[11] On the Cairo manuscripts see *Cairo ENL Catalogue* (in Arabic) I, in which they are listed by accession numbers, and II, especially under 3.1.2 (azimuth tables of Ibn Yūnus), 3.1.2 (tables of al-Maqsī), and 3.1.10 (edition of al-Bakhāniqī); as well as *Cairo ENL Survey* (in English), no. B59/3.1.1 (Ibn Yūnus), C15/3.1.2 (al-Maqsī), and C28 (al-Bakhāniqī). For Istanbul manuscripts some catalogues have been prepared after I worked on the manuscripts, but I have not seen them. For manuscripts from other catalogued collections references to the catalogues are given *ad loc*.

[12] See also nn. 4:31 and 10:2.

[13] *Berlin Catalogue*, pp. 206-207.

Fig. 4.2a: The title folio of the Berlin manuscript, in which the azimuth tables are attributed to Ibn Yūnus. The tables that follow are, however, a disordered mixture of tables of a (which correspond to the title), then t and T as functions of h and λ. See also **Figs. II-5a-b**. This was the only manuscript of the corpus known to Carl Schoy when he worked on Ibn Yūnus in the early years of the 20th century. [From MS Berlin Ahlwardt 5753, fol. 1r, courtesy of the Deutsche Staats-bibliothek, Berlin.]

Fig. 4.2b: The title-folio of the Dublin manuscript. The decorated title, illegible in this photograph, announces only the azimuth tables of Ibn Yūnus, and the double tables of the time since sunrise and time before midday which follow are in fact in a different hand. Compare **Figs. I-2.1.1a** and **I-5.1.1a**. This did not stop me in the early 1970s from blithely assuming that the whole lot were by Ibn Yūnus. I had ordered a microfilm of the manuscript because Bernard Goldstein had told me it had been catalogued as a "new" copy of the *Ḥākimī Zīj*. [From MS Dublin CB 3673, fol. 121r, courtesy of the Chester Beatty Library.]

for 8° and 9° (fols. 11r-11v) the function is $t(h,\lambda)$; and for 52° to 83° (fols. 12v-20r) we are dealing with $T(h,\lambda)$! On fol. 20v there is a table of $t_a(\lambda)$, which was published by Carl Schoy,[14] and on fol. 21r, a table of $1/\tilde{h}(\lambda)$. An additional folio at the end of the manuscript contains an illegible title and some instructions on a calculation in spherical astronomy.

MS Cairo TR 191:
This is a copy of part of the corpus written in 1215 H [= 1800/01] by ʿAbd al-Bāriʾ al-ʿAshmāwī (also the copyist of MS Cairo DM 108 – see below). The title reads *Jadāwil faḍl al-dāʾir min*

[14] Schoy, *Gnomonik der Araber*, p. 53.

qibal al-irtifāʿ ḥisāb ... Ibn Yūnus, and most of the manuscript consists of a complete set of tables of $t(h,\lambda)$. These are followed by tables of the functions:

$$H,\ s,\ n,\ T_q,\ z_{a(12)},\ r,\ 2N,\ Z_{(12)},\ t_q,\ \sigma,\ \alpha_\phi,\ \alpha',\ \alpha_s \text{ and } \alpha_r\ .$$

The twilight tables are based on parameters 19° and 17°, the qibla tables on q = 53° and the *salām* is assumed to be 1° before daybreak. The manuscript concludes with an undated catalogue of the equatorial coordinates of 60 stars, copied by al-ʿAshmāwī in 1217 H [= 1802/03].

MS Dublin CB 3673:[15]

This manuscript (annoyingly foliated 121r-1v as for a Latin manuscript) is a virtually complete copy of the Cairo corpus, incorrectly catalogued as the *Ḥākimī Zīj*. The title on fol. 121r reads: *Kitāb al-Samt li-bn Yūnus*, but this refers only to the first part of the tables: see **Fig. 4.2b**. This folio also contains some notices of possession, a floriated *ṭughrā* signature, and the date of copying, 772 H [= 1371]. The tables are copied in the distinctive elegant hand of Muhammad ibn Muhammad ibn Muḥammad ibn ʿAbd al-Qawī al-Qurashī known as Ibn al-Kattānī – see the notes to MS Cairo MM 72 below. This part of the manuscript contains a complete set of tables of $a(h,\lambda)$, with additional entries for altitudes 84° to 88° (fols. 121v-96r) (see **Fig. 5.1.1a**), and these are followed by tables of

$$h_q\ (q = 52°),\quad h_0,\quad d \quad\text{and}\quad h_v\ .$$

(The entries in the table of h_v decrease to 0;41° at λ = 270°. If Ibn al-Rashīdī compiled this table, why is it presented here together with the uncorrected azimuth tables that al-Bakhāniqī states Ibn al-Rashīdī had corrected?)

The remaining part of this manuscript contains tables of $T(h,\lambda)$ and $t(h,\lambda)$ (fols. 82v-15r) and various other functions for timekeeping and the prayers, also copied in an elegant hand, but not that of Ibn al-Kattānī: see **Figs. 2.1.1a** and **II-4.7a**. This part of the manuscript has no separate title page but begins with the instructions on the use of the tables (fols. 83r-83v), in which the name of al-Maqsī has been altered to that of Ibn Yūnus. The minor functions tabulated (fols. 15v-3r) are:

$$t_a,\ d,\ H,\ h_q\ (q{=}53°),\ T_a,\ \tilde{h},\ \psi,\ h_0,\ D,\ t_a,\ \delta,\ r\ (h_r{=}20°),\ s\ (h_s{=}16°),$$
$$h_{a=60°},\ h_{a=30°},\ t_q\ (q{=}53°),\ \text{arc Sin}\ (x),\ r\ (h_r{=}19°),\ s\ (h_s{=}17°),\ 2N,\ n,\ 2N,\ r\ (h_r{=}20°),$$
$$d,\ H,\ \delta,\ \psi,\ h_0,\ 1/\tilde{h}$$

Fol. 82r has been filled with odd items such as notices of possession, verses of poetry, and a list of Graeco-Coptic numerals, given with their Arabic *abjad* equivalents.[16] The manuscript ends with some astrological writings in a different hand (fols. 2r-2v).

MS Escorial ár. 924,7:[17]

This is an incomplete copy of Ibn Yūnus' azimuth tables dating from the 14th (?) century. The title on fol. 30r reads: *Kitāb al-Samt li-bn Yūnus wa-Kitāb al-Ẓill maḥlūl daqīqa daqīqa*, "Azimuth Tables of Ibn Yūnus, and Cotangent Tables computed for each Minute of Argument". Below this is the curious note: *fī ʿilm al-rumūz li-fiqh al-majmaʿ*, "on the science of secret signs for the under-standing of the community (?)". Fols. 30v-63v contain azimuth tables for solar

[15] *Dublin CB Arabic Catalogue*, p. 73. The date of copying is misread as 672 H.
[16] On these see, for example, Ritter, "Griechisch-koptische Ziffern". See also n. 4:21.
[17] *Escorial Catalogue*, pp. 32-33.

altitudes from 1° to 78°. The remainder of the azimuth tables, and the Cotangent tables, are missing from the manuscript.

MS Cairo DM 108:
This is a fairly complete copy of the main Cairo corpus as edited by al-Bakhāniqī. It was copied by ʿAbd al-Bāriʾ (ibn) Naṣr al-ʿAshmawi (fol. 74v) in 1218 H [= 1803] (see also MS Cairo TR 191 above) (see **Fig. II-4.11a**). The title folio gives the title as *Kitāb Ghāyat al-intifāʿ fī maʿrifat al-dāʾir wa-faḍlihi wa-'l-samt min qibal al-irtifāʿ* and identifies the author as Ibn Yūnus. The manuscript contains al-Maqsī's introduction followed by al-Bakhāniqī's notes (see **Fig. II-5.6c**), a complete set of tables of the functions (T,t,a) (see **Fig. II-5.6b**), followed by various prayer-tables (see below), as well as some calendrical tables tables not found in the other main copies of the corpus. The latter part of al-Maqsī's introductions, al-Bakhāniqī's notes, and the first five folios of tables are in the older and more elegant hand of the copyist of MS Cairo DM 777 (see below). The parameters for twilight in this fragment are 20° and 16°.

Following the main set of tables are some calendrical tables for tables for finding the date in the Hijra calendar for the years 1521-1680 in the Coptic calendar [= 1219-1383 H], and tables of the following functions:

$$H, D, t_a, h_a, T_a, h_q, t_q, h_{q*}, 2N, n, s, r, \sigma, \Delta D, \tilde{h}, 2D^h, \tilde{h}, \tau \text{ and } t_{q*} .$$

The twilight tables are based on parameters 19° and 17°, the qibla tables on q = 53° and q* = 37°, and the *salām* is taken to be 1° before daybreak. See **Fig. II-4.11a**.

MS Cairo Azhar falak 4382:
This is a carefully-copied manuscript of tables of t(h,λ) (fols. 1v-35r) and a(h,λ) (fols. 36v-70v), and also of a(h) for certain altitudes at the equinoxes and solstices (fol. 71r) and ψ(λ) together with $h_0(\lambda)$ (fol. 71v). All of the tables are in the same 14th-century hand. The hour-angle tables are preceded by a page on which the title *Kitāb Faḍl al-dāʾir* is mentioned but no author is identified. The azimuth tables, which contain entries for arguments h = 1° to 83°, are preceded by a page on which the title *Kitāb al-Samt* is mentioned and Ibn Yūnus named as author.

MS Cairo MM 137:
This manuscript, which was copied about 1500, contains a complete set of Ibn Yūnus' azimuth tables bearing the title *Kitāb al-Samt*. The altitude argument runs beyond 83° to 86° (**4.4**). Information about the solar altitude for which the azimuth changes direction is given in the margins of the sub-tables.

MS Dublin CB 4078 (new):
This is a complete set of carefully-copied tables of T(h,λ) and t(h,λ) for Cairo dating from *ca*. 1450. The title appears to read *Kitāb al-Dāʾir wa-faḍluhu*, and there is no mention of the compiler or date of copying.

MS Gotha A 1410 (new):[18]
This consists mainly of a complete set of Ibn Yūnus' azimuth tables with entries for altitudes

[18] *Gotha Catalogue*, III, p. 62.

up to 89°. The copyist was ʿAlī ibn Muḥammad ibn ʿAlī al-Dalāmī[19] and the date of completion 841 H [= 1437/38]. An undated notice of possession names the owner as Jalāl al-Dīn Muḥammad al-Ramlī, *muwaqqit* at the the Umayyad Mosque (in Damascus). There are additional tables of h_q ($q = 53°$) and h_v (with entry 0;41° for $\lambda = 270°$). The remaining tables were originally from a different manuscript and consist of an incomplete anonymous set of double-argument lunar equation tables, attributed elsewhere to Ibn Yūnus.[20]

MS Cairo MM 184 (new):
This contains tables (fols. 7v-10r, copied *ca*. 1700) of the following functions:

$$h_q,\ \delta*,\ h_v,\ B \text{ and } C\ ,$$

as well as a table of the difference between sunrise and sunset at Mecca and Cairo, such as is contained in MS Cairo MM 204. The table of h_q is based on $q = 53°$ and the entry for h_v for $\lambda = 270°$ is 0;41°.

MS Cairo DM 190,2 (new):
The tables (fols. 25r-33r) listed below were copied *ca*. 1550 and begin with a page of notes on the values of ΔD for each sign (max. 62, min. 32), the (Coptic) numbers for the *warrāqīn*,[21] and a statement that the sun moves 18 miles on its sphere each minute. The tables display the following functions:

$$D,\ t_a,\ h_a,\ T_a,\ s,\ n,\ r,\ \delta,\ h_0,\ \alpha_s,\ \alpha_\sigma,\ \alpha_\phi \text{ and } \alpha'\ .$$

MS Cairo DM 954,2:
This manuscript, copied *ca*. 1475, contains an incomplete set of tables from the corpus (fols. 2r-10v) with no accompanying title-page. The functions tabulated are:

$$t_q,\ h_{q*},\ t_{q*},\ \tau,\ \sigma,\ h_v,\ Z_{(12)},\ h_{a=60°},\ \tilde{h},\ z_{a(12)},\ h_{a=30°},$$
$$\alpha',\ (D+s),\ \delta,\ d,\ H,\ 2N,\ \tilde{h} \text{ and } h_q\ .$$

The tables relating to the qibla are based on $q = 53°$ and the table of h_v has 0;41° at $\lambda = 270°$.

MS Cairo DM 53 (new):
This contains a complete set of tables of $T(h,\lambda)$, $t(h,\lambda)$ and $a(h,\lambda)$ with a triplet of entries for each pair of arguments. The colophon indicates that the copyist was the Azhar *muwaqqit* ʿAbd al-Fattāḥ ibn Muḥammad al-Dulajī and that the manuscript was completed in the Nile Valley town of *Jirjā al-ʿarīḍa* (Girga) in 1117 H [= 1705]. The first folio bears the title *Kitāb al-Dāʾir wa-faḍluhu wa-'l-samt* and the tables are preceded by al-Maqsī's introduction and al-Bakhāniqī's notes. In the introduction the parameters advocated for twilight are 19° and 17°. At the end of the manuscript are some absurd instructions on the use of the tables for latitudes other than 30°.

MS Cairo DM 45:
This contains a collection of spherical astronomical tables in different hands and in considerable disorder. The various parts of the manuscript date from about 1700. The first function tabulated, as indicated on the title folio, is the oblique ascensions of the ascendant at the beginning of

---

19 On al-Dalāmī and other manuscripts copied by him see *Cairo ENL Catalogue*, I, p. 704.
20 See King, "Lunar Equation Table" (n. 4:4), p. 132.
21 See already n. 4:16.

the afternoon prayer “according to the new observation”, *i.e.* based on obliquity 23;30° (**7.1**). The tables which follow are for finding the entry of the sun into each zodiacal sign and are due to Riḍwān Efendī, as stated (**7.10**). The next tables display the functions (T,t,a) for ϕ = 30°, a triplet of entries being given for each pair of arguments (h,λ) but only for altitude arguments 21° to 57°. These are followed by tables of H for ϕ = 30° (ε = 23;35°), as well as for eight different values of ϕ between 32° and 45° (ε = 23;30°). Of particular interest is an odd folio of tables (fol. 27) of an unspecified function f(h,λ) tabulated for arguments h = 24° and 25° and each degree of solar longitude: the function is in fact t(h,λ) for ϕ = 32° (**6.7**). The manuscript also contains an incomplete sexagesimal multiplication table.

MS Cairo MM 204:
This manuscript, copied in 1052 H [= 1642/43], contains an unusual set of tables displaying the following functions (fols. 80v-81r and 86r):

$$\mathrm{Tan}_5\ \Delta,\ \mathrm{Tan}_5\ \delta(\lambda),\ G(\theta),\ h_a(H)\ \text{and}\ \mathrm{Cot}_{12}\ h$$

for each degree of argument. The table of G is labelled “table for finding the hour-angle with ease”: see further **I-6.9.3** (illustrated). Another table in this set displays the difference between sunrise or sunset at Mecca and Cairo for each degree of λ, such as is also found in MS Cairo MM 184 (see above).

MS Cairo MM 58,2:
This was copied 1450 by Ibn Abi ’l-Fatḥ al-Ṣūfī and contains tables (fols. 2v-12v) of the following functions:

$$\alpha_\phi\ (\phi{=}31°,\ \text{Alexandria}),\ \alpha_\sigma,\ \alpha,\ \alpha_s,\ n,\ h_a,$$
$$\lambda(\delta)\ (\Delta\delta = 0;15°),\ h_v,\ h_a(H),\ \text{and}\ 2D^h\ (\Delta\lambda = 0;6°)\ ,$$

as well as some astrological tables. The table of h_v has 0;41° at λ = 270°.

MS Gotha A1402 (new):[21a]
This manuscript (date ?) bears the title *Risāla fī ’l-falak li-’l-Ṣūfī,* “Treatise on astronomy by al-Ṣūfī (that is, al-Maqsī)”, and contains a complete set of tables of T(h,λ) preceded by an introduction introduced in the name of al-Maqsī (written al-Maqdisī, as if he or his forbears were from Jerusalem, a curious mistake for a Cairene copyist, who must have known the suburb of al-Maqs) and followed by an incomplete and disordered set of various other tables for timekeeping. These additional tables are numbered and are bound in the manuscript now as follows:

$$1{:}\ t_a;\quad 2{:}\ D;\quad 3{:}\ T_q;\quad 4{:}\ 1/\tilde{h};\quad 5{:}\ h_q;\quad 23{:}\ \mathrm{Tan}\ \delta;$$
$$24{:}\ \mathrm{Tan}_{60}\ \theta;\quad 9/10{:}\ d\ \text{and}\ \delta;\quad 11{:}\ h_q\ (=\text{no. }5);\quad 21{:}\ \mathrm{Vers}\ \theta;\quad \text{and}\quad 22{:}\ \mathrm{Sin}\ \theta\ .$$

It is clear that there were originally some two dozen additional tables. The first four appear to be original to al-Maqsī’s set (**5.4**). All of the qibla tables are based on q = 53°. The table of $\delta(\lambda)$ with values to three digits for each degree of λ is based on that of Ibn Yūnus in the *Ḥākimī Zīj.* The table of Tan $\delta(\lambda)$, also giving values to three digits, is not identical to that of al-Marrākushī, but the entry 26;11,41 for λ = 90° is used by al-Maqsī in his treatise on sundials.[22] The three trigonometric tables give values to three digits for each degree of arc. In the introduction the parameters advocated for twilight are 20° and 16°.

[21a] *Gotha Catalogue*, III, pp. 56-57.
[22] See n. 4:6.

MS Gotha A1411 (new):[22a]
This consists of an anonymous almanac (fols. 1r-6v) followed by four pages of tables from the corpus in a different hand (fols. 7r-8v). As well as a table of h_q (q = 53°) and α_ϕ for Cairo (the values are those of al-Marrākushī), these extra pages contain a table of values of the azimuth of the qibla for each degree of latitude and longitude difference from Mecca. This table, which is based on an approximate formula and is attributed to Yūsuf ibn al-Damīrī, is of a kind found in other sources that I have discussed elswhere (**4.7** below). However, no such qibla tables are attested in any of the other known copies of the Cairo corpus.

MS Istanbul Kılıç Ali Paşa 684 (new):[23]
This is an apparently unique copy of Ibn al-Kattānī's set of tables, beautifully copied in his own hand and dated 768 H [= 1361]. It is a valuable source of information on the authorship of some of the tables in the Cairo corpus. See further **5.5**.

MS Leipzig 817 (new):[24]
This 16th-century manuscript bears no title, but contains an incomplete set of tables of $t(h,\lambda)$ for Cairo (fols. 1v-32v, h = 1°, 2°, ... , 68°), followed by an incomplete copy of the instructions (fols. 33r-35r, the first page is missing), and a complete set of tables of $T(h,\lambda)$ for Cairo (fols. 35v-67r). Neither tables nor text, which are all written in a single hand, bear the name of any compiler or copyist. The parameters for twilight mentioned in the introduction are 19° and 17°.

MS Istanbul Nuruosmaniye 2903 (new):
This bears no title and begins with an anonymous introduction (due to al-Maqsī). The parameters advocated for twilight are 20° and 16° but the copyist added in the margin that 19° and 17° were preferable. The remainder of the manuscript consists of a complete set of tables of (T,t,a) as functions of (h,λ), copied in a different hand by Khalīl al-Ḥarīrī in 1089 H [= 1678].

MS Istanbul Nuruosmaniye 2925 (new):
This undated manuscript begins with tables of the functions $2D^h$ and $\tilde{h}$ and continues with al-Maqsī's introduction and al-Bakhāniqī's notes. The main tables display (T,t,a) as functions of (h,λ) and are complete. These are followed by tables of:

$$h_v,\ \mathrm{Cos}\ \delta,\ B,\ h_q,\ t_q,\ h_{q*},\ t_{q*} \text{ and } Z_{(12)}\ .$$

The table of h_v (with 0;41° for λ = 270°) is stated to have been computed by Ibn al-Rashīdī.

MS Paris BNF ar. 2520,2:
See **8.1**.

MS Cairo MM 43:
Various tables from the Cairo and Damascus corpora, copied in the hand of [ʿAlī ibn Muḥammad al-Dalāmī] *ca*. 1450. See **6.14** and **10.2**.

[22a] *Gotha Catalogue*, III, p. 63.
[23] Overlooked in Rosenfeld & İhsanoğlu, *MAIC*, no. 735.
[24] *Leipzig Catalogue*, p. 264.

MS Cairo MM 64:
A beautifully-executed copy of Ibn Yūnus' azimuth tables, dating from about 1400. The title folio is missing, and there is no colophon. The manuscript begins with the last page of a set of tables displaying the solar longitude for each day of a four-year period in the Coptic calendar. For each month of the Coptic year four columns display the solar longitude in signs, degrees and minutes for each of the four years, labelled *sana rubʿ*, *sana niṣf*, *sana niṣf wa-rubʿ*, *sana kabīs*. Only the tables for Baʾūna, Abīb and Misrā are contained in the manuscript now. The entry for Baʾūna 1 (first year) is Gemini 12;16°. The altitude argument in the azimuth tables runs up to 89° and there is a marginal note which attempts to rationalize this (**4.4**). For each altitude above 36° the meridian altitude corresponding to the last longitude argument for which there is an entry for the azimuth is given to three digits and is called *ghāyat irtifāʿ hātayn al-darajatayn ruʾūs al-jadwal,* "the meridian altitude for these two solar longitudes (whose signs are given) at the head of the table".

MS Cairo DM 444:
This manuscript contains a complete set of tables of T(h,λ) copied in 858 H [= 1454] in the elegant hand of Abu 'l-Yumn Muḥammad ibn Muḥammad ibn ʿArab al-Shabībī (or al-Shayyibī).[25] It bears a notice of possession (fol. 3r) by the Cairo *muwaqqit* ʿAbd al-Raḥmān al-Ṭūlūnī[26] dated 1046 H [= 1636/37]. The title is given as *Kitāb Ghāyat al-intifāʿ fī maʿrifat al-dāʾir min al-falak min qibal al-irtifāʿ li-ʿarḍ lām shamāl li-'l-Maqsī*, and the tables are preceded by al-Maqsī's introduction (with parameters 20° and 16° advocated for twilight). al-Maqsī's four additional tables are not contained in this manuscript; indeed the tables of T conclude with the words: *tamma wa-kamala wa-l-'llāh al-ḥamd*, "it is completed and fulfilled and praise be to God". The tables of T(h,λ) end on the recto side of the last folio and the verso is blank except for a notice of sale dated 997 H [= 1588/89]. The tables for 17° and 19° bear instructions for finding the duration of twilight. See **Figs. I-2.1.1b-c** and **II-5.6a**.

MS Cairo DM 690:
This consists mainly of two incomplete sets of tables of (T,t,a) bound in disorder. The handwriting in both cases is neat and can be dated to about 1500. The last folio of tables (for altitude arguments 40°-42°) is copied in the later hand of Ḥasan al-Ṭablāwī *ca*. 1700.[27] There is no title folio. The tables for altitudes 19° and 17° bear instructions for finding the duration of morning and evening twilight.

MS Cairo DM 777:
This contains an incomplete set of tables of T(h,λ) and t(h,λ) on facing pages, for values of h from 2° to 46° and then 76° to 83°. There is no title folio and no compiler is mentioned. The tables are copied in the elegant hand of the copyist of MS Cairo MM 241 of Ibn al-Mushrif's auxiliary tables (**6.9**) and can thus be dated *ca*. 1500; see also the notes on MS Cairo DM 616 below.

---

[25] On other manuscripts copied by Abu 'l-Yumn see *Cairo ENL Catalogue*, I, p. 714.

[26] On ʿAbd al-Raḥmān al-Ṭūlūnī see *Cairo ENL Survey*, no. D16, and on his library see **V-10**.

[27] Close to 20 scientific manuscripts copied by him *ca*. 1700 are listed in *Cairo ENL Catalogue*, I, p. 688 (also n. 4:29). See also *Cairo ENL Survey*, no. D56, on some tables for constructing sundials apparently computed by him, as well as İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, p. 347, no. 212.

MS Cairo DM 616:
This consists of four parts, the first two forming a complete set of tables of (T,t,a) preceded by al-Maqsī's introduction and al-Bakhāniqī's notes. The title page and tables for altitude arguments 7° and above are written in the same hand and dated (10)98 H [= 1686/87]. The title is given as *Kitāb Kifāyat al-waqt fī maʿrifat al-dāʾir wa-faḍlihi wa-'l-samt* and is attributed to Abu 'l-ʿAbbās Aḥmad ibn ʿUmar al-Ṣūfī, that is, al-Maqsī. Part of the introduction and the tables for altitude arguments 1° to 7° (fols. 2r-9v) are in an older hand datable to about 1500, which is in fact the same hand as that of MSS Cairo MM 241 and Cairo DM 777 (see above). The third part of the manuscript, copied about 1600], consists of a set of prayer-tables displaying values of the functions:

$$H, D, h_a, t_a, T_a, d, n, 2N, \sigma, \alpha_\sigma, s \text{ and } r\ ,$$

side by side for each degree of λ starting with $\lambda = 1°$. The fourth part of the manuscript contains some tables for finding the longitude of the horoscopus and upper midheaven from the oblique ascensions of the horoscopus.

MS Cairo DM 651,5 (new):
This source, copied *ca*. 1700, contains (fols. 59r-64v) a set of tables of various functions from the corpus tabulated together, namely:

$$H, D \text{ and } N, 2D \text{ and } 2N, t_a \text{ and } T_a, s \text{ and } r, h_a \text{ and } z_{a(12)}\ ,$$

as well as tables of α_ϕ and $\tilde{h}$ and a table of geographical coordinates.

MS Cairo DM 739:
This is in the hand of the copyist of the title folio of MS Cairo DM 616 (see above) and can hence be dated to about 1685. It bears the title *Kitāb Kifāyat al-waqt fī maʿrifat al-dāʾir wa-faḍlihi wa-'l-samt*, attributed to Abu 'l-ʿAbbas Aḥmad ibn ʿUmar al-Ṣūfī, that is, al-Maqsī. It contains al-Maqsī's introduction followed by al-Bakhāniqī's notes and a complete set of tables of the functions (T,t,a). These tables end on the recto side of the last folio (fol. 64). On the verso there are displayed values of a(h,λ) only, for altitude arguments 84° to 88°, and values of a(h) at the solstices.

MS Cairo DM 776, fols. 1v and 10r-43r:
This contains a complete set of tables of T(h,λ), apparently copied in 981 H [= 1573/74]. The title folio, which is in the hand of an owner Muhammad Sinār / Sannār (?) al-Aḥmadī,[28] gives this date of copying and the title as *al-Dāʾir li-'rtifāʿ al-shams*. No compiler is mentioned and there is no introduction. The tables for arguments 16° and 20° are marked as suitable for twilight by the original copyist, and are marked *b-ṭ-l* for *baṭal(a)*, "is no good", in a different hand. The tables for 17° and 19° are marked in another different hand as suitable for twilight according to "the new observations", a phrase which usually refers to the Samarqand observations of Ulugh Beg, but here may refer to the opinion of the Cairo editor of the Samarqand *zīj*, namely, Ibn Abi 'l-Fatḥ al-Ṣūfī.

[28] He is to be identified with the Muḥammad Sinār / Sannār who was a muwaqqit at ṭhe Aḥmadī (from Sayyid Aḥmad Badawī) Mosque in Tanta, on whom see *Cairo ENL Survey*, no. D108; and also İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 625-626, no. 460.

MS Cairo DM 778:
This was copied by Ḥasan ibn ʿAlī al-Ghamrī and appears to date from *ca*. 1500. It contains a disordered incomplete set of hour-angle tables followed by various prayer-tables. The title is given as *Kitāb Faḍl al-dāʾir* and no compiler is mentioned. Following the main set there are tables of:

$$D,\ t_a,\ T_a,\ s,\ h_a,\ r \text{ and } \sigma\ .$$

The twilight tables are based on parameters 19° and 17° and the *salām* is assumed to be 1° before daybreak. These prayer-tables are copied in the same hand as the hour-angle tables, but the colophon occurs at the end of the latter.

MS Cairo DM 786:
An incomplete copy of the tables of (T,t,a) for altitude arguments 45° to 83°. The manuscript bears neither title nor colophon but is copied in the distinctive and untidy hand of Ḥasan al-Tablāwī[29] and can thus be dated *ca*. 1700.

MS Cairo DM 1101:
A late copy of Ibn Yūnus' azimuth tables, perhaps copied *ca*. 1785, and entitled *Jadwal al-samt al-maḥlūl daqīqa daqīqa*. Entries are given for altitude arguments up to 89°.

MS Cairo DM 1108,9 (fols. 35v-59r):
An unique copy, dated 1053 H [= 1643/44], of a set of azimuth tables for Cairo arranged so that for each degree of solar longitude the solar altitude is entered vertically. The altitude arguments are arranged in columns of up to 30 entries, and the meridian altitude is given to three digits below the last integral altitude argument for certain degrees of solar longitude. The last southern azimuth in each solar longitude sub-table is written in red ink and the corresponding solar altitude argument is labelled *darajat al-intiqāl*.

MS Cairo DM 1109:
A complete set of tables of (T,t,a) copied in 1128 H [= 1716] by Ḥasan ibn ʿAbdallāh, nicknamed *al-Qiṭṭ al-miskīn*, perhaps meaning "Poor Pussy". There is no introduction and the title *al-Dāʾir wa-faḍluhu wa-ʾl-samt ḥisāb Ibn Yūnus al-Miṣrī li-ʿarḍ Miṣr* is a later addition. On fol. 1r there is a list of pilgrim stations on the road from Cairo to Mecca with latitudes and qiblas (see **8.4** and **10.9**).

MS Cairo DM 1224:
An incomplete copy of the tables of the functions (T,t,a) bearing the title *Jadāwil al-dāʾir wa-faḍl al-dāʾir wa-ʾl-samt min qibal al-irtifāʿ* and attributed to Ibn Yūnus. There are some simple instruments on the use of the tables but no other introduction. The manuscript was copied by ʿAbd al-Bāriʾ al-ʿAshmāwī (see MSS Cairo TR 191 and DM 108) and can hence be dated *ca*. 1800. The tables end with those for altitude argument 51°.

MS Cairo K 4044:
This copy of the main tables in the corpus is dated 1308 H [= 1890/91]. The work is divided into two parts, the first being entitled *Kitāb sharḥ ʿalā faḍl al-dāʾir li-l shaykh Abi ʾl-ʿAbbās*

[29] See n. 4:27.

al-Maqsī. This refers to al-Maqsī's introduction which precedes a complete set of tables of T(h,λ), not t(h,λ) as one might have expected from the title. The second part of the manuscript is written in a different hand but also dated 1308 H. It is entitled *Kitāb Maḥlūl al-samt li-bn Yūnus li-ʿarḍ lām shamāl* and contains a set of azimuth tables for altitude arguments up to 83°. The last folio of the manuscript contains tables of the solar azimuth at the equinoxes and solstices and the functions ψ(λ) and h_0(λ).

MS Princeton Yahuda 861,1:
See **8.1**.
MS Cairo ṬJ 367,5, fols. 35r-40r (new):
Copied *ca*. 1450, this contains part of an introduction and part of a solar longitude table, followed by tables of the functions:

$$r,\ H,\ h_q,\ h_0,\ \psi,\ d \text{ and } \delta$$

from the corpus. The work bears the title *Natījat al-afkār fī aʿmāl al-layl wa-'l-nahār* and is attributed to the 15th-century Egyptian astronomer ʿIzz al-Dīn al-Wafāʾī. However, the prayer tables are not mentioned in the introduction (see further **5.7**). The same introduction and solar tables, followed by a much more extensive set of prayer-tables for different latitudes, is contained in MS Princeton Yahuda 861,1 (**8.1**).

4.3 Tables of standard spherical astronomical functions

Some copies of the corpus contain a number of spherical astronomical functions of the kind standard in Islamic *zījes*. The notes which follow give a clearer picture of those tables than was possible in my original analysis.

(a) Solar declination, δ(λ)
MSS Dublin CB 3673, Gotha A1402 and Cairo DM 153 are the only copies of the corpus in which tables of δ(λ) occur, but in most manuscripts there are tables of the related function H(λ) – see (c) below. The entries in these tables of δ(λ) are given to three digits for each degree of λ′ and contain several variants from the corresponding entries in the *Ḥākimī Zīj,* which are more accurate.

(b) Half excess of daylight, d(λ)
The tables of d(λ) to two digits in the Chester Beatty manuscript are *not* based on the corresponding table in the *Ḥākimī Zīj*, which has entries to three digits: about one-sixth of the entries in the table in the former source are in error by ±1 in the second digit. See further (d) below on the related tables of D(λ).

(c) Meridian altitude, H(λ)
The errors in a table of H(λ) will reflect those in the underlying values of δ(λ) since $H = \bar{\phi} + \delta$. Furthermore, the values for $\delta > 0$ and $\delta < 0$ can be compared to distinguish computational errors from copyists' errors. Now the table of H in MS Dublin CB 3673, fol. 14v, contains only one error of the former kind: for λ = 15° we find H = 65;58°, and for λ = 195° we find

H = 54;2°, which means that δ(15°) was taken as 5;58° instead of the accurate value 5;57°. If this table were based on Ibn Yūnus' declination table in the *Ḥākimī Zīj* the entries would all be correct, since of his entries for integral values of λ only seven are in error by only ±1 in the *third* sexagesimal digit. However, it is possible that Ibn Yūnus' accurately-computed entry 5;56,37° for δ(15°) was misread as 5;57,37° and rounded to 5;58°. In MS Dublin CB 3673, fols. 4r and 11r, however, it is clearly 5;56,37°.

I do not think that al-Maqsī could have computed δ(λ) accurately to two digits. al-Marrākushī's declination table (computed to two digits) contains some nine errors of -1 in the second digit, but it seems that he compiled this table by truncating values to three digits found in an earlier ʿIrāqī source.

(d) Semi diurnal arc, D(λ)

The tables of D(λ) to two digits in MSS Dublin CB 3673 and Cairo DM 108 were compiled from the table of δ(λ) to two digits in the Dublin manuscript, using the simple relation D = 90° + d (d $\gtreqless$ 0 as δ $\gtreqless$ 0). As noted in (b) above this table of δ(λ) is *not* derived from Ibn Yūnus' carefully-computed table in the *Ḥākimī Zīj*, which gives entries to three digits. al-Bakhāniqī does not state that al-Maqsī computed either d or D but al-Maqsī does refer to a table of D(λ) in his introduction to his tables of T(h,λ). I suspect that al-Maqsī did compute these two tables himself and that he used his table of D(λ) to compute T(h,λ) from some hour-angle tables by Ibn Yūnus. Perhaps al-Bakhāniqī had his own table of D(λ) which he had used to make minor alterations in the tables of t(h,λ) starting from al-Maqsī's tables of T(h,λ). See further (e) below.

(e) Nocturnal arc, 2N(λ)

The tables of 2N(λ) in MS Dublin CB 3673, fols. 5v and 6v, are based on different tables of D(λ). The values of 2N on fol. 6v in the Dublin manuscript are fairly closely related to those of D on fol. 12r. The values of 2N on fol. 5v are based on a set of values of D or d which is significantly different from those on fol. 12r and also from the more accurate set of values of d in MS Paris BNF ar. 2513, fol. 53v, of the *Muṣṭalaḥ Zīj*. The table of length of darkness n(λ) (see **4.10**) on fol. 6r of the Dublin copy is based on the table of 2N(λ) on fol. 6v.

(f) Length of seasonal day-hours in degrees, $\tilde{h}$(λ)

This is derived by dividing 2D by 12. In the *Ḥākimī Zīj* Ibn Yūnus computes it to four digits and the third digit of his entries is generally accurate. In MS Dublin CB 3673, fol. 13r, the table of $\tilde{h}$(λ) to two digits contains 5 errors of ±1 in the second digit, so that it was not derived from the *Ḥākimī* values by rounding.

(g) Factor for converting equatorial degrees to seasonal day-hours, 1/$\tilde{h}$(λ)

The table of 1/$\tilde{h}$(λ) to two digits in MS Dublin CB 3673, fol. 3r, contains some 40 errors of ±1 in the second digit. It was probably prepared by al-Maqsī using rounded values of $\tilde{h}$. In the introductions to MSS Dublin CB 3673 and Cairo DM 108, both of which sources contain the table, al-Maqsī states that he has included tables of t_a(λ) and l/$\tilde{h}$(λ) "at the end of the book" (**5.4**). In the Dublin copy a marginal note by the side of the table of l/$\tilde{h}$(λ) states "this is the

end of the book", but in the Cairo copy there are four tables following the one of $1/\tilde{h}(\lambda)$. In MS Berlin Ahlwardt 5753, fols. 20v and 21r, however, immediately following an incomplete set of tables of $T(h,\lambda)$, there are others of $t_a(\lambda)$ and $1/\tilde{h}(\lambda)$ which conclude the work.

(h) Length of daylight in equatorial hours, $2D^h(\lambda)$

The table of $2D^h(\lambda) = 2D(\lambda)/15$ to two digits occurs only in MS Cairo DM 108, fol. 75v, of the main sources. There are four errors of ±1 in the second digits of the entries, and that for arguments which differ from those for which $\tilde{h}(\lambda) = 2D(\lambda)/12$ is in error. The function is not tabulated in the *Ḥākimī Zīj and* is not referred to in al-Maqsī's introduction to the corpus. al-Bakhāniqī states that it was he himself who added this table (**5.6**).

(i) Normed right ascensions, $\alpha'(\lambda)$

My original implication that the tables of $\alpha'(\lambda)$ in MSS Hartford, Conn., Theological Seminary 621 and Paris BNF ar. 2553 of the prayer-tables of al-Lādhiqī (**7.8**) were taken from the *Ḥākimī Zīj* is incorrect. In his *Zīj* Ibn Yūnus computes $\alpha(\lambda)$ carefully to three digits. The table of $\alpha'(\lambda)$ to two digits in MS Leiden Or. 143 of the *Zīj* is not derived from this: it is either spurious or taken from an earlier work of his. On the other hand, the table of $\alpha'(\lambda)$ to three digits in MS London BL Or. 3624 of the Yemeni *Mukhtar Zīj* is based on the *Ḥākimī* table of $\alpha(\lambda)$, and the table with entries to two digits in MS Cairo MM 43 appears to be based on this. Likewise, al-Marrākushī (**6.7**) tabulated $\alpha'(\lambda)$ carefully to three digits: his values differ from those of Ibn Yūnus and are in general slightly less accurate. The values of $\alpha'(\lambda)$ in the Hartford and Paris BNF ar. 2553 copies of al-Lādhiqī's tables can *not* be derived from either of Ibn Yūnus' or al-Marrākushī's three-digit values by rounding and they are not identical with those to two digits in MS Leiden Or. 143.

(j) Oblique ascensions for Cairo, $\alpha_\phi(\lambda)$ for $\phi = 30°$

The tables of $\alpha_\phi(\lambda)$ to two digits in the Hartford and Paris copies of al-Lādhiqī's prayer-tables contain numerous errors, which fact immediately distinguishes them from the corresponding table in the *Ḥākimī Zīj,* since it contains but two small errors. In fact, these tables are al-Marrākushī's. On the other hand, the table of $\alpha_\phi(\lambda)$ in the *Natīja* attributed to al-Wafā'ī (**8.1**), which contains a few copyist's errors, appears to have been taken from the *Ḥākimī Zīj.*

(k) Solar altitude in the prime vertical, $h_0(\lambda)$

The *Ḥākimī Zīj* contains a table of $h_0(\lambda)$ carefully computed to three sexagimal digits. In MS Dublin CB 3673, fol. 85v, of the corpus contains this same table with numerous copyist's errors, but it is two other tables of $h_0(\lambda)$ to two digits in the Chester Beatty manuscript, also occurring in other copies of the corpus, which merit comment. Firstly note that:

$$h_0(\lambda) = \text{arc Sin } \{ R \text{ Sin } \delta(\lambda) / \text{Sin } \phi \} = \text{arc Sin } \{ 2 \text{ Sin } \delta(\lambda) \} \quad \text{for} \quad \phi = 30° .$$

Since also:

$$\text{Sin } \delta = \text{Sin } \varepsilon \text{ Sin } \lambda / R ,$$

we have:

$$h_0 = \varepsilon \text{ for } \lambda = 30° .$$

Now in the table of $h_0(\lambda)$ in MS Dublin CB 3673, fol. 3v, the entry for $\lambda = 30°$ is 24;34°, which is clearly absurd. The entry for $\lambda = 29°$ has been miscopied as 23;49° instead of 22;49,(32)°: the 24 in 24;34° appears to have been "fixed up" and the 34 can be explained as a copyist's error for 35°. Nevertheless, the entry for $\lambda = 31°$ is 24;21°. (Even in the table to three digits in MS Dublin CB 3673, fol. 85v, the entry for 30° is miscopied as 23;45,0°.) These same errors occur in the Hartford and Paris copies of al-Lādhiqī's prayer-tables (**7.8**), but in MS Princeton Yahuda 861,1 of the *Natīja* attributed to al-Wafāʾī (**8.1**) they have been corrected. In the table of $h_0(\lambda)$ on fol. 12v of the Dublin manuscript, the entries in the third column have been further garbled, apparently by someone who believed that the maximum value of h_0 was 53;46° rather than the accurate value 53;9°. Again, in the prayer-tables of al-Minūfī (**7.1**) most of the standard functions are recomputed for $\varepsilon = 23;30°$: his table of h_0 is an exception, since he simply plagiarized an earlier source. The entries in his table for $\lambda =$ 30° and 90° are again 24;34° and 53;9°. Finally, in the prayer-tables of al-Fawānīsī (**7.5**), the values of h_0 corresponding to $\lambda = 30°$ and 90° are respectively 23;47° and 53;21°, both inaccurate.

(l) Solar rising azimuth, $\psi(\lambda)$

This function is defined by:

$$\psi(\lambda) \;=\; \text{arc Sin } \{ \text{ R Sin } \delta(\lambda) \;/\; \text{Cos } \phi \} \,,$$

and so for $\phi = 30°$ is slightly more difficult to compute than $h_0(\lambda)$ (see (k) above). In the *Ḥākimī Zīj* Ibn Yūnus computes $\psi(\lambda)$ carefully to thirds. His table occurs again in MS Dublin CB 3673, fol. 84r, and those on fols. 12v and 3v of the same manuscript, computed to two digits, are slightly less accurate than can be obtained from his values by rounding. Nevertheless the errors are much smaller than those in the associated tables of h_0.

(m) Arc Sines

The table of arc Sin (x) in MS Dublin CB 3673, fol. 8r, contains some errors which are identical to those in the corresponding entries in the tables in MS London BL Or. 3624, fol. 169v, of the Yemeni *Mukhtār Zīj*, a work containing numerous tables of Ibn Yūnus and other errors which are not. The errors in the table in the Chester Beatty manuscript also differ from these in al-Marrākushī's table of arc Sines (**6.7**). It is possible to derive arc Sines accurately to two sexagesimal digits from Ibn Yūnus' extensive Sine tables in MS Berlin Ahlwardt 5752 (Landberg 1038), copied 895 H [= 1490], but we do know that Ibn Yūnus used Sine tables less accurate than those in the *Ḥākimī Zīj* in his early work.

The minor tables in the corpus can now be subdivided according to their provenance as follows:

$h_0(\lambda)$ and $\psi(\lambda)$ (3 digits)	Ibn Yūnus
$H(\lambda)$	based on Ibn Yūnus
$\delta(\lambda)$, $d(\lambda)$, $D(\lambda,)$, $2D^h(\lambda)$, $2N(\lambda)$, $\tilde{h}(\lambda)$, $\alpha'(\lambda)$	anonymous
$h_0(\lambda)$ and $\psi(\lambda)$ (2 digits)	anonymous
$1/\tilde{h}(\lambda)$	al-Maqsī
$2D^h(\lambda)$	al-Bakhāniqī

$\alpha_\phi(\lambda)$ ($\phi = 30°$) al-Marrākushī
arc Sin (x) anonymous

The tables of $h_0(\lambda)$ and $\psi(\lambda)$ to three digits were appended to Ibn Yūnus' *Kitāb al-Samt* (**5.2**). The table of $1/\tilde{h}(\lambda)$ was appended to al-Maqsī's tables of time since sunrise (**5.4**). al-Bakhāniqī appears to have compiled the table of $2D^h(\lambda)$ and added those of $\alpha'(\lambda)$ and $\alpha_\phi(\lambda)$ ($\phi = 30°$) to his edition of the corpus (**5.6**). The identity of the person who compiled the remaining tables remains obscure.

4.4 Tables of solar azimuth

These, called *Kitāb al-Samt* when copied separately, display the solar azimuth $a(h,\lambda)$ measured from the prime vertical, for each integral degree of solar altitude h up to the maximum 83° and each degree of solar longitude λ' ($\delta \gtrless 0$).

In each of MS Berlin Ahlwardt 5753, Dublin CB 3673, Escorial ár. 924,7, Cairo Azhar *falak* 4382, Cairo MM 137 and Gotha A 1410, the title *Kitāb al-Samt* is given and Ibn Yūnus is named as the compiler (see **Figs. 4.2a-b**). al-Bakhāniqī and al-Khaṭāʾī (**5.6** and **6.8**) also say that Ibn Yūnus compiled the azimuth tables. I see no reason to doubt this. In the *Ḥākimī Zīj* there are tables of $a(\lambda)$ for altitudes 30° and 35° with values for each degree of λ in 12 columns of 30 entries (**5.1**): no reason is given for choosing these two altitudes. The two tables in the *Zīj* are slightly more accurately computed than the corresponding tables in the *Kitāb al-Samt*.[30]

It seems that Ibn Yūnus' azimuth tables for altitudes 80° to 83° were either not completed or were lost at some stage in their transmission. Some anonymous with a very limited knowledge of spherical astronomy computed a set for 80° to 89° (extant in MS Gotha A 1410; MSS Dublin CB 3673 and Cairo MM 137 have entries up to 88° and 86°: see **Fig. I-5.1.1a**). A corrected set for 80° to 83° was compiled by Ibn al-Rashīdī (**4.1.3**) and these azimuth tables were used by al-Bakhāniqī (MSS Cairo DM 108 and DM 53) and were also copied separately (MS Cairo Azhar *falak* 4382). I must withdraw my previous statement that al-Maqsī "plagiarized" Ibn Yūnus' azimuth tables,[31] since now it is clear that al-Bakhāniqī put together the tables of T, t and a, and the reference to azimuth tables in the introduction to MS Dublin CB 3673 is probably not original to al-Maqsī's introduction. Ibn Yūnus tabulated certain other functions along with the azimuth – see **5.2**.

In the margin of the tables in some copies of the corpus, *e.g.*, MSS Dublin CB 3673 and Escorial ár. 924,7, for altitudes 1° to 53° it is stated for which solar longitudes the azimuth changes from north to south for that particular altitude. In MS Berlin Ahlwardt 5753 the change in direction is more crudely indicated. In the margin of the tables for altitude 54° in the Chester Beatty manuscript we find the comment that "there are no more azimuths in the north for this altitude and those above it". When, for certain longitudes, the sun does not attain a given altitude, the entries in the table are left blank. For altitudes of about 60° and above, the entries for more than one altitude can be tabulated on one folio. In the Escorial manuscript, for a given

[30] Both are edited in King, "Astronomical Timekeeping in Medieval Cairo", pp. 354-355.
[31] *Ibid*., p. 364. See also the text to nn. 4:12 and 10:2.

altitude, the meridian altitude is given for the longitude whose azimuth is closest to 90°. Thus, in the tables for altitude 37°, the last significant entry in the right-hand side of the table is an azimuth of 87;3° for Sagittarius 17° and Capricorn 13°. The next and final entry is given as 0;0°, which does not mean due east or west, but that the sun does not attain the altitude 37° for Sagittarius 18° or Capricorn 12°. The meridian altitude is accurately stated as 37;3,22° for the solar longitudes having azimuth 87;3°. The purpose of giving this detail is not fully clear to me. However, in the unique MS Cairo DM 1108,9, copied in 1053 H [= 1643/44], the solar azimuth is displayed as a function a(λ,h), with values a(h) for each value of λ, and the values of H(λ) are given for each λ. It seems unlikely that Ibn Yūnus' original tables were in this form, not least because he tabulated a(h,λ) for two altitudes in the *Ḥākimī Zīj* (**5.1**).

By virtue of the format of the tables no values are shown for the equinoxes. However, in some copies of the corpus, such as MS Cairo Azhar *falak* 4382, fol. 71r, there is a table displaying a(h) at the equinoxes for each degree of h from 1° to 60° (= $\bar{\phi}$) and also for each 0;5° of h between 36;5° and 36;25° (= $\bar{\phi}$-ε) at the winter solstice and again for each 0;5° of h between 83;5° and 83;30° and also for 83;33° and 83;35° (= $\bar{\phi}$+ε) at the summer solstice. al-Bakhāniqī (**5.6**) states that this table was computed by Ibn al-Rashīdī (**4.1.3**).

4.5 Tables of time since sunrise and the hour-angle

These, often called *Kitāb al-Dāʾir* and *Kitāb Faḍl al-dāʾir* when tabulated separately, display T(h,λ) and t(h,λ) for each integral degree of h up to the maximum 83° and each degree of λ′ (δ ≷ 0). There are no such tables in the *Ḥākimī Zīj*. In MSS Dublin CB 3673, Dublin CB 4078, Leipzig 817 and Cairo DM 777, the tables of both functions for a given altitude generally occupy two facing pages of the manuscript. In MSS Cairo TR 191 and Cairo Azhar *falak* 4382 only t is presented, and in MS Gotha A1402 only T. In MS Berlin Ahlwardt 5753 the functions are tabulated separately. In MSS Cairo DM 108, Cairo DM 53, Istanbul Nuruosmaniye 2903 and 2925, Cairo DM 690 and DM 616, triplets of values (T,t,a) are given for each pair of arguments (h,λ), where a is the solar azimuth. See **Figs. I-2.1.1a-b**, **2.1.1c**, **II-5-4b** and **5.6a**.

These tables of T and t are less accurately computed than the azimuth tables. In general the entries are less accurate for higher altitudes, as a result of the nature of the tabulated functions. When the three functions are tabulated side by side, for altitudes over 80° the number of longitude arguments for which entries are given for T and t differs slightly from the number for which entries are given for a: see **Fig. 5.6b**.

The values of T and t are related by the values of D(λ) tabulated elsewhere in the corpus (**4.3d**). al-Maqsī (**5.4**) claims to have computed the tables of T(h,λ) himself. Each of al-Karakī, Ibn al-Kattānī and al-Bakhāniqī (**9.4, 5.5** and **5.6**) attribute these tables to him. Ibn al-Kattānī claims to have computed the tables of t(h,λ) from al-Maqsī's tables of T(h,λ) and al-Bakhāniqī claims to have done the same, adding that his entries "may differ in some cases from previous calculations".

I am now of the opinion that al-Maqsī did compile the tables of T(h,λ) and Ibn al-Kattānī compiled the tables of t(h,λ). The fact remains that in order to tabulate T(h,λ) one must first compute t(h,λ): I suspect that al-Maqsī used an earlier set of tables of either t(h,λ) or T(h,λ)

compiled by Ibn Yūnus which are no longer extant in their original form and were not available to astronomers after al-Maqsī: see further **5.2** and **5.3**.

4.6 Tables of solar altitude in specific azimuths

Two tables of the function h(a,λ) for a = 30° and 60° and each degree of λ′ (δ ≷ 0) occur in MSS Dublin CB 3673, Cairo DM 954,2 of the Cairo corpus. The second table alone is also found in MS Cairo DM 153,2, of the corpus, and the first table alone is found in MS Cairo DM 620 of al-Qaymarī's tables for the afternoon prayer (**7.3**). The functions are labelled *al-irtifāʿ idhā kāna 'l-samt lām/sīn*. In MS Cairo DM 153 Ibn Yūnus is specifically identified as the compiler and indeed the same tables occur along with others based on different azimuths in the *Ḥākimī Zīj* (**5.1**), where Ibn Yūnus describes the way in which they can be used to determine the meridian. The tables in each of the above-mentioned copies of the corpus are arranged symmetrically in the same way as the azimuth and time tables, whereas those in the *Zīj* give values of h for each degree of λ in 12 columns of 30 entries.

The Cairo corpus also contains tables of solar altitude in the azimuths of the qibla and the rising sun at the winter solstice: see **4.7** and **4.8**.

4.7 Tables of the solar altitude and the time of day when the sun is in the azimuth of the qibla

The tables in the corpus relating to the qibla are based on the assumption that the qibla at Cairo-Fusṭāṭ is 53° E of S. This value is rounded from the 53;0,6° and 53;0,17° derived by Ibn Yūnus in his *Zīj*, and it was used at least by the astronomers for the qibla in Cairo throughout the medieval period. (Other directions were used by different interest groups, such as the azimuth of the rising sun at the winter solstice: see **4.8**.) Ibn Yūnus' discussion of the qibla in his *Zīj* concludes with a table of the solar altitude in the azimuth of the qibla, $h_q(\lambda)$, based on the parameter q = 52°, computed "some time ago". The value 53° is based on a different value of the longitude difference between Cairo and Mecca, but no further information is provided about the way in which this longitude difference was measured or what coordinates were used to derive the value q = 52°.

The table of $h_q(\lambda)$ in MS Dublin CB 3673, fol. 86v, of the corpus is based on the parameter q = 52° and is arranged with values for each degree of λ, that is, in 12 columns of 30 entries. There are numerous corrupt readings, as can be seen by checking entries symmetrically-placed with respect to the solstices or by comparing the table with the more accurate one in the *Ḥākimī Zīj*. This table is not contained in any other copies of the corpus, although al-Marrākushī (**6.7**) tabulated $h_q(\lambda)$ for each 30° of λ′ and six of the seven values, which are based on q = 52°, are the same as the corresponding entries in the *Ḥākimī Zīj*. The Dublin manuscript (fol. 14r), and several other copies of the corpus contain a table of $h_q(\lambda)$ for q = 53°, symmetrically arranged like the azimuth and time tables, but less accurately computed: see **Fig. 4.7a**.

A number of sources, including the Dublin manuscript, contain a table of $t_q(\lambda)$, the hour-angle when the sun is in the direction of Mecca, also symmetrically arranged. This is based on q = 53° and is not carefully computed. Using the values of solar altitudes for this azimuth

Fig. 4.7a: The table of the hour-angle when the sun is in the azimuth of the qibla at Cairo. On the left is a table for the duration of evening twilight, based on a value of 17° for the angle of solar depression below the horizon. [From MS Dublin CB 3673, courtesy of the Chester Beatty Library.]

Fig. 4.7b: The tables for the solar altitude and hour-angle when the sun is perpendicular to the qibla. [From an unidentified Cairo manuscript, courtesy of the Egyptian National Library.]

as given, for example, on fol. 14r of the Dublin manuscript, together with the tables of $t(h,\lambda)$ in the corpus, I derive more accurate values of t_q than those in this table of $t_q(\lambda)$. A table of $T_q(\lambda)$, measuring the interval from sunrise to the moment when the sun is in the azimuth of the qibla, is found only in MS Dublin CB 3673, and the values in it are easily derived from the tables of $t_q(\lambda)$ and $D(\lambda)$.

In a few sources, including MS Cairo DM 108, there are tables for the solar altitude $h_{q*}(\lambda)$ and the corresponding hour-angle $t_{q*}(\lambda)$ when the sun is in the direction perpendicular to the qibla (*ʿalā tarbīʿ al-qibla*). These tables are based on q = 53° and are less carefully computed than those of h_q and t_q: see **Fig. 4.7b**.

The table of h_q (q = 52°), which occurs in only one source, is certainly due to Ibn Yūnus. The tables of h_q and h_{q*} (q = 53°) are also due to him, if my interpretation of al-Bakhāniqī's remarks (**5.6**) is correct. The table of T_q appeared already in al-Maqsī's set compiled in the

late 13th century (see **5.4**) and was probably based on an earlier table of t_q. The table of t_{q*} appears to be a later addition.

One copy of the corpus, MS Gotha A1411, contains a table displaying the qibla as a function of the latitude and longitude differences from Mecca. Values of $q(\Delta\phi,\Delta L)$ are given for each degree of each argument from 1° up to 20° and are based on the approximate formula:

$$\text{Tan } q \quad = \quad R \text{ Sin } \Delta L \text{ / Sin } \Delta\phi \text{ .}$$

The table is attributed to Yūsuf ibn al-Damīrī, an individual as yet unidentified.[32] However, the same table exists in several other manuscripts, all of Egyptian provenance, and in one of these the compiler is named as Yūsuf al-Damīrī and in another as the 9th-century Baghdad astronomer al-Khwārizmī. I have discussed this table and related approximate methods for determining the qibla in a separate study.[33]

4.8 A table for orienting ventilators

Several copies of the corpus contain a table displaying the solar altitude, h_v, in the azimuth of the ventilator (*al-bādahanj*), for each 1° of λ' ($\Delta \gtreqless 0$). The table is based on an azimuth of 27;30° (S of E), that is, the azimuth of the rising sun at the winter solstice. It is understood that the back of the base of the ventilator should be aligned in this direction and that the front should be exposed to the winds from the north east. See **Figs. VIIb-3.1** and **4.1**.

We know from medieval travel accounts that ventilators were a common feature on the roofs of the multi-storey houses of al-Qāhira and Fusṭāṭ; those few which survive in Cairo are late Ottoman constructions. The table of $h_v(\lambda)$ in the Cairo corpus, as well as other information on ventilators in medieval Egyptian astronomical sources, inform us that in Mamluk and early Ottoman times it was the custom to erect ventilators in this astronomically-defined direction. But the qibla of the first Muslim settlers in Egypt was towards winter sunrise, so that, for example, the Mosque of ʿAmr in Fusṭāṭ faces this direction. In addition, the entire medieval city of al-Qāhira is, by hydrographical convenience and human design accompanied with a good deal of luck, aligned in this direction. The ventilators were simply aligned with the roughly-orthogonal street-plan of the medieval city, and they were in fact open to a direction which is not entirely appropriate for gathering favourable winds.

The entries in the table of $h_v(\lambda)$, which are given to two digits, are not as accurately computed as those in Ibn Yūnus' tables of $h(a,\lambda)$ (**4.4**). Many of the entries are in error by about -0;5°, which would suggest that the value used for $h_e(a)$ (**4.6** and **5.2**) was in error by this amount. In certain sources, e.g., MSS Cairo MM 58 and Istanbul Nuruosmaniye 2925, the entries in the sixth column decrease to 0;41° rather than to 0;0° as one would expect, since the sun rises in this azimuth when $\lambda = 270°$. We can be certain that the correction to 0;41° represents an attempt to account for the displacement of the visible horizon from the true horizon as a result of refraction (see further **4.11**). Yet not all later astronomers included this adjustment: the

---

32 In 2001 a set of tables of double-argument planetary (but no lunar) equation tables compiled by him was located in MS Istanbul Selim Ağa 728,2, fols. 38r-181r, copied in the 14th century. These merit detailed investigation.

33 King, "Earliest Qibla Methods", pp. 133-141, 110-112, and especially p. 112.

corresponding table of h_v in MS Cairo DM 190 of al-Lādhiqī's prayer-tables, for example, has entries which decrease to 0;0°. The difference would hardly affect the efficiency of the ventilators, which were not happily oriented perpendicular to winter sunrise anyway.

al-Bakhāniqī (**5.6**) implies that this table was appended to Ibn Yūnus' azimuth tables, but in MS Istanbul Nuruosmaniye 2925 it is stated that the table (now with entry 0;41° at the winter solstice) was computed by Ibn al-Rashīdī (**5.2**). In MSS Dublin CB 3673 and Gotha A 1410, however, the table of h_v (again with entry 0;41°) is appended to sets of azimuth tables uncorrected by Ibn al-Rashīdī, which suggests an earlier compiler, perhaps Ibn Yūnus. Certainly there were ventilators in Cairo-Fusṭāṭ already in the 12th century, if not in the 10th. About the year 1200 the ʿIrāqī scholar ʿAbd al-Laṭīf al-Baghdādī visited Cairo and wrote: "one hardly sees any houses without ventilators". On the significance of the ventilators and their orientation for understanding the layout of the medieval city see **VIIb**.

4.9 Tables for determining the time of the afternoon prayer

The tables in the corpus relating to the afternoon prayer are based on the standard definition that the permitted time for the prayer commences when the gnomon shadow exceeds its midday minimum by the length of the gnomon, and ends at sunset. The most commonly-tabulated functions are:

$$h_a(\lambda),\quad t_a(\lambda)\quad \text{and}\quad T_a(\lambda)\ ,$$

labelled respectively *irtifāʿ awwal waqt al-ʿaṣr*, *al-dāʾir bayna 'l-ẓuhr wa-'l-ʿaṣr* and *mā bayn al-ʿaṣr wa-'l-ghurūb* and representing the solar altitude and the hour-angle at the beginning of the prescribed interval and the time remaining until sunset: see **Fig. 5.4b**. In MS Cairo TR 191 only there are tables of:

$$z_{a(60)}(\lambda)\quad \text{and}\quad z_{a(12)}(\lambda)$$

instead, which measure the shadow at the time for the prayer. Clearly, at least one of these quantities, easily determined from the corresponding shadow at midday, Z, by addition of one gnomon length, would need to be calculated in the process of deriving h_a.

al-Maqsī claims to have computed the table of $t_a(\lambda)$ himself (**5.4**). The entries in the table for $h_a(\lambda)$ in the corpus differ slighly from those in the corresponding tables in the *Muṣṭalaḥ Zīj* and in the auxiliary tables of al-Khaṭāʾī (**6.6** and **6.8**).

4.10 Tables for twilight

The tables of $r(\lambda)$ and $s(\lambda)$, measuring the duration of morning and evening twilight, are labelled respectively *ḥiṣṣat al-fajr wa-huwa min ṭulūʿ al-fajr ilā ṭulūʿ al-shams* and *ḥiṣṣat al-shafaq wa-huwa min al-ghurūb li-maghīb al-shafaq al-aḥmar*. These titles indicate only that morning twilight lasts from daybreak to sunrise and evening twilight from sunset to the disappearance of the red twilight glow. The underlying values of the solar depression are 19° for morning twilight and 17° for evening twilight and the two tables are derived from the tables of $T(h,\lambda)$ in the corpus (**4.3**) using:

$$r(\lambda)\ =\ T(19°,\lambda^*)\quad \text{and}\quad s(\lambda)\ =\ T(17°,\lambda^*)\ .$$

MS Dublin CB 3673 contains these tables – see **Fig. 4.7a** – and also another pair based on the parameters 20° and 16°, both marked *ḍaʿīf*, "weak", in a later hand, suggesting that someone objected to these parameters.

Ibn Yūnus discussed the determination of twilight in the *Ḥākimī Zīj*[34] but did not present any tables for twilight. An interesting question that cannot yet be answered definitively, is whether he himself computed any tables for twilight, using either the parameter 18° mentioned in the *Ḥākimī Zīj* or the parameters 20° and 16° attributed to him by Najm al-Dīn al-Miṣrī (**2.5**) three centuries after his time. al-Maqsī (**5.4**) did not include any twilight tables in his set but advocated the parameters 20° and 16° in his introduction. Ibn al-Kattānī (**5.5**) included twilight tables based on parameters 19° and 16° in his set and also additional tables for morning twilight based on 20°, which parameter he says was used by recent scholars (presumably he was referring to al-Marrākushī and al-Maqsī), and 18°, which he says was used by earlier scholars (presumably referring to Ibn Yūnus). The parameters 19° and 17° were favoured by most later *muwaqqit*s in Egypt and Syria from the 15th century onwards. Various tables for twilight purporting to be adjusted for refraction at the horizon occur in late Egyptian prayer-tables in the tradition of al-Minūfī (**7.1**).

The table in the corpus displaying the duration of darkness, n(λ), labelled *jawf al-layl*, was compiled using the simple relation:

$$n(\lambda) \;=\; 2N(\lambda) - s(\lambda) - r(\lambda) \qquad (h_r = -19°,\;\; h_s = -17°).$$

As indicated by the title of the table in MS Dublin CB 3673, darkness lasts from the beginning of the prayer at nightfall to the "true dawn" (*wa-huwa mā bayn al-ʿishāʾ al-ākhira wa-ṭulūʿ al-fajr al-ṣādiq*).

The tables of r(λ), s(λ) and n(λ) are the main twilight tables in the corpus, but in various late copies such as MSS Cairo DM 108 and Princeton Yahuda 861,1 (**8.1**), as well as in the prayer-tables of al-Lādhiqī (**7.8**), other tables derived from these are presented either to facilitate timekeeping relative to the times of nightfall and daybreak or to serve the determination of various twilight-related religious institutions. These I now describe.

Firstly, tables of $\alpha_s(\lambda)$, labelled *maṭāliʿ al-shafaq*, "the (oblique) ascensions of (the ascendant at) nightfall", occur in various late Egyptian prayer-tables, notably those of al-Lādhiqī, and are based on the relation:

$$\alpha_s(\lambda) \;=\; \alpha_\phi(\lambda_H) \;=\; \alpha_\phi(\lambda^*) + s(\lambda)\ ,$$

where H denotes the ascendant at nightfall and the parameter for s is 17°. Tables of $\alpha_r(\lambda)$, labelled *maṭāliʿ al-fajr*, "the (oblique) ascensions of (the ascendant at) daybreak", are less common in late Egyptian prayer-tables, the function $\alpha_\sigma(\lambda)$ (see below) generally being preferred. The underlying relation is:

$$\alpha_r(\lambda) \;=\; \alpha_\phi(\lambda_H) \;=\; \alpha_\phi(\lambda) - r(\lambda)\ ,$$

where H denotes the ascendant at daybreak and the parameter for r is 19°. I have no clues as to the identity of the individual who first compiled these tables. However, the earliest reference to the two functions in the sources known to me is in the treatise by Najm al-Dīn al-Miṣrī on spherical astronomy (MS Milan Ambrosiana 227a (C49), Ch. 24: see **2.5**), written *ca*. 1325. He prescribes procedures that are equivalent to the above formulae.

[34] See King, *Ibn Yūnus*, III.16.

A group of tables occuring in MS Cairo DM 108 of the corpus, as well as in the prayer-tables of al-Lādhiqī, is associated with two activities called *salām* and *ṭafy*. It was the custom in medieval Egypt to light candles (*qanādīl*) on the minarets of mosques during the nights of Ramaḍān: the term *ṭafy* refers to the extinction of these candles 20 minutes before daybreak, which may have been the signal to the faithful that the day's fasting should begin. (In Egypt today the beginning of the fast is called the *imsāk* and is also 20 minutes before daybreak.) The *salām* was a special call of the muezzin, a few minutes before daybreak, invoking blessings on the Prophet Muḥammad. It is clear that the *muwaqqit*s in medieval Cairo were not of one mind concerning the precise time of the *salām*. From the sources investigated below it appears that the *salām* was variously timed to begin 1°, 1;30° or 2° (that is, 4, 6 or 8 minutes) before daybreak: see **Fig. 4.10** for a table based on a parameter of 1°.

The table in MS Cairo DM 108 and in most copies of al-Lādhiqī's prayer-tables which is entitled "time from sunset to the *salām*, two (equatorial) degrees before daybreak" is based on the relation:

$$\sigma(\lambda) \quad = \quad 2N(\lambda) - r(\lambda) - 2^\circ \qquad\qquad (h_r = -19^\circ)$$

In MS Paris ar. 2553 of al-Lādhiqī's tables the corresponding entries are 1° more, so that the *salām* is assumed to be at 1° before daybreak. In a later anonymous set of this function preserved in MS Cairo DM 57, computed to three digits for each 0;6° of λ by linear interpolation (**7.2**), the entries are based on those in MS Paris ar. 2553. Whilst this means that the *salām* is 1° before daybreak, the title of the tables states that it is 2° before. On the other hand, a note by the title states that the *shaykh* who compiled the tables was in error because the time-difference is actually 1° not 2°.

A related table is found only in al-Lādhiqī's prayer-tables. It is entitled "(oblique) ascensions of (the ascendant at the time of) the *salām*, two (equatorial) degrees before daybreak". The title also states that if one subtracts three degrees from the entries the result will be the "ascensions of the *ṭafy*". An entry is given for each degree of λ in 12 columns of 30 entries. If H is the ascendant at daybreak, then:

$$\alpha_\sigma(\lambda) \quad = \quad \alpha_\phi(\lambda_H) \quad = \quad \alpha_\phi(\lambda) - r(\lambda) - 2^\circ \qquad (h_r = -19^\circ)\ .$$

In MS Istanbul Reisülküttap Mustafa Efendi 582 the table of α_σ is based on the assumption that the *salām* is 1° before daybreak. In later related tables of this function in MSS Cairo DM 157, datable *ca*. 850, and DM 158, datable *ca*. 1600, values are given for each 0;3° or 0;6° of λ, respectively, generated by linear interpolation. In these two sources the entries are larger than those in the Hartford copy by 0;30° and 1°, respectively.

Tables of the function τ(λ), *al-dāʾir li-ʾl-ṭafy fī Ramaḍān ʿalā anna baynahu wa-bayn al-fajr hāʾ daraj*, "time from sunset to the *ṭafy* in Ramaḍān, 5° before daybreak", occur only in MSS Princeton Yahuda 861,1 and Cairo DM 108 amongst the sources investigated. In the former the function tabulated is:

$$\tau(\lambda) \quad = \quad 2N(\lambda) - r(\lambda) - 5^\circ \qquad\qquad (h_r = -19^\circ)\ ,$$

as one might expect. In the latter the function tabulated is in fact:

$$\tau'(\lambda) \quad = \quad 2N(\lambda) - r(\lambda) - 2\Delta D(\lambda) \qquad (h_r = -19^\circ)$$

where the values of ΔD, a correction for refraction at the horizon, are those attributed elsewhere to Ibn Yūnus (**4.11**). Note that this formula includes a correction for both horizons. This table

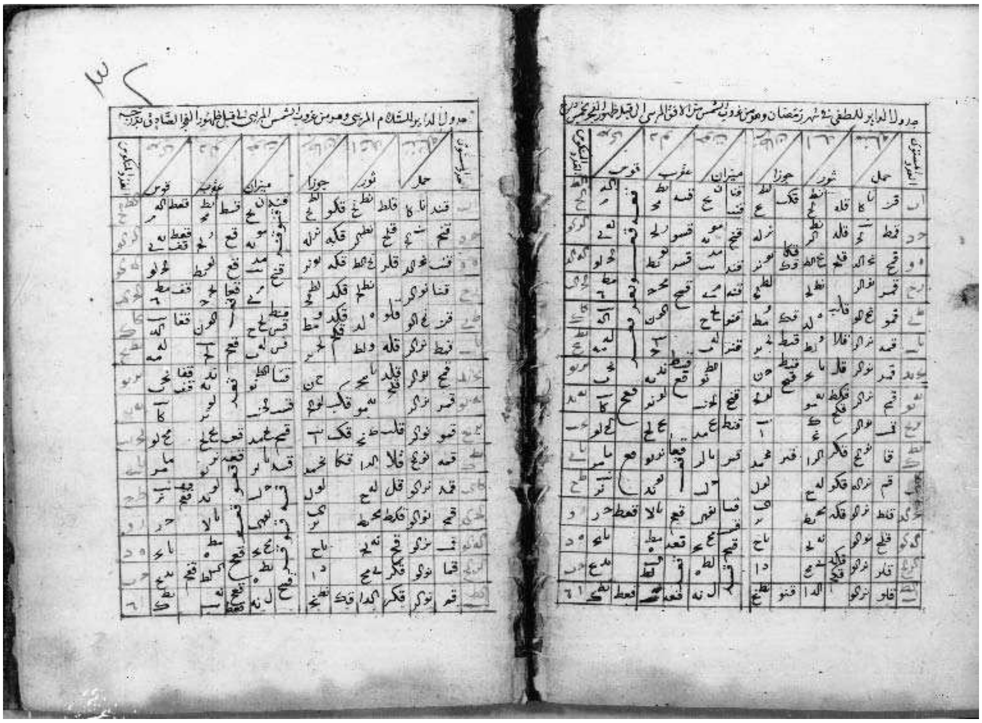


Fig. 4.10: On the right is a table displaying the time from sunset until the moment for extinguishing the candles in the lanterns on the minarets (*al-ṭafy*) in Ramaḍān, taken as 5°, that is, 20 minutes, before daybreak. On the left is a table displaying the corresponding time to the benediction on the Prophet (*al-salām*), taken as 1°, that is, 4 minutes, before daybreak. The entries in the second table are, of course, 4° more than the corresponding entries in the first. Both tables are adjusted using the "difference minutes" for the time of sunset over the visible (*marīʿ*) horizon. [From an unidentified Cairo manuscript of the Cairo corpus, courtesy of the Egyptian National Library.]

is the only one in MS Cairo DM 108 which is adjusted, silently at that, for refraction at the horizon.[35]

Tables of the function $\alpha_\tau(\lambda)$, labelled *maṭāliʿ al-ṭafy*, "the (oblique) ascensions (of the ascendant at the time) of the *ṭafy*", occur in MS Cairo DM 153 and Princeton Yahuda 861,1 of the sources investigated, as well as in MS Istanbul Reisülküttap Mustafa Efendi 582 of al-Lādhiqī's prayer-tables. Now the table of α_τ in MS Princeton Yahuda 861,1 is not based on the table of $\tau(\lambda)$ in MS Cairo DM 108. Rather, the tabulated function is:

$$\alpha_\tau(\lambda) \;=\; \alpha_\phi(\lambda) - r(\lambda) - 5° \qquad (h_r = -19°)\ ,$$

as one would expect. The table in this Istanbul manuscript is in fact based on:

$$\alpha_\tau(\lambda) \;=\; \alpha_\phi(\lambda) - r(\lambda) - 5° - \Delta D(\lambda) \qquad (h_r = -19)\ ,$$

[35] In "Astronomical Timekeeping in Medieval Cairo", p. 371, I noted that the entries would correspond to the relation one would expect (namely, that used in the Princeton manuscript) only if the value used for h_r was about -20;15°. The structure of this anomalous table is now explained.

using the values of ΔD attributed elsewhere to Ibn Yūnus. This formula includes a correction ΔD only for the western horizon, but there is no indication of any correction at all in the title of the table. Furthermore, the accompanying table of $\alpha_\sigma(\lambda)$ in the same manuscript is not corrected for horizontal refraction.

I have not found any references to functions relating to the *salām* and *ṭafy* in the Egyptian astronomical literature. In MS Princeton Yahuda 861,1 (**8.1** and **9.5**) there is a table giving the time from sunset to the *salām* for Jerusalem, but the underlying time is daybreak rather than a few minutes before. Shihāb al-Dīn al-Ḥalabī, writing in Damascus, stated that daybreak (*al-fajr*) is the time of the prayer-call (*al-ādhān*) in Ramaḍān and of the *salām* (written *s-l-m*) in the other months of the year (**11.2**). In MS Damascus Ẓāhiriyya 7564 (**11.11**) there are some cryptic notes about the time for the *salām* as practiced in Aleppo.

4.11 A table of corrections for refraction at the horizon

In MS Cairo DM 108 of the corpus, dating from the early 19th century, there is a table entitled *daqāʾiq al-ikhtilāf*, literally, "minutes of difference", here denoted by ΔD(λ): see **Fig. 4.11a**. The function is tabulated to two sexagesimal digits for each degree of λ. In some manuscripts of al-Lādhiqī's prayer-tables (**7.8**), such as MS Cairo DM 190, values of the same function to one digit are given for each zodiacal sign, along with the remark that it measures the time in minutes (*daqāʾiq*) of an equatorial degree taken by the sun to move between the true horizon (*al-ufq al-ḥaqīqī*) and the visible horizon (*al-ufq al-marʾī*).

This table in MS Cairo DM 108 also occurs in the prayer-tables of the early-16th-century astronomer Muḥammad al-Minūfī (MS Cairo DM 107, but not MS Cairo DM 177). al-Minūfī modified each of the tables associated with horizon phenomena in the prayer-tables of the Cairo corpus to account for the effect of refraction at the horizon (**7.1**). The same table occurs in MS Alexandria 4441J of the prayer-tables of the 18th-century *muwaqqit* Ibn Abī Rāya (**7.9**). Although al-Minūfī's table of ΔD(λ) is the earliest table of the function in any of the known sources, the underlying theory, at least that part of it which is in reasonable conformity with physical reality, appears, as I shall now attempt to show, to date from the 10th century.

The entries in the tables of ΔD(λ) in MSS Cairo DM 108 and DM 107, and Alexandria 4441J were calculated by linear interpolation between the values at the equinoxes and the solstices. The significant entries, in minutes of an equatorial degree, are the following:

EQ: 47 SS: 62 WS: 32 .

The attribution of these values to Ibn Yūnus is claimed by al-Minūfī. In the introduction to his prayer-tables (*e.g.*, MS Cairo DM 107, fol. 7v), he states that Ibn Yūnus had found the difference minutes to be 47 at the equinoxes, increasing by 5 for each sign up to the summer solstice and decreasing by 5 for each sign to the winter solstice. We may presume that al-Minūfī had access to a work by Ibn Yūnus that is no longer extant, for none of his known works contains any mention of refraction. In an earlier, 15th-century Egyptian source, these same values are associated with the 14th-century Damascus astronomer Ibn al-Shāṭir (**5.7**), but this association is less likely.

Now only part of the theory described by al-Minūfī and reflected in the tables makes sense.

Fig. 4.11a: A table for the time taken by the sun to move between the true and visible horizons. [From MS Cairo DM 108, fol. 74v, courtesy of the Egyptian National Library.]

It seems reasonable to assume that Ibn Yūnus would have considered the visible horizon to be at a certain arc of depression below the true horizon. If this is so, the above equinoctial and solstitial values are easily shown to be mutually inconsistent. On the other hand, if the three values are the results of observations made at the equinoxes and solstices, then only the equinoctial value is reasonable. But it seems much more likely that the solstitial values were simply computed from the equinoctial value, which was possibly derived by observation. In any case, it is worthwhile to investigate the three values further.

Consider **Fig. 4.11b**, which represents a portion of the western sky about the west-point in plane cross-section. The sun sets over the horizon at an angle $\bar{\phi}$. AB represents the true horizon and A′B′ the visible horizon at an arc of depression Δh below the true horizon. The problem is to find to a first approximation how long the sun takes to move between these two horizons. XX′ represents the path of the centre of the sun at the equinoxes and YY′ and ZZ′ its path at the summer and winter solstices, respectively. Clearly:

$$XX' = YY' = ZZ' = \Delta h \sec \phi \ .$$

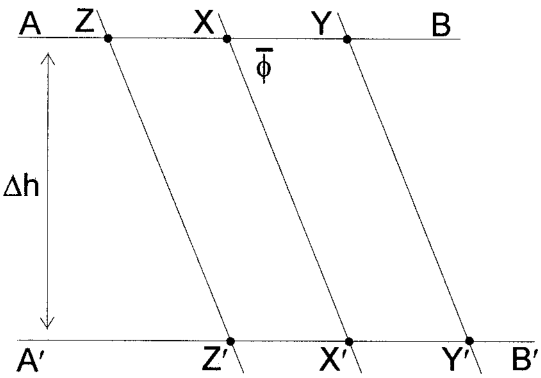


Fig. 4.11b: A plane projection of the visible and true horizons (AB and A′B′), and the solar day-circles at the equinoxes (XX′), the summer solstice (YY′) and the winter solstice (ZZ′). Not to scale.

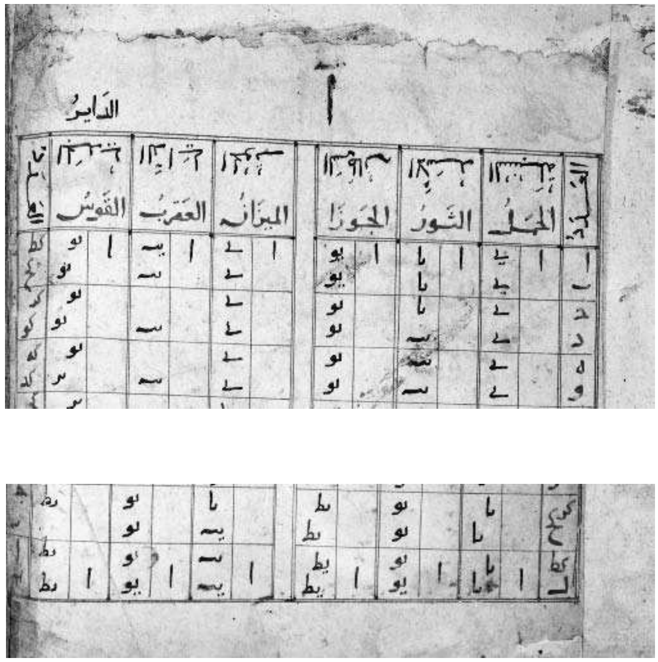


Fig. 4.11c: The table of the time since sunrise for altitude 1° from the corpus. [From MS Dublin CB 3673, courtesy of the Chester Beatty Library.]

Now the time taken by the sun to move from X to X′ is measured by the corresponding arc of the celestial equator. We assume, to a first approximation, that the difference minutes at the equinoxes, ΔD_0, equal $\Delta h \sec \phi$. Since Ibn Yūnus, according to al-Minūfī, took these as 0;47° for latitude 30°, we have:

$$\Delta D_0 \;=\; \Delta h \sec 30° \;=\; 0;47° \;,$$

whence:

$$\Delta h \;\approx\; 0;40° \;=\; {}^2/_3° \;.$$

This, then, is an approximate value of the vertical displacement of the horizon due to refraction corresponding to the value for ΔD_0 attributed to Ibn Yūnus. Note, however, that the value 0;41° is used in the table of h_v (**4.8**) to represent the apparent altitude of the sun when its centre is on the true horizon. The modern value for this displacement is 0°35′.[36]

The solstitial values attributed to Ibn Yūnus are, in the light of modern knowledge, less reasonable. A distance of 0;47° on either of the solstitial day-circles corresponds approximately to an arc of:

$$0;47° \sec \varepsilon \;=\; 0;51°$$

on the celestial equator and it is clear that *the time taken by the sun to move between the two horizons is the same at both solstices*. This time is some 0;4° more than the corresponding time at the equinoxes, according to the above calculation. Note too that, if we assume a value

[36] For a modern discussion see Smart, *Spherical Astronomy* (n. 1:37), pp. 69-70, *etc.*

of 0;40° for Δh, the corresponding times ΔD can be found from tables of $T(h,\lambda)$ in the same way as the duration of twilight (**4.10**). Thus:

$$\Delta D(\lambda) \quad = \quad T({}^{2}/_{3}{}^{\circ},\lambda^{*}) \quad \approx \quad {}^{2}/_{3}\ T(1^{\circ},\lambda^{*})\ .$$

From the table of $T(h,\lambda)$ for 1° (see **Fig. 4.11c**) we can read off the time taken by the sun to move 1° below the horizon, namely:

EQ: 70 SS/WS: 79 ,

from which it follows that the time taken by the sun to move ${}^{2}/_{3}{}^{\circ}$ below the horizon is approximately:

EQ: 47 SS/WS: 53 .

Since the variation in ΔD is relatively small, it is quite reasonable to interpolate linearly between the equinoctial and solstitial values of ΔD for intermediate solar longitudes.

Ibn Yūnus was a master of spherical astronomy, and I deem it unlikely that he would make such an elementary error as to give radically different values of ΔD at the solstices. Rather, I suspect that he originally stated that *the correction was 47 minutes at the equinoxes and increased about 5 minutes to maxima at the solstices*, which would be virtually correct. The assertion of al-Minūfī that Ibn Yūnus had stated that *the correction was 47 minutes at the equinoxes and changed by 5 minutes for each sign with a maximum at the summer solstice and a minimum at the winter solstice* is then simply garbled. My hypothesis has no substantiation in the manuscripts examined thus far and my investigations of the various Arabic manuscript sources dealing with refraction at the horizon contribute little to our understanding of the problem as treated by Ibn Yūnus and (mis)construed by his successors.

The theory of refraction at the horizon associated with Ibn Yūnus as interpreted by al-Minūfī was adopted by the latter's son ʿAbd al-Qādir al-Minūfī to serve all latitudes (**8.2**), using the approximation:

$$\Delta D(\phi,\lambda) \quad \approx \quad \phi\ /\ 30^{\circ} \cdot \Delta D(30^{\circ},\lambda)\ .$$

Four sources have been identified in which values based on this formula are presented, namely, for Mecca, Crete, Damascus and Istanbul (**6.10**, **8.8**, **11.13** and **14.9**). No Muslim astronomer is known to have noted that the underlying equinoctial and solstitial values were mutually inconsistent.

4.12 The titles and instructions

The "instructions" accompanying the table in MS Dublin CB 3673 and the slightly modified version in MS Cairo DM 108 contain virtually no information about the way in which the tables were intended to be used in practice, nor do they reveal how the tables were computed. In the Dublin copy the title *Kitāb Ghāyat al-intifāʿ* ... , which may be (and has been) rendered "Very Useful Tables ... ", is given to the whole corpus. Also al-Kattānī called his tables *Kitāb Nihāyat al-intifāʿ* ... , which may be rendered "Extremely Useful Tables ... ". Otherwise the various copies of the tables of $T(h,\lambda)$, $t(h,\lambda)$ and $a(h,\lambda)$ bear the titles *Kitāb al-Dāʾir*, *Kitāb Faḍl al-dāʾir* and *Kitāb al-Samt*, or various combinations, or they bear no titles at all.

The introduction begins with pious statements about God as the Creator of the Universe. The Qurʾānic verse X.5/5:

> "(God) made the sun as a shining light and made the moon a luminary and measured out its mansions"

is continued thus (fol. 83r:5):

> "for perfecting prognostications about what He orders and what He effects."

Then the verse XXXVII.6/5:

> "(God) adorned the lower heavens with the stars"

is continued (MS CB 3673, fol. 83r:5-6):

> "and the times are known by what rises and sets."

Since, states the author, astronomical timekeeping (*ʿilm al-mawāqīt*) was ordained by God, he thought he would prepare these table to be the "limit of usefulness" (*ghāyat al-intifāʿ*). The instructions proper are translated in full below, using MSS fols. 83r-83v of the Dublin manuscript and fols. 2v-3r of the Cairo one. The paragraph numbers are my own. §9 does not occur in copies of the introduction that are not followed by the azimuth tables. §10a appears to be original to al-Maqsī's introduction, whereas §10b, whose highly sophisticated contents merit special commentary, was probably inserted by someone more competent that al-Maqsī. The instructions read:

> 1 To determine the time since sunrise up to any instant you wish, enter the solar longitude in the table of the instantaneous altitude and you will find the required time, if the solar altitude is in the east. If it is in west, subtract the value in the table from the diurnal arc and the remainder will be the time up to that instant. God – may He be exalted – knows best. Example: Suppose the sun is in 21° of Taurus and the instantaneous altitude is 30°. We find 35;38°[a] in the table: this is the time from sunrise to the instant in question. If the altitude is in the west, subtract 35;38° from the diurnal arc, which is 201;46°[b]: the remainder is 166;8°, which is the required time.
>
> 2 To determine how much of the day has passed and how many hours of daylight are left, divide the time since sunrise by the number of degrees corresponding to a seasonal day-hour for the day in question, or by 15 if you want equinoctial hours. The result will be the number of hours of daylight that have passed. Subtract the seasonal hours from 12 and the equinoctial hours from the number of hours of daylight in question: the remainder will be the number of hours of daylight left. Example: Suppose the sun is in 11° of Aries and the instantaneous altitude is 45° in the east. We find the time since sunrise for this altitude as before: it is 53;19°[c]. We divide this by the number of degrees corresponding to a seasonal day-hour, which is 15;24°[d], and the result is [3][e];27. These are the hours of daylight that have passed; subtract them from 12 and the remainder, which is the number of hours of daylight left, is 8;33. If we divide the time since sunrise by 12, the quotient is [3;33][f] and this is the number of equinoctial hours that have elapsed. We subtract these from the number of equinoctial hours of daylight on the day in question, which is 12;20, and the remainder is 8;47. This is the number of hours of daylight remaining. *I have included a table at the end of the book in which the entry corresponding to the solar longitude is multiplied by the time since sunrise to give the seasonal day-hours passed since sunrise.*
>
> 3 To determine (the altitude of the sun for) the seasonal and equinoctial hours, enter the solar longitude in the tables of the time since sunrise for each altitude and turn

through table after table until you find a time as large as the degrees corresponding to the seasonal or equinoctial hours. Interpolate if necessary and the result with be the required altitude. Example: Suppose the sun is in the last degree of the sign of Gemini and we want the altitude at the end of the first seasonal hour. We enter the solar longitude in the tables, turning through table after table, and find opposite the longitude in the table for (altitude argument) 13° a time of 16;24°[g]. We subtract this from the number of degrees in one seasonal hour, which is 17;26°[h], and the remainder is 1;2°. We divide this by the difference between the time [in the tables for altitude 13° and for altitude 14°, which difference is 1;15°. The ratio of one to the other is 0;50, which we add to the first altitude][i] and so the required altitude is 13;50°. If we want the altitude after one equatorial hour, we enter the solar longitude in the tables and we find opposite it 13;56°. We subtract this from 15° and the remainder is 1;4°, which we divide by the difference between the time for altitude 11° and that for 12°, which is 1;15°. The ratio of the one to the other is 0;51, which we add to the first altitude, obtaining 11;51° for the required altitude.

4 To determine the ascendant at the eastern horizon, add the time since sunrise to the oblique ascensions of the solar longitude and the result will be (the oblique ascensions of) the ascendant at that time. Example: Suppose the sun is in 18° of Scorpio and the instantaneous altitude is 25°. We take the time since sunrise for this altitude and find it to be 33;20°[j]. We add this to the oblique ascensions of the sun, namely, 235;52°[k], and the sum is 269;12°. We take the longitude corresponding to this: the result is 16;18°, which is the degree of Sagittarius that is rising.

5 To determine the hour-angle at the beginning of the afternoon prayer, you should know that *there is a table for this included at the end of the book*. You enter the solar longitude as argument and read off the required value.

6 *To determine the semi diurnal arc, you enter the solar longitude in the appropriate table* and read the required value.

7 To determine the duration of morning twilight, which is the remainder of the nocturnal arc to sunrise, you enter the longitude directly opposite the solar longitude on the ecliptic in the table of the time since sunrise for altitude 19° and the value you find will be the required time.

8 To determine the time from sunrise to the moment when the sun is in the azimuth of the qibla, enter the solar longitude in the appropriate table and read off the required value.

9 To determine the azimuth, proceed as with the tables of time. The direction of the azimuth is south if the sun is in the southern zodiacal signs, or if it is the northern signs and the altitude is greater than the altitude in the prime vertical; otherwise, it is north. God knows best.

10a Numerous interesting facts about the operations of timekeeping by day and night will be revealed to anyone who examines these tables closely. However, in this book we have chosen to keep the discussion short.

10b Certain interesting facts will be revealed to anyone who examines these tables closely. For example, if the solar altitude equals the declination, then both the azimuth

and the time since sunrise are equal to the complement of the other, Further, if you take the declination as altitude and you know the complement of the time since sunrise for the solar longitude, then, if the latitude is 30°, it will be the half excess of daylight. Numerous other facts and points of interest relating to night and day also become evident. God – may He be exalted – knows best."

[a] As on fol. 54v. [b] As on fol. 12r. [c] As on fol. 39v. [d] Fol. 13r has the more accurate value 15;25°. [e] Text: 4 (*sic*). [f] Text: 4;13 (*sic*). [g] As on fol. 71v. [h] As on fol. 13r. [i] [ ] missing in the MS. [j] As on fol. 59v. [k] This is accurate, and is the value given in the *Ḥākimī Zīj*. MS Hartford TS 621, p. 21, has 235;51°.

These instructions (with the exception, perhaps, of §10b) were written by al-Maqsī, not by Ibn Yūnus as I stated in my first study of the Cairo corpus.[37] They were intended to precede his tables of T(h,λ) discussed in **5.4**. This explains why there is no mention here of the hour-angle t(h,λ) and most of the minor functions tabulated in the corpus. Nevertheless the style differs markedly from that of al-Maqsī's introduction to his tables for sundial construction.[38]

In the new source MS Gotha A1402 the instructions are in al-Maqsī's name and are followed by his tables of T(h,λ). In MS Cairo DM 53 they also bear al-Maqsī's name and are followed by al-Bakhāniqī's arrangement of the tables of (T,t,a). In MS Cairo DM 108 the introduction ends with al-Maqsī's name. However, in MS Dublin CB 3673, which was the major source of my first study of the Cairo corpus, the instructions are introduced in the name of Ibn Yūnus. It is now apparent that the earlier scholar's name was inserted by the copyist: it is preceded by a badly-written phrase with Arabic letters improperly pointed, which I originally read as *anwaʾ al-nās* and, with considerable stretching of the imagination, interpreted as meaning something like "most proficient of astrologers". However, the phrase must be a corruption of Abu 'l-ʿAbbās, the *kunya* of al-Maqsī. The copyist of the Chester Beatty manuscript was of the opinion that Ibn Yūnus deserved more credit that al-Maqsī for the tables which he was copying: that copy does indeed contain more tables by Ibn Yūnus than any other known source. He omitted al-Maqsī's name and inserted that of Ibn Yūnus, not realizing, perhaps, that Abu 'l-ʿAbbās was al-Maqsī's *kunya* rather than that of Ibn Yūnus. This copyist also included the remarks of §10b above, which imply that the author had before him tables of *both* a(h,λ) and T(h,λ). I do not think that either al-Maqsī or al-Bakhāniqī would have noticed these subtleties. Rather, they are reminiscent of Ibn Yūnus' remark in Ch. 15 of the *Ḥākimī Zī*j (MS Leiden Or. 143, p. 335:17-20):[39]

"If for a particular locality there are no (tables of) oblique ascensions available but there is a table of the azimuth computed for the first point of Aries giving values for each degree of solar altitude up to 24°, then enter the solar declination as argument in this table and the corresponding value in the table will be the half excess of daylight."

In al-Maqsī's introduction the parameters advocated for morning and evening twilight were probably originally 20° and 16°. This is the case in MSS Cairo DM 108 and Gotha A1402,

[37] King, "Astronomical Timekeeping in Medieval Cairo", pp. 376-377.
[38] See n. 4:6.
[39] King, *Ibn Yūnus*, III.15.1(b).

but in MS Dublin CB 3673 only the value for morning twilight is mentioned, and in MS Cairo DM 53 the parameters 19° and 17° are proposed. On the other hand, MS Dublin CB 3673 is the only copy of the corpus containing twilight tables based on parameters 20° and 16° (although tables based on the more common parameters 19° and 17° are also included in this manuscript). See further **4.10**.

In the introduction to the tables in the Chester Beatty manuscript separate tables of:

$$\tilde{h}(\lambda),\ t_a(\lambda),\ D(\lambda) \text{ and } T_q(\lambda)$$

are specifically mentioned. In the introduction in MS Cairo DM 108 only separate tables of:

$$\tilde{h}(\lambda),\ D(\lambda) \text{ and } T_q(\lambda)$$

are specifically mentioned. In both sources there is mention of the tables of $a(h,\lambda)$, but this seems to be a later addition to al-Maqsī's original introduction. In **5.4** I discuss al-Maqsī's introduction and tables further.

CHAPTER 5

THE DEVELOPMENT OF THE MAIN CAIRO CORPUS

5.0 Introductory remarks

It is not yet possible to give a completely clear picture of the development of the main Cairo corpus described in **Ch. 4**. New manuscripts of tables from the corpus could add considerably to our knowledge and it is not too much to hope that some will turn up in future investigations of uncataloged manuscript collections.

5.1 The tables for spherical astronomy in the *Ḥākimī Zīj*

The celebrated Fatimid astronomer Ibn Yūnus (**I-2.1.1** and **II-4.1.1**) tabulated numerous spherical astronomical functions in his *Ḥākimī Zīj*.[1] The title-folio of the Leiden manuscript is shown in **Fig. 5.1**. It is written in the hand of Ibn Abi 'l-Fatḥ al-Ṣūfī, and bears a notice of ownership by Taqi 'l-Dīn ibn Maʿrūf, indicating that it once formed part of his library at the short-lived observatory in Istanbul.[2] No other *zīj* contains so many tables of this kind; Ibn Yūnus thus began a trend which was to distinguish the later traditions of astronomical timekeeping in Egypt and Syria. The tables in the *Zīj* which have values computed to minutes, seconds or thirds [here indicated by [1], [2] and [3]) for each 1° of λ (or h, where relevant), occasionally for each 0;10° (here indicated by *). The underlying parameters, where relevant, are:

$$\phi = 30;0^\circ \quad \text{and} \quad \varepsilon = 23;35^\circ .$$

The tables display the following functions:

$\delta(\lambda)$* [2]; $\alpha(\lambda)$ [2]; $\alpha'(\lambda)$ [1]; Sin $\alpha(\lambda)$ [3]; Tan $\delta(\lambda)$ [3];
$\alpha_\phi(\lambda)$ [1] (for each 1° of ϕ from 1° to 48°);
$d(\lambda)$ [3] and Sin $d(\lambda)$ [3];
$\psi(\lambda)$ [2] and Sin $\psi(\lambda)$ [2]; $h_0(\lambda)$ [2]; $k(h)$ [2];
$a(\lambda)$ [1] for two different altitudes; $h(\lambda)$ [1] for ten different azimuths;
and $h_q(\lambda)$ [1] ($q = 52°$).

Of particular importance for the development of the main Cairo corpus was the inclusion of tables of $a(h,\lambda)$ for two values of h (**I-5.1.1**), and $h(h,\lambda)$ for ten values of a (**I-4.7.1**); such tables are not known from earlier sources. The tables of:

---

[1] On Ibn Yūnus see n. **I**-2:3. King, *Ibn Yūnus*, deals specifically with the spherical astronomy in the *Ḥākimī Zīj*.

[2] See also King, "Astronomy in Fatimid Egypt", p. 499, and also Ünver, *Istanbul Observatory* (in Turkish), figs. 41-42, for the marks of ownership on other manuscripts from Taqi 'l-Dīn's library.

Fig. 5.1: The title folio of the *Ḥākimī Zīj*, containing biographical information on the author in the hand of the late-15th-century Cairene astronomer Ibn Abi ʿl-Fatḥ al-Ṣūfī (**I-9.10**). On the right is a notice of possession of the Istanbul astronomer Taqi ʿl-Dīn (see **Fig. I-1.0a**). [From MS Leiden Or. 143, fol. 1r, courtesy of the Universiteitsbibliotheek.]

$$\psi(\lambda),\ h_0(\lambda);\ k(h),\ \text{and}\ h_q(\lambda)$$

are also the earliest known tables of their kind. Also significant is the inclusion of the various auxiliary functions, notably:

Sin $\alpha(\lambda)$ (**I-7.2**); Tan $\delta(\lambda)$ (**I-7.1.3**); Sin $d(\lambda)$;
$k(h)$ (**I-8.1.1**); and Sin $\psi(\lambda)$ (**I-8.3.1**) .

There are also tables of Sin $\psi(\lambda)$ [3] and k(h) [2] for ϕ = 33;25°, serving Baghdad and perhaps an indication of the source of part of his inspiration.

5.2 Ibn Yūnus' solar azimuth tables

Ibn Yūnus compiled a work called *Kitāb al-Samt* containing a complete set of tables of $a(h,\lambda)$. Subsequently the entries for h ≥ 80° were lost and they were recomputed by an anonymous incompetent who included tables for 80° ≤ h ≤ 89° even though the maximum solar altitude at Cairo is only about $83^1/_2$°. In the 14th century the tables for 80° ≤ h ≤ 83° were recomputed more carefully by Ibn al-Rashīdī (**4.4**). al-Bakhāniqī used the better set for his edition of the corpus (**5.6**), extant in MSS Cairo DM 108, Cairo DM 53, Istanbul Nuruosmaniye 2903 and 2925, Cairo DM 690 and Cairo DM 616, but the uncorrected tables also survive in MSS Dublin CB 3673, Cairo MM 137 and Gotha A 1410 of the *Kitāb al-Samt*. In these three sources, entries are given up to 88°, 86° and 89°, respectively: see **Fig. I-5.1.1a**. The corrected tables of the *Kitāb al-Samt* were also copied separately, as in MS Cairo Azhar *falak* 4382.

Some copies of the *Kitāb al-Samt* have tables of h_v, h_q (q = 52° or 53°), h_{q*} (q = 53°) and a(h) (λ' = 0° and 90°) appended, as well as ψ and h_0. MS Dublin CB 3673 contains a set of uncorrected tables of $a(h,\lambda)$ as well as of h_v, h_q (q = 52°), ψ and h_0. MS Gotha A 1410 contains the uncorrected azimuth tables and also tables of h_q (q = 53°) and h_v. MS Cairo Azhar *falak* 4382 contains the azimuth tables and a(h) (λ' = 0° and 90°). MSS Berlin Ahlwardt 5753 and Escorial ár. 924,7 of the azimuth tables are incomplete. The tables of h_q (q = 52°), and ψ and h_0 (both to three digits), were taken from the *Ḥākimī Zīj*. The rather carelessly computed table of h_v usually has the entry 0;41° for the winter solstice rather than zero, to correct for refraction at the horizon. The remark in MS Istanbul Nuruosmaniye 2925 that the table of h_v (which has the entry 0;41°) is due to Ibn al-Rashīdī may mean either that he computed the table or that he introduced the modification for the winter solstice. If I interpret al-Bakhāniqī correctly as asserting that the table of h_{q*} (q = 53°) was due to Ibn Yūnus, then it is reasonable to suppose that Ibn Yūnus also computed h_q (q = 53°), although these tables for q = 53 ° are less accurate than his table for q = 52°.

5.3 A lost work of Ibn Yūnus?

I conjecture that Ibn Yūnus also compiled a small set of prayer-tables for the latitude of Cairo-Fusṭāṭ, rather like those for Mecca preserved in MS Cairo MM 68 (**6.10**), and containing tables of such functions as:

$$H, D, h_a, t_a, r \text{ and } s\ .$$

Perhaps it was in such a work that he proposed the equinoctial and solstitial values for the correction ΔD due to refraction at the horizon. MS Cairo MM 68 contains a set of these corrections for Mecca.

al-Bakhāniqī (**5.6**) appears to state that at the end of a copy of Ibn Yūnus' azimuth tables used by him there had been added (*uḍyafa*) tables of the functions:

$$t_q, H, h_a, z_a, t_a, T_a, \tilde{h}, r \text{ and } s .$$

These might have been Ibn Yūnus' prayer-tables and I should expect that the table of t_q would be based on the parameter q = 53° and that the parameters underlying the tables of r and s would be 20° and 16°.

The use of the Arabic passive *uḍyafa*, "it was added," is a little curious in this context, and the verb should be feminine, *i.e. uḍyafat*, since it refers to the noun *jadāwil*, "tables". Note that the same Arabic word written *ʾḍyf* can also be read *uḍīfu*, "I am adding", which does not fit syntactically, but the word could be a corruption of *aḍiftu*, "I added", which makes more grammatical sense. This would mean that it was it was al-Bakhāniqī who compiled the prayer-tables, which seems very unlikely. He lived in the mid-14th century when prayer-tables for Damascus had already been compiled by individuals such as al-Mizzī (**9.2**), apparently using the model of the Egyptian tables.

As noted above (**2.5**), Najm al-Dīn al-Miṣrī attributes the twilight parameters 20° and 16° to Ibn Yūnus. He is clearly quoting a work compiled after the *Ḥākimī Zīj*, since there the parameter 18° is given for both morning and evening. Likewise ʿAbd al-Qādir al-Minūfī (**8.2**) states that Ibn Yūnus presented his values for the effect of refraction at the horizon in a work "other than his *Zīj*". This suggests that Ibn Yūnus compiled either a treatise in which these topics were discussed, or, more likely, a small set of tables of r(λ) and s(λ) based on parameters 20° and 16 ° and instructions on how to correct these for the effect of refraction at the horizon.

I have located three different tables of the function $h_a(\lambda)$ for Cairo (*cf.* **4.9**) and consider it likely that Ibn Yūnus compiled one of them. I also think the table of $t_a(\lambda)$ in the corpus is due to Ibn Yūnus. The supposition that Ibn Yūnus tabulated r(λ) and s(λ) explains why al-Maqsī merely indicated the way in which these functions could be derived from the tables of T(h,λ). I suspect that al-Maqsī's table of $T_q(\lambda)$ (q = 53°) is based on a table of $t_q(\lambda)$ computed by Ibn Yūnus.

In the *Ḥākimī Zīj* Ibn Yūnus presents several methods for finding T and t, of which one is suitable for tabulating T(h,λ). He prescribes:

$$T(h,\lambda) \;=\; d(\lambda) + \text{arc Sin } \{ \text{ Sin } h \cdot G(\lambda) - \text{Sin } d(\lambda) \} ,$$

where:

$$G(\lambda) \;=\; R^2 / \{ \text{ Cos } \delta(\lambda) \text{ Cos } \phi \} .$$

Note also that:

$$t(h,\lambda) = \text{arc Cos } \{ \text{ Sin } h \cdot G(\lambda) - \text{Sin } d(\lambda) \} ,$$

although this is not explicitly stated by Ibn Yūnus. However, he did carefully tabulate both d(λ) and Sin d(λ). He also discussed the important function G(λ) at great length, but did not tabulate it in the *Ḥākimī Zīj*. A table of:

$$G'(\lambda) \;=\; R / \text{Cos } \delta(\lambda)$$

occurs in MS Paris BNF ar. 2513, fol. 62v, of the 13th-century Egyptian *Muṣṭalaḥ Zīj* (**6.6**) and the entries are badly garbled, which suggests successive careless copying from an earlier source. A simpler formula for tabulating t(h,λ) not explicitly stated in the *Ḥākimī Zīj* is:

$$\text{Cos } t(h,\lambda) \;=\; \text{Sin } h \cdot G(\lambda) - \text{Sin } d(\lambda) .$$

This follows immediately from another formula that Ibn Yūnus does use,[3] namely:

[3] King, *Ibn Yūnus*, III.15.3c and e.

$$\text{Vers } t(h,\lambda) = \text{Vers } D(\lambda) - \text{Sin } h \cdot G(\lambda) .$$

Again t(h,λ) can be found from a(h,λ) by using another formula expounded by Ibn Yūnus,[4] namely:

$$t = \text{arc Sin } \{ \text{Cos } h \cdot \text{Cos } a / \text{Cos } \delta \} .$$

It seems very unlikely that Ibn Yūnus would have used this indirect method of finding t(h,λ), even though he had tabulated a(h,λ) and possibly also the reciprocal of Cos δ(λ). al-Bakhāniqī states that he added to the corpus a table called *al-dāʾir al-āfāqī*, of which a reasonable translation would be "table for finding time for all latitudes", but he says that the table was for finding *the solar longitude* by a method of Ibn Yūnus. Suppose the illustrious astronomer had compiled a set of tables of the function:

$$p(h,\lambda) = \text{Sin } h \cdot G(\lambda).$$

To do this it would be reasonable to first tabulate:

$$n \cdot G(\lambda) , \qquad n = 1, 2, \dots , R \ (= 60)$$

so that if Sin h = x;y then:

$$p(h,\lambda) = x \cdot G(\lambda) + y \cdot G(\lambda) / R ,$$

and the multiplication is transformed into an addition.[5]

Again, Najm al-Dīn's enormous table of T(H,h,D) for all latitudes was compiled in Cairo not long after al-Maqsī's table of T(h,λ). No *ṭaylasān* tables of T(H,h) are known to have been compiled specifically for the latitude of Cairo, but it is appropriate to mention that Ibn Yūnus had access to the *Zīj*es of the Banū Amājūr, one of which contained such a table for Baghdad (**3.2**). My current hypothesis is that Ibn Yūnus compiled a set of tables for timekeeping which inspired (a) Najm al-Dīn al-Miṣrī to compile his universal table for timekeeping; (b) al-Maqsī to compile his table of T(h,λ) for Cairo; and, in addition, (c) Abu 'l-ʿUqūl to compile his tables of T(h,λ) for Taiz.

5.4 al-Maqsī's tables of time since sunrise

The tables for timekeeping called *Kitāb al-Dāʾir* compiled by al-Maqsī (**4.5**) consisted of a complete set of tables of T(h,λ) and four additional tables of:

$$D(\lambda),\ t_a(\lambda),\ T_q(\lambda)\ (q = 53°) \text{ and } 1/\tilde{h}(\lambda) .$$

The tables were preceded by an introduction in which each of these functions was specifically mentioned (**4.12**). MS Gotha A1402 (**4.1**) is the closest manuscript to al-Maqsī's original set. It contains his introduction and a complete set of tables of T(h,λ) followed by the four smaller tables in the order noted above. The parameters advocated for twilight in the introduction are 20° and 16°. The other tables which follow the one of $1/\tilde{h}(\lambda)$ are to be regarded as later additions to al-Maqsī's set. In MS Berlin Ahlwardt 5753 (**4.1**) there are tables of $t_a(\lambda)$ and $1/\tilde{h}(\lambda)$ immediately following a set of tables of T(h,λ): see **Figs 5.4a-b**. However, in all other sources investigated T(h,λ) is tabulated either together with t(h,λ), or together with t(h,λ) and a(h,λ) in al-Bakhāniqī's arrangement of the corpus.

---

[4] *Ibid.*, III.15.3d.
[5] See n. **I**-1:22.

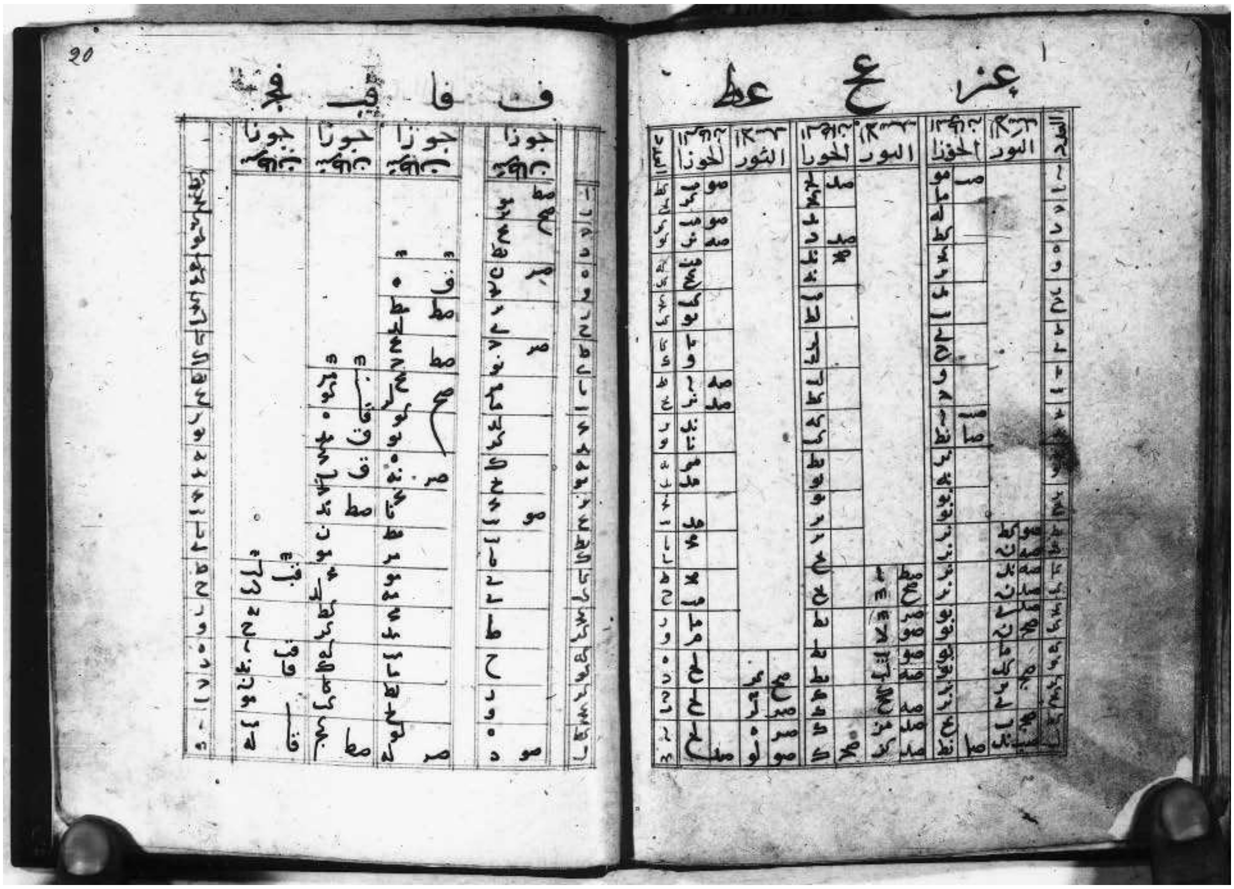


Fig. 5.4a-b: (a) The tables of time since sunrise for values of the solar altitude from 77° to 83° as found in the Berlin manuscript, and (b) the tables of $t_a(\lambda)$ and $1/\tilde{h}(\lambda)$ which immediately follow. These were the only tables of the corpus known to Carl Schoy and mentioned in his *Gnomonik der Araber* (1923), p. 53. [From MS Berlin Ahlwardt 5753, fols. 19v-20r and 20v-21r, courtesy of the Deutsche Staatsbibliothaek, Berlin.]

al-Maqsī uses the phrase *raʾaytu an uḥarrir*, "I thought I would compile / edit / prepare" when introducing his tables. He makes no reference to any earlier tables of the same kind. In the introduction to his sundial tables he uses the verb *ḥasaba*, "to compute", and mentions various of his predecessors and contemporaries, with the notable exception of al-Marrākushī who compiled similar tables (**2.7**). It seems to me probable that al-Maqsī compiled his tables of $T(h,\lambda)$ by using his own table of $D(\lambda)$ and some earlier hour-angle tables. I suspect also that his table of $t_a(\lambda)$ is not original and that his table of $T_q(\lambda)$ was derived from an earlier table of $t_q(\lambda)$: I think Ibn Yūnus was compiler of both the tables of $t_a(\lambda)$ and $t_q(\lambda)$. Finally, al-Maqsī's table of $1/\tilde{h}(\lambda)$, which is rather carelessly computed, was probably compiled from rounded values of $D(\lambda)$.

5.5 Ibn al-Kattānī's hour-angle tables and prayer-tables

MS Istanbul Kılıç Ali Paşa 684 is a unique copy of an extensive set of tables compiled by Ibn al-Kattānī (**4.1.4**). The manuscript was executed in Cairo in the author's own elegant hand and is dated 768 H [= 1366]. MS Cairo MM 72, preserved in Cairo, contains a set of solar altitude tables copied in 747 H [= 1346/47] by Ibn al-Kattānī but not necessarily computed by him (**6.9**, illustrated).

The Kılıç Ali Paşa manuscript is of considerable importance for the present study. The work it contains is entitled *Kitāb Nihāyat al-intifāʿ fī maʿrifat faḍl al-dāʾir min al-falak min qibal al-irtifāʿ*, "Extremely Useful Book for Finding the Hour-angle from the (Solar) Altitude", and was patronized by Burhān al-Dīn Ibrāhīm al-Battanūnī.[6] In his introduction to the tables Ibn al-Kattānī states that when he saw the tables of time since sunrise (*jadwal al-dāʾir min al-falak*) of al-Maqsī, he got upset about the fact that the format of the tables caused the upward arguments to run only from 0 to 29 and he wanted to make both vertical arguments run to 30°. He gives an example for $\lambda = 360°$ and $h = 60°$ and computes $T = 90°$, remarking that al-Maqsī gives the value 85;1°, and there is a difference of 4;59°. Ibn al-Kattānī's naïveté is well illustrated by the above: al-Maqsī's value 85;1°, the first entry in his tables of $T(h,\lambda)$ for $h = 60°$, is for $\lambda = 1°$ rather than the equinox. Ibn al-Kattānī goes on to say that most of the astronomical works he knew of dealt with the hour-angle rather than the time since sunrise and this fact was what inspired him to prepare the present work. The major part of the tables which follow consists of a complete set of tables of the hour-angle $t(h,\lambda)$ derived from al-Maqsī's tables of $T(h,\lambda)$. The entries are the same as those in the hour-angle tables in the main Cairo corpus (**4.5**) except that there are 31 rows of entries on each page rather than 30, both arguments running from 0° to 30°. Thus Ibn al-Kattānī has contrived to include the values for the equinoxes in the hour-angle tables: of course, the other entries in the top row merely duplicate the corresponding entries in the bottom row.

The remainder of the tables of Ibn al-Kattānī consists mainly of a set of prayer-tables, having basically the same entries as the Cairo corpus, but Ibn al-Kattānī's distinctive format. The functions which he tabulates are:

[6] I have not identified this individual.

h_q and t_q ($q = 53°$), H, h_a, D, $\tilde{h}$, t_a, T_a, r (see below), s, ψ, δ^*, α', α_ϕ, α_r, $\Delta\alpha$ and α^* .
Three different tables are given for r, the duration of morning twilight, based on parameters 20°, "according to recent scholars" (*raʾy al-mutaʾakhkhirīn*), 18°, "according to earlier scholars" (*raʾy al-mutaqaddimīn*), and 19°, "according to the compiler (of these tables)" (*raʾy wāḍiʿihi*). In each of these three tables the entries in the first row other than the equinoctial value are *not* the same as the corresponding entries in the last row: that is, Ibn al-Kattānī – may God forgive him – has interpolated between the values for, say, arguments 1^s 30° (= 2^s 0°!) and 2^s 1°, to find the value for 2^s 0°. One table is given for s, the duration of morning twilight, and the underlying parameter, which is not stated, is 16°; it does not display the absurdities of the tables for evening twilight. The table of ψ, the solar rising amplitude, is computed to three sexagesimal digits and is taken originally from the *Ḥākimī Zīj*, although this is not stated. It contains numerous copyist's errors.
The remaining tables consist of a solar longitude table giving positions in signs, degrees and minutes for each day of the Coptic year 1081 Diocletian [= 1364] (the entry for Tūt 1 is Virgo 13;55°) and two tables displaying the names of stars culminating at daybreak and culminating at nightfall for each degree of solar longitude.

Ibn al-Kattānī seems to have been the first to compile a complete set of hour-angle tables for Cairo, but it is very curious that his name is nowhere associated with the numerous other copies of the hour-angle tables which I have inspected. I doubt that any of his prayer-tables are original. Only in MS Istanbul Kılıç Ali Paşa 684 are there tables for Cairo with Ibn al-Kattānī's format, and later Egyptian astronomers, with good reason, prefered the original format of Ibn Yūnus. I know of only one other table for Cairo which displays the equinoctial value: in the margin of MS Gotha A 1410, fol. 36r, by the side of a table of h_q ($q = 53°$) the equinoctial value 46;13° is given. This is the same value given by Ibn al-Kattānī.

5.6 al-Bakhāniqī's edition of the corpus

Of prime importance for our investigations is a note by al-Bakhāniqī (**5.1**) which he added to al-Maqsī's introduction (**5.4**) when preparing his own edition of the corpus. The distinguishing feature and virtually sole innovation of this edition is that the values of the time since sunrise, hour-angle and azimuth are tabulated together, a triplet of entries (T,t,a) being given for each pair of arguments (h,λ): see **Fig. 5.6a-b**. al-Bakhāniqī's note is contained, for example, in MSS Cairo DM 108, Cairo DM 53, Istanbul Nuruosmaniye 2925, Cairo DM 616 and Cairo DM 739. The version in MS Cairo DM 53 is incomplete and rather corrupt. I present an annotated translation of the version in MS Cairo DM 108 (see **Fig. 5.6c**). (The texts in the other manuscripts offer no variants of consequence.) The most important sections, which are omitted in MS Cairo DM 53, are in italics in the translation.

> "The wretched slave of God Aḥmad ibn Muḥammad ibn Aḥmad al-Azharī known as al-Bakhāniqī said: I have arranged this book so that under each zodiacal sign there are three tables. The first of these gives the time since sunrise by al-Maqsī – may God have mercy on him – and the second gives the hour-angle. *The values of this latter function may differ in some cases from previous calculations*:[a] I calculated them by using the corresponding values (of the time since sunrise and half diurnal arc).[b] The

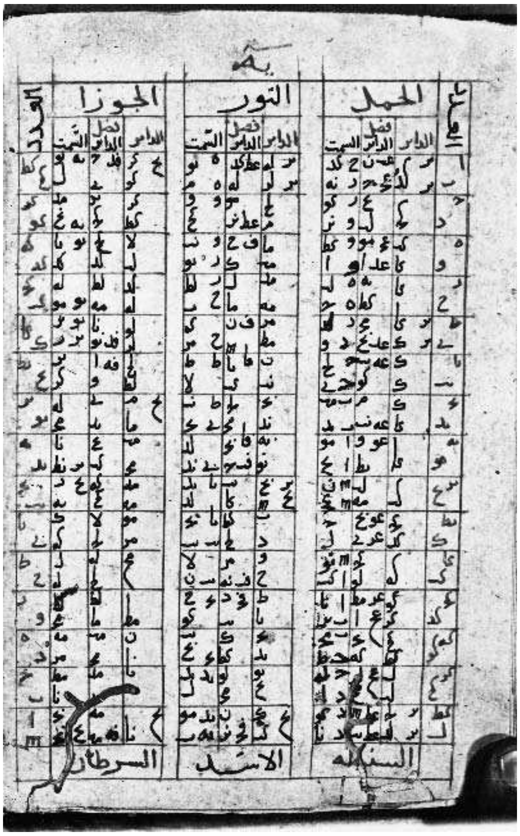


Fig. 5.6a: In al-Bakhāniqī's edition of the main tables of the Cairo corpus, the time since sunrise, the time till midday and the azimuth are tabulated side by side, as a function of the solar altitude, here 15°. [From MS Cairo DM 444, courtesy of the Egyptian National Library.]

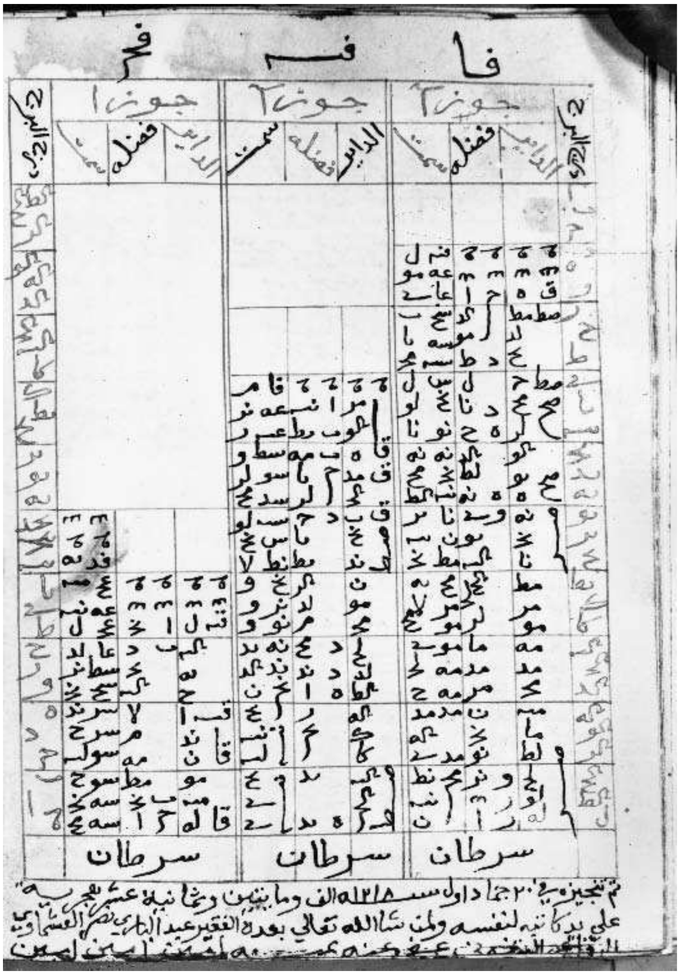


Fig. 5.6b: The last page of tables showing the problems with the triplets of entries for t and T, as compared with those for a: see **4.5**. [From MS Cairo DM 108, fol. 65v, courtesy of the Egyptian National Library.]

third function is the azimuth for the altitude in question computed by the Shaykh Shams al-Dīn Abū ʿAbdallāh Muḥammad ibn Ibrāhīm ibn ʿAbdallāh al-Rashīdī – may God have mercy upon him[c] – correcting the errors introduced by repeated copying of the azimuth tables of Abu 'l-Ḥasan[d] ibn Yūnus – may God have mercy on him. *He*[e] *had preceded these with two tables on a single page, one giving the solar rising azimuth and the other the solar altitude in the prime vertical. (He also included) tables of the solar altitude in the azimuth of the ventilator and in the azimuths of the qibla (S.E.) and the perpendicular direction (S.W.), on the basis that its azimuth (measured from the prime vertical) was 37°, and an additional complementary*[f] *table for the azimuth at the equinoxes and solstices. To these there was added*[g] *at the end of the book the tables which al-Maqsī had at the end of his book,*[h] *as well as the hour-angle when the sun is in the azimuth of the qibla, the solar meridian altitude, the solar*

Fig. 5.6c: The text of al-Bakhāniqī's remarks on the corpus. [From MS Cairo DM 108, fols. 3v-4r, courtesy of the Egyptian National Library.]

altitude at the beginning of the afternoon prayer and its Cotangent, the time from midday to the beginning of the afternoon prayer and the time remaining till sunset, and also the number of equatorial degrees in the seasonal day-hours, and the duration of morning and evening twilight, (all) as functions of solar longitude.

I arranged these tables in a satisfactory fashion,[i] *and added to them others giving the normed right ascensions, regular right ascensions and oblique ascensions for latitude 30°, as well as tables of Cotangents to base 60, 12 and 7.*[j] If you enter these with the altitude you obtain the horizontal shadow of a vertical gnomon, and if you enter with the complement you obtain the vertical shadow of a horizontal gnomon. If you take one-half of the Cotangents to base 12 you obtain *Cotangents to base 6*, and if you take one-half of one-sixth of the entries you get the function which, with the declination as argument, is multiplied by the same function, that is the Cotangent, of the equinoctial meridian altitude or the Tangent of its complement (to find the Sine of of the half excess of daylight).[k] *(I also added) a table of the number of equatorial hours of daylight for latitude 30°.*

If you enter the solar longitude under the appropriate solar altitude in the tables of the time since sunrise and hour-angle you get for eastern altitudes the time since sunrise and the time till midday, and for western altitudes the time till sunset and the time since midday. The sum of the time since sunrise and the hour-angle is the semi diurnal arc. If you add the time from midday to the semi diurnal arc you get the time to sunset for eastern altitudes and the time since sunrise for western altitudes. As for the azimuth, it will be northern if (the solar longitude) on the given page is before the longitude at which the azimuth changes direction (from north to south), and it will be southern if the solar longitude is after this particular longitude. For altitudes in the east the azimuth moves from north to south and for altitudes in the west the opposite is the case. But God knows better.

In this book (I have also included) the universal table which I computed for finding the time: this is for obtaining the (hour-angle) [text: solar longitude] *according to the admirable method of Ibn Yūnus. I know of no easier method, and he had a treatise (on the subject) which was unrivalled.*[l] *I also computed a star catalogue, in which the declinations, right ascensions, and degrees of transit can be used in all localities, but the arcs of visibility, oblique ascensions of their risings and settings, as well as their meridian altitudes, are for the latitude 30° only. The text (explaining their use) is at the end of this book.*[m] God knows the truth best and, knowing all our works, is our best Judge."

Notes:

(a) This sentence implies that al-Bakhāniqī had seen some tables of $t(h,\lambda)$ computed for Cairo but preferred to prepare a new set.

(b) The expression *bi-'l-taṭābuq* means using the relation:

$$T(h,\lambda) = D(\lambda) - t(h,\lambda)$$

to compute the entries.

(c) The text has "them both" rather than "him", and the phrase should probably occur after the name of Ibn Yūnus, and refer to both him and al-Rashīdī. On the other hand, it may refer to al-Rashīdī and his father, but his grandfather's name is also mentioned.

(d) The text has incorrectly Abu 'l-Ḥusayn.

(e) This could refer to either Ibn Yūnus or Ibn al-Rashīdī – the latter seems the more likely.

(f) The Arabic term *jāmiʿ* seems to indicate that this supplementary table together with the main azimuth tables display the solar azimuth for *all* longitudes.

(g) I read the Arabic as *uḍayfa*, but since the subject is plural the word should be *uḍyafat*. Note that al-Bakhāniqī did not use the word *aḍāfa*, which would refer to Ibn al-Rashīdī.

(h) These are the tables of the functions D, t_a, T_q and $1/\tilde{h}$ – see below.

(i) This seems a reasonable rendering of *rattabtuhā tartīban*. The word *ʿajīb* generally means "wonderful / admirable / strange".

(j) Called *sittīnī*, *iṣbaʿ* and *qadam*, "sexagesimal, digit, and foot", respectively. These expressions are standard.

(k) This procedure is equivalent to the formula:

$$\mathrm{Sin}\ d = {}^{1}/_{12} \cdot \mathrm{Tan}_{60}\ \Delta\ \mathrm{Cot}_{12}\ \bar{\phi}\ ,$$

Δ being the declination of a star or the sun, and $\bar{\phi}$ being the co-latitude. See **I-7.0** and **7.1**.

(l) The text has *darajat al-shams* "solar longitude" rather than "hour-angle", a curious error. al-Bakhāniqī may have been referring to the function:

$$G(\lambda) \;=\; R^2 \,/\, \{\ \mathrm{Cos}\ \delta(\lambda)\ \mathrm{Cos}\ \phi\ \}$$

discussed by Ibn Yūnus in Ch. 15 of the *Ḥākimī Zīj* (see also **I-6.7.1**).[7] The term *risāla* means "treatise" or "method" in medieval scientific Arabic: it probably refers to the passage in the *Ḥākimī Zīj.*

(m) al-Bakhāniqī's star catalogue and notes thereon have not been identified in any of the known manuscripts of the corpus.

Thus it appears that al-Bakhāniqī rearranged the tables of Ibn Yūnus and al-Maqsī as follows. Taking Ibn Yūnus' tables of a(h,λ) corrected by Ibn al-Rashīdī (**5.2**), al-Maqsī's tables of T(h,λ) (**5.3**), and Ibn al-Kattānī's tables of t(h,λ) (**5.4**), he then tabulated the values (T,t,a) as triplets for each pair of arguments (h,λ). This arrangement of the tables is found for example, in MSS Cairo DM 108, Cairo DM 53, Istanbul Nuruosmaniye 2925, Cairo DM 616, and Cairo DM 739. If I understand al-Bakhāniqī properly he is saying that Ibn al-Rashīdī added the tables of:

$$\psi,\ h_0,\ h_v,\ h_q,\ h_{q*} \text{ and } a(h) \text{ for } \lambda = 0°,\ 90°,\ 270°$$

before the azimuth tables, and that after the azimuth tables he had put the tables which al-Maqsī had at the end of his book, namely, tables of:

$$D,\ t_a,\ T_q\ (q = 53°), \text{ and } 1/\tilde{h}\ ,$$

as well as the tables of:

$$t_q,\ H,\ h_a \text{ and } z_a,\ t_a \text{ and } T_a,\ 2D^h,\ r \text{ and } s\ .$$

But if Ibn al-Rashīdī had indeed added all these tables, why is he not generally acknowledged as the author of all the minor tables in the corpus? His name is specifically mentioned in the tables of the corpus only in relation to the two tables of $h_v(\lambda)$ and a(h) at the equinoxes and solstices.

To al-Maqsī's introduction al-Bakhāniqī added the note translated above, describing what he had done to the tables. The five manuscripts listed above contain this modified introduction as well as the tables of (T,t,a), but MS Cairo DM 108 also contains a variety of other tables. In al-Bakhāniqī's note in that copy (DM 108), but not in MS Cairo DM 53, he states that he included tables of α, α' and α_ϕ (ϕ = 30°). These are not contained in MS Cairo DM 108, but in later sets of prayer-tables for Cairo we find, for example, Ibn Yūnus' tables of α' (MS Cairo MM 43) and α_ϕ (ϕ = 30°) (MS Princeton Yahuda 861,1), and al-Marrākushī's tables of α_ϕ (ϕ = 30°) (MSS Hartford TS 621 and Paris BNF ar. 2507). al-Bakhāniqī further asserts that he included tables of the Cotangent function to bases 60, 12, and 7: these are likewise not contained in MS Cairo DM 108. He also mentions Cotangents to base 6 (*cf.* **2.9**).[8] Finally, al-Bakhāniqī states that he himself added a table of $\tilde{h}$ to the corpus (extant in MS Cairo DM 108) as well as a star catalogue and a table for finding the hour-angle (if my correction "hour-angle" for

[7] King*, Ibn Yūnus*, III.15.3-4.
[8] See n. **I**-1:37.

"solar longitude is correct), which may have been some kind of auxiliary table (both not extant in the known sources).

5.7 al-Wafāʾī's prayer-tables and notes on refraction at the horizon

MS Cairo ṬJ 367,5, fols. 35r-40r, copied about 1450, contains a short introduction and a solar longitude table followed by tables of the functions:

$$r, H, h_q, h_0, \psi, d, \text{and } \delta$$

from the corpus. The work bears the title *Natījat al-afkār fī aʿmāl al-layl wa-'l-nahār* and is attributed to the 15th-century Egyptian astronomer al-Wafāʾī (**1-9.9**). The solar table (fol. 37v) is not complete and the prayer-tables which follow (fols. 38r-40r) are in the same hand but are not mentioned in al-Wafāʾī's introduction. If the title, which mentions timekeeping by day and night, is original, it seems probable that the prayer-tables were included in the work by al-Wafāʾī himself. If it is indeed the case then it was al-Wafāʾī who set the trend followed by later Egyptian astronomers such as al-Lādhiqī, al-Ikhṣāṣī, Ibn Abī Rāya and Riḍwān Efendī (**7.8-10**), of presenting updated tables for calendar conversion and finding the solar longitude for a given date, and including selected tables from the already available corpus.

MS Cairo DM 157,2, copied *ca*. 1450, contains a set of tables of the function $\alpha_\sigma(\lambda)$, the oblique ascension at the time of the *salām*, computed for each six minutes of argument with entries given to minutes. These tables are specifically attributed to al-Wafāʾī. A similar set of tables for the time of nightfall is attributed in MS Cairo DM 1218, copied *ca*. 1500, to Muḥammad al-Ghazāwī, a *muwaqqit* in the Muqassam (?) Mosque in Cairo, whose name is new to the modern literature. MS Cairo MM 209,1 contains a set of normed right ascensions, $\alpha'(\lambda)$ (ε = 23;35°), with values for each minute of argument attributed to Ibn al-Mushrif (**I-9.8** and **II-6.15**), copied in 873 H [= 1468/69] by Ibn Abi 'l-Fatḥ al-Ṣūfī (**6.15**) from a copy by the author dated 848 H [= 1444/45]. Apart from the earlier ascension tables of Baylak al-Qipjāqī (**6.4**), all other Islamic tables of this kind, which are a particularly Egyptian phenomenon, and in which values for each degree of solar longitude are stretched to each six, three, or even one minute of argument, date from the period of Ibrāhīm ibn Qāyitbāy and ʿAbd al-Qādir al-Minūfī, namely the late 16th and early 17th centuries. See further **7.2**.

MSS Cairo MM 181,2 (fols. 4v-5r, copied 1030 H [= 1620/21], TR 303,2 (fol. 33v, copied 1266 H [= 1849/50]), and MM 155,2 (fol. 23v, *ca*. 1700, defective) are copies of a short treatise (*nubdha*) attributed to al-Wafāʾī (at least in the first source) concerning the *daqāʾiq al-ikhtilāf*, that is, the the time taken by the sun to cross between the visible and true horizons (**4.11**). The first two sources are displayed in **Fig. 5.7**. In the treatise it is stated that the values:

EQ: 47 SS: 62 WS: 32

were derived by the Damascus astronomer Ibn al-Shaṭīr (**9.3**). This is curious for the following reasons:

(1) the Minūfīs, father and son, attributed these values to Ibn Yūnus (**7.1** and **8.2**);

(2) Ibn al-Shaṭīr is known to have written that he had observed the half arcs of visibility of the sun and stars and found that they were greater than the arcs found by calculation by an amount exceeding $^2/_3$° (**9.3**), but is not otherwise known to have written more

↑
Fig. 5.7a: Notes attributed to al-Wafāʾī on the time taken by the sun to travel between the true and visible horizons. [From MS Cairo MM 181,2 (fols. 4v-5r), courtesy of the Egyptian National Library.]

→
Fig. 5.7b: The same notes abridged and without the attribution to al-Wafāʾī, but note that in both these Cairo manuscripts the theory is attributed to the illustrious Ibn al-Shāṭir. [From MS Cairo TR 303,2 (fol. 33v), courtesy of the Egyptian National Library.]

on the subject. There is no reference to the subject in Ibn al-Shaṭīr's prayer-tables or in his introduction to them (see again **9.3**).

(3) If Ibn al-Shaṭīr had indeed computed a set of values of the "difference minutes" for the zodiacal signs he would surely have computed them for Damascus, rather than Cairo. If he had computed them for Cairo, he would surely also have computed them for Damascus. But the later Syrian tables for $\Delta D(\lambda)$ are based on the values for Cairo adapted for Damascus (**11.13**).

(4) If al-Wafāʾī was indeed responsible for these notes, I find it curious that he did not refer to the "difference minutes" in any of his other known works, not least, in his treatise and tables for timekeeping discussed above.

The treatise explains how the "difference minutes" are to be used, and concludes with a table displaying for each zodiacal sign the following quantities, all in minutes of arc:

(1-4) the amounts to be added to the solar longitude at midday to find the solar longitude at the time of the *ʿaṣr* prayer, sunset, nightfall, and the time of the *salām.*

(5) The time taken by the sun to move between the two horizons, labelled *daqāʾiq ikhtilāf al-āfāq al-marʾiyya.*

Similar tables are contained in most of the copies of al-Lādhiqī's prayer-tables for Cairo (**7.8**).

5.8 Sibṭ al-Māridīnī's notes on interpolation in timekeeping tables

Sibṭ al-Māridīnī (**2.10**), in his treatise on sexagesimal arithmetic entitled *Raqāʾiq al-ḥaqāʾiq fī ḥisāb al-daraj wa-'l-daqāʾiq*, "The Finest Truths concerning Sexagesimal Arithmetic", discusses interpolation[9] in tables for timekeeping. The relevant passage (I have used MS Gotha A1390, fol. 33v) translates as follows:

> "The conclusion of this work deals with interpolation (*taʿdīl mā bayn al-saṭrayn*). The tables of the semi diurnal arc, the half excess of daylight, solar rising amplitude, ascensions, and the like, are generally computed for each integral degree of argument although they may also be computed for each minute. Certain tables are generally computes for each minute, such as the (Co)tangent, Sine and Versed Sine. Entries in these are usually given for argument differences of a minute. Likewise the tables of the inverse (Co)tangent, Sine and Versed Sine also have entries for each minute of argument. In tables of this sort there is no need to interpolate... ."

It is of interest that he mentions tables of spherical astronomical functions and trigonometric functions computed for each minute of argument: see below.

Sibṭ al-Māridīnī then gives examples of how to use linear interpolation in a table of, say, $D(\lambda)$ for non-integral λ. He also shows how to interpolate in double-argument tables, quoting an example from "his teacher", that is, the celebrated Ibn al-Majdī (1359-1447).[10] To find the solar azimuth when the altitude is 19;24° and the solar longitude is 53;20° he interpolates in

[9] See n. **I**-1:21.

[10] On Ibn al-Majdī see Suter, *MAA*, no. 432; *Cairo ENL Survey*, no. C62; and also King & Kennedy, "Ibn al-Majdī's Tables", pp. 48-49.

the azimuth tables for altitudes 19° and 20° as follows. The relevant entries which he cites (all accurate) are:

	h = 19°	20°
λ = 53°	11; 2°	10;31°
54°	11;20	10;49

(as in Ibn Yūnus' tables), and our author computes what he calls *al-taʿdīl al-awwal* and *al-thānī*, "the first and second interpolations":

$$a(19°, 53;20°) = 11;8° \text{ and } a(20°, 53;20°) = 10;37° ,$$

from which:

$$a(19;24°, 53;20°) = 10;56° .$$

He states that the accurate value of a is 10;55,36°, which is just one second too high, and calls this kind of interpolation *taʿdīl al-tajyīb*. He makes no mention of the author of the tables.

Sibṭ al-Māridīnī's remark that certain tables for timekeeping had entries for each minute of argument (solar longitude) is confirmed by the tables of Ibn al-Mushrif, Ibrāhīm ibn Qāyitbāy and al-Wafāʾī (**5.7**). The Sine and Cotangent tables attributed to Ibn Yūnus give values also to each minute, but to five sexagesimal digits.[11] By the 15th century such tables were standard in the major centres of astronomy in the Islamic world, the most popular ones being the Sine and Cotangent tables from the *Zīj* of Ulugh Beg.[12] However, there was also an independent Egyptian tradition of tables of the standard trigonometric functions with entries to three digits for each minute of argument, of which several manuscripts survive: see **Fig. I-1.0e** for an example.[13]

---

[11] King, *Ibn Yūnus*, III.10.5, and *idem*, *Mecca-Centred World-Maps*, p. 166.
[12] Schoy, *Trigometrie des al-Bīrūnī*, pp. 92-100; see also King, *Mecca-Centred World-Maps*, p. 165.
[13] These are listed in *Cairo ENL Survey*, no. C137.

CHAPTER 6

OTHER EARLY EGYPTIAN TABLES FOR TIMEKEEPING

6.0 Introductory remarks

Virtually no Egyptian astronomical works survive from the period between Ibn Yūnus and the middle of the 13th century. (This accounts, for example, for the importance of the treatises by Ibn al-Ḥammāmī discussed in **2.4**.) For the history of astronomy in Egypt during that period we have little more than biographical references to various individuals, and al-Maqrīzī's account of the observatory that was built in Cairo in the 12th century.[1] From late 13th-century Cairo we have the A-Z on spherical astronomy and instruments by Abū ʿAlī al-Marrākushī, the treatise on sundial construction by al-Maqsī, and a *zīj* called *al-Zīj al-Muṣṭalaḥ*. al-Marrākushī's treatise enjoyed considerable popularity in later centuries in Egypt, Syria, and Turkey. I discuss his spherical astronomical tables in **6.7**. In his encyclopedia *Irshād al-qāṣid*, the early-14th-century scholar Ibn al-Akfānī states that the two books on *mīqāt* that were used (in Egypt) in his time were a short work *Nafāʾis al-yawāqīt* and the longer work *Jāmiʿ al-mabādiʾ wa-'l-ghāyāt* of Abū ʿAlī al-Marrākushī.[2] He does not mention the author of the first work, but it may be that the *Durar al-yawāqīt* described in **3.2** is identical with, or at least related to the *Nafāʾis al-yawāqīt*. Ibn al-Akfānī also mentions that the closest astronomical observations to his time were those conducted under Hulagu by Naṣīr al-Dīn al-Ṭūsī and his associates, and that in Egypt the yearly ephemerides were prepared from a *zīj* which had been compiled from a number of other *zīj*es and which was called the *Muṣṭalaḥ Zīj*. In **6.6** I discuss the spherical astronomical tables found in the two extant versions of this work. Various Egyptian astronomers compiled auxiliary tables for facilitating the solution of problems in spherical astronomy. The historical context of the monumental table of Najm al-Dīn al-Miṣrī (**6.5**) is still something of a mystery, even though François Charette has produced a detailed study of the table itself and gathered all the known material relating to its author. It seems that the tables of al-Khaṭāʾī (**6.8**) do not, after all, and as one might have expected, precede those of al-Khalīlī of Damascus (**10.3**). al-Khalīlī's tables were also used in Egypt (**6.14**). All of the other Egyptian auxiliary tables postdate those of al-Khalīlī (**6.15**). Egyptian astronomers, in particular Ibn al-Rashīdī, compiled timekeeping tables for the latitudes of Mecca and Jerusalem (**6.11** and **6.12**).

6.1 A 13th-century muezzin's manual

MS Oxford Bodley Or. 133 is a beautifully-illustrated manuscript comprising mainly an astrological work *Kitāb al-Bulhān* (*sic* for *al-Burhān* ?) by an otherwise unknown ʿAbd al-Ḥasan

[1] This activity is described in Sayılı, *The Observatory in Islam*, pp. 167-175.
[2] Ibn al-Akfānī, *Irshād al-qāṣid*, p. 5. See also Charette, *Mamluk Instrumentation*, p. 12.

ibn Aḥmad ibn ʿAlī ibn al-Ḥasan al-Iṣfahānī and two other shorter works in poetry and calligraphy.[3] A treatise on astronomical timekeeping, written in a different hand but datable to the 14th century, has been bound in the manuscript (fols. 94v-130r) after the astrological work. It is entitled *Kitāb al-Durar wa-'l-yawāqīt fī ʿilm al-raṣad wa-'l-mawāqīt*, "Book of Pearls and Sapphires on Astronomical Observations and Timekeeping". On the title pages (fols. 94v-95r) the author's name has been partially deleted, but the following letters are still legible: *Abī A... ... ibn ...d ibn Aḥmad*, al-.ā... al-... (perhaps, *al-Shāfiʿī al-madhhab*, referring to his adherance to the Shāfiʿī legal school) *al-Miṣrī al-waṭn* (the Egyptian). This name, however, does not correspond exactly with the author's name which appears on fol. 97v, namely: [Abu] 'l-ʿAbbās Aḥmad ibn Abī ʿAbdallāh Muḥammad ibn Aḥmad. The copying of the manuscript was completed in 734 H [= 1334], and the name of the copyist was Abu 'l-Ḥasan ʿAlī ibn (?) Muḥammad ibn (?) Baṣīr (?). The original work can be dated to the early 13th century (see below).

The treatise is a complete *muwaqqit*'s manual for timekeeping by the lunar mansions and regulating the prayer-times approximately (see also **V-5**). It may be that Ibn al-Akfānī, writing in the early 14th century, was referring to this work when he stated that the book on simple timekeeping which was currently most widely used in Egypt was called *Nafāʾis al-yawāqīt* (**6.0**). The treatise begins with a set of prayer-tables for Cairo (fols. 98r-104r) of a kind not attested in any other known Egyptian source: see **Fig. 6.1**. For each day of the Coptic year and corresponding date in the Syrian calendar the lunar mansion rising at daybreak is indicated and following functions are tabulated:

λ	*darajat al-shams*
H	*ghāyat al-irtifāʿ*
$Z_{(12)}$	*ẓill al-zawāl*
h_a	*irtifāʿ al-ʿaṣr*
2D	*qaws al-nahār*
2N	*qaws al-layl*
h_q	*samt al-qibla*
$z_{q(12)}$ (= $\mathrm{Cot}_{12}\ h_q$)	*ẓill al-samt*
r	*ḥiṣṣat al-fajr*.

The values of H, 2D and 2N are given in degrees and minutes, and the remaining functions are displayed to the nearest degree or digit. From the values of H the parameters are found to be:

$$\phi = 30° \text{ (Cairo) and } \varepsilon = 23;35° .$$

All of the functions are approximate. The values given for H for each 30° of λ starting from the winter solstice are:

36;25° 40;8 48;4 60;0 70;36 79;56 83;35 ,

but starting from the summer solstice are:

83;35° 80;0 72;6 60;0 48;4 40;20 36;25 .

The values of δ vary in each of the four quadrants of the ecliptic! In the text the author states

[3] On the astrological work see Ullmann, *Die Natur- und Geheimwissenschaften im Islam*, pp. 344-345; Sezgin, *GAS*, VII, pp. 24-25; and the detailed study of the art-historical aspects in Carboni, *Kitāb al-bulhān*.

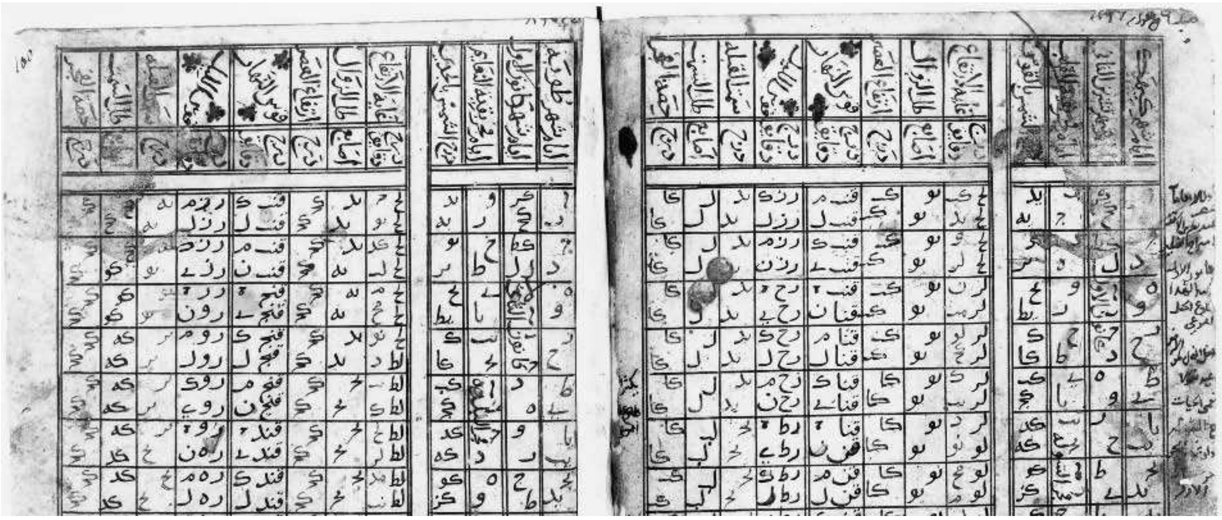


Fig. 6.1: The prayer-tables for the months of Kayhak and Ṭūba in the *Kitāb al-Durar wa-'l-yawāqīt*. [From MS Oxford Bodley Or. 133, fols. 99v-100r, courtesy of the Bodleian Library.]

that δ varies by 12°, 8°, and 4° for each 30° of λ measured from the equinox, but he has not used these figures in his tables. The scheme underlying the table of 2D, which is outlined elsewhere in the text, is that the function increases by 0;30° for each degree of Aries, 0;20° for each degree of Taurus, and 0;10° for each degree of Gemini, *etc*. This explains, for example, the extremal values 150;0° and 210;0° (accurately, 150;48° and 209;12°). The function $Z_{(12)}$ is made to assume values:

16 13 10 7 4 2 1;21

for each 30° of solar longitude starting from the winter solstice (entries less than 2 are given to two digits). The underlying scheme is clearly very crude, although the (accurate) value 1;21 for the summer solstice was probably copied from another source. The accurately computed shadow lengths are:

16;16 14;26 10;38 6;56 4;0 2;03 1;21

The values of h_a for each 30° of λ starting at the winter solstice are:

21 25 26 31 37 40 42 ,

compared with the accurate values:

23 24 28 33 37 40 42 .

In the text the author advocates the approximate rule (*cf.* **2.9**):

$$h_a \approx {}^1/_2\, H + {}^1/_{10}\, (83° - H) ,$$

which gives values:

23 24 27 32 37 40 42 .

The tabulated function h_q assumes values:

13 19 35 45 58 71 80

for each 30° of λ starting at the winter solstice. The corresponding values in Ibn Yūnus' table of h_q ($q = 52°$) and the other table of h_q in the Cairo corpus ($q = 53°$) round to:

14	19	31	47	63	75	80
13	18	30	46	62	75	80 ,

respectively. Using the approximate formulae for h_q found in the treatises of Najm al-Dīn al-Miṣrī and al-Bakhāniqī (**2.5** and **2.9**), namely:

$$h_q \approx 47° + 1;25\ \delta \quad \text{and} \quad h_q \approx 47° + 1;24\ \delta:$$

we obtain values:

14	18	31	47	63	76	80 .

Again, using linear interpolation between the extremal values 13 and 80, we obtain:

13	24	35	47	58	69	80 .

Thus the tabulated values correspond to none of these approximations.

No other known Egyptian prayer-tables display the function $z_{q(12)}$ (= $\mathrm{Cot}_{12}\ h_q$), using which one can determine the qibla from the gnomon shadow.

The tabulated values of r are:

22	21	21	24	25	25	27

for each 30° of λ starting at the winter solstice. It is difficult to explain these values. For parameters 18°, 19°, and 20°, the corresponding accurate values are respectively:

22	22	21	21	22	24	25,
24	23	22	22	23	26	27
25	24	23	23	25	27	28 .

Note that the compiler realized that r displays a secondary maximum at the winter solstice.

Most of the remainder of the treatise and the other tables are for timekeeping by the lunar mansions. Some of the tables resemble those found in a later Yemeni almanac (**12.5**). Thus there are tables displaying the names of the lunar mansions rising, culminating and setting at nightfall, and at one-third, one-half, two-thirds and three-quarters of the night, as well as at daybreak, for each 13 days of the Coptic year. Other tables display the 13th divisions of the lunar mansions rising at daybreak and the solar longitude for each day of the Coptic year (the entry in the latter for Tūt 1 is Virgo 14°). The longitudes of 23 prominent stars are given; these correspond to the early 13th century (*e.g.* the longitude of Aldebaran is 2^s 29;37°), enabling us to date the compilation of the treatise to the early 13th century.

6.2 Prayer-tables attributed to Ibn Qudāma

MS Damascus Ẓāhiriyya 10732, fols. 3r-3v, copied *ca*. 1600, contains some simple prayer-tables for an unspecified latitude with values of the functions:

$$D,\ t_a,\ T_a,\ s,\ r,\ n \text{ and } 2N$$

to one degree for each zodiacal sign. The tables occur on a single folio that may be related to the text that immediately precedes and follows, namely, simple material on the lunar mansions and shadow lengths attributed to the late-12th-/early-13th-century legal scholar and theologian of Jerusalem, Ibn Qudāma.[4] Whether or not this is the case, the values in the table can be derived from those in the main Cairo corpus.

[4] On Ibn Qudāma see the article "Ibn Ḳudāma" in EI_2 by George Makdisi. On the shadow scheme associated with him see **III-8.1**.

6.3 Prayer-tables in an anonymous Egyptian almanac

MS Cairo DM 187 (74 fols., copied *ca*. 1850) is the only known copy of an anonymous almanac based on the Coptic calendar of the same kind as the *Calendar of Cordova*,[5] and accompanied by various simple calendrical and astrological tables. This almanac merits detailed investigation. I am unable to ascertain the date of its compilation, but have noted a list of planetary apogees amongst the tables, as follows:

Saturn	Jupiter	Mars	Sun (and Venus)	Mercury
245°173°	125°	83°	202°	

These correspond very closely to those in the 9th-century Abbasid *Mumtaḥan Zīj*, which are as follows for the year 199 Yazdigird [= 831]:[6]

244;30°	172;32	128;33	82;39	201;0

On the other hand, some prayer-tables contained in this work – see **Fig. 6.3** – do not, I think, predate the 14th century. These display values of the following functions for each five days of the Coptic month:

D (*al-ẓuhr*), t_a (*al-ʿaṣr*), T_a (*al-maghrib*), s (*al-ʿishāʾ*), n (*jawf al-layl*), r (*al-fajr*)

Values are given in degrees and also in hours (misspelled *saʿāt* !) and minutes, and correspond to those in the main Cairo corpus. The twilight tables are based on 19° and 17°, which are not attested in Egyptian sources before the 14th century.

At the end of the almanac there are various notes, including one expounding some approximate formulae for timekeeping, of which the first three are not attested elsewhere. These are as follows:

$t_a \approx {}^4/_7\, D$, $s \approx {}^1/_8\,(2N)$, $r \approx {}^1/_7\,(2N)$ and $h_a \approx {}^1/_2\, H + {}^1/_2 \cdot {}^1/_6\,[90° - {}^1/_2\, H]$.

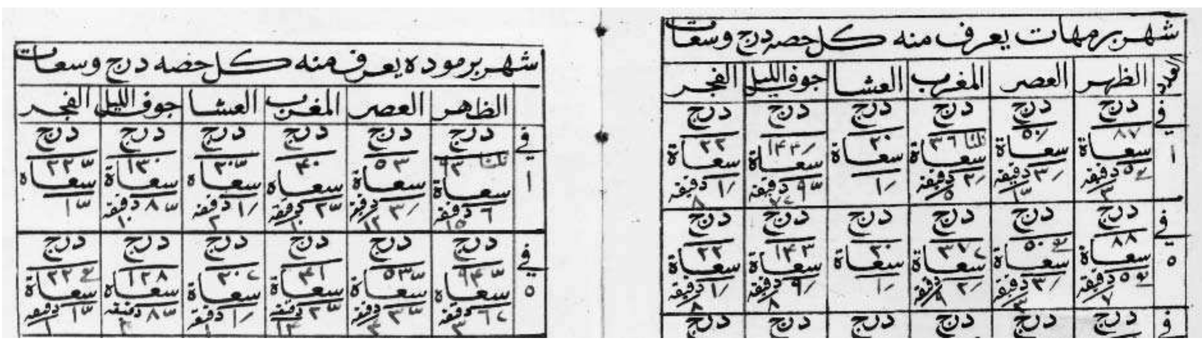


Fig. 6.3: The part of the prayer-tables in an anonymous Egyptian almanac serving the months of Baramhāt and Barmūda. [From MS Cairo DM 187, courtesy of the Egyptian National Library.]

[5] This is published with translation in Charles Pellat, *Calendrier de Cordoue*. On the shadow schemes in that work and also those in MS Cairo DM 187 see **III-5.1** and **9.7a**.

[6] On the *Mumtaḥan Zīj* see n. **I**-4:7. The apogees given are extracted from Caussin, *Table Hakémite* (cited in n. **I**-2:3), pp. 234 ff.

6.4 Baylak al-Qipjāqī's tables of ascensions

MS Cairo MM 41, copied in 1191 H [= 1777/78], contains a set of tables of the right ascensions for each minute of argument computed to three digits, attributed to the well-known scholar Baylak ibn ʿAbdallāh al-Qipjāqī[7] and stated to have been compiled in 674 H [= 1274/75]. The manuscript contains 41 folios of tables. There is an introduction by Sayf (?) al-Dīn Ibn al-Mushrif (**I-9.10**).

6.5 Najm al-Dīn al-Miṣrī's universal table for timekeeping

The tables of either of the two functions t(h,λ) or T(h,λ) which were compiled for particular latitudes each contain over 10,000 entries. To compile a table displaying either function for each degree of latitude would be an enormous undertaking. Nevertheless, around the year 1325 the Egyptian astronomer Najm al-Dīn al-Miṣrī (**2.5**) compiled a table displaying for any latitude the time since rising of either the sun or any non-circular star as a function of the solar or stellar altitude. This remarkable table is the largest known table from the entire medieval period and contains over 440,000 entries.

MSS Cairo MM 132 and Oxford Marsh 676 (Uri 944 = 995) are two halves of a unique copy of Najm al-Dīn's tables, copied *ca*. 1325 probably by the author. The arguments which one feeds into the main table to find the time since rising of any celestial body for any latitude are H, h, and D: for details see **I-2.6.1**. The table can also be used for solving all problems of spherical astronomy for any latitude: for details see **I-9.3***. Other tables are included to facilitate the determination of H and D for any latitude, starting from the solar longitude λ or stellar declination Δ. Solar tables based on Ibn Yūnus' parameters, and are a star catalogue giving the equatorial coordinates of 367 stars, are also provided.

The existence of Najm al-Dīn's tables raised certain questions. What, if any, was his relationship with al-Maqsī and al-Marrākushī? What, again if any, was his relationship with the *Muṣṭalaḥ Zīj* ? What were his sources? One surely does not just sit down to compile a table containing over close to half a million entries which works for all latitudes, without having seen smaller tables for specific latitudes. If Najm al-Dīn preceded al-Maqsī, as I believe he did, then there were not yet any tables for T(h,λ) available for Cairo. No tables of T(H,h), the so-called *ṭaylasān zīj*es, are known to have been compiled for Cairo, although they had been used by astronomers elsewhere for centuries. I suspect that Najm al-Dīn's inspiration for his main table might have come from some tables of Ibn Yūnus which are no longer extant. Suppose, for example, that Ibn Yūnus had compiled a table of:

$$G(H,D) \;=\; \text{Vers } D \,/\, \text{Sin } H \;,$$

for each degree of both arguments. The values of T(H,h,D) can then be generated using the simple formulae:

[7] On Baylak see the article by Martin Plessner in *DSB*, and *Cairo ENL Survey*, no. C9. The unique manuscript of a treatise on mechanical clocks by Riḍwān al-Khurasānī (MS Istanbul Köprülü 949 – see *Cairo ENL Survey*, no. C1) was copied by Baylak in 658 H [= 1260]; this treatise is now published – see Hill, *Studies*, p. xix, n. 4.

$$\text{Vers } t(H,h,D) = (\text{Sin } H - \text{Sin } h) \cdot G(H,D) \quad \text{and} \quad T(H,h,D) = D - t(H,h,D)\ .$$

It seems to me that certain errors in the tables of h(D,T) for Cairo (**6.9**) which I believe were also computed by Najm al-Dīn point to the fact that they are based on an auxiliary table computed by someone else. For example, the entries for T = 104° and 105° in the table for D = max D = 104;36° are respectively 83;30° and 83;31° when they should be 83;34° and 83;35° (= max H).

It would not surprise me if further research on Egyptian astronomical manuscripts revealed the existence of a table of G(H,D) computed by Ibn Yūnus. Tables of related functions compiled by later Egyptian astronomers have been located in the manuscript sources: see **I-6.7-9**.

What did surprise me in 1982 was the discovery in MS Dublin CB 102,1 of Najm al-Dīn's instructions on the use of his table as a universal auxiliary table. Only in 1998 was an account of this published, namely by François Charette:[8] see the summary in **I-9.3***.

6.6 The spherical astronomical tables in the *Muṣṭalaḥ Zīj*

The *Muṣṭalaḥ* ("Technical") *Zīj* appears to have been the most popular *Zīj* from the 13th to the 15th centuries (**6.0**). The 17th-century Turkish bibliographer Ḥājjī Khalīfa attributes it to an individual named Muḥammad ibn Muḥammad al-Fāriqī,[9] on whom I have no further information, and he notes that it is based on the *Zīj* of Ibn Yūnus. Two recensions of this work are known to me, MSS Paris BNF ar. 2513 from the 13th century and Paris ar. 2520 from the 14th century, and both are incorrectly attributed to Ibn Yūnus on their title folios. In the former, the *zīj* is entitled *Kitāb al-Taʿlīm fī waḍʿ al-taqwīm,* "Book of Instruction on Finding Planetary Positions", and in the latter *Kitāb al-Zīj al-Muṣṭalaḥ*. The two copies are by no means identical, but their relationship to each other is firmly established by the similarity of the introductory text and the planetary tables, if not the spherical astronomical tables.

The planetary tables in the two Paris manuscripts are a motley collection lifted from various sources, including the *Zīj* of Ibn al-Aʿlam,[10] the *Zīj*es of Ibn Yūnus, the *Zīj* of al-Kammād,[11] and a *zīj* whose name is variously written as *Shāhī, Shāwī,* and *Shāmī,* and which may be the *Shāhī Zīj* of Ḥuṣām al-Dīn Sālār.[12] There are also tables in both manuscripts taken from Abbasid sources such as *Mumtaḥan Zīj* (solar eclipse table, MSS ar. 2513, fol. 48r, and ar. 2520, fols. 58v-59r),[13] the *Zīj* of Ḥabash (auxiliary tables for spherical astronomy, MS ar. 2520, fols. 70r-71r),[14] and another early *zīj,* perhaps of Andalusī origin (lunar crescent visibility table for the seven climates based on Indian theory, MS ar. 2513, fol. 71v).[15] MS ar. 2520 seems to be closer to the original than MS ar. 2513. In the former the mean motion tables run from

[8] Charette, "Najm al-Dīn's Monumental Table".
[9] On the *Muṣṭalaḥ Zīj* see n. **I**-6:14.
[10] On Ibn al-Aʿlam see Sezgin, *GAS*, VI, pp. 215-216, and now Kennedy, "Ibn al-Aʿlam's Tables"; Tihon, "L'astronome Alim"; and Mercier, "Ibn al-Aʿlam's Parameters".
[11] On al-Kammād see Suter, *MAA*, no. 487 and Kennedy, "*Zīj* Survey", nos. 5, 66, 72.
[12] On Ḥusām al-Dīn Salār see Suter, *MAA*, no. 482, and Kennedy, "*Zīj* Survey", no. 32.
[13] On these tables see Kennedy & Faris, "Yaḥyā b. Abī Manṣūr on Solar Eclipses".
[14] On Ḥabash see n. **I**-9:1.
[15] King, "Tables for Lunar Crescent Visibility", pp. 197-207, and **VIa-7**.

630 H [= 1232] and in the latter from 750 H [= 1349]. In MS ar. 2513, fol. 11r (but not in MS ar. 2520) there is a numerical example of the determination of the solar longitude for the year 682 H [= 1283]. I suspect that further investigation of these and related sources will reveal a connection with the scholar Ibn al-Lubūdī, who lived in the mid 13th century and is known to have moved from Damascus to Cairo.[16] He is also known to have compiled two *zīj*es, one of which was based on the *Shāhī Zīj*. At this stage of the research it is perhaps not amiss to mention an anonymous Yemeni *zīj* preserved in MS Paris BNF ar. 2523, copied in the late 14th century, which was compiled in Taiz in 775 H [= 1374] and contains planetary tables from the *Ḥākimī, Shāhī* and *Muntaḥal* (?) *Zīj*es.[17] Thus the combination of *Ḥākimī* and *Shāhī* tables is attested in both Egypt and the Yemen, but we also know that the Yemeni astronomers of the 14th century were familiar with the *Muṣṭalaḥ Zīj*.[18]

The potential value of each of these manuscripts for furthering our knowledge of Islamic astronomy is obvious, and all of them deserve detailed study. My present purpose is to briefly note certain of the spherical astronomical tables which occur in the two copies of the *Muṣṭalaḥ Zīj* and relate to our subject. Most of them are not found in any other Egyptian sources currently known to me.

Geographical coordinates

The latitude of Cairo is taken as 30;0° in the spherical astronomical tables in the *Muṣṭalaḥ Zīj*. However, MSS Paris ar. 2513 and 2520 contain two different sets of geographical tables, the former also with values of q,[19] as is shown by the following entries:

Source	2513, fols. 87v-88v			2520, fols. 82v-83r	
Locality	ϕ	L	q	ϕ	L
Cairo	30;0°	55;0°	52;30°	*29;55°*	54;55°
Damascus	33;27	60;0	39;50	33;30	60;0
Jerusalem	32;0	57;50	43;30	32;10	57;20
Mecca	21;20	67;0	-	21;30	67;0

Note that the distinctive value 29;55° for the latitude of Cairo was also used by al-Marrākushī (**6.7**), and that the distinctive value 33;27° for the latitude of Damascus was also used by al-Mizzī (**9.2**). Accurately computed for the coordinates displayed in MS ar. 2513, the three qibla values are respectively 54;2°, 28;51°, and 39;42° using the exact formula and 54;4°, 30;8°, and 40;43° using the standard approximate formula. Thus the stated qibla values are wretchedly inaccurate, or based on other coordinates.[20] al-Marrākushī (**6.7**) states that the qibla value 52;30° for Cairo was widely accepted there, and it may be that it was conceived as a happy compromise between Ibn Yūnus' first and second values, 52° and 53°.

[16] On Ibn al-Lubūdī see n. 9:10.
[17] See n. 6:9.
[18] King, *Astronomy in Yemen*, pp. 36-37.
[19] Overlooked in King, *Mecca-Centred World-Maps*, pp. 76-84, where, however, several other wretched Egyptian tables of this kind are discussed.
[20] *Ibid*., p. 80. See also **9.1**.

Obliquity of the ecliptic

In MS ar. 2520, fol. 72r, in what purports to be the text of the *Zīj* (there is no corresponding section in MS ar. 2513) it is stated that "*al-Sharīf*", which is the common designation of the 10th-century Baghdad astronomer Ibn al-Aʿlam (see above) and is used to refer to him elsewhere in MS ar. 2520, found the solstitial meridian altitudes of the sun to be 80;10° and 33;6° and derived the value 23;33° for the obliquity. The corresponding latitude is 33;27°, and so the observation must have been made at Damascus (see **9.2**). However, Ibn al-Aʿlam is not known to have visited Damascus. Ibn Yūnus (MS Leiden Or. 143, p. 223) attributes the value 23;34,2° for the obliquity to him.[21]

In the various tables in MSS ar. 2513 and 2520 (see below) the four values ε = 23;30°, 23;33°, 23,35° and 23;51,20° are used. In general we can associate the first with the *Īlkhānī Zīj* (and perhaps the *Shāhī Zīj*), the second with the *Mumtaḥan Zīj* and the *Zīj* of Ḥabash (and Ibn al-Aʿlam?), the third with Ḥabash again as well as al-Battānī and Ibn Yūnus, and the fourth with Ptolemy.

Trigonometric and spherical astronomical tables

In MS ar. 2520 we find the following tables of this kind:

(1) Tables of Sin θ, $\mathrm{Tan}_{60}\ \theta$, Cot_{12} h, and Cot_7 h (fols. 67v-69r). Values are given to three digits for each degree of argument.

(2) A table of $\delta(\lambda)$ with values to three digits for each 0;6° of λ (fols. 50v-51v). The table is based on ε = 23;30°, and it is stated that it was taken from the *Īlkhānī Zīj.*
Tables of $H(\lambda)$ and $2D^h(\lambda)$ for ϕ = 30° with values to three digits for each degree of λ (fols. 63v-64r), the first based on ε = 23;30° and the second on ε = 23;35°.
A table of $\mathrm{Tan}_{60}\ \delta(\lambda)$, labelled *fuḍūl al-maṭāliʿ li-'l-arḍ kullihā*, "ascensional differences for all the earth", with values to three digits for each degree of λ. The underlying value of ε is 23;33,0°. See further **I-7.1.9**.

(3) Tables of $\alpha(\lambda)$, $\alpha'(\lambda)$, and $\alpha_\phi(\lambda)$ (λ = 30°) with values to two digits for each degree of λ (fols. 60v-63r). These tables are based on ε = 23;35°; the first two appear to be derived from Ibn Yūnus' table of $\alpha(\lambda)$ to three digits, and the third is the same as his table in the *Ḥākimī Zīj.*

(4) Anonymous auxiliary tables based on ε = 23;33°, in fact due to Ḥabash (fols. 70r-71r) – see **I-9.1**.

MS ar. 2513 contains the following spherical astronomical tables:

(1) A table of $\mathrm{Tan}_{60}\ \theta$ to three digits for each degree of argument, which differs from the one in MS ar. 2520.

(2) A table of $\delta(\lambda)$ with values to three digits for each degree of λ (fol. 35r), based on ε = 23;33,0°. But for minor variants this is the same as the table in the Mosul recension of the *Mumtaḥan Zīj* preserved in MS Escorial ár. 927.
A table of $H(\lambda)$ for ϕ = 30°, with values to three digits for each degree of λ (fol. 49r), based on ε = 23;35,0°. The underlying values of $\delta(\lambda)$ are those of the *Ḥākimī Zīj.*

[21] See King, "Earliest Muslim Geodetic Measurements", pp. 226-227, on Ibn al-Aʿlam's determination of the latitude of Baghdad as 33;21°.

A table of $\delta^*(\lambda) = 90° + \delta(\lambda)$ to two digits (fol. 55r), which is based on $\varepsilon = 23;35°$, but on values of $\delta(\lambda)$ which are less accurate than those of Ibn Yūnus.

A table of $\text{Tan}_5\ \delta(\lambda)$ to three digits based on $\varepsilon = 23;35°$ (fol. 81r). This table differs from that of al-Marrākushī (**6.7**) but is the same as that in MS Cairo MM 43 , fol. 42r, of a set of anonymous spherical astronomical tables for Cairo. See further **I-7.1.8** on this table and **II-6.14** on the others in MS Cairo MM 43.

(3) A table of $d(\lambda)$ for $\phi = 30°$, with values to two digits for each degree of λ (fol. 53v), based on the values to three digits in the *Ḥākimī Zīj.*

A table of the maximum half-excess of daylight in equinoctial hours, max $d^h(\phi)$, for each degree of ϕ from 1° to 66°, with values to three digits (fol. 57v). The table is based on Ptolemy's value of the obliquity, $\varepsilon = 23;51,20°$.

Tables of $\alpha'(\lambda)$ and $\alpha_\phi(\lambda)$ ($\phi = 30°$) (fols. 46v-47r), as in MS ar. 2520.

(4) A table of $h_a(H)$ to two digits for each degree of H (fol. 89v).

Also, a table of $h_a(\lambda)$ with entries to two digits for each degree of λ (fol. 54r). The entries differ slightly from those in the main corpus and the auxiliary tables of al-Khaṭāʾī (**4.9**).

(5) Auxiliary tables for timekeeping by the sun, based on $\phi = 30°$ and $\varepsilon = 23;35°$ (fols. 53v and 62v). The functions tabulated are:

$$\text{arc Sin } \{ \text{ Sin } \delta(\lambda) \sin \phi \ / \ R \ \} \text{ and } R \ / \text{ Cos } \delta(\lambda) \ .$$

See further **I-6.7.1** and **I-6.10.3** on these.

6.7 al-Marrākushī's spherical astronomical tables for Cairo

al-Marrākushī's *summa* on spherical astronomy and instruments was the most important single work on this subject to be compiled in medieval times (see already **2.7** and **6.0**). It was also the most influential, if only in Egypt, Syria and Turkey. I here describe only the tables presented by al-Marrākushī and discuss his remarks on the qibla at Cairo and on the twilight.

The treatise begins with a discussion of calendars and trigonometry, with approximate tables, as well as a table displaying the solar longitude for the year 1275/76 and a star catalogue for 1282. al-Marrākushī was of the opinion that the obliquity of the ecliptic oscillated between 23;53° and 23;33° and that there was an associated oscillatory motion of the equinoxes on the ecliptic. For the present discussion it suffices to note that in all of his tables which are based on a value of the obliquity he used $\varepsilon = 23;35°$.

al-Marrākushī tabulated $\delta(\lambda)$ to two digits for each degree of λ (I.183). Nine of his ninety entries are in error by -1 in the second digit, which suggests he rounded the entries in a table of $\delta(\lambda)$ to three digits. He used the approximate value $\text{Sin } \varepsilon = \text{Sin } 23;35 \approx 24$ instead of the more accurate value 24;0,18 in his numerical example. He also tabulated $\delta_2(\lambda)$, the second declination, in similar fashion (I.184). al-Marrākushī then tabulated $\text{Tan } \delta(\lambda)$ for $\varepsilon = 23;35°$ to three digits for each degree of λ (I.185) – see **I-7.1.7**. His entries are again considerably less accurate than those in the corresponding table in the *Ḥākimī Zīj*. In particular he uses $\text{Tan } \delta(90°) = \text{Tan } 23;35° = 26;11;40$, a value which can be obtained from interpolating between correct values of Tan 23° and Tan 24°. Ibn Yūnus has the more accurate value 26;11,33,(23).

al-Marrākushī also tabulated $\lambda(\delta)$ to two digits for each 0;15° of δ up to 23;35° (I.186/187).

Two rather unusual tables (see further **I-7.1.7**) presented by al-Marrākushī (I.209-210) display the functions:

$$f(\lambda) = {}^{1}/_{12}\ \mathrm{Tan}_{60}\ \delta(\lambda) \quad \text{and} \quad g(\Delta) = {}^{1}/_{12}\ \mathrm{Tan}_{60}\ \Delta$$

to three and two digits respectively for each degree of arguments λ and Δ. The purpose of these tables is to determine the half excess of daylight using only another of Cotangents to base 12, thus:

$$\mathrm{Sin}\ d(\lambda) = f(\lambda) \cdot \mathrm{Cot}_{12}\ \bar{\phi} \quad \text{and} \quad \mathrm{Sin}\ d(\Delta) = g(\Delta) \cdot \mathrm{Cot}_{12}\ \bar{\phi}\ .$$

The use of these functions reflects a strong sentimental attachment to the base 12, which was characteristic of the *muwaqqits* of medieval Egypt.

al-Marrākushī's next tables in the treatise display $d(\phi,\lambda)$ for each 6° of both arguments (I.214-215). His entries for $\phi = 30°$ are slightly different from the corresponding entries in the main Cairo corpus. His tables of $\alpha'(\lambda)$ to three digits for each degree of λ (I.222-225) are less accurate than those which can be derived from the corresponding values of $\alpha(\lambda)$ in the *Ḥākimī Zīj.* He also tabulates $\alpha_\phi(\lambda)$ to degrees and minutes for each 6° of both arguments ϕ and λ (I.230-235), whereas Ibn Yūnus tabulated them for each 1° of both arguments. al-Marrākushī does, however, include a table of $\alpha_\phi(\lambda)$ for $\phi = 30°$ with entries for each degree of λ in degrees and minutes (I.238-240), which again are less accurate than those in the *Ḥākimī Zīj*. Another table of al-Marrākushī's displays the length of the longest day in hours, minutes, and seconds for each degree of ϕ up to 66° (I.246).

al-Marrākushī presents a number of tables scattered throughout the remainder of his treatise which relate the solar altitude and shadow to the time of day. All of the entries in these tables are to two digits, and the functions of time have T = 1, 2, ... , 5 or 6^{sdh} as arguments and sometimes also the time of the afternoon prayer. The tables which are independent of terrestrial latitude are of the functions:

$h(T,H)$	$H = 5°, 10°, \ldots, 90°$	(I.457)
$z_{(12)}(T,H)$	$H = 5°, 10°, \ldots, 90°$	(I.253)
$z'_{(12)}(T,x)$	$x = Z'_{(12)} = 1, 2, \ldots, 12$	(I.254)
$z_{(12)}(T,x),\ z'_{(12)}(T,x)$	$x = Z_{(12)} = 1, 2, \ldots, 36$	(I.256-257)
$h_a(H)$, Sin $h_a(H)$, $z_a{}'_{(12)}(H)$	$H = 5°, 10°, \ldots, 90°$	(I.271)

The tables specifically for $\phi = 30°$ are of the functions:

$h(T,\lambda)$	$\lambda = 270°, 300°, \ldots, 90°$	(I.454)
$z_{(12)}(T,\lambda),\ z'_{(12)}(T,\lambda)$	$\lambda = 270°, 280°, \ldots, 90°$	(I.428-429, 436-437)
$h_0(\lambda),\ \psi(\lambda)$	$\lambda = 30°, 60°, 90°$	(II)
$h_q(\lambda)$	$\lambda = 270°, 300°, \ldots, 90°$	(II)

Finally there is a table displaying $h(T,\lambda)$ for each *equinoctial* hour from 1 to 6 and each 30° of λ. Each of these small tables for timekeeping is of limited practical use. See further **I-4.2.4**. The remaining tables in the treatise are specifically intended for use with sundials, and do not concern the present study.

Geographical coordinates and qibla

In his lists of geographical coordinates (I.202-204, 315-317) al-Marrākushī has the following entries for Mecca and Cairo:

Mecca	ϕ: 21;0°	L: 77;0°
Cairo	29;55°	64;50° (I.316) / 64;54° (I.320)

The latitude of Cairo he claims to have measured himself; however, this value is also attested in earlier sources.[22] Also, in most of his numerical examples he uses the standard value 30;0° (*e.g.*, I.260, 303, 306, 433) rather than 29;55° (*e.g.* I.320, 339). He computes the qibla for Cairo using coordinates (I.320-322):

Mecca	ϕ: 21°	L: 77°
Cairo	30°	65°

to be 37;3° measured from the prime vertical (q = 52;57°). Ibn Yūnus' value 53,0,17°, derived from the same latitudes and longitude difference, is much closer to the accurately-computed value 53;0,23°. al-Marrākushī then computes the qibla at Cairo for his own coordinates (with 64;50° for the longitude of Cairo), and finds it to be 36;18° or 36;19° (q = 53;42° or 53;41°). The first of these values is in fact accurately computed. He notes that if one uses 21;40° for the latitude of Mecca the azimuth (measured from the prime vertical) will be smaller, adding that the qibla value 37;30° (q = 52,30°) (Sédillot has 35;30° here, which must be a copyist's error) which is "widely accepted in Cairo" is "quite incorrect". This value q = 52;30° occurs again in the geographical tables in MS Paris BNF ar. 2513 of the *Muṣṭalaḥ Zīj* (**6.6**), and was probably intended as a happy compromise between Ibn Yūnus' first and second values, 52° and 53°. Finally, it is worth noting that al-Marrākushī's table of $h_q(\lambda)$ (see above) is based on parameter q = 52°, although this is not stated. Further, six of al-Marrākushī's seven values of h_q are the same as those of Ibn Yūnus in the *Ḥākimī Zīj*.

As I have already noted (**2.7**), al-Marrākushī advocated the parameter 20° and 16° for morning and evening twilight. He presents several numerical examples, and computes r to be 23;16°, 28;29°, and 24;54° (Sédillot has incorrectly 24;15°) at the equinoxes, summer and winter solstices, and s to be 18;33°, 22;13°, and 20;4°. Now in MS Dublin CB 3673 of the main Cairo corpus there are tables of r(λ) and s(λ) based on these parameters (**4.10**). No values are given for the equinoxes because of the format of the tables but the solstitial values given, each of which is accurately computed, are respectively:

r: 28;28° and 24;54° and s: 22;13° and 20;4° .

6.8 al-Khaṭāʾī's auxiliary tables for timekeeping by the sun

Muḥammad ibn al-Amīr Fakhr al-Dīn ʿUthmān al-Khaṭāʾī is an individual whose name is new to the modern literature; his father was a Mamluk prince who flourished around 1450 (**I-6.15.1**). His auxiliary tables, which contained about 825 entries, are partially extant in MS Vatican Borg. ar. 217,2 (fols. 6r-7v), copied *ca*. 1500. They are less general in their application than other later Egyptian and Syrian tables, being specifically intended for finding the hour-angle from the solar altitude at the latitude of Cairo. The functions tabulated are related to those of al-Khalīlī's minor set of auxiliary tables, but the two sets may have been compiled inde-

[22] See Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 111, and now King, "Geography of Astrolabes", p. 9 and n. 23.

pendently. al-Khaṭāʾī also tabulated $h_a(\lambda)$, and a(h) at the equinoxes. His table of the first function is more accurate than the ones in the main corpus and the *Muṣṭalaḥ Zīj* (**4.4** and **6.6**). See further **I-6.15.1**.

6.9 Anonymous solar altitude tables for Cairo

MS Cairo MM 72, copied in 747 H [= 1346/47] by Ibn al-Kattānī (**5.5**), contains a curious set of tables preceded by an introduction of which unfortunately only the last page remains. The function tabulated is not explained in the surviving portion of the introduction but in fact it is the solar altitude for values of the time since sunrise and different values of the semi diurnal arc, that is, h(D,T). Values of this function to two digits are given for the domains:

$$D = 104;36°,\ 104°,\ 103°,\ \ldots,\ 76°,\ 75;24° \quad \text{and} \quad T = 1°,\ 2°,\ \ldots,\ [2D]\ ,$$

where [2D] is the largest integer less than 2D. The underlying parameters are:

$$\phi = 30;0° \text{ (Cairo) and } \varepsilon = 23;35°\ ,$$

as in the main Cairo corpus. (Note that 104;36° and 75;24° are the extremal values of D for these parameters.) For each double page of tables serving a value of D, a value of the function $t_a(D)$ is given alongside the argument D at the head of the tables. The altitude of the sun at each seasonal and equinoctial hour of daylight is also displayed for each value of D at the side of the main tables. I suspect that these tables may be by Najm al-Dīn al-Miṣrī (**6.5**): more information is in **I-4.4.1** (illustrated).

6.10 Anonymous prayer-tables for Mecca

MS Cairo MM 68, copied *ca*. 1500, contains an anonymous set of prayer-tables computed for latitude 21°, that is, for Mecca.[23] The work bears no title and no date, but I do not doubt that it was compiled in Egypt. Establishing the identity of the compiler of these tables would probably solve some of the remaining problems associated with the main Cairo corpus. The format of the tables in this Cairo manuscript is the same as that of the main corpus, and the functions tabulated are:

$$H,\ D,\ h_a,\ t_a,\ T_a,\ 2N,\ (2n\ \text{-}r),\ r,\ s,\ \alpha_s \text{ and } \alpha_\phi\ .$$

The underlying value of ε is 23;35° and the tables are rather accurately computed.

The tables for latitude 21° in the corpus of prayer-tables for all latitudes preserved in MS Princeton Yahuda 861,1 (**8.1**) are quite different from these, being very carelessly computed. The table for α_ϕ in the Cairo manuscript is identical with that of Ibn Yūnus for latitude 21° in MS Leiden Or. 143 of the *Ḥākimī Zīj*, containing the same slight errors. The tables for twilight, that is, of the functions r and s, are based on parameters 19° and 17°, and do not display the error made by al-Mizzī in his tables for Damascus (**9.2**). MS Cairo MM 68 also contains a table of a function called *al-aṣl al-muṭlaq*, “the absolute base”, defined by:

$$B(\delta) \quad = \quad \text{Cos}\ \delta\ \text{Cos}\ \phi\ /\ R\ .$$

Values are given to two digits for each 0;5° of argument up to δ = ε. (See further **I-6.4.4**.)

[23] *Cairo ENL Survey*, no. C40.

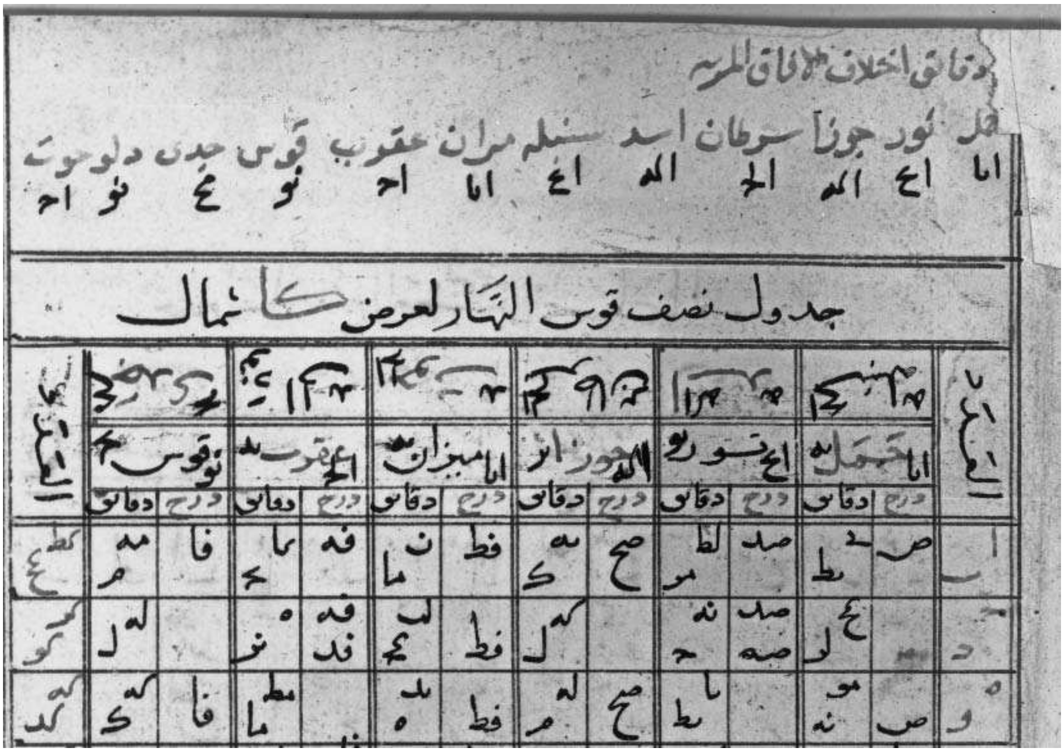


Fig. 6.10: The values above the table of the half arc of daylight represent the time taken by the sun to move between the true and the visible horizons. [From MS Cairo MM 68, courtesy of the Egyptian National Library.]

At the top of the table of D(λ) is a set of numbers for each of the signs (see **Fig. 6.10**) . They are labelled *daqāʾiq ikhtilāf al-āfāq al-marʾiyya*, "the "difference minutes" of the visible horizons", and it is clear that they are intended to represent the effect of refraction at the horizon on the arc of daylight, that is 2(ΔD), twice the "difference minutes" of the Cairo astronomers (**4.11**). The values given for the twelve signs starting with Aries are as follows:

1;11° 1;18 1;25 1;33 1;25 1;18 1;11 1;3 0;56 0;48 0;56 1;3

Thus the equinoctial value is intended to be 1;11° and the values for the summer and winter solstices are respectively 0;22° more and 0;23° less. The equinoctial value corresponds to an angle of depression below the horizon of approximately:

$$(^1/_2 \cdot 1;11°) \cos 21° = 0;33° ,$$

but this is not what is intended. Consider the values of ΔD for $\phi = 21°$ which are given by al-Minūfī (**8.2**). These yield the following rounded values for 2(ΔD), "according to al-Minūfī" (A) and "according to earlier scholars" (B):

A:	1;6	1;13	1;20	1;27	1;20	1;13	1;6	0;59	0;52	0;45	0;52	0;59
B:	1;6	1;14	1;21	1;28	1;21	1;14	1;6	1;0	0;53	0;46	0;53	1;0

These values are close to those given in the Cairo manuscript. Actually, however, if we multiply Ibn Yūnus' values by a factor of 2 • 22;40°/30°, rather than 2 • 21°/30°, which is the factor underlying al-Minūfī's values, we obtain the set:

1;11 1;19 1;26 1;34 1;26 1;19 l;11 1;3 0;56 0;48 0;56 1;3 .

Now these correspond very closely to those for Mecca in the Cairo manuscript. It remains only to explain the number 22;40°. Perhaps the values of 2(ΔD) were originally intended for ϕ = 21;40°,[24] another widely-used medieval value for the latitude of Mecca, and a careless error was made in the computation.

6.11 Ibn al-Rashīdī's prayer-tables for Mecca

MS Cairo Sh 89,4 (fols. 29v-32v), copied in 1025 H [= 1616], is the only known copy of the introduction to a set of prayer-tables for latitude 21°, that is, Mecca, by Ibn al-Rashīdī (**4.1**).[25] The tables are no longer contained in the manuscript but they are described in the text. The latitude 21° is specifically mentioned, as well as the parameters 19° ands 17° for twilight (*cf.* **6.10**). Ibn al-Rashīdī mentions ten tables of the following functions:

$$T(h) \text{ and } t(h) \text{ for } \lambda = 0°,\ H,\ D,\ h_a,\ t_a,\ h_b,\ T_a,\ s,\ r,\ \alpha_\phi\ .$$

The first table has its counterpart in the tables for Jerusalem in MS Princeton Yahuda 861,1 which I suspect were also computed by Ibn al-Rashīdī (**8.1** and **9.5**).

Added in proof: My notes to these tables indicate that MS Leiden UB Or. 2805, which I have not consulted, may contain the same tables.

6.12 Anonymous timekeeping tables for Jerusalem

Two disordered copies of timekeeping tables for Cairo, MSS Cairo DM 45 and Cairo DM 153, contain odd folios copied in the same hand *ca*. 1650 from a set of tables of the hour-angle t(h,λ) computed for

$$\phi = 32;0° \text{ (Jerusalem) and } \varepsilon = 23;35°\ .$$

The tables have the same format as the Cairo corpus and are probably of Egyptian provenance.[26] The entries are identical with the corresponding entries in al-Karakī's tables of t(λ,h) (**9.4**). Since al-Karakī states in his introduction that the 14th-century Egyptian astronomer Ibn al-Rashīdī had compiled a set of tables of t(h,λ) for an unspecified latitude I suspect these tables for Jerusalem are by Ibn al-Rashīdī. See also **I-2.1.5**.

6.13 al-Bakhāniqī's tables for the afternoon prayer at different latitudes

MS Cairo MM 33,2 (fols. 6v-7r), penned *ca*. 1600, is the only copy known to me of a table for the afternoon prayer attributed to al-Bakhāniqī, the editor of the main Cairo corpus (**4.1.5** and **5.6**). The table is entitled *jadwal maʿrifat faḍl dāʾir al-ʿaṣr li-ruʾūs al-burūj*, "table for finding the hour-angle at the beginning of the afternoon prayer at the first points of the zodiacal signs". The function tabulated is:

$$t_a(\phi,\lambda)$$

---

[24] King, "Earliest Muslim Geodetic Measurements", pp. 225-226.
[25] See *Cairo ENL Survey*, no. C39/3.1.5.
[26] *Ibid*.

for argument domains:

ϕ = 10°, 11°, ... , 50° and λ = 90°, 60°, 30°, 0°, 210°, 240°, 270° ,

and entries are given in degrees and minutes. See also **VIb-7** (illustrated).

6.14 Four Egyptian copies of al-Khalīlī's universal auxiliary tables

MS Cairo MM 43 is an Egyptian manuscript, perhaps dating from around 1600, which contains al-Khalīlī's universal auxiliary tables (**10.5**), complete with his introduction but without mention of his name. The functions f_ϕ and g_ϕ are tabulated for values of ϕ up to 50° rather than 55° and the function G is tabulated for values up to 55 rather than 59. These auxiliary tables are preceded by tables of:

$\delta(\lambda)$ (ε = 23;31°) and $h_a(H)$

to two digits for each degree of argument, and are followed by a mixed bag of anonymous spherical astronomical tables. The following functions are tabulated to two digits for each integral unit of argument unless otherwise stated:

(1) $B(\lambda)$ (ϕ = 30° and ε = 23;55°), $\Delta\lambda$ = 0;15°
(2) max $D(\phi)$ ($\varepsilon \approx$ 23;30°), to hours, minutes and seconds, ϕ = 1°, 2°, ... , 66°
(3) $\alpha_\phi(\lambda)$ (ε = 23;35°), ϕ = 6°, 12°, ... , 66°, $\Delta\lambda$ = 6°
(4) arc Sin (x)
(5) $\alpha(\lambda)$
(6) arc Tan_{12} (x)
(7) $H(h_a)$, h_a = 1°, 2°, ... , 45° (for use on quadrants)
(8) $\psi(\Delta)$ (ϕ = 30°), Δ = 1°, 2°, ... , 60°
(9) $\lambda(\delta)$, δ = 1°, 2°, ... , 23°, 23;15°, 23;30°, 23;35°
(10) arc Cot_{12} (x) (garbled)
(11) $h_a(H)$, ΔH = 0;30°
(12) Cot_{12} h, Δh = 0;30°
(13) $^1/_{12}$ Tan Δ
(14) $^1/_{12}$ Tan $\delta(\lambda)$ (ε = 23;35°), 3 digits
(15) Vers θ, 3 digits
(16) Sin θ, 3 digits
(17) $\delta(\lambda)$ (ε = 23;35°)
(18) $\alpha_\phi(\lambda)$ (ε = 23;35°, ϕ = 21°, 24°, 36°)
(19) $\alpha(\lambda)$ (ε = 23;35°)
(20) $\Delta\alpha_n(\phi)$ (ε = 23;35°), ϕ = 1°, 2°, ... , 67°, n = 1, 2, ... , 12
(21) $G(\Delta)$ (ϕ = 30°), Δ = 1°, 2°, ... , 60°
(22) $d(\Delta)$ (ϕ = 30°), Δ = 1°, 2°, ... , 60° .

All of the tables are anonymous except no. 20 which is attributed to Ibn Yūnus. There is also a list of geographical coordinates and qiblas[27] and tables for marking curves on astrolabes. On tables nos. 1, 8, 13-14, and 21 see further **I-6.4.3**, **5.6.1**, **7.1.7** and **6.8.1**, respectively.

[27] Overlooked in King, *Mecca-Centred World-Maps*, pp. 76-86.

Three other Egyptian copies of al-Khalīlī's universal auxiliary tables are in MSS Cairo MM 98,2, copied *ca*. 1700, Princeton Yahuda 861,2, and Cairo DM 758, copied *ca*. 1650. In the first the tables are anonymous. In the second the auxiliary tables have been bound in the midst of the tables of the *Natīja al-kubrā* (**8.1**): here they are incorrectly entitled *Faḍl dāʾir al-Khalīlī*, "al-Khalīlī's hour-angle (tables)". In the third the tables also bear the title *Kitāb Faḍl al-dāʾir* and are attributed to al-Khalīlī. Between the introduction and the tables themselves is an anomalous table of $h_v(\lambda)$ for Cairo attributed to Ibn al-Rashīdī (**4.8**), with a remark that for other latitudes the azimuth of the *bādahanj* is also the rising point of Capricorn (!): see **Fig. VIIb-4.1**.

Finally, MSS Cairo DM 644,1, copied *ca*. 1700, and Istanbul S. Esad Efendi Medresesi 119,2, date ?, are two copies of an anonymous Egyptian set of auxiliary tables entitled *Fatḥ al-Karīm al-Bāqī fī maʿrifat al-dāʾir wa-faḍlihi āfāqī*, "The Victory of God, the Noble and Eternal, for Finding the Time Since Sunrise and Hour-Angle for all Latitudes". These tables were lifted from al-Khalīlī's auxiliary tables for timekeeping by the sun (**10.3**). See further **I-9.11**.

6.15 The auxiliary tables of Ibn al-Mushrif, al-Māridīnī, al-Wafāʾī, and al-Ṣūfī

At least five Egyptian astronomers compiled sets of auxiliary tables for solving the standard problems of spherical astronomy.[28] Tables of this kind containing a few hundred entries were first devised by Ḥabash and Abū Naṣr in the 9th and 10th centuries, but the most outstanding example is the set containing over 13,000 entries compiled by al-Khalīlī in the 14th century (**10.7**). Ḥabash's tables were known in Egypt: they are referred to by Ibn Yūnus and were incorporated into one recension of the 13th century *Muṣṭalaḥ Zīj* (**6.6**). al-Khalīlī's universal auxiliary tables, as well as his minor set, were also known in Egypt (**6.14**). I have surveyed the development of Islamic auxiliary tables, including the Egyptian examples noted below, in **I-9**.

Jamāl al-Dīn ʿAbdallāh ibn Khalīl al-Māridīnī (d. 1406) was one of the leading astronomers in Cairo and/or Damascus in his time (**I-9.6**). His auxiliary tables, which contain 8,100 entries, offer little practical advantage to the user because of the simple nature of the functions tabulated. They are contained in MS Paris BNF ar. 2525,1 (fol. 1v-16v), copied *ca*. 1450, where they precede an incomplete set of prayer-tables for all latitudes (**8.1**), and also MS Cairo K 4026 (late). See further **I-9.6**.

Abū Bakr ibn Ismāʿīl known as Ibn al-Mushrif worked in Cairo in the first half of the 15th century, and his name is new to literature.[29] MS Cairo MM 241, penned *ca*. 1450, is a unique copy of a set of auxiliary tables compiled by him containing over 15,000 entries. The tables take advantage of the mathematical equivalence of the problems of determining the hour-angle and the azimuth from solar or stellar altitude. See further **I-9.8**.

ʿIzz al-Dīn ʿAbd al-ʿAzīz ibn Muḥammad al-Wafāʾī (d. *ca.* 1470) was one of the leading astronomers in Egypt in the mid-15th century (**I-9.9** and **II-5.7**). His auxiliary tables are extant

[28] See already King, "Universal Solutions from Mamluk Egypt and Syria", of which a new version is in **VIb**.
[29] On Ibn al-Mushrif (**I-9.8**) see *Cairo ENL Survey*, no. C43. The individual with the same name mentioned in King, *Astronomy in Yemen*, no. 12, is a Yemeni astronomer from the early 14th century.

in MSS Vatican Borg. ar. 217,1 (fols. 1v-5v) and Istanbul Nuruosmaniye 2921,2 (fols. 22r-26v), and contain about 1,500 entries. They are based on the function $G(\lambda)$ discussed at length by Ibn Yūnus, and, like those of Ibn al-Mushrif and al-Khalīlī, exploit the mathematical equivalence of the problems of determining the hour-angle and azimuth from solar or stellar altitude. See further **I-9.9**.

Ibn Abi 'l-Fatḥ al-Ṣūfī (d. *ca.* 1495) was another of the leading astronomers in Cairo in the late 15th century.[30] He prepared an extensive *zīj* based on those of Ibn al-Shāṭir (*ca.* 1350) and Ulugh Beg (*ca.* 1430), adapting the tables for the geographical coordinates of Cairo. Nevertheless in his own solar tables which precede his auxiliary tables in MS Oxford Seld. Supp. 101 (Uri 1040) he prefered to use the parameters of Ibn Yūnus derived five centuries previously. (The title-folio of MS Leiden Or. 143 of the *Ḥākimī Zīj* bears biographical information on Ibn Yūnus in the hand of al-Ṣūfī: see **Fig. 5.1**) al-Ṣūfī's auxiliary tables contain over 15,000 entries and are more suitable for finding the hour-angle from solar or stellar altitudes for any latitude than for finding the azimuth. See further **I-9.10**.

6.16 al-Asyūṭī's prayer-tables for Assiut

MS Cairo DM 188,1 (fols. 1r-10r), copied *ca*. 1700, contains a set of prayer-tables entitled *Nuzhat al-abṣār fī aʿmāl al-layl wa-'l-nahār li-ʿarḍ kāf-zāy*, "Delights of the Eyes for Astronomical Operations at Latitude 27°", that is, for the city of Assiut. On the title page it is stated that the tables were prepared by Muḥammad (ibn ʿAbd al-Qādir ibn Dallāl) al-Wafāʾī (al-Asyūṭī),[31] a student of Muḥammad ibn (Abī) al-Fatḥ al-Ṣūfī (**6.15**).

The functions tabulated are the following, with identical format to the tables in the Cairo corpus:

$$\delta,\ H,\ d,\ D,\ \tilde{h},\ 2N,\ \psi,\ h_0,\ Z_{(12)},\ z_{a(12)},\ t_a,\ T_a,\ r,\ s,\ (2N\text{-}r),\ (r{+}D) \text{ and } \alpha_\phi\ .$$

al-Asyūṭī was not a bad student: his tables are rather carefully computed and are more accurate than the tables for latitude 27° in the *Natīja* attributed to al-Wafāʾī (**8.1**). However, al-Asyūṭī preferred to use 23;35° for the obliquity rather than the more up-to-date value 23;30,(17)° of Ulugh Beg. His tables of r and s are based on parameters 19° and 17° respectively, and the former is more accurately computed than the latter. His tables of α_ϕ ($\phi = 27°$) and δ are less accurate than those in the *Ḥākimī Zīj* (**5.1**). On the title folio of MS Cairo DM 188 there is a note stating the approximation:

$$\max d(\phi) \approx {}^1\!/_2\ \phi\ .$$

al-Asyūṭī would doubtless have disapproved of this: his own tables give the accurate value 12;51° for max $d = d(90°)$ ($\phi = 27°$), and the value one can derive with this approximate formula, namely 13;30°, is in error by almost $^2/_3$°.

---

[30] On Ibn Abi 'l-Fatḥ al-Ṣūfī see n. **I**-9:30.

[31] On al-Asyūṭī see *Cairo ENL Survey*, no. C101.

6.17 Ibn Ṭūghān's table for determining the times of moonrise and moonset

MS Cairo MM 167,4 (fols. 158v-160r), copied in 989 H [= 1581], contains a short introduction and table of two pages by Jamāl al-Dīn Yūsuf ibn Ṭūghān al-Qiṭṭajī, otherwise known to us as the author of a treatise on astrology.[32] The table, displayed in **Fig. 6.17**, enables the user to determine the time between sunset and moonset from the nocturnal arc, entered horizontally, and the days elapsed since conjunction, entered vertically. Entries are given in degrees and minutes for each 6° of nocturnal arc and each day from 1 up to 16. If more than 14 days of the month have passed the instructions tell us to subtract 14 from the number of days passed since conjunction and enter the remainder to find the time passed since sunset when the moon will rise. The author acknowledges that the underlying procedure is approximate, and in fact the entries f(2N,n) are determined by simple relation:

$$f = \eta \cdot 2N/180 \cdot n ,$$

where η is the mean daily relative motion of the sun and moon, 12;53°.

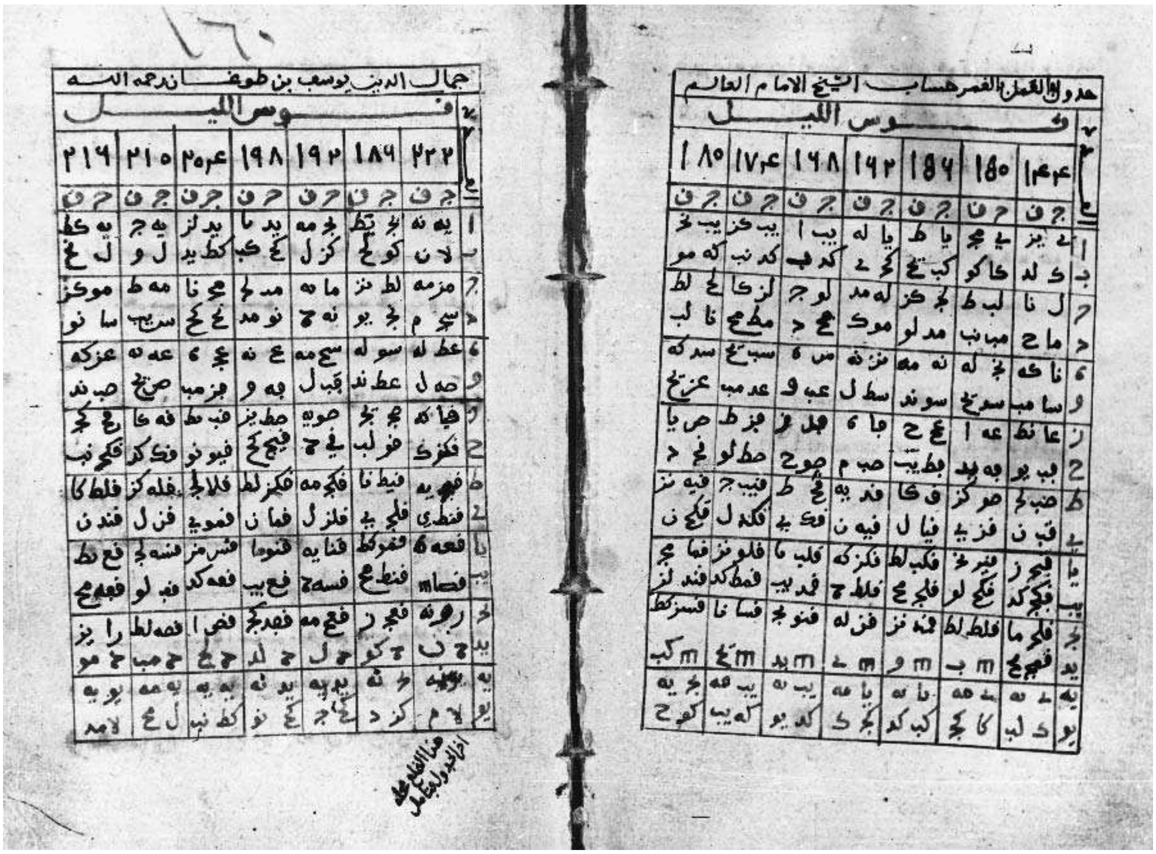


Fig. 6.17: A unique table for finding the times of moonrise and moonset. Note that the first column on the left-hand page should be the last column, as noted by the copyist. [From MS Cairo MM 167,4, fols. 159v-160r, courtesy of the Egyptian National Library.]

[32] On Ibn Ṭūghān see *Cairo ENL Survey*, no. C92; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 228-229, no. 108.

CHAPTER 7

LATE MODIFICATIONS TO THE MAIN CAIRO CORPUS

7.0 Introductory remarks

From a mathematical point of view it was difficult for the astronomers of medieval Egypt to improve in any way upon the tables in the main Cairo corpus, these being in general very carefully computed, and the changes in the obliquity of the ecliptic over the centuries had an effect on the prayer-times that was negligible. Nevertheless, one Egyptian astronomer endeavoured to modify at least the prayer-tables in the corpus for a more up-to-date value of the obliquity (**7.1**). Certain *muwaqqit*s with time on their hands succumbed to the temptation to stretch some of the tables in the corpus to display values for each minute of solar longitude instead of each degree (**7.2**). Others rearranged the format of the tables and added information about timekeeping by the stars (**7.5**). Yet others were content to prepare some simple tables for finding the solar longitude for a date in the Hijra calendar and supplement these with prayer-tables lifted from the corpus (**7.7-9**).

From a practical point of view it was clearly more convenient to display the times of prayer in almanacs with values arranged for each day of the year (**7.4**). For those who did not like tables at all, the times of prayers for each day of the year were written out in words (**7.6**, **7.7** and **7.10**). The Egyptian astronomers also modified the tables of the corpus so that the entries were expressed in hours and minutes according to the Ottoman convention that sunset is 12 o'clock (see **7.11** and **7.12-13**; on the convention see **14.0**).

7.1 al-Minūfī's prayer-tables

Shams al-Dīn Muḥammad ibn Nāṣir al-Dīn al-Minūfī was a *muwaqqit* at the Ghawriyya *madrasa* in Cairo[1] who lived in the mid 16th century and compiled a set of prayer-tables in which he sought to modify the tables of the main Cairo corpus. Complete copies of these are MSS Cairo DM 470 and 467, the former, in 50 fols., penned *ca*. 1570 by the author's son ʿAbd al-Qādir (**7.2** and **8.2**), the second in 1144 H [= 1731/32]. The tables are entitled *ʿAnwān al-muhimmāt fī taḥrīr al-awqāt*, "The Epitome of Important Matters in Fixing Prayer-Times".

In the introduction the author claims that the tables are computed according to the "new observations" and that he has reworked the tables of "the *Natīja*" according to the new value of the obliquity, meaning the value 23;30(,17)° found by the astronomers of Ulugh Beg in

[1] Muhammad al-Minufī is not mentioned in the modern bio-bibliographical sources before *Cairo ENL Survey*, no. C120; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 187-188, no. 88 (listing mainly Cairo manuscripts). The suggestion in King, "Astronomical Timekeeping in Medieval Cairo", p. 373, n. 68, that he might be identical with Muhammad ibn Abi 'l-Fatḥ al-Ṣūfī (see **6.15**) is incorrect.

Samarqand. In fact, only a few of the simpler tables have been recomputed. It would be interesting to know precisely what al-Minūfī meant by *Natīja*: perhaps the complete set of prayer-tables for Cairo appended to those of T, t and a in the edition of the main corpus by al-Bakhāniqī (**5.6**)?

The other modification which al-Minūfī introduced was to incorporate corrections for refraction at the horizon in all tables involving a period of time measured from sunrise or sunset. In the introduction to his tables he mentions that because of the nature of light rays, an observer can see more than half of the celestial sphere. According to Ibn al-Haytham, he says, a person standing at the equator $3^1/_2$ cubits above the surface of the earth sees 0;4,26 more than half of the celestial sphere. I have been unable to trace this statement in any treatise associated with the illustrious 11th-century physicist (see, however, **9.3**, also **14.9**). The remark makes little sense, especially in the light of what follows. al-Minūfī proceeds to state that Ibn Yūnus fixed the interval between the moment when the centre of the sun is on the true horizon and the moment when its centre is on the apparent horizon as 0;47° at the equinoxes, increasing 0;5° for each sign to the summer solstice and decreasing similarly to the winter solstice. These are the corrections ΔD which are tabulated in certain late copies of the Cairo corpus (**4.11**), including al-Minūfī's prayer-tables (see below). al-Minūfī's son ʿAbd al-Qādir wrote at length about this function ΔD and tabulated it for all latitudes (**8.2**).

al-Minūfī tabulates most of the functions found in the main corpus, modified as follows. For those functions which involve horizon phenomena the function is tabulated for the sun on the true horizon and then again for the sun on the apparent horizon, the second table having the correction ΔD taken into account. Another feature that is not found in the main corpus is the instructions presented in marginal notes concerning the exact time of day for which to determine the solar longitude to use as argument in the tables. Thus, for example, there is a table of t_a, the time from midday to the afternoon prayer: the instructions are to enter the true longitude of the sun at the time of the afternoon prayer as argument. In the table of T_a, the time from the afternoon prayer to sunset, the instructions indicate that the true longitude of the sun at sunset should now be the argument and the value corrected with the "difference minutes" and the "minutes of the solar radius" to find the time when the sun has disappeared over the apparent horizon. This table is followed by another in which the "difference minutes" are taken into account, and one need only correct for the solar radius to obtain the value required. There is no table of the "minutes of solar motion" between midday and the other prayer-times to facilitate the computation of the argument, such as is found in a note by al-Wafāʾī (**5.7**) and al-Lādhiqī's tables (**7.8**).

al-Minūfī's work contains tables of the following functions (those adjusted for the difference minutes are asterisked):

$$D,\ D^*,\ t_a,\ h_a,\ T_a,\ T_a^*,\ H,\ h_v,\ d,\ \delta,\ T_q,\ t_q,\ Z_{(12)},\ h_q,\ \psi,\ h_0,\ 2D^h,\ 2D^{*h},\ \tilde{h},\ 2D^{*h},$$
$$s,\ r,\ (r{+}D),\ (D{+}s),\ 2N,\ 2N^*,\ n,\ n^*,\ \sigma,\ \sigma^*,\ \tau,\ \Delta D,\ \alpha_s,\ \alpha_\tau,\ \alpha',\ \alpha \text{ and } \alpha_f\ .$$

The work concludes with a star table giving equatorial coordinates for the year 961 H [= 1551].

The functions d (and hence D, 2N, $2D^h$, $\tilde{h}$, *etc.*), δ (and hence H), ψ, h_a and t_a, and α' and α_ϕ (ϕ = 30°) have been recomputed for ε = 23;30°. The functions h_q, T_q, t_q, h_0, r and s, and h_v, have been taken from the main corpus, being based on ε = 23;35°. Functions such as n (= 2N-(r+s)) are based on values computed with different parameters.

al-Minūfī's knowledge of spherical astronomy was such that he reproduced the garbled entries:

$$\psi(60°) = 23;29° \text{ and } h_0(30°) = 23;34°$$

from earlier tables. Both these values should be the same as ε for latitude 30° (*cf.* **4.3**), that is, 23;30° for $\psi(60°)$ and 23;35° for $h_0(30°)$. Furthermore, he did not realize that Ibn Yūnus' values for the difference minutes were mutually inconsistent. al-Minūfī's efforts to improve the corpus were hardly successful, but he was the only Egyptian *muwaqqit* to attempt to radically modify it.

Another work on timekeeping by al-Minūfī is a treatise entitled *Naẓm al-yawāqīt fī taḥrīr aʿmāl al-mawāqīt,* "Organizing the Gems concerning the Correction of the Operations of Timekeeping", and is extant in MS Cairo MM 235,1 (fols. 1r-7r), copied *ca*. 1600 by his son ʿAbd al-Qādir. In this al-Minūfī mentions the "difference minutes" and the way in which they should be used, and cites the *Tanqīḥ al-manāẓir*, the well-known work on optics by the late-13th-/early-14th-century Baghdad scholar Kamāl al-Dīn al-Fārisī,[2] rather than Ibn Yūnus or Ibn al-Haytham. See further **8.2**.

7.2 The prayer-tables of ʿAbd al-Qādir al-Minūfī and Ibrāhīm ibn Qāyitbāy

ʿAbd al-Qādir al-Minūfī[3] was the son of Muḥammad al-Minūfī who modified the main Cairo corpus (**7.1**). He wrote on the subject of refraction of the horizon (**8.2**) and also compiled some prayer-tables in which the tables of various functions, mainly those tabulated by his father, are stretched by linear interpolation to display values for smaller argument increments. In this he was following an earlier tradition of al-Wafāʾī and Ibn al-Mushrif (**5.7**).

MS Cairo DM 1101 was copied in the hand of al-Minūfī-Jr. in 974 H [= 1566/67] and contains tables of the functions:

$$D + \Delta D,\ 2D^h,\ t_a \text{ and } s$$

with values to three digits for each 0;3° of λ. MS Cairo MM 66, copied in the previous year by al-Minūfī-Jr., contains tables of $\alpha'(\lambda)$ with values to three digits for each minute of argument, and MS Cairo MM 45, copied *ca*. 1625 by Ibrāhīm ibn Qāyitbāy (see below) contains similar tables with entries to five digits (lifted from Ulugh Beg), both attributed to ʿAbd al-Qādir al-Minūfī.

Ibrāhīm ibn Qāyitbāy was a student of Shams al-Dīn al-Minūfī who worked in Cairo *ca*. 1625.[4] Several manuscripts preserved in Cairo contain prayer-tables computed and also copied by him, similar to those of ʿAbd al-Qādir al-Minūfī. The functions tabulated in the various manuscripts are as follows:

$$t_a \text{ (DM 34 and DM 152)},\ \alpha_s \text{ (DM 153,1 and DM 740,2)},\ \Delta D \text{ (DM 682)},$$
$$T_a + \Delta D \text{ (DM 152) and } D + \Delta D \text{ (DM 33)}$$

See **Fig. 7.2b** for a sample of the first of these, and **Figs. 7.2a** and **c** for their title-folios. Some

---

[2] On Kamāl al-Dīn see the article by Roshdi Rashed in *DSB*. See also n. 8:9.

[3] On ʿAbd al-Qādir al-Minūfī see Suter, MAA, no. 479 (confused); *Cairo ENL Survey*, no. C121; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 217-220, no. 97.

[4] Ibrāhīm ibn Qāʾitbāy is not mentioned in the modern sources besides *Cairo ENL Survey*, no. C122, and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 264-265, no. 130 (lists only Cairo manuscripts).

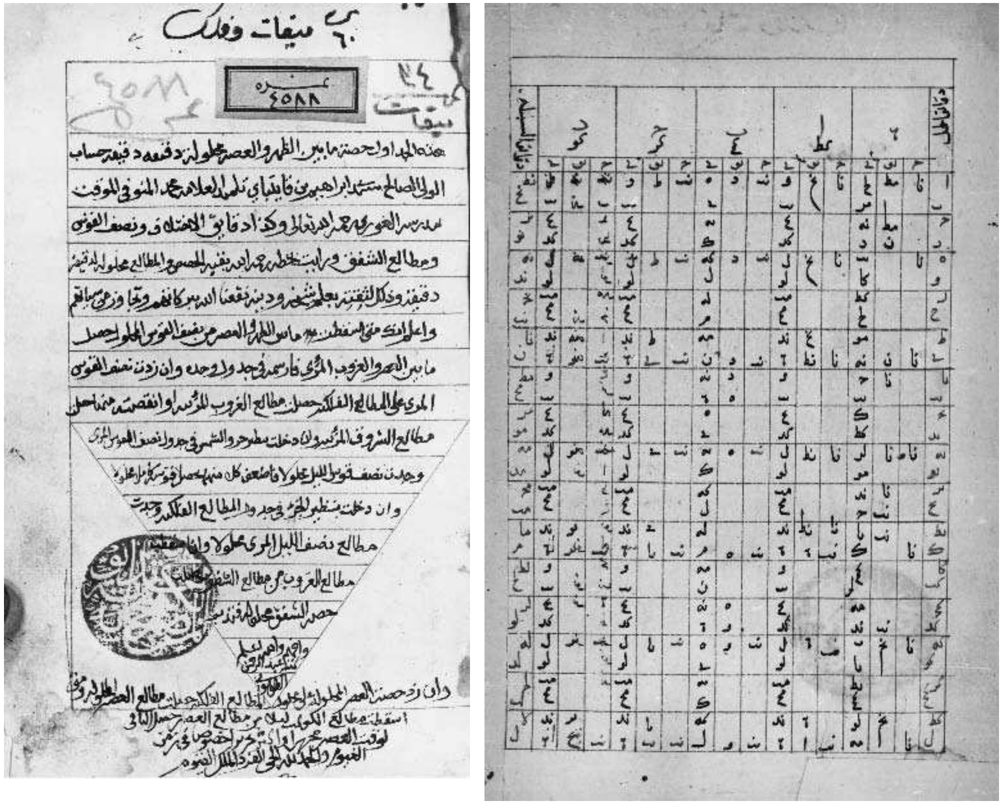


Fig. 7.2a-b: The title-folio and an extract from the "extended" tables of the hour-angle at the beginning of the ʿ*aṣr*. This manuscript belonged to the celebrated 17th-century astronomer and muwaqqit ʿAbd al-Raḥmān al-Ṭulūnī: see **V-10**. [From MS Cairo DM 34, courtesy of the Egyptian National Library.]

Fig. 7.2c: The title-folio of Ibrāhīm ibn Qāyitbāy's "extended" tables of the "difference minutes". This manuscript also belonged to al-Ṭulūnī. [From MS Cairo DM 682, courtesy of the Egyptian National Library.]

other anonymous prayer-tables of the same kind are preserved in various other Cairo manuscripts. For example, MSS Cairo DM 57,1 and 151,2, originally one codex of seven folios copied *ca*. 1600, contain a set of tables of σ (*al-dāʾir li-ʾl-salām*) for each 0;6° of λ, based on the assumption that the *salām* is 2° before daybreak, and MSS DM 545, 158 and 423, copied about the same time, contain tables of α_σ (*maṭāliʿ al-salām*) for each 0;3° of λ based on the assumption that the *salām* is 1° before daybreak.

7.3 al-Qaymarī's tables for the afternoon prayer

MS Cairo DM 620,9 (fols. 58r-59r + 64r-72v), copied *ca*. 1450, contains a set of tables entitled *jadwal al-bāqī li-ʾl-ʿaṣr*, "table of time remaining until the afternoon prayer", and attributed to an individual called al-Qaymarī, on whom I have no further information. The function tabulated is simply:

$$\tau_a(h,\lambda) = t_a(\lambda) - t(h,\lambda)$$

and values are given for each integral degree of h such that $h_a(\lambda) \leq h \leq H(\lambda)$ for each degree of λ. An extract is shown in **Fig. 7.3**. See **11.3** on two similar sets of tables for Damascus.

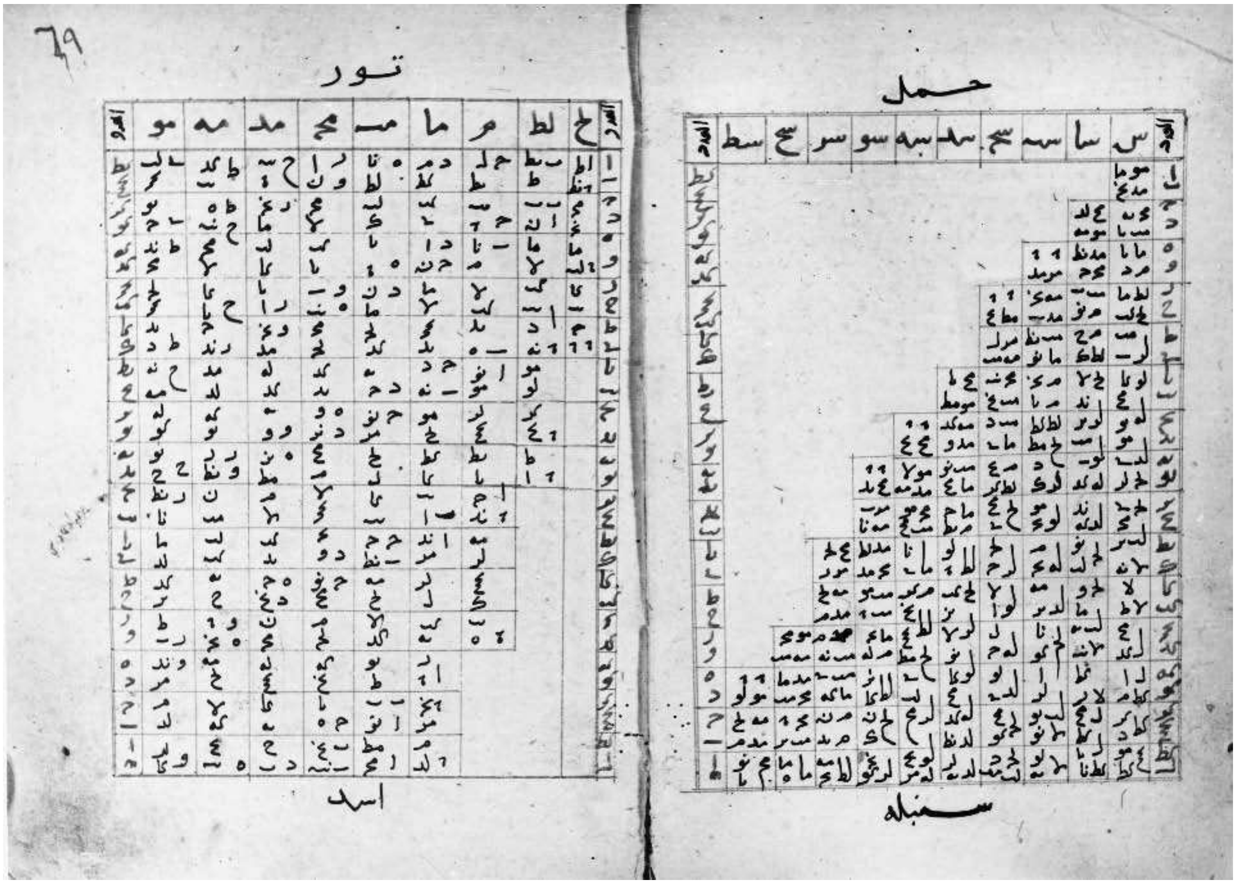


Fig. 7.3: An extract from al-Qaymarī's tables, serving Aries and Taurus / Virgo and Leo. [From MS Cairo DM 620,9, fols. 68v-69r, courtesy of the Egyptian National Library.]

These tables are followed (fols. 72r and 72v) by others of $\psi(\lambda)$ and $h_0(\lambda)$ (the entries, which are given to two digits, are not corrupt as in other such tables in the Cairo corpus – see **4.3**) and $h(a,\lambda)$ for $a = 30°$ (**4.6**).

7.4 Ibn Abi ’l-Khayr al-Ḥusnī’s prayer-tables

MSS Cairo TR 114 (74 pp.) and ZK 154 (91 fols.), both from *ca*. 1700, are two copies of a treatise entitled *Itḥāf al-ḥabīb bi-maʿrifat al-tawqīʿāt wa-’l-awqāt wa-’l-qibla bi-’l-taqrīb* by the 16th-century Egyptian astronomer Muḥammad ibn Abi ’l-Khayr al-Ḥusnī.[5] In this work al-Ḥusnī has included the main prayer-tables of the main corpus in the text of his treatise. For each day of the Coptic year he gives the values of the functions:

$$D,\ t_a,\ T_a,\ s,\ n \text{ and } r\ ,$$

with values given to two digits and written in *abjad* notation. See **8.3** on some sophisticated tables for twilight by al-Ḥusnī.

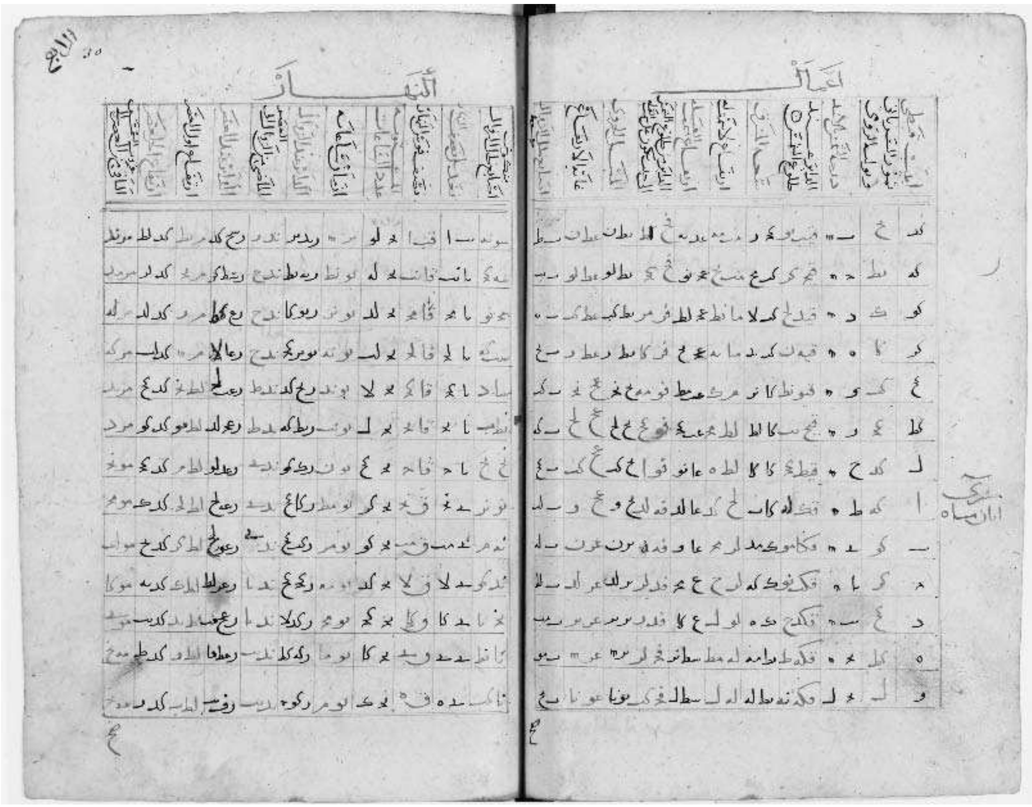
7.5 al-Fawānīsī’s prayer-tables for Cairo

MSS Paris BNF ar. 2545, copied *ca*. 1500 in an elegant hand, and Oxford Seld. Supp. 99 are copies of an extensive set of prayer-tables entitled *Natījat al-afkār fī aʿmāl al-layl wa-’l-nahā*r, “The Result of Thoughts about the Operations of Timekeeping by Night and Day”, the same title used by al-Lādhiqī (**7.8**). These were compiled by a late-16th-century Egyptian astronomer named Muḥammad ibn ʿUmar ibn Ṣiddīq ibn ʿUmar al-Bakrī al-Fawānīsī.[6] It is not difficult to show that very little of consequence in this at first sight imposing work is original.

In the introduction to the tables the author notes certain approximate rules for determining the standard functions of *mīqāt*, but makes no mention of the fact that similar rules had been formulated by his predecessors Najm al-Dīn al-Miṣrī and al-Bakhāniqī – see **2.5** and **2.9** above. Each of al-Fawānīsī’s rules is intended for latitude 30°, except those for functions h_a and H, of which he incorrectly states that they work well for latitudes between 0° and 40°. His rules, outlined in words in the text, are the following:

(a)	$d \approx {}^1/_2\,(\delta + {}^1/_4\,\delta)$	
(b)	$\psi \approx \delta + {}^1/_6\,\delta$	
(c)	$h_q \approx 47° + \delta + {}^1/_4\,\delta + {}^1/_6\,\delta$	
(d)	$T_q \approx 57° + \delta + {}^1/_2\,\delta + {}^1/_6\,\delta$	
(e)	$t_a \approx 52° + {}^1/_2 \cdot {}^1/_6\,\delta$	$(\delta > 0)$
	$t_a \approx 52° - [\,\lvert\delta\rvert/4 + \lvert\delta\rvert/6\,]$	$(\delta < 0)$
(f)	$h_a \approx {}^1/_2\,H + {}^1/_{10}\,[\,\max H - H\,]$	
(g)	$h_b \approx {}^1/_2\,h_a + {}^1/_{10}\,[\,\max H - h_a\,]$	

[5] On Ibn Abi ’l-Khayr al-Ḥusnī see Suter, *MAA*, no. 511; Brockelmann, *GAL*, SII, p. 485; *Cairo ENL Survey*, no. C124; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, pp. 255-262, no. 127.

[6] On al-Fawānīsī see Suter, *MAA*, no. 475; Brockelmann, *GAL*, II, p. 469, and SII, p. 485; *Cairo ENL Survey*, no. D10; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 237-238, no. 111.

(h)	$r \approx 21;0° + {}^1/_8\,\delta + {}^1/_2 \cdot {}^1/_6\,\delta$	$(\delta > 0)$
	$r \approx 21;0° + {}^1/_2 \cdot {}^1/_8\,\lvert\delta\rvert$	$(\delta < 0)$
(i)	$s \approx 18;30° + {}^1/_8\,\delta$	$(\delta > 0)$
	$s \approx 18;30° + {}^1/_2 \cdot {}^1/_8\,\lvert\delta\rvert$	$(\delta < 0)$

al-Fawānīsī states that his formula for r is based on the parameter 18°, rather than 20° which was the commonly-accepted value in his day. On the other hand, he notes, his formula for s is based on the commonly-used parameter 16° rather than 18°, which had been used before his time. Some of the same approximations are given in a short anonymous treatise on time-keeping extant in MS Escorial ár. 961,3 (fols. 13v-16v), copied in 863 H [= 1459], that is, before the time of al-Fawānīsī (see **2.5**). In an another anonymous source, MS Cairo DM 181,3, (fols. 43r-46v), copied *ca*. 1800, his rule for t_a appears together with two rules for the duration of twilight (for $\delta > 0$ only?) which differ from both his rule and that of al-Bakhāniqī, namely:

$$r \approx 22° + {}^1/_5\,\delta \quad \text{and} \quad s \approx 20° + {}^1/_2 \cdot {}^1/_8\,\delta + {}^1/_6\,\delta\ .$$

These are clearly based on parameters 19° and 17°, respectively.

The tables which follow this introduction are not based on these approximate formulae. For each day of the Coptic year al-Fawānīsī tabulates the corresponding date in the Syrian calendar, the solar longitude (usually to the nearest degree, occasionally to the nearest half-degree), and then some 19 functions relating to spherical astronomy and the prayer-times – see the extract in **Fig. 7.5a**. The entries for a given day are written across a pair of facing pages: such an opening of the manuscript serves 13 days, grouped according to which of the 28 lunar mansions rises at daybreak. For Tūt 1 the solar longitude is given as Virgo 13° and this date is the third day during which the 11th mansion al-Kharāthān rises at dawn. The functions tabulated by al-Fawānīsī for each day are the following:

α_ϕ	"the amount of rotation of the celestial sphere (from the equinox) at sunrise", *i.e.*, oblique ascension (*al-dāʾir ʿinda ṭulūʿ al-shams*)
ψ	solar rising amplitude (*saʿat al-mashriq*)
h_0	solar amplitude in the prime vertical (*al-irtifāʿ alladhī lā samt lahu*)
h_q	solar altitude in the azimuth of the qibla (*irtifāʿ samt al-qibla*)
T_q	time from sunrise to the moment when the sun is in the azimuth of the qibla (*al-dāʾir min ṭulūʿ al-shams ilā ḥayth takūn al-shams ʿalā samt al-qibla*)
δ	solar declination (*al-mayl al-juzʾī*)
H	solar meridian longitude (*ghāyat al-irtifāʿ*)
$Z_{(12)}$	horizontal meridian shadow to base 12 (*aṣābiʿ ẓill al-zawāl mabsūṭ*)
$Z'_{(12)}$	vertical meridian shadow to base 12 (*aṣābiʿ ẓill al-zawāl mankūs*)
d	equation of half daylight (*taʿdīl niṣf al-nahār*)
D	semi diurnal arc (*niṣf qaws al-nahār*)

←
Fig. 7.5a: An extract from al-Fawānīsī's prayer-tables for day-time, displaying 19 different functions for each day of the Coptic year, here Āb 24 to Misrā 6. [From MS Paris BNF ar. 2545, fols. 29v-30r, courtesy of the Bibliothèque Nationale de France.]

Fig. 7.5b: An extract from al-Fawānīsī's prayer-tables for night-time, serving the end of Abīb and the beginning of Misrā. The tables have a very distinctive format. [From MS Paris BNF ar. 2545, fols. 30v-31r, courtesy of the Bibliothèque Nationale de France.]

$2D^h$	number of equinoctial hours of daylight (*ʿadad sāʿātihi al-mustawiya*)
$\tilde{h}$	number of equatorial degrees in one seasonal hour of daylight (*azmān sāʿātihi*)
α'	"the amount of rotation at midday", *i.e.*, the normed right ascensions (*al-dāʾir ʿind al-zawāl*)
t_a	time from midday to the afternoon prayer (*al-māḍī min al-zawāl ila ʾl-ʿaṣr*)
α_a	"the amount of rotation at the afternoon prayer", *i.e.*, the oblique ascensions of the horoscopus at that time (*al-dāʾir ʿind al-ʿaṣr*)
h_a	solar altitude or the beginning of the afternoon prayer (*irtifāʿ awwal al-aṣr*)
h_b	solar altitude at the end of the afternoon prayer (*irtifāʿ ākhir al-ʿaṣr*)
T_a	time remaining at the beginning of the afternoon prayer until sunset (*al-bāqī min al-ʿaṣr ilā ghurūb al-shams*)

With the following reservations, the values are essentially those of the main Cairo corpus:

(a) The values of h_0 are extremely corrupt. The entries for solar longitude 30° and 90° are 23;47° and 53;21° rather than the correct values 23;35° and 53;9° – see **4.3**. The entry 53;21° is probably to be explained as a scribal error for 53;9°: the Arabic notations for 21 (*kāf-alif*) and 9 (*ṭāʾ*) are quite similar.

(b) The values of h_q are likewise rather corrupt. They are based on q = 52° and are ultimately derived from Ibn Yūnus' table of h_q in the *Ḥākimī Zīj* (**5.1**). However, a deliberate attempt has been made to adjust the entries near the summer solstice so that the reading for λ = 90° is 80;39° rather than 80;2°. Nevertheless al-Fawānīsī's values of T_q can be derived by subtracting the values of t_q (q = 53°) from those of D in the main corpus!

(c) There are no tables of the functions $\mathrm{Tan}_{12}\, h_a(\lambda)$ or $\alpha_a(\lambda)$ in the known manuscripts of the corpus. The former can be found directly from h_a or by using:

$$\mathrm{Tan}_{12}\, h_a \cdot z_{a(12)} = 144 \ ,$$

and the latter is defined by:

$$\alpha_a = \alpha' + t_a \ .$$

Some tables of α_a for Cairo apparently based upon ε = 23;30° and computed by Riḍwān Efendī are contained in MS Cairo DM 45 – see **7.10**.

(d) There are no tables of the function $h_b(\lambda)$ in the known manuscripts of the corpus. al-Fawānīsī's values are fairly accurate, but the computation of h_b from H is straightforward. The only known tables of the corresponding time $t_b(\lambda)$ computed for Cairo are in the later source MS Istanbul Kandilli 424 – see **7.11**.

For the nights corresponding to each day of the Coptic year al-Fawānīsī tabulates eight functions, namely:

(1)	$\alpha_\phi(\lambda^*)$	"the amount of rotation at sunset", *i.e.*, the oblique ascensions of the ascendant at sunset (*al-dāʾir ʿinda ʾl-ghurūb*)
(2)	$\alpha_s(\lambda) + 2°$	"the amount of rotation at nightfall with two degrees added to the ascensions", *i.e.*, the oblique ascensions of the ascendant at nightfall plus 2° (*al-dāʾir ʿinda ghiyāb al-shafaq bi-ziyādat darajatayn ʿala ʾl-maṭāliʿ*)
(3)	$2N(\lambda)$	nocturnal arc (*qaws al-layl bi-kamālihi*)
(4)	$2N^h(\lambda)$	length of night in equinoctial hours (*ʿadad al-sāʿāt al-mustawiya*)
(5)	$\tilde{h}'(\lambda)$	length of the seasonal night-hours in degrees (*azmān al-sāʿāt*)

(6)	$2N(\lambda)$ - $r(\lambda)$	time from sunset to daybreak (*al-māḍī min ghurūb al-shams ilā ṭulūʿ al-fajr*)
(7)	$\alpha'(\lambda) + 180°$	"the amount of rotation at midnight", *i.e.*, the oblique ascensions of the ascendant at midnight (*al-dāʾir ʿind niṣf al-layl*)
(8)	$\alpha_r(\lambda)$	"the amount of rotation at daybreak", *i.e.*, the oblique ascensions of the ascendant at daybreak (*al-dāʾir ʿinda ʾl-fajr*)

Approximate values of $r(\lambda)$ and $s(\lambda)$ are given at the top of each pair of facing pages. These are expressed to the nearest 0;30° and are based on angles of depression of 18° and 16°, respectively. They are intended to be used for all the nights served by the given pair of pages. The 2nd, 6th, and 8th functions are based on these approximate values of r and s. al-Fawānīsī did not copy the tables of α_r and α_s in the corpus because these are based on parameters 19° and 17°. He gives no explanation why he adds 2° to the values of $\alpha_s(\lambda)$: perhaps he was not really convinced that 16° was a better parameter for the evening twilight than 18°. He also names the stars which are culminating at nightfall, midnight, and daybreak, or gives what is perhaps intended to be their distance from the meridian at these times, for each night of the year. (See **5.5** for a different set of solar tables compiled by Ibn al-Kattānī.)

The anonymous tables in MS Oxford Hyde 32 (Nic. 284), fols. 69v-81r, are related to those of al-Fawānīsī. The following functions are tabulated for each day of the Coptic year:

$$\lambda,\ Z_{(12)},\ H,\ h_a,\ \alpha',\ D,\ t_a,\ T_a \text{ and } 2N\ .$$

For the corresponding nights the altitudes of three prominent stars in the eastern and western sky at daybreak are given, as well as the values of various functions relating to nightfall and daybreak.

7.6 Two almanacs for the years 1015 and 1044 H

MS Paris BNF ar. 2571 contains an almanac for Cairo for the year 1015 H [= 1606/07], by Aḥmad ibn Muḥammad al-Ḥusnī,[7] entitled *Taqwīm al-sana al-qamariyya*, literally, "Ephemeris for the Lunar Year". al-Ḥusnī begins his almanac by paying tribute to his teacher, the Ottoman astronomer Jamāl al-Dīn Abū Dāʾūd Sulaymān (ibn Ḥamza), otherwise known as Ibn Bakhshīsh,[8] and also in passing mentions the Ottoman Sultān Aḥmad Khān (reg. 1603-1617).

The almanac contains calendrical information and remarks on the nature of the lunar crescent at first visibility for each month of the year. al-Ḥusnī states that the lunar crescent whose appearance marks the beginning of the year will be seen on a Sunday evening (*laylat al-ithnayn* or "night of Monday"). The first day of the year would thus be a Monday, which would correspond to April 30, 1606 (Julian calendar): modern tables give Muḥarram 1 as Sunday, April 29 for that year. For the first and 15th day of the lunar month al-Ḥusnī also gives the values of:

$$D,\ t_a,\ T_a,\ s,\ n \text{ and } r$$

written out in words, and expressed in equatorial degrees, with parts of a degree expressed as simple fractions. In Ramaḍān the time τ is also shown, based on the assumption that the

[7] This al-Ḥusnī is not mentioned in the modern sources.
[8] On Ibn Bakhshīsh see Azzawi, *History of Astronomy in Iraq*, pp. 181; *Cairo ENL Survey*, no. C117; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 194-198, no. 95.

ṭafy occurs 5° before daybreak. The solar longitude on Muḥarram 1 is taken as 48°, which is reasonable.

The entries in al-Ḥusnī's prayer-tables for a given day are generally mutually inconsistent. The values of the functions t_a, s and r are rounded from the corresponding entries in the main Cairo corpus, but those for D, T_a and n have been haphazardly adjusted by what was probably intended to be a correction for horizon phenomena. al-Ḥusnī's values of the *niṣf qaws al-nahār al-sharʿī*, "semi diurnal arc according to religious law", are in excess of the geometric semi diurnal arc by approximately the following amounts:

VE: 0;3° SS: 0;55 AE: 0;51 WS: 0;43

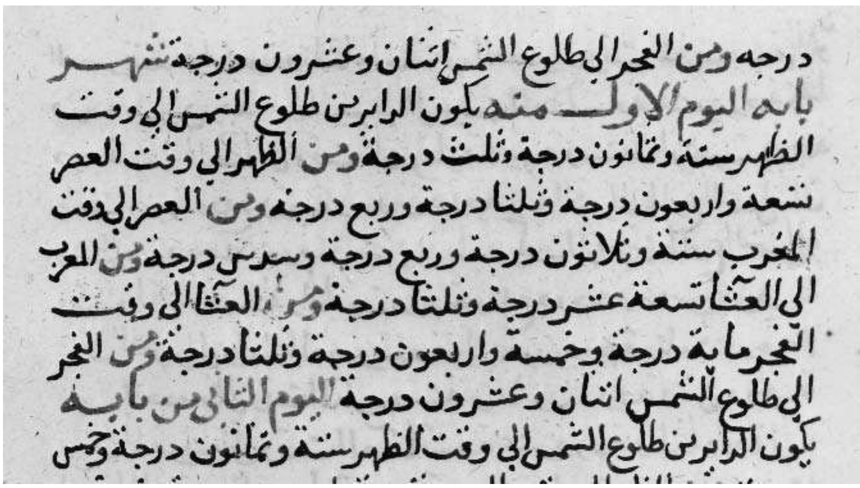
He gives no explanation of these corrections.

MS Cairo ṬM 87 contains a similar almanac for the year 1044 H [= 1634/35] attributed to Muḥammad al-Kutubī,[9] which I have not investigated.

7.7 al-ʿAjmāwī's prayer-tables for Cairo

ʿAbd al-Qādir ibn Aḥmad al-ʿAjmāwī al-Azharī[10] was a *muwaqqit* in the *madrasa* of Sulṭān Ḥasan in Cairo who compiled a set of prayer-tables entitled *Nuzhat al-nāẓir fī maʿrifat mā bayn al-awqāt min al-dawāʾir*, "The Delights of Knowing the Intervals between the Prayer-times". MS Paris BNF ar. 2578,2, dated 1024 H [= 1616], and also Cairo ZK 287, 203 fols., penned 1056 H [= 1646/47], are copies of this work, containing the prayer-times *written out in words* for each day of the Coptic year. An extract is displayed in **Fig. 7.7**. Values are given for the functions:

$$D,\ t_a,\ T_a,\ s,\ n \text{ and } r\ ,$$



درجه ومن الفجر الى طلوع الشمس اثنان وعشرون درجة شهر
بابه اليوم الاول منه يكون الدائر من طلوع الشمس الى وقت
الظهر ستة وثمانون درجة وثلث درجة ومن الظهر الى وقت العصر
تسعة واربعون درجة وثلثا درجة وربع درجة ومن العصر الى وقت
المغرب ستة وثلاثون درجة وربع درجة وسدس درجة ومن المغرب
الى العشا تسعة عشر درجة وثلثا درجة ومن العشا الى وقت
الفجر مائة درجة وخمسة واربعون درجة وثلثا درجة ومن الفجر
الى طلوع الشمس اثنان وعشرون درجة اليوم الثاني من بابه
يكون الدائر من طلوع الشمس الى وقت الظهر ستة وثمانون درجة وخمس

Fig. 7.7: An extract from al-ʿAjmāwī's prayer-tables written as text. [From MS Paris BNF ar. 2578, fol. 46r, courtesy of the Bibliothèque Nationale de France.]

[9] On Muḥammad al-Kutubī see *Cairo ENL Survey*, no. D27; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, p. 276, no. 141.

[10] On al-ʿAjmāwī see Brockelmann, *GAL*, SII, p. 1018, and *Cairo ENL Survey*, D11.

and are those of the main Cairo corpus with the equatorial minutes rounded so that they can be expressed as simple fractions.

7.8 The prayer-tables of al-Lādhiqī and al-Ikhṣāṣī

A very popular set of Egyptian prayer-tables, extant in numerous manuscript sources, is attributed to an individual named Shams al-Dīn Muḥammad ibn Muḥammad al-Lādhiqī.[11] The title of the tables is usually *Natījat al-afkār fī aʿmāl al-layl wa-'l-nahār*, "The Result of Reflection on Matters of Night and Day", although the title *Bughyat al-nafs ... ,* "The Desire of the Soul ... ", also occurs and relates to the same work.

I have examined several manuscripts of al-Lādhiqī's prayer-tables, including MSS Cairo DM 198, Cairo DM 61, Hartford Theological Seminary 621, Cairo DM 190 and Paris BNF ar. 2553 (other manuscripts are listed below). Each contains an introduction on calendars, consisting of ten sections and a conclusion, all of no great interest. al-Lādhiqī mentions, however, that he compiled his solar tables for the longitude of Cairo, 54;55°, a value also used by al-Marrākushī (**6.7**). The first source, MS Cairo DM 198, contains no tables at all. The second and third contain a table of solar positions for Cairo for each day of the Syrian year. This table and other simple calendrical tables, followed by various others from the main corpus, namely:

$$H,\ d,\ D,\ 2N,\ \alpha',\ \psi,\ h_0,\ h_a,\ t_a,\ T_a,\ r,\ s,\ n,\ \sigma,\ \alpha_\sigma \text{ and } \alpha_s\ .$$

The fifth source, MS Cairo DM 190, contains these and also tables of h_q and h_v, the first based on $q = 53°$ and the latter with entry 0;0° at $\lambda = 270°$.

A very simple table occuring in most of the copies of al-Lādhiqī's work which I have consulted gives the corrections to be added to the solar position at noon in order to find the solar longitude at:

(1) the beginning of the afternoon prayer;
(2) sunset;
(3) nightfall;
(4) the time of the *salām*, and
(5) sunrise.

Values of these corrections in minutes are given for each zodiacal sign, but their use is only fully exploited by al-Minūfī (**7.1**). In al-Lādhiqī's tables the difference minutes ΔD (according to the theory associated with Ibn Yūnus) and the mean solar motion in a day are also tabulated. An earlier table of this kind is attributed to al-Wafāʾī (**5.7**).

Other manuscripts of al-Lādhiqī's tables that have come to my attention are MSS Gotha A1399, Istanbul (Millet Genel) Ali Emiri 2743, Istanbul Haci Beşir Ağa 674,2, Istanbul Reisülküttap Mustafa Efendi 582, Cairo DM 307, Cairo MM 14, 15, 16, Cairo MM 25, Cairo MM 221, 222, 223, 224, 225, and Istanbul UL A 4082,1 (fols. 1r-20v). There is no mention of Ibn Yūnus, or of al-Maqsī, in any of the manuscripts consulted. It seems that the prayer-tables were not "plagiarized" by al-Lādhiqī himself, but were simply used widely together with

[11] On al-Lādhiqī see Suter, *MAA*, no. 519; Brockelmann, *GAL*, SII, p. 1023; *Cairo ENL Survey*, no. C132; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 229-230, no. 109, and p. 760.

his table for solar longitude by later *muwaqqit*s. In MS Berlin Ahlwardt 5765 [= Wetzstein 1149, fols. 85v-88v] his introduction is followed by a set of prayer-tables for the latitude of Nablus (**11.7**). I have no information on the life of al-Lādhiqī, but his name indicates that his family was at one time associated with Lattakia in Syria.

A similar work, with a slightly different introduction and different arrangement of the tables is attributed to Muḥammad ibn Haykal al-Ikhṣāṣī, a *muwaqqit* at the Azhar Mosque in Cairo.[12] al-Ikhṣāṣī's tables are preserved in MSS Cairo DM 60, 480, 739, and MM 81.

7.9 Ibn Abī Rāya's prayer-tables

MS Alexandria 4441J is the only copy known to me of a set of prayer-tables for Cairo attributed to an individual called Ibn Abī Rāya, a *muwaqqit* at the Muʾayyad Mosque in Cairo at the beginning of the 18th century.[13] The work is entitled *Zuhrat al-afkār fī aʿmāl al-layl wa-'l-nahār*, "The Beauty of Reflection on the Operations of Timekeeping by Night and Day", and begins with an introduction of 13 chapters. A table of solar longitude for each day of the Coptic year (with Virgo 16;12° for Tūt 1) is is said to be computed for longitude 54;55° (**7.8**).

The other simple calendrical, astrological and solar tables in this work do not concern the present study, save to mention one table displaying the lunar mansion corresponding to the position of the sun, such as is found in MS Istanbul Bağdatli Vehbi Efendi 887 of the tables of al-Ḥakīm al-Lādhiqī for Lattakia (**11.9**). This particular table is said to be computed for the year 1117 H [= 1705/06] by Ibn Abī Rāya.

The main set of prayer-tables is preceded by individual tables of ψ (computed for $\varepsilon = 23;30°$ but having 23;29° for $\lambda = 60°$) and h_0 (computed for $\varepsilon = 23;35°$ but having 23;54° for $\lambda = 30°$) (see **4.3** on this problem), as well as garbled tables of Cos $\delta(\lambda)$ (called *jayb tamām al-mayl*) and $C(\lambda)$ (called *buʿd al-quṭr li-'l-jayb* (*sic*)) (see **I-6.2** and **6.4**). The main tables display 16 functions side by side for each degree of λ beginning with $\lambda = 1°$. The first and third of these, D and D′, are called respectively *niṣf qaws al-nahār al-sharqī* and *al-gharbī*, semi diurnal arc in the east and in the west. The former is simply the semi diurnal arc and the latter the semi diurnal arc corrected for refraction at the horizon. The last function tabulated, which I denote by [D(λ) - min D], is called *ziyādat al-nahār*, and measures the excess of the length of daylight over its minimum at the winter solstice; I have not found this elsewhere. The underlying value of ε is 23;35°, and the tables are lifted from those in the main corpus rather than those of al-Minūfī (**7.1**). The 16 functions tabulated are:

$D, H, D', h_a, t_a, T_a, s, n, r, 2N, h_q, \Delta D, Z_{(12)}, (2N\text{-}r), (D+2N\text{-}r)$ and $[D - \min D]$.

The work concludes with a set of tables of:

$$\alpha', \alpha_s, \alpha_r, \alpha_\phi \text{ and } \alpha^* ,$$

as well as a star catalogue displaying the equatorial coordinates of about 285 stars for an unspecified date.

---

[12] On al-Ikhṣāṣī see *Cairo ENL Survey*, no. D3; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 348-349, no. 215.

[13] Ibn Abī Rāya is new to the modern literature.

7.10 Riḍwān Efendī's prayer-tables

The celebrated astronomer and author Riḍwān Efendī al-Falakī,[14] who died in 1710, compiled a lengthy volume on timekeeping called *Dustūr uṣūl ʿilm al-mīqāt wa-natījat al-naẓr fī taḥrīr al-awqāt*, "Statutes of the Principles of Timekeeping and the Results of Reconsidering the Prayer-times". I have consulted MSS Cairo Azhar *falak* 8216, Berlin Ahlwardt 5710, Istanbul S. Esad Efendi Medresesi 119,1, Istanbul Nuruosmaniye 2781, Cairo TR 116 and TR 276 of this work. Apparently, the only original part is a set of tables for determining the times of the sun's entry into each of the signs: the treatment of the problems of spherical astronomy in the introduction is standard and all the prayer-tables which follow have been lifted from earlier sources.

These manuscripts contain all of the prayer-tables from the main corpus, and Riḍwān Efendī has made no effort to recompute even the simplest of them for the value of the obliquity used in his *Zīj*, namely, 23;30,17°. This *Zīj al-Riḍwānī*, or *al-Zīj al-Mufīd*, of which I have examined the copy preserved in MS Princeton Garret Hitti 1004, is based entirely on that of Ulugh Beg, which in Cairo was available in Arabic in the recension of Ibn Abi 'l-Fatḥ al-Ṣūfī (**6.15**). MS Cairo DM 45 contains some tables computed by Riḍwān Efendī giving the oblique ascensions, α_a, at the beginning of the interval for the afternoon prayer, "according to the new observation". The "new" observation of Ulugh Beg was almost three centuries prior to Riḍwān's activity in Cairo.

7.11 Miscellaneous anonymous prayer-tables for Cairo

MS Cairo DM 200 (7 fols., *ca*. 1750) contains a set of tables of the functions:

$$D,\ t_a,\ T_a,\ s,\ n,\ r, \text{ and the solar longitude } \lambda\ ,$$

tabulated side by side for each day of the Syrian calendar (equivalent dates in the Coptic calendar are also given). Values are given to two digits. MS Cairo DM 531,1 contains sets of sub-tables displaying values of the first six functions to one digit for each five days of the Coptic calendar.

MS Cairo DM 46 (17 fols., *ca*. 1750) contains a motley collection of prayer-tables for Cairo, some with entries to two digits taken from the main corpus, and others recomputed for Ulugh Beg's values of the obliquity, 23;30,17°, with entries to three digits. Four of the tables display the "latitude of visible climate", ν, and its complement, $\bar{\nu}$, which is the altitude of the pole of the ecliptic, as well as $\mathrm{Cos}_{60}\ \nu$ and $\mathrm{Tan}_{60}\ \nu$, with values to three digits for each degree of λ.[15] The tables that were not lifted from the corpus display functions:

$$h_b,\ T_a,\ T_b,\ (T_a{+}D),\ (T_b{+}D),\ \tau_{a/b},\ 2N,\ n,\ (2N\text{-}r) \text{ and } h_q\ ,$$

and the tables that were lifted from the corpus display functions:

$$2N,\ n,\ s,\ r,\ t_q,\ t_{q*},\ T_q,\ h_{a=30°} \text{ and } h_{a=60°}\ .$$

No author is associated with any of them.

---

[14] On Riḍwān Efendī see n. **I**-6:6.
[15] See n. 3:14 on this concept.

MS Istanbul Kandilli 424, copied in 1164 H (= 1750/51), contains (fols. 1v-4r) an anonymous set of tables displaying the longitude of the astrological houses for Cairo as a function of the ascensions of the horoscopus, computed "according to the Ptolemaic method",[16] and (fols. 5r-6v) an anonymous set of Ottoman-type prayer-tables for Cairo. I know of no other copies of either set. The prayer-tables display seven functions side by side for each degree of solar longitude beginning with Aries, and the times are given in equinoctial hours and minutes according to the Ottoman convention (**14.0**). The tables are entitled *jadwal sāʿāt ʿaqrab al-sāʿāt*, literally, "tables of the hours of the clock-hand". The functions tabulated are the following:

s′	*ʿishāʾ*	the time of nightfall
j′	*ādhān Ramaḍān*	the time of the call to prayer in Ramaḍān
r′	*fajr*	daybreak, the time of morning prayer
R′	*shurūq*	the time of sunrise
m′	*ẓuhr*	the time of the midday prayer
a′	*ʿaṣr awwal*	the time of the first afternoon prayer
b′	*ʿaṣr thānī*	the time of the second afternoon prayer.

A note by the title states that the tables are computed for the setting of the upper limb of the sun over the apparent horizon and the crossing of the entire solar disc across the meridian. It is stated that these modifications take into consideration the correction for the difference between the true and the apparent horizons and the radius of the sun, and that the tables are based on the parameters of Ulugh Beg (*i.e.*, $\varepsilon = 23;30,17°$).

Comparison of the tables with a set of Ottoman-type prayer-tables computed for $\phi = 30;0°$ and $\varepsilon = 23;30,17°$ and not modified for horizon phenomena reveals the following:

a) The tables of r′ and s′ are based on parameters 19° and 17°, and the time j′ is always 20^m before r′.

b) The times m′, a′, and b′ are modified by a correction of approximately:

EQ: 4^m SS: 6^m WS: 3^m ,

and the times r′ (and hence j′) and R′ are modified by double these amounts. The radically different amounts of this correction for the solstices suggest that the compiler used the corrections for refraction associated with Ibn Yūnus (**4.11**). Indeed if we add 0;15° (an approximate value of the time for half the solar disc to set over the horizon) to Ibn Yūnus' values we obtain

$$\begin{array}{ll} \text{EQ:} & 0;47° + 0;15° = 1;2° \approx 4^m \\ \text{SS:} & 1;2° + 0;15° = 1;17° \approx 5^m \\ \text{WS:} & 0;32° + 0;15° = 0;47° \approx 3^m . \end{array}$$

(c) It is difficult to confirm whether or not the compiler of the tables introduced a correction of about $+0;15° \approx +1^m$ to the time m′ in order to ensure that the whole solar disc had crossed the meridian at the time of the midday-prayer.

I have not investigated whether or not most of the entries in these tables can be derived from the prayer-tables of al-Minūfī (**7.1**). No earlier tables of t_b for Cairo are attested yet, but the function h_b was tabulated by al-Fawānīsī (**7.5**).

[16] On the houses see n. **I**-3:2.

7.12 al-Khāwānikī's prayer-tables

Some manuscripts of al-Kutubī's tables (**7.14**), such as MSS Cairo ṬM 88 and ṬM 142, contain a set of prayer-tables for Cairo computed by the 18^{th}-century Egyptian astronomer Ramaḍān ibn Ṣāliḥ al-Khāwānikī.[17] The following functions are tabulated in hours and minutes according to the Ottoman convention, with values for each day of the Coptic year:

s′, i′, r′, R′, m′, a′ and b′ .

The time of the *imsāk*, i′, is 20^m before daybreak, r′. The table for b′ occurs only in the first source. I have not been able to check whether these tables are identical to those in MS Istanbul Kandilli 424 (**7.11**).

7.13 Ḥasan al-Jabartī's prayer-tables

MS Cairo TR 346, penned *ca*. 1875, is the only copy known to me of a set of prayer-tables for Cairo attributed to Ḥasan al-Jabartī (d. 1774), father of the celebrated historian ʿAbd al-Raḥmān al-Jabartī.[18] The tables are entitled *Mawqiʿ ʿaqrab al-sāʿa murattaba ʿala 'l-shuhūr al-Qibṭiyya*, "The Place where the Watch-hand Falls (at the Times of Prayer), arranged according to the Coptic Months", and display the times:

s′, i′, r′, R′, m′ and t_a

for each day of the Coptic year. The time of the *imsāk*, i′, is 20^m before daybreak, r′.

7.14 al-Kutubī's timekeeping tables

MSS Cairo DM 149, ṬJ 811,8, ṬM 88 and ṬM 142, DM 812, DM 1103 and 1104 contain a set of tables for timekeeping prepared in the year 1150 H [= 1737/38] by ʿAbd al-Laṭīf al-Dimashqī, known as al-Kutubī.[19] These display for the latitude of Cairo the times in hours and minutes according to the Ottoman convention, before and after midday when the sun has a given altitude and longitude, for each degree of both arguments: see further **I-2.1.8** and the extract illustrated there The entries are doubtless based on the entries in the tables of $t(h,\lambda)$ and $T(h,\lambda)$ in the main corpus (**4.5**). al-Kutubī states in his introduction that in his calculations he has taken into consideration the apparent radius of the solar disc and the "difference minutes", as well as the increase in solar longitude from midday to the time in question.

---

[17] On al-Khawānikī see Brockelmann, *GAL*, II, pp. 471-472, and SII, p. 48; Azzawi, *History of Astronomy in Iraq*, pp. 320-321; *Cairo ENL Survey*, no. D78; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 418-426, no. 279.

[18] On Hasan al-Jabartī see Brockelmann, *GAL*, II, p. 472, and SII, p. 487; *Cairo ENL Survey*, no. D91; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 472-479, no. 314. On his son see the article "al-Djabartī" by David Ayalon in EI_2.

[19] On al-Kutubī (**I-2.1.8**) see *Cairo ENL Survey*, no. D76; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 427-429, no. 281.

CHAPTER 8

OTHER LATE EGYPTIAN TABLES FOR TIMEKEEPING

8.0 Introductory remarks

In the sequel I discuss a set of late Egyptian prayer-tables for for each degree of latitude between Mecca and Istanbul (**8.1**), as well as sets of such tables specifically for use in Alexandria, Damietta, Rosetta and Crete (**8.5-8**). These are all clearly inspired by the tables in the main Cairo corpus. I also discuss ʿAbd al-Qādir al-Minūfī's writings on refraction at the horizon and his tables purporting to display the time taken by the sun to pass from the true horizon to the visible horizon for all latitudes (**8.2**), and Ibn Abi ʾl-Khayr al-Ḥusnī's tables for computing the duration of twilight for all latitudes (**8.3**). al-Nabatītī's prayer-tables for the pilgrim route between Cairo and Mecca (**8.4**) have unfortunately not survived. The much later tables of Muḥammad Ḥifẓī serve all localities of consequence in Egypt and the Hejaz (**8.10**). These, and also the set for Cairo by ʿAbd al-Majīd Efendī printed in 1895 (**8.9**), bring our discussion of one thousand years of astronomical timekeeping in Egypt to a close.

8.1 Anonymous prayer-tables for all latitudes

MS Princeton Yahuda 861,1, penned *ca*. 1600, contains a set of prayer-tables computed for each integral degree of latitude between 21° (Mecca) and 41° (Istanbul). The tables are entitled *al-Natīja al-kubrā*, which may be rendered "Universal Prayer-tables". (The term *natīja* means "calendar" or "prayer-tables" in late medieval scientific Arabic.) On the title-folio the tables are attributed to ʿIzz al-Dīn al-Wafāʾī, a *muwaqqit* at the Muʾayyad Mosque in Cairo who died about 1470.[1] This attribution is highly doubtful, for reasons which will become clear below, but at least some of the tables are due to him. The manuscript is carelessly copied in an untidy hand, and is bound in some disorder. According to the title folio al-Wafāʾī's tables also included a set of auxiliary tables, but these are those of al-Khalīlī – see **10.7** – and are described as such in their sub-title. al-Wafāʾī did compile some auxiliary tables of his own, which are extant in another source – see **6.15**.

The manuscript begins with a short introduction on the use of the table displaying the solar longitude for each day of the Coptic year. This is stated to have been computed for longitude 55° by al-Wafāʾī (the entry for Tūt 1 is Virgo 14;39°). A star catalogue showing the right ascension of 72 stars is likewise attributed to al-Wafāʾī. Neither solar longitude table nor star catalogue is dated. There follow two sets of prayer-tables, the first for Cairo, and the second for all latitudes, including that of Cairo.

[1] On al-Wafāʾī see n. **I**-9:29.

In the first set of prayer-tables the functions:

δ, H, D, h_a, t_a, T_a, 2N, σ, α_σ, s and r

are tabulated for latitude 30°. The entries are those of the main Cairo corpus, but the format differs. For each pair of zodiacal signs these functions are displayed in columns across two facing pages of manuscript. The use of this format was perhaps inspired by the earlier prayer-tables of al-Khalīlī – see **10.6**. However, this first set of tables begins with Aries and Virgo, whereas al-Khalīlī's tables begin with Capricorn and Sagittarius.

In the second set most of the eight functions:

D, H, h_a, t_a, r, s, n and α_ϕ

are given for latitudes (no localities are mentioned):

21° (Mecca), 24° (Medina), 30° (Cairo), 31° (Alexandria), 32° (Jerusalem),
33;30° (Damascus and Bagdad), 34° (Tripoli and Homs) and 36° (Aleppo).

These are ostensibly based on obliquity 23;35°. *For other integral latitudes between 21° and 41° the functions D and H are given.* The function D is also tabulated for latitude 15° (Yemen). Other standard functions are given for various latitudes, without any pattern: for example, s is given for latitude 22° but not r, and for latitude 27° t_a but not h_a. It is not difficult to show that virtually all the 150 tables were lifted from other sources. My analysis reveals the following:

(a) The tables of D(λ) for all latitudes except 30° are the same as those of Najm al-Dīn al-Miṣrī in MS Oxford Marsh 676 (Uri 944 = 995) (**6.5**), where D(λ) is tabulated for each value of ϕ from 1° to 89°. This also explains the presence of the odd table of D(λ) for latitude 15°.

(b) The entries in the tables for 21° are quite different from the ones in MS Cairo MM 68 (**6.10**), being carelessly computed. The tables of t_a and D in the Princeton manuscript also occur in MS Istanbul Köprülü 1619 of an anonymous set of prayer-tables for Cairo.

(c) The 23 different functions tabulated for latitude 30° are taken from the main Cairo corpus.

(d) The 15 different functions for latitude 32° are taken from an independent source. There are tables of h_q and T_q for this latitude, based on a particular value of q: the only other tables for the qibla in the *Natīja* are those for latitude 30°. Also, the tables of r and s are based on parameters 20° and 16° rather than 19° and 17° as in the tables for other latitudes. I suspect that these tables were originally computed by Ibn al-Rashīdī or al-Karakī – see **9.5** for a further discussion.

(e) The tables for latitude 33;30° are those of al-Khalīlī (**10.6**) and are based on a value of 23;31° for the obliquity.

(f) The tables for latitude 34° are the same as those for Tripoli in MS Damascus Ẓāhiriyya 4893 (**11.6**).

Certain of the tables are particularly wretched: in the one giving $h_a(\lambda)$ for latitude 39°, for example, it appears that there are three different functions tabulated rather than one! Some of these errors are due to copyists, but the fact that most of them are original is confirmed by examination of a second source for these tables.

MS Paris BNF ar. 2525,2 (fols. 17r-36v), copied *ca.* 1450, contains part of a beautifully-

copied set of anonymous prayer-tables for various latitudes. This section of the manuscript is unrelated to those which precede and follow and is clearly incomplete. It begins with a table of α_ϕ for latitude 25°, and continues with tables of:

$$D, H \text{ and } \alpha_\phi$$

for latitudes 26° to 29° (except that for 27° t_a is tabulated rather than H). There follow tables of 24 different functions computed for the latitude of Cairo. The tables of D and H or t_a for latitudes 26° to 29° are virtually the same as those in the Princeton manuscript, being carelessly computed. On the other hand, those of α_ϕ in the Paris manuscript are fairly accurately computed: except for copyist's errors (particularly for ϕ = 25°), they are the same as the corresponding tables in the *Ḥākimī Zīj*. All of the tables for latitude 30° are the same as those in the main Cairo corpus.

Other functions tabulated in the Princeton manuscript are the following:

(a) "meridian altitude for all latitudes": this is simply $\delta^*(\lambda) = 90°+ \delta(\lambda)$ tabulated for two digits for each degree of λ. Note that $H(\lambda,\phi) = [90°+ \delta(\lambda)] - \phi$.

(b) "altitude for the afternoon prayer for all latitudes": this is simply $h_a(H)$ to two digits for each degree of H from 1° to 90°.

(c) "rising times of the signs for different latitudes": the rising times $\Delta\alpha$ for each pair of zodiacal signs are given to two digits for each degree of ϕ from 1° to 60°. The table is based on ε = 23;35°, and contains numerous errors. In MS Cairo MM 58 (**I-5.5.1**) there is a similar table based on ε = 23;51,20° (Ptolemy's value), which is attributed to Ibn Yūnus: the same table occurs in the Yemeni astronomical miscellany in the manuscript in a private collection in Sanaa (**12.4**), without the false attribution.

(d) "time since sunrise and hour-angle at the equinox for latitude 31°": entries are given for both functions for each degree of solar altitude from 1° to 58°. The table is based on latitude 32°, not 31° as stated.

There is also an anonymous table of the latitudes of the stations on the Syrian pilgrim route to Mecca. The entries are corrupt but were clearly lifted from the table in al-Khalīlī's Damascus corpus (**10.9**).

The celebrated Egyptian historian al-Jabartī (d. *ca*. 1825)[2] attributes two works entitled *al-Natīja al-kubrā* and *al-Natīja al-ṣughrā* to the astronomer Riḍwān Efendī (**7.9**), adding that these works were well known and in common use in his time. These two titles could refer to sets of prayer-tables (*natīja*) for all latitudes (*kubrā*, literally, "larger") and for one latitude (*ṣughrā*, literally, "smaller"). No universal prayer-tables attributed to Riḍwān survive in the manuscript sources. His prayer-tables for Cairo survive in numerous copies, several of which are in his own hand (**7.10**). The possibility that Riḍwān was the compiler of the wretched tables in MS Princeton Yahuda 861,1 cannot be excluded, but I consider it unlikely.

8.2 ʿAbd al-Qādir al-Minūfī's tables of corrections for refraction at the horizon at all latitudes

MS Istanbul Nuruosmaniye 2929,4 (fols. 22r-23v) contains two sets of tables compiled by ʿAbd al-Qādir al-Minūfī (see **7.2** on his father) in 975 H [= 1567] displaying the correction for

[2] Quoted in Dorn, "Drei arabische Instrumente", pp. 32-33.

refraction at the horizon for all latitudes (*daqāʾiq ikhtilāf mā bayn al-ufq al-ḥaqīqī wa-ʾl-marʾī li-jamīʿ al-ʿurūḍ*).[3] The first set (A) is "according to the opinion of ʿAbd al-Qādir ibn Muḥammad al-Minūfī", and the second set (B) is "according to the opinion of earlier astronomers" but also computed by al-Minūfī. Both tables display the function $\Delta D(\lambda,\phi)$ for arguments:

$$\lambda = 270°, 300°, 330°, 0°, 30°, 60°, 90° \quad \text{and} \quad \phi = 1°, 2°, \ldots, 60°.$$

Entries are given to four digits in A and three in B. No indication is given in the brief introduction to the tables of precisely what al-Minūfī thought he was doing when he compiled these tables. The two sets are based on the assumption that ΔD varies linearly with ϕ, so that the entries are merely proportional parts of the entries for $\phi = 60°$. The entries for $\phi = 30°$ in the two sets are:

	$\lambda = 270°$	$\lambda = 0°$	$\lambda = 90°$
A:	0;32,54,0	0;47,59,30	1;3,5,0
B:	0;32,0	0;47,0,0	1;2,0,0

Thus al-Minūfī's new tables (set A) represent an extension of his puerile attempt to "improve" the values attributed to Ibn Yūnus in other sources.

The notion that $\Delta D \propto \phi$ is not without importance for the development of astronomical timekeeping in Islam. We find in other sources sets of values of ΔD for the latitudes of Mecca, Crete, Damascus, and Istanbul, apparently based or intended to be based on this assumption. See further **6.10**, **8.8**, **11.13** and **14.9**.

MS Dublin CB 4067 contains the preface to a work entitled *Ḥadaqat an-nāẓir fī ʾkhtilāf al-manāẓir*, "The Pupil of the Eye of the Observer of Parallax (!)", by ʿAbd al-Qādir al-Minūfī. In his preface the author mentions that the work contains an introduction, seven chapters, and a conclusion, but the preface is followed by several pages of religious invocations and repetitions of the name of the Prophet. al-Minūfī Jr. states that he wishes to discuss the problem of the true and visible celestial circles using rigorous geometrical proofs. His father had told him that the scholar Aḥmad (*sic*) ibn Yūnus had fixed the difference between the two horizons (*mā bayna ʾl-ufuqayn*) as:

EQ: 47^m SS: 62^m WS: 32^m

and that ʿAlāʾ al-Dīn Ibn al-Shāṭir (**9.3**) fixed them as 2^m more than Ibn Yūnus, *i.e.*,

EQ: 49^m SS: 64^m WS: 34^m .

al-Minūfī Jr. could not understand how Ibn al-Shāṭir had arrived at these values. He states that the only earlier astronomer who discussed this matter was Abu ʾl-Ḥasan (*sic*) ibn al-Haytham,[4] who stated that a person of height $3^1/_2$ cubits could see 0;4,26 more than half of the celestial sphere. al-Minūfī Jr. could not understand what this meant, but he knew that Ibn al-Haytham wrote two large volumes on parallax (!) which he had not seen and in which perhaps Ibn al-

[3] On ʿAbd al-Qādir al-Minūfī see Suter, MAA, no. 479, and Brockelmann, GAL, SII, p. 486 (both confused); and *Cairo ENL Survey*, no. C121.

[4] On Ibn al-Haytham see the article in *DSB* by Abdelhamid I. Sabra. Prof. Sabra assures me that this subject is not discussed in Ibn al-Haytham's *Optics*. I have examined a manuscript in Oxford of a short treatise by Ibn al-Haytham on the fact that the amount of the celestial sphere that is visible is more than half, but it does not contain the numerical value given by al-Minūfī. However, several other manuscripts of two versions of this are listed in Sezgin, *GAS*, VI, p. 260.

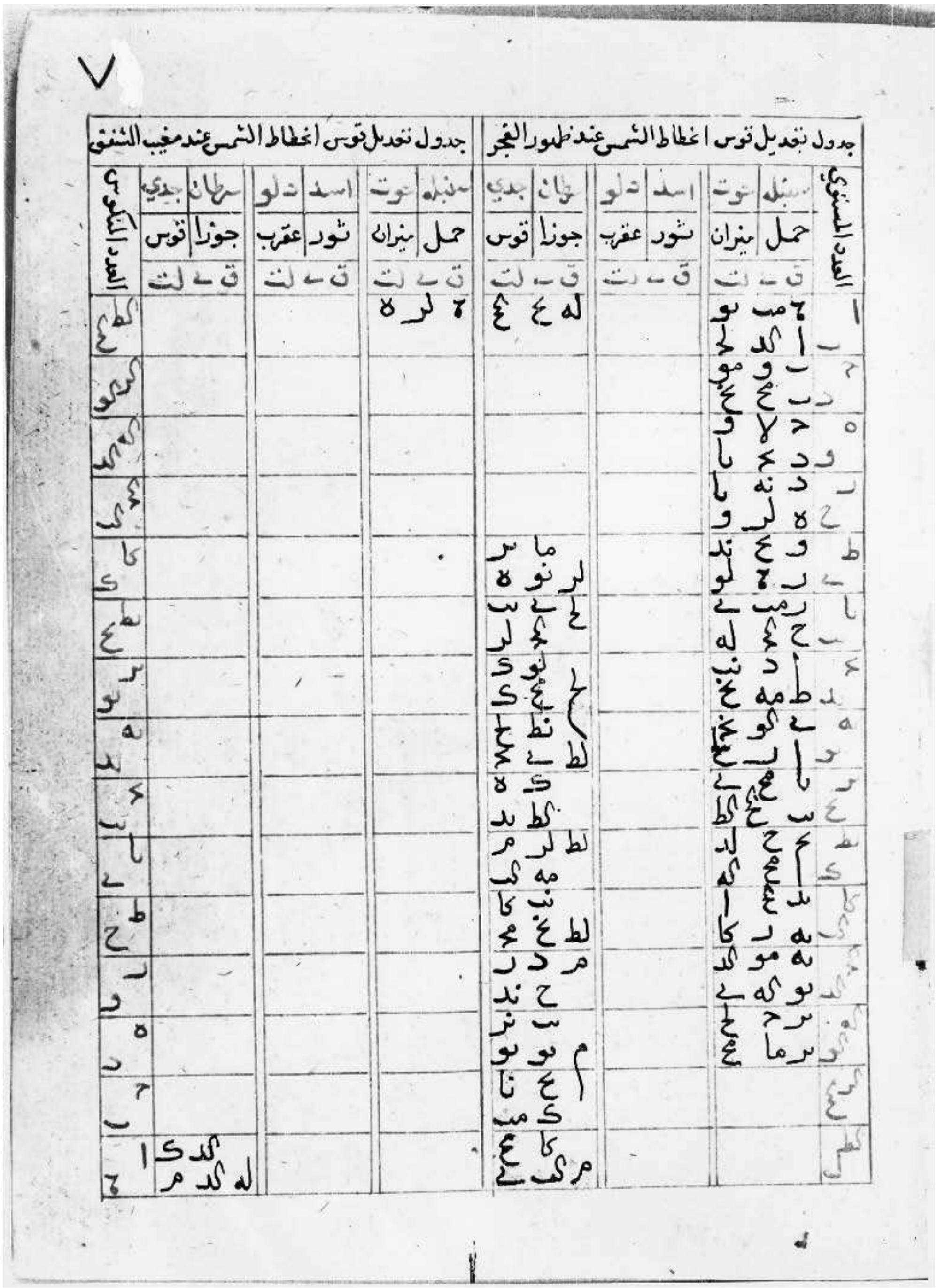


Fig. 8.2: Some mysterious tables in ʿAbd al-Qādir al-Minūfī's treatise *Rafʿ al-khilāf*. They purport to display the "correction" to the arc of depression of the sun at daybreak and nightfall as a result of the effect of refraction at the horizon, but most of the entries have been left blank, and it is not clear that our author knew what he was doing. [From MS Cairo MM 123, fol. 7r, courtesy of the Egyptian National Library.]

Haytham had explained the matter. However, he adds that Samawʾal Yaḥyā ibn ʿAbbās al-Maghribī, in Part I of Book V of his work entitled *Kitāb al-Tahdhīb fī ṣināʿat al-nujūm*, had made a table showing the amount of the celestial sphere visible for altitudes above the surface up to 12,000 cubits, apparently using Ibn al-Haytham's method.[5] al-Minūfī Jr. mentions that Abu 'l-Ḥasan (*sic*) ibn ʿAlī ibn ʿUmar al-Marrākushī (**2.7**) was asked in Rajab 694 H [= May/June 1295] whether one could find the amount of the celestial sphere one could see standing on the surface of the earth, and that he replied in the affirmative and had given geometrical and numerical demonstrations which ʿAbd al-Qādir approved. Unfortunately, our author's preface concludes without giving any more historical information.

Another treatise by ʿAbd al-Qādir al-Minūfī is preserved in MS Cairo MM 123, copied *ca.* 1600. This is entitled *Rafʿ al-khilāf fī ʿamal daqāʾiq al-ikhtilāf,* "The Removal of the Difference of Opinion on Operations with the Difference Minutes", and in it he mentions both Ibn Yūnus and Ibn al-Haytham again. It is not clear to me that al-Minūfī Jr. really knew what he was talking about, but certainly the entire tradition of which he is at the tail end merits detailed investigation. Some incomplete tables are found in this manuscript, whose purpose escapes me: see **Fig. 8.2**.

8.3 Ibn Abi 'l-Khayr al-Ḥusnī's twilight tables for all latitudes

MS Cairo DM 1108,3 (fols. 5v-10v, copied in 1052 H [= 1642/43]) is a unique copy of a treatise entitled *Kashf al-karabāt fī taḥqīq masāʾil yaḥtāj ilayhi ṭālib ʿilm al-awqāt*, which means something like "The Exposure of Apprehensions concerning the Proper Handling of Methods by the Student of Timekeeping", by Abu 'l-Khayr al-Ḥusnī (see already **7.4**). Part of the introduction is devoted to a discussion of the "difference minutes" in which the values:

47 52 57 62 57 52 47 42 37 32 37 42

for each sign beginning with Aries are attributed to Ibn al-Shāṭir (see **9.3**). The remaining part of the introduction is devoted to a discussion of the determination of the duration of twilight by computation, without reference to any observations.

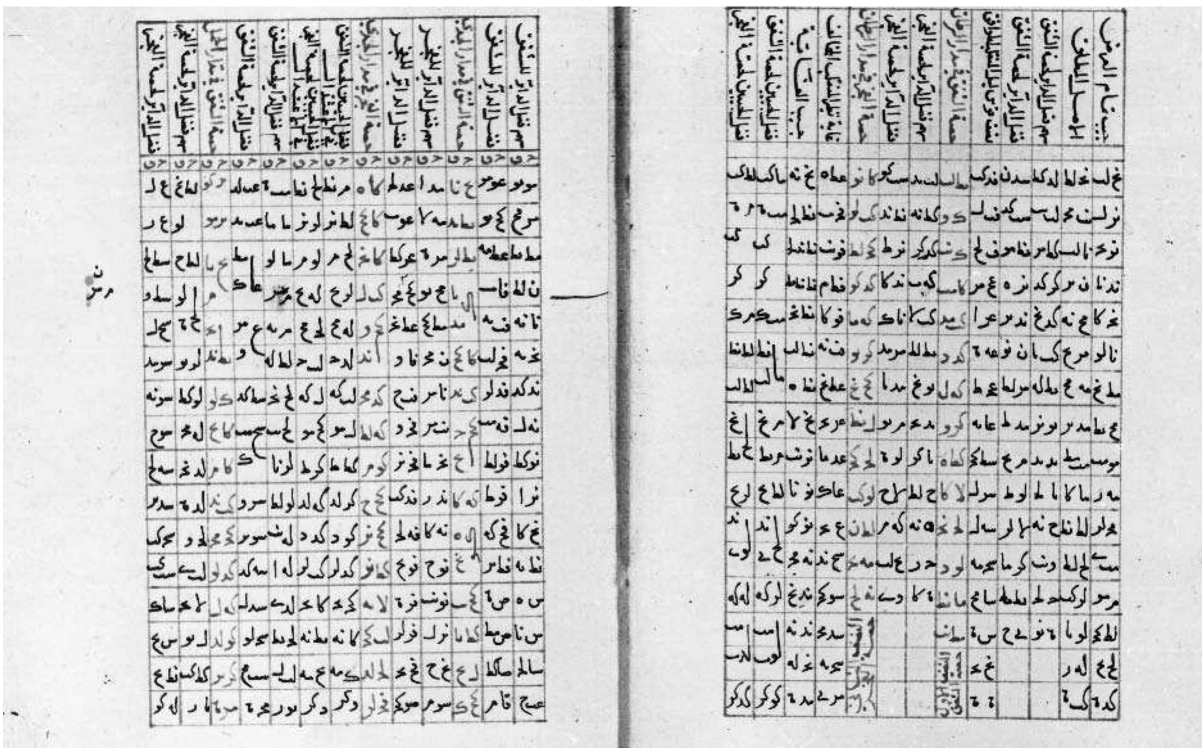
The work concludes with a set of tables displaying the duration of morning and evening twilight r and s for each half-climate at the solstices and equinoxes. al-Ḥusnī first gives values of:

$$\phi \text{ and } \mathrm{Tan}_{60}\ \phi$$

for each half-climate and then for the solstices gives values of:

$$\mathrm{Sin}\ d,\ d,\ D,\ D^h\ .$$

[5] On Samawʾal al-Maghribi (Suter, *MAA*, no. 302, and *Cairo ENL Survey*, no. G9), an extremely important scholar of 12th-century Iran, see the article in *DSB* by Adel Anbouba, where his astronomical works are unfortunately overlooked. On the introduction to his astronomical treatise see Rosenthal, "Al-Asṭurlâbî and as-Samawʾal on Scientific Progress". After this study was completed (the first time round), I obtained microfilms of the Oxford and Leiden manuscripts of Samawʾal's astronomical works and found that there is indeed a table such as was described by al-Minūfī. al-Samawʾal's astronomical works merit a separate publication, which will contain much that is new to the history of Islamic science. See already Sezgin, *GAS*, VI, pp. 65-66. Parts of the texts were investigated in my seminar on Arabic scientific manuscripts at Frankfurt during 2000-01. See also the text to n. **V**-10:12.



Then for each of the solstices and the equinoxes, as well as for the solar longitude for which the sun passes through the zenith (*i.e.*, such that $\delta(\lambda) = \phi$), he gives values of the quantities:

$$\text{Sin } H^*,\ [\text{Sin } H^* - 19°],\ [\text{Sin } H^* - 17°],\ B,\ \text{Vers } t_r,\ \text{Vers } t_s,\ r \text{ and } s\ ,$$

where t_r and t_s are the hour-angles at daybreak and nightfall, thus illustrating the steps by which r and s were calculated. The tables are shown in **Fig. 8.3**. See further **VIb-19** and also **II-11.14** on the tables of ʿAbdallāh al-Ḥalabī for Aleppo.

8.4 al-Nabatītī's prayer-tables for the pilgrimage to Mecca

MS Gotha A1412, which was lost from the Thüringsche Landesbibliothek, Gotha, during World War II, contained a set of four pages of prayer-tables compiled by ʿAbd al-Munʿim al-Nabatītī for the year 1041 H [= 1631/32].[6] His prayer-tables, of which no other copy has come to my attention, were intended to be used by pilgrims bound for Mecca, probably on the route from Cairo. The Gotha catalogue records the introduction, which translates as follows:

> "I computed these useful tables, according to the latest observations, for finding the times of prayer (*ḥiṣaṣ al-awqāt*) by day and night at the stations on the pilgrim road during the lunar year 1041."

Several other Arabic sources contain lists of geographical coordinates and qiblas for the stations on the pilgrim roads from Cairo or Damascus to Mecca.[7] As an example, I cite here the tables in MS Paris BNF ar. 2560, fol. 164v, which apparently form part of the *zīj* entitled *al-Rawḍ al-ʿāṭir* by the Damascus astronomer Ibn Zurayq (*fl. ca.* 1400).[8] These display the latitudes of some 32 stations on the pilgrim route from Damascus to Mecca, and the latitudes, longitudes, and qiblas of 15 stations on the pilgrim route from Cairo to Mecca. For the few stations listed in both tables the latitudes given are generally different. See also **10.9**.

8.5 Anonymous timekeeping tables for Alexandria

MS Cairo TR 354, copied *ca.* 1700, contains about 140 pages of tables, including a complete set of tables of the functions:

$$t(h,\lambda),\ \ T(h,\lambda),\ \text{ and }\ a(h,\lambda)$$

←
Fig. 8.3: The tables of Abu 'l-Khayr al-Ḥusnī for determining the length of twilight for all latitudes. The tables are of particular interest in that the compiler presents all of the auxiliary functions which he used. [From MS Cairo DM 1108,3, fols. 9r-10v, courtesy of the Egyptian National Library.]

---

[6] *Gotha Catalogue*, III, p. 63. On ʿAbd al-Munʿim al-Nabatītī see İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 308-309, no. 167, citing the Gotha manuscript and a short treatise on timekeeping extant in Berlin. I have seen a copy of an almanac by ʿAbd al-Munʿim al-Nabatītī for the year 1073 H [= 1662/63], consisting mainly of simple calendrical tables, but I forget where. He is not to be confused with ʿAbd al-Qādir al-Nabatītī (*Cairo ENL Survey*, no. D28, and İhsanoğlu *et al.*, *op. cit.*, I, p. 279, no. 143), author of a treatise on the agricultural and meteorological aspects (*tawqīʿāt*) of the Syrian calendar, extant in MS Cairo MM 108,1, or the latter's son, ʿAlī ibn ʿAbd al-Qādir al-Nabatītī, author of a treatise on the algebra of inheritance and others on simple timekeeping (*Cairo ENL Survey*, no. D28, and İhsanoğlu *et al.*, *op. cit.*, I, pp. 286-287, no. 151).

[7] Some examples are mentioned in King, *Mecca-Centred World-Maps*, pp. 84-86.

[8] On Ibn Zurayq see n. 11:4.

computed to two digits for each degree of both arguments. The underlying parameters are found by inspection to be:

$$\phi = 31° \text{ (Alexandria) and } \varepsilon = 23;35° .$$

The maximum solar altitude argument is thus 82° rather than 83° as in the main Cairo corpus. The three functions are tabulated side by side as in al-Bakhāniqī's edition of the Cairo corpus and in MS Cairo MM 71 of al-Khalīlī's and Shihāb al-Dīn al-Ḥalabī's tables for Damascus (**5.6**, **10.5** and **11.2**). Following these extensive tables is a single table of h_0 based on the parameters $\phi = 31°$ and $\varepsilon = 23;30°$.

On the title-page it is stated that the tables are due to Ibn Yūnus, but the person who wrote this knew so little about the illustrious Egyptian astronomer that he called him Ibn Yūnus al-Mawṣilī (*i.e.* from Mosul), confusing him with an ʿIrāqī scientist also named Kamāl al-Dīn ibn Yūnus.[9] It is easily shown that the tables are not due to Ibn Yūnus the Egyptian.

Firstly, the tables are generally less accurately computed than those in the Cairo corpus. Secondly, the values of t and T in MS Cairo TR 354 are related by those in the table of $D(\lambda)$ for $\phi = 31°$ found in MS Princeton Yahuda 861,1 of the prayer-tables for all latitudes, which are ultimately due to Najm al-Dīn al-Miṣrī (**8.1** and **6.5**). Again, the tables of $T(h,\lambda)$ for altitudes 19° and 17° in MS Cairo TR 354 show suspicious resemblance to the tables of $r(\lambda)$ and $s(\lambda)$ for $\phi = 31°$ in MS Princeton Yahuda 861,1.

MS Cairo ṬM 216,3 (fols. 14r-18v, 23r-87r, copied *ca.* 1800), contains a different set of tables of:

$$T(h,\lambda) \text{ and } t(h,\lambda)$$

for parameters:

$$\phi = 31° \quad \text{and} \quad \varepsilon = ?.$$

The tables are incomplete and bound in disorder. Tables of the two functions are copied on facing pages.

MS Cairo DM 1207,1 (fols. 1r-57v) is a copy of a set of prayer-tables for latitude 31° prepared in 1168 H [= 1754/55] by Ḥusayn, a *muwaqqit* in Rosetta, from a copy in the hand of the author Riḍwān, dated 1090 H [= 1679/80]. The tables are not related to those in MS Cairo TR 354 discussed above. The work begins with a brief introduction with calendrical, solar, and astrological tables, followed by a set of geographical tables (fol. 18r) in which we find the following entries for various cities in the Delta:

	L	ϕ
Cairo	54;55°	30; 0°
Damietta	53;50	31;25
Maḥalla	54;30	31; 0
Alexandria	51;54	30;58
Rosetta	52;50	31;20
(Mecca	67; 0	21; 0)

The prayer-tables (fols. 21v-35r) display the following functions:

$$\delta,\ d,\ \psi,\ h_0,\ h_q,\ Z_{(12)},\ H,\ D,\ D'',\ h_a,\ t_a,\ T_a,\ s,\ r,\ 2N,\ n,\ B,\ C,\ h_q,\ \alpha',\ \alpha_\phi,\ \alpha_s \text{ and } \alpha_r .$$

[9] On Kamāl al-Dīn ibn Yūnus see n. 7:2 above. Similar confusion also occurred in the 20th century: see King, "Ibn Yūnus and the Pendulum", pp. 43-47.

Values appear to be based on $\phi = 31°$ (ε uncertain), although the table of $Z_{(12)}$ and the second table of h_q are lifted from the Cairo corpus and are based on $\phi = 30°$ (with $q = 37°$). The tables for twilight are based on parameters 19° and 17° but the tables are not the same as the corresponding ones in MSS Cairo TR 354 and Cairo ṬM 216. The first table of h_q is based on parameters $\phi = 31°$, and I was not able to investigate the underlying value of q. The function D″ represents the time between midday and when the sun has set over the visible horizon and appears to be computed using:

$$D'' = D + \Delta D + 0;16° ,$$

where ΔD are the "difference minutes" for Cairo (**4.11**). The work concludes (fols. 37v-57r) with an undated star catalogue.

MS Cairo ZK 324, copied *ca*. 1800, is another set of prayer-tables for latitude 31° containing the same introduction, calendrical tables, and most of the prayer-tables that are in MS Cairo DM 1207. MS Cairo DM 1211, copied in 1101 H [= 1689/90] in the hand of Riḍwān Efendī, contains some of these prayer-tables, namely:

$$Z_{(12)}, H, h_a, t_a, T_a, s , r, 2N, n , \alpha', \alpha_\phi \text{ and } \alpha_s ,$$

with a note by the first that it is for $\phi = 30°$ not 31°.

MS Cairo ṬM 216, fols. 19r-22v, copied *ca*. 1700, contains another set of prayer-tables for $\phi = 31°$ displaying functions:

$$D, t_a, T_a, h_a, s, r \text{ and } n .$$

Clearly Riḍwān Efendī had some input in certain of these tables, but I am not inclined to attribute the main tables of (T,t,a) for latitude 31° to him without new evidence.

8.6 Quṭb al-Dīn al-Maḥallī's prayer-tables for Damietta

MS Cairo ṬM 241,1 (fols. 1r-33r), copied 1245 H [= 1829/30], and several other inferior copies preserved in the Egyptian National Library, contains a set of prayer-tables for the latitude of Damietta compiled by the Egyptian astronomer Quṭb al-Dīn al-Maḥallī in 1088 H [= 1677/78].[10] The following functions are displayed to two digits for each degree of λ:

$$H, \delta, d, D, h_a, t_a, T_a, s, r, n, \alpha_\phi, \alpha_r \text{ and } \alpha' ,$$

and based on the parameters:

$$\phi = 31;25° \text{ and } \varepsilon = 23;35° ,$$

with 19° and 17° for twilight. Two different conventions are used for the argument λ. In the tables of T_a, α_ϕ, α_r and α', the arguments are as in the main Cairo corpus. In those of H, δ, d, D, t_a, s, r and n, the vertical arguments are still 1 to 30 downwards and 0 to 29 upwards, but Aries 1° corresponds to $\lambda = 0°$. Thus the first entry in the table is for the vernal equinox and no entries are given for the solstices. The upward argument is absurd, and implies that Virgo 29° corresponds to the autumnal equinox. The hapless soul who arranged these tables in this way so that the first entry was for $\lambda = 0°$ rather than $\lambda = 1°$ inadvertently rearranged the table of h_a twice, since the first entry is for $\lambda = 359°$! The entries for δ and d (and also

[10] On Quṭb al-Dīn al-Maḥallī see *Cairo ENL Survey*, no. D37; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 312-313, no. 173 (lists other new sources).

H and D) are fairly accurately computed, but the table of α_ϕ is carelessly computed. The tables of t_a and T_a, which have two different formats, show corresponding error patterns, but some of the entries in each are garbled.

In an Egyptian manuscript which in 1939 was in the Institut für Geschichte der Medizin und der Naturwissenschaften in Berlin, according to a description by Willy Hartner,[11] there were some tables giving the solar azimuth as a function of the altitude for an unspecified latitude (**I-5.1.4**), as well as a table of h_0 for ϕ = 31;25° (not found in MS Cairo DM 106 – see **I-4.8.7**) and another relating to the azimuth of Mecca, probably of the function h_q. Perhaps the azimuth tables were computed for latitude 31;25°, which was the standard value for Damietta. Elsewhere in the former Berlin manuscript there were some other spherical astronomical tables for this same latitude, as well as a table of the corrections for refraction at the horizon – Hartner's *Minuten des Abstandes,* this last table being attributed to an individual identified as al-Hindī.[12]

8.7 Anonymous prayer-tables for Rosetta

MS Cairo DM 1207,2 (fols. 57v-58v, copied *ca*. 1850) – see **Fig. 8.7** – contains a set of tables of the functions:

2N, D and d ,

computed for latitude ϕ = 31;20° (Rosetta). The tables are copied in an atrocious hand and are very carelessly computed. Consider, for example, the following values of d compared with the correctly computed values d′ (for ε = 23;35°):

λ	15°	30	45	60	75	90
d	3;51°	7;43	10;17	12;51	14; 8	15;25
d′	3;38°	7; 8	10;21	13; 0	14;47	15;25

Note that the value given for argument 30° is about one-half that given for 90°, and that the values for 15°, 45°, and 75° were probably derived from those for 30°, 60°, and 90° by linear interpolation. See also **11.9** for another example of simplistic interpolation of this kind.[13]

8.8 Yūsuf Kilārjī's prayer-tables for Crete

al-Jamālī Yūsuf ibn Yūsuf, Kilārjī (*major domo*) of Ḥasan Āghā, was a student of Riḍwān Efendī who became *rūznāmejī* in Cairo and later worked in Mecca and the Yemen; he died in 1153 H [= 1741/42].[14] One of his works was a set of prayer-tables for Crete, extant in the unique copy MS Cairo DM 834, 70 fols., copied in 1120 H [= 1708/09], perhaps by the author. In his introduction Yūsuf Kilārjī states that he prepared these tables because there were none

---

[11] Hartner & Ruska, "Katalog", pp. 55-56.

[12] Possibly to be identified with Shams al-Dīn Muḥammad al-Hunayd(ī), on whom see *Cairo ENL Survey*, nos. C133-134 (though see also no. D41 on a "real" (?) Indian working in Cairo, albeit in mathematics rather than astronomy); and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, p. 116, no. 55.

[13] See n. **I**-1:21.

[14] On Yūsuf Kilārjī see Dorn, "Drei arabische Instrumente", p. 35; *Cairo ENL Survey*, no. D61; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 412-415, no. 275.

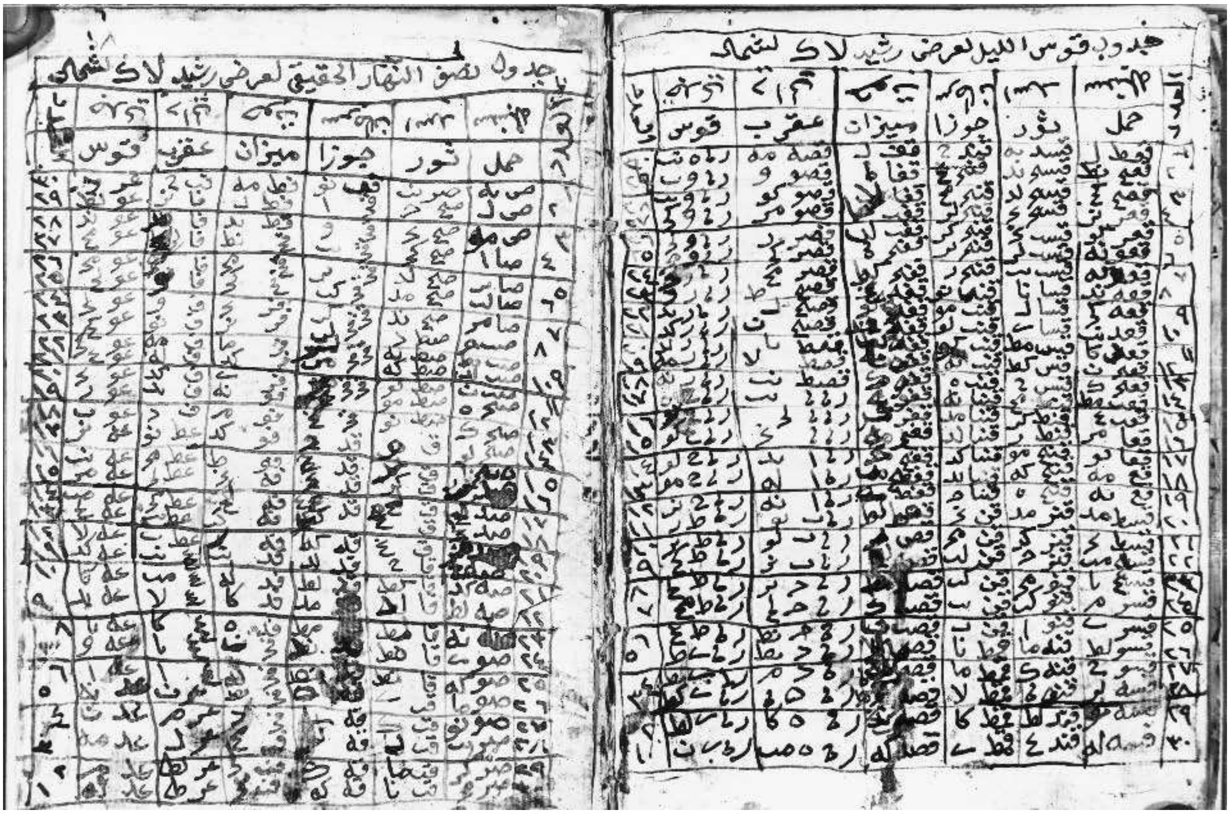


Fig. 8.7: Some messy prayer-tables for Rosetta, displaying the duration of night and half the "true" length of daylight. [From MS Cairo DM 1207,2, courtesy of the Egyptian National Library.]

available for Crete. He mentions the traditions relating to refraction at the horizon attributed to Ibn al-Haytham and Ibn Yūnus and refers to a universal table displaying $\Delta D(\phi)$, although without mentioning al-Minūfī (**8.2**). His introduction ends with the poem on the prayer-times which he attributes to al-Shāfiʿī (**2.1**). After some tables for calendar conversion and solar longitude and velocity Yūsuf Kilārjī presents tables with the format of the main Cairo corpus of the following functions, mostly with values to three digits:

$$\delta,\ d,\ \psi,\ h_0,\ B,\ C,\ h_q,\ H,\ D,\ \Delta D,\ (D+\Delta D),\ h_a,\ t_a,\ T_a,\ Z_{(12)},\ Z_{(7)},\ z_{a(7)},\ z_{b(7)},$$
$$(r+D),\ s,\ r,\ 2N,\ n,\ D^h,\ 2D^h,\ \tilde{h},\ 2N^h,\ t_a{}^h,\ T_a{}^h,\ s^h,\ r^h \text{ and } (2N\text{-}r)^h\ .$$

These are followed by tables with entries for each five days for the functions:

$$s^h,\ (2n\text{-}r)^h,\ r^h,\ 2N^h,\ (r+D)^h,\ t_a{}^h,\ T_a{}^h \text{ and } 2D^h\ .$$

The work concludes with tables of the functions:

$$\alpha_\phi,\ \alpha_\phi(\lambda^*),\ \alpha_s \text{ and } \alpha_r\ ,$$

and a star catalogue for 1120 H [= 1708/09] as well as an unusual kind of table displaying the solar longitude for which various stars become visible in the west. The parameters underlying the prayer-tables are:

$$\phi = 35;30° \text{ (Crete) and } \varepsilon = 23;30,17°\ .$$

The maxima and minima for ΔD are 1;13,12,0 and 0;37,52,0. The corresponding values of ΔD for $\phi = 35;30°$ according to the theory of ʿAbd al-Qādir al-Minūfī (**8.2**) are 1;13,22 and 0;37,52°. The parameters used for twilight are 19° and 17°. The azimuth of the qibla underlying the table of h_q has not been investigated. See also **14.10** on some Ottoman-type prayer-tables for the latitude of Khania in Crete.

8.9 ʿAbd al-Majīd Efendī's prayer-tables for Cairo and al-Rashīdī's table for finding the qibla

MS Cairo Azhar *falak* 18082 consists of a set of prayer-tables for Cairo printed there in 1312 H [= 1894/95], and compiled by ʿAbd al-Majīd Efendī.[15] These display the times of daybreak, sunrise, midday, the afternoon prayer, sunset and nightfall in *ifranjī*, that is, European time. At the end of this work there is a list of geographical coordinates and qiblas of several cities, giving the value 134;32,58° (from the north) as the qibla for Cairo. It is stated that these values are taken from a book by Ismāʿīl Pāshā al-Falakī.[16] This qibla is about $1^1/_2$° north of the actual (*i.e.*, modern) qibla for Cairo, but is about $7^1/_2$° south of the qibla favoured by the medieval Egyptian astronomers (q = 53°) (**4.7**).

Following this list of coordinates is a discussion of the use of a compass for finding the qibla. Then ʿAbd al-Majīd Efendī presents a table for determining the qibla which he states was compiled by Muḥammad al-Rashīdī al-Falakī.[17] This purports to give the time of each day of the Coptic year when the sun indicates the azimuth of Mecca. The times are given according to the Ottoman convention that sunset is 12 o'clock (**14.0**), and the underlying azimuth of Mecca which gives the closest fit to the entries in the table (maximum: $4;19^h$, mean: $3;40^h$, and minimum: $3;0^h$) is about 54°, measured from the meridian. Thus it appears that al-Rashīdī simply converted the table of t_q in the Cairo corpus to hours and minutes, Turkish time.

8.10 Muḥammad Ḥifẓī's prayer-tables for Egypt and the Hejaz

MSS Cairo K 3792-3802, copied *ca.* 1900, consist of eleven small booklets of prayer-tables compiled by Muḥammad Efendī Ḥifẓī, an official of the Egyptian Customs in ʿAbbāsiyya, Cairo: see **Fig. 8.10a-b**.[18] The tables display the times:

r′ (*fajr*), R′ (*shurūq*), ι′ (*ʿīd*), z′ (*zawāl*), m′ (*ẓuhr*),
a′ (*ʿaṣr awwal*), b′ (*ʿaṣr thānī*), S′ (*maghrib ifranjī*), s′ (*ʿishāʾ*)

Generally, separate tables are given for "Arab" (*ʿarabī*), *i.e.*, "Turkish", and local mean time (*ifranjī*). Where the time for the *ʿīd* prayer is given it is usually between 20 and 30^m after sunrise. The localities served by these tables are Cairo ($\phi = 30°$); Assiut (27°); Qusayr (26;7°); Sohag

[15] On ʿAbd al-Majīd al-Falakī see Azzawi, *History of Astronomy in Iraq*, p. 325. He does not seem to be listed in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*.
[16] On Ismāʿīl Pāshā al-Falakī see Azzawi, *op. cit.*, pp. 328-329. He does not seem to be listed in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*.
[17] This al-Rashīdī is apparently not mentioned in the modern sources.
[18] See *Cairo ENL Survey*, no. D194.

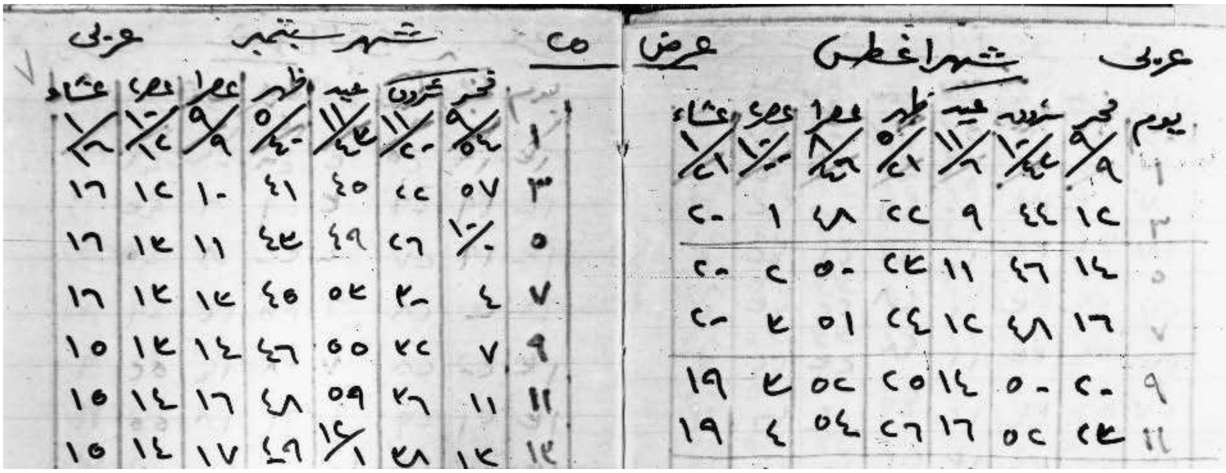


Fig. 8.10a: An extract from Muḥammad Efendī Ḥifẓī's prayer-tables for latitude 25° and Turkish time (*ʿarabī*), serving August and September. The entries are in minutes, with the appropriate hour indicated above a diagonal slash in the first entry of each column. [From MSS Cairo K 3792-3802, courtesy of the Egyptian National Library.]

Fig. 8.10b: These tables display the shadow at the *ẓuhr* for every two days of the year, also for latitude 25°. The entries are in feet and decimal fractions thereof. [Same source.]

and Akhmim (26;30°); Qus (26°); Edfu and Esna (25°); Medina (both 24° and 24;10°); *Markaz al-Durr* (?) (23°); Korosko (*Markaz al-sidr*) (22;30°); Wadi Halfa (22°); Mecca (both 21;28° and 21;30°); and Jabal al-Ṭūr (in the Sinai) (28;5°). Some other funtions are tabulated for certain latitudes.

CHAPTER 9

EARLY SYRIAN TABLES FOR TIMEKEEPING

9.0 Introductory remarks[1]

The first Muslim astronomer known to have worked in Damascus (as well as Baghdad and Samarra) was Ḥabash al-Ḥāsib (*ca*. 850).[2] He wrote at least three zījes, one of which, called *al-Zīj al-Dimashqī*, "the Damascus *Zīj*", survives and has been studied by Ted Kennedy and Marie-Thérèse Debarnot.[3] Ḥabash did work of considerable interest, and is to be regarded as the most original astronomer of the early Islamic period, as well as one of the most prolific. His early work in mathematical astronomy shows considerable Indian influence, but this is supplanted by Greek influence and then – he lived for 100 years – we find him solving higly complicated problems which went beyond the interests of earlier traditions. His work was in a sense experimental for the Islamic world was not yet ready for some of his achievements, some of which had far less influence in later Islamic astronomy than they shoud have had. Thus, for example, he compiled a set of tables for constructing horizontal sundials for a range of latitudes (**I-4.1.1**); he compiled the first set of universal auxiliary trigonometric tables for solving problems of spherical astronomy for all latitudes (**I-9.1**); he was the first to tabulate the standard trigonometric functions that we know today; he conceived an astrolabe based on a projection preserving azimuth and distance to the celestial pole;[4] and he was the first to solve the qibla problem accurately by the so-called "method of the *zīj*es", the mathematics of which underlies the cartographic grids for world-maps centred on Mecca preserving direction and distance to the centre, known from three Safavid examples.[5]

Shortly after the time of Ḥabash, we find al-Battānī (*ca*. 910) making serious observations at Raqqa and compiling a *zīj* which, as a competent epitome of Ptolemaic mathematical astronomy with a distinct Islamic flavour, was to exert far more influence in medieval and Renaissance Europe than it actually did in the Islamic world.[6] This work marks the successful appropriation and synthesis of Ptolemaic planetary astronomy and Greek and Indian spherical astronomy and trigonometry, a process to which Ḥabash had been the main contributor.

The only astronomers know to have prepared *zīj*es in Syria between the time of Ḥabash and al-Battānī and the 14th century, when Damascus became the leading centre of astronomy in the Islamic (and indeed in the entire) world are:

---

[1] I have attempted an overview of astronomy in Syria in *Paris IMA 1993-94 Catalogue*, pp. 386-395. See also my "Astronomy of the Mamluks".

[2] On Ḥabash see n. **I**-9:1.

[3] Kennedy, "*Zīj* Survey", pp. 151-154, and Debarnot, "*Zīj* of Ḥabash".

[4] See Kennedy & Lorch & Kunitzsch, *Melon Astrolabe*.

[5] See King, *Mecca-Centred World-Maps*, esp. pp. 61-64 and 241, and also **VIIc**.

[6] On al-Battānī see n. **I**-4:11.

(1) Ibn al-Dahhān (*ca*. 1170) who worked in the service of the Ayyubid ruler Ṣalāḥ al-Dīn (Saladin);[7]
(2) Muʾayyad al-Dīn al-ʿUrḍī (*ca*. 1250), who later assisted in the observations at Maragha in Iran under the direction of Naṣīr al-Dīn al-Ṭūsī;[8]
(3) Muḥyi 'l-Dīn al-Maghribī (*ca*. 1250), who also went on to work in Maragha;[9] and
(4) Ibn al-Lubūdī (*ca*. 1250), who worked in both Damascus and Cairo.[10]

The *zīj*es of Ibn al-Dahhān, al-ʿUrḍī and Ibn al-Lubūdī are no longer extant, and the *zīj* which al-Maghribī compiled in Damascus, called *Tāj al-azyāj*, "Crown of the *Zīj*es", survives but has not been studied in detail. A fragment from the *Zīj* of Ibn al-Dahhān preserved in a Yemeni manuscript is discussed in **9.1**. The recovery of al-ʿUrḍī's *Zīj* would contribute to our knowledge of the development of Islamic planetary theory, since al-ʿUrḍī's work influenced Ibn al-Shāṭir (see below). Likewise a detailed study of the *Zīj* of al-Maghribī and his later *zīj*es prepared at Maragha, would also contribute towards clarifying our knowledge of astronomy in Syria and its influence on the Maragha school. Important for our understanding of the influence of Andalusī and Maghribī influence in Syria is the recension for Damascus of the *Zīj* of the 13th-century Tunisian astronomer Ibn Isḥāq (**13.1**).

In the 14th century a number of astronomers flourished in Syria, mostly associated with the Umayyad Mosque in Damascus.[11] Ibn al-Sarraj, who worked in Aleppo apparently without institutional backing, specialized in devising new astronomical instruments.[12] Foremost amongst Damascus scholars were Ibn al-Shāṭir, who made substantial contributions to planetary theory,[13] and al-Mizzī[14] and al-Khalīlī,[15] whose interests were in astronomical timekeeping. Each of these 14th-century astromomers devised different kinds of quadrants and other devices for solving problems in spherical astronomy.[16] There was especial interest in instruments which would provide solutions for all latitudes (**VIb**). Both Ibn al-Shāṭir and al-Mizzī had studied astronomy in Alexandria before taking up their positions at the Umayyad Mosque, and the Egyptian influence is apparent in their interests and achievements.

al-Khalīlī's extensive tables for timekeeping (**Ch. 10**), which were used in one form or another from the 14th to the 19th century, replaced an earlier set compiled by al-Mizzī (**9.2**). Ibn al-Shāṭir also prepared some prayer-tables (**9.3**). Another student of al-Mizzī was al-Karakī,

---

7 On Ibn al-Dahhān see n. 9:17.

8 On al-ʿUrḍī see Suter, *MAA*, pp. 147 and 154; Kennedy, "*Zīj* Survey", no. 42; *Cairo ENL Survey*, no. C170; and Saliba, "13th-Century Observational Notebook". Saliba has also edited the text of his treatise on theoretical astronomy (Beirut, 1990).

9 On al-Maghribī see n. **I**-5:8.

10 On Ibn al-Lubūdī see Suter, *MAA*, no. 365, Kennedy, "*Zīj* Survey", nos. 86 and 87, and Azzawi, *History of Astronomy in Iraq*, pp. 155-156; and King, "Astronomy of the Mamluks", pp. 533-534.

11 For a survey of their achievements in astronomical timekeeping see King, "Astronomical Timekeeping in Medieval Syria".

12 On Ibn al-Sarrāj (Suter, no. 508 (confused), and *Cairo ENL Survey*, no. C26) and his instruments see King, "The Astronomical Instruments of Ibn al-Sarrāj", published for the first time in *idem*, *Studies*, B-IX, and also **X-5.2**. See also Charette & King, *The Universal Astrolabe of Ibn al-Sarrāj* (forthcoming).

13 See n. **I**-2:15.

14 On al-Mizzī see n. 9:21.

15 On al-Khalīlī see n. 10:1.

16 On some of these see Schmalzl, *Geschichte des Quadranten*, and **VIb-8**, as well as Charette, *Mamluk Instrumentation*.

who worked in Jerusalem and adapted the tables of Ibn al-Rashīdī to form the main Jerusalem corpus (**9.4**).

Damascus was razed to the ground by the Mongols under Timurlang in 1401. There was little creative astronomical activity of any consequence in Syria thereafter: the centre of Islamic planetary astronomy moved to Samarqand, and the centre of astronomical timekeeping moved back to Cairo for a century or two before it finally moved to Istanbul.

9.1 A fragment from the *Zīj* of Ibn al-Dahhān

Fakhr al-Dīn Abū Shujāʿ Muḥammad ibn ʿAlī known as Ibn al-Dahhān, who worked in Damascus *ca*. 565 H [≈ *ca*. 1170], was an astronomer associated with the celebrated ruler Ṣalāḥ al-Dīn (Saladin).[17] He compiled a *zīj*, which is unfortunately no longer extant. However, a fragment survives in the Sanaa miscellany of al-Afḍal (**12.4**), and consists of a set of tables giving the values of the following functions at the summer solstice:

$$d,\ d^h,\ H,\ Z_{(12)} \text{ and } z_{a(12)}$$

for some 120 cities.[18]: see **Fig. 9.1**. Values are given to two digits. The underlying value of ε is 23;35°. The relation between the last two functions is simply:

$$z_a = \text{Cot } h_a = \text{Cot } H + 12 = Z + 12.$$

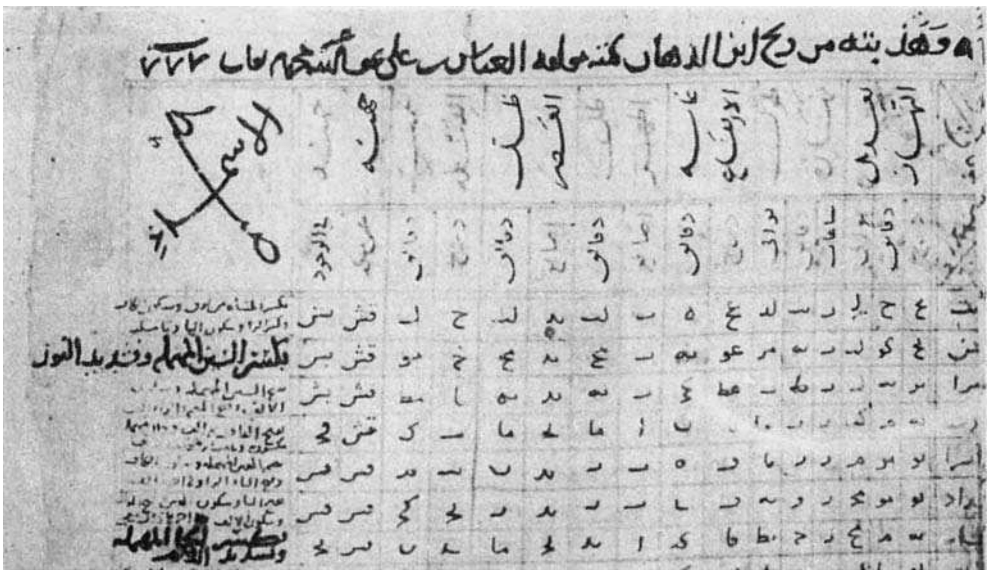


Fig. 9.1: An extract from the beginning of Ibn al-Dahhān's tables. Notice that the place-names are not visible on this page. A reconstruction of the entire table and the geographical coordinates underlying it would be possible and even worthwhile; but where are the volunteers? [From Varisco & Smith, eds., *al-Afḍal's Anthology*, p. 151, courtesy of Professor Daniel Varisco.]

[17] On Ibn al-Dahhān see Suter, *MAA*, no. 310; Kennedy, "*Zīj* Survey", no. 89; and King, *Astronomy in Yemen*, p. 37. Add to II n. 10:17: This Ibn al-Dahhān is not to be confused with ʿImād al-Dīn ibn al-Dahhān, chief of the astronomers in Mosul *ca*. 1285, for a reference to whom see Cook, "Muslim Material on Comets and Meteors", p. 147, and Rada, "Comets in Arabic Literature", p. 45.

[18] See now the facsimile in Varisco & Smith, eds., *al-Afḍal's Anthology*, pp. 151-154.

The azimuth of the qibla measured from the meridian is also given,[19] as well as the vowelling of the Arabic names of the cities. Not all of the city names are visible on the microfilm which I have used, but there seems to be no entry for Mecca (as one can see from the qibla-values, if not the names). Sample significant entries for other cities (values of ϕ are calculated from those of H) are:

Locality	d	max H	[ϕ]	q
Cairo	14;3°	84;20°	[29;15°]	54;39°
Jerusalem	16;5	80;35	[33;0]	41;21
Damascus	16;40	80;5	[33;30]	28;29

Ibn al-Dahhān is known to have visited Cairo, but his value for the latitude of that city, confirmed by the value of d (so that it is not a scribal error for 29;55°), is too small by $^3/_4$°.[20] Likewise his value for the latitude of Jerusalem is too large by about 1°. (However, a later qibla table compiled for Jerusalem is based on the parameter 41;30°, rather than on values which can be derived using a more accurate latitude for Jerusalem (see further **9.5**).) Of the Islamic geographical tables investigated by the Kennedys (**1.3**), only the table in al-Bīrūnī's *Qānūn* has the same latitudes for these three cities as Ibn al-Dahhān. However, Ibn al-Dahhān's qibla values do not correspond to those that can be computed from al-Biruni's coordinates using either the exact or the standard approximate formulae.

9.2 al-Mizzī's hour-angle tables and prayer-tables for Damascus

The astronomer Shams al-Dīn Muḥammad ibn Aḥmad ibn ʿAbd al-Raḥīm al-Mizzī (1291-1343), whose family originated from a suburb of Damascus, studied under Ibn al-Akfānī in Alexandria, and then moved back to Damascus where he eventually worked in the Umayyad Mosque.[21] He was a fine instrument-maker, and several instruments of his construction and treatises on their use have survived.[22] He also compiled some tables for timekeeping. The main set appears to be no longer extant, but, according to his student al-Karakī (**9.4**), it consisted of tables of the hour-angle $t(h,\lambda)$ computed for latitude 33;27°, that is, Damascus: see **I-2.1.3**. al-Mizzī's minor set of tables survives in MS Cairo MM 62, copied *ca*. 1400. The tables are not accompanied by any introduction, and the functions tabulated are:

$$\delta,\ H,\ Z_{(7)},\ d,\ D,\ \tilde{h},\ 2D^h,\ 2N,\ h_0,\ \psi,\ \alpha,\ \alpha_\phi,\ h_a,\ t_a,\ T_a,\ r \text{ and } s\ .$$

The entries are arranged with the format of the main Cairo corpus (see **Fig. 9.2**), and are based on al-Mizzī's distinctive parameters:

$$\phi = 33;27° \text{ and } \varepsilon = 23;33°\ ,$$

on which see below. They are very accurately computed, although al-Mizzī used an interpolation system in the fourth columns of his tables of r and s which failed to take account of the actual behaviour of these functions for solar longitudes in Libra and Pisces. This means

[19] This table was advertently overlooked in King, *Mecca-Centred World-Maps*.
[20] On the unhappy value 29;15° see King, "Geography of Astrolabes", p. 9 and n. 23.
[21] On al-Mizzī see n. **I**-2:13.
[22] Mayer, *Islamic Astrolabists*, pp. 61-62. See also *Paris IMA 1993-94 Exhibition Catalogue*, p. 438, no. 333, on one of his quadrants.

Fig. 9.2: An extract from al-Mizzī's prayer-tables showing the sub-tables for the time from the beginning of the *ʿaṣr* to sunset and the duration of evening twilight (based on a solar depression of 17°). [From MS Cairo MM 62, fols. 8v-9r, courtesy of the Egyptian National Library.]

that the corresponding entries in his hour-angle tables would also be in error. The 15th-century astronomer Jamāl al-Dīn al-Māridīnī[23] states that al-Mizzī used parameters 20° and 16° for morning and evening twilight, but the tables of r and s in this Cairo manuscript are based on 19° and 17°.

MS Florence Laurenziana 24 is the only known copy of a recension of the *Īlkhānī Zīj* for Mosul, prepared by Dāʾūd al-Mawṣilī.[24] On fols. 1r-1v there is an incomplete set of tables that are unrelated to the *zīj* of al-Mawṣilī. On fol. 1r there is a simple table for finding Easter and another of $\delta(\lambda)$ with entries to two digits for each degree of λ and based on $\varepsilon = 23;33°$. On fol. 1v there is the first page of a set of prayer-tables for an unspecific latitude. Entries are given to two digits for each degree of λ from 1° from 30°. The functions tabulated are:

$$H,\ D,\ \psi,\ h_0,\ h_a,\ t_a,\ 2D^h,\ \tilde{h},\ s \text{ and } r\ ,$$

and the values are those of al-Mizzī for Damascus.

The latitude 33;27° for Damascus is attested in MS Paris BNF ar. 2513, fols. 87v-88v, of the Egyptian 13th-century *Muṣṭalaḥ Zīj*, and from MS Paris BNF ar. 2520, fol. 72r, of another rescension of the same work we learn that both parameters were derived by observations made by an individual referred to as *al-Sharīf*, to be identified with the 10th-century Baghdad astronomer Ibn al-Aʿlam (**6.6**). Ibn al-Shāṭir also used these parameters in his early work: they are found, for example, in Chs. 18 and 114 of his treatise on the "complete quadrant" (MS Damascus Ẓāhiriyya 3098). In the year 765 H [= 1363/64] Ibn al-Shāṭir derived a new pair of parameters, for which al-Khalīlī computed a new corpus of tables for Damascus (**Ch. 10**) to replace those of al-Mizzī.

[23] I forget where I read this, but surely in an instrument treatise by al-Māridīnī (**6.15**).

[24] On the *Īlkhānī Zīj* see Kennedy, "*Zīj* Survey", no. 6. Dāʾūd al-Mawṣilī, not mentioned in the modern literature, states that he was asked to prepare the recension by *al-ṭabīb* (the medic) ʿĪsā and that it took him twenty days to translate the introduction into Arabic.

9.3 Ibn al-Shāṭir's prayer-tables for Tripoli (?)

The Damascus astronomer Ibn al-Shāṭir (1306-1375/80), author of two *zīj*es and an extensive treatise on theoretical astronomy in which he explained his new models for the sun, moon, and planets,[25] also compiled a set of prayer-tables for latitude 34°, possibly intended for the new Mamluk city of Tripoli (accurate latitude 34°27′). The evidence for this is contained in a short treatise attributed to him, preserved in MS Leiden Or. 1001, fols. 108r-113r, which was originally intended as an introduction to the tables. The tables themselves are extant in a different manuscript (see below).

Ibn al-Shāṭir begins his introduction by stating that one of his friends asked him to prepare a set of prayer-tables for latitude 34°. The functions:

$$H,\ Z_{(7)},\ D,\ 2N,\ h_a,\ t_a,\ T_a,\ r,\ s,\text{ and } t_q$$

are specifically mentioned (fol. 110v:16-19). Ibn al-Shāṭir's description of the method of finding the value of a given function for a particular solar longitude indicates that the functions were tabulated side by side. Thus they would resemble the tables of al-Khalīlī in appearance, rather than those of Ibn Yūnus and al-Mizzī. He also notes how to compute:

$$\delta,\ d,\ \tilde{h},\ 2D^h \text{ and } z_{a(7)}$$

from the entries in the tables, indicating that he did not actually tabulate these functions.

MS Cairo DM 1170,2 (fols. 11r-22v), copied about 1700, is the only known copy of an anonymous set of prayer-tables for latitude 34° which answer the description of Ibn al-Shāṭir's introduction in the Leiden manuscript: see **Fig. 9.3**. The tables on fols. 14v-19r are preceded by an introduction, of which the beginning is missing, and by calendrical and solar longitude tables (fols. 11r-14r), and they are followed by a star catalogue for the end of the year 1033 H [= 1624] and an almanac (fols. 19v-22v). According to a note in the margin of the star catalogue (fol. 21r) the entries in this, and probably also the other introductory tables, were adopted by Muḥammad ibn al-Ḥulwānī[26] for 1033 H [= 1623/24] from another set compiled by ʿAlāʾ al-Dīn ibn Nāṣir al-Din al-Ṭarābulusī, a *muwaqqit* at the Umayyad Mosque in Damascus, for the end of 984 H [= 1577]. Values of the following functions:

$$H,\ Z_{(7)},\ D,\ 2D^h,\ h_a,\ t_a,\ T_a,\ 2N,\ s,\ r,\ n \text{ and } t_q\ ,$$

computed to two digits, are displayed side by side for each degree of λ beginning with Capricorn 1°. The underlying parameters are:

$$\phi = 34;0° \text{ and } \varepsilon = 23;31°.$$

I have not been able to investigate the parameters used for twilight or the qibla. Additional tables display the functions:

$$d,\ \alpha_\phi \text{ and } \alpha',\ s \text{ and } r$$

for the same ϕ and ε. It seems to me most likely that this collection of tables are the prayer-tables of Ibn al-Shāṭir.

Added in June, 2002: Whilst finalizing the illustrations for this study I realized that in the 1970s I had not included mention of the tables for latitude 34°, specifically intended for Tripoli,

[25] On Ibn al-Shāṭir see the references cited in n. **I**-2:15.

[26] On Muḥammad ibn Ibrāhīm al-Ḥulwānī al-Ḥusnī al-Shāfiʿī see *Cairo ENL Survey*, no. D22; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 280-281, no. 146.

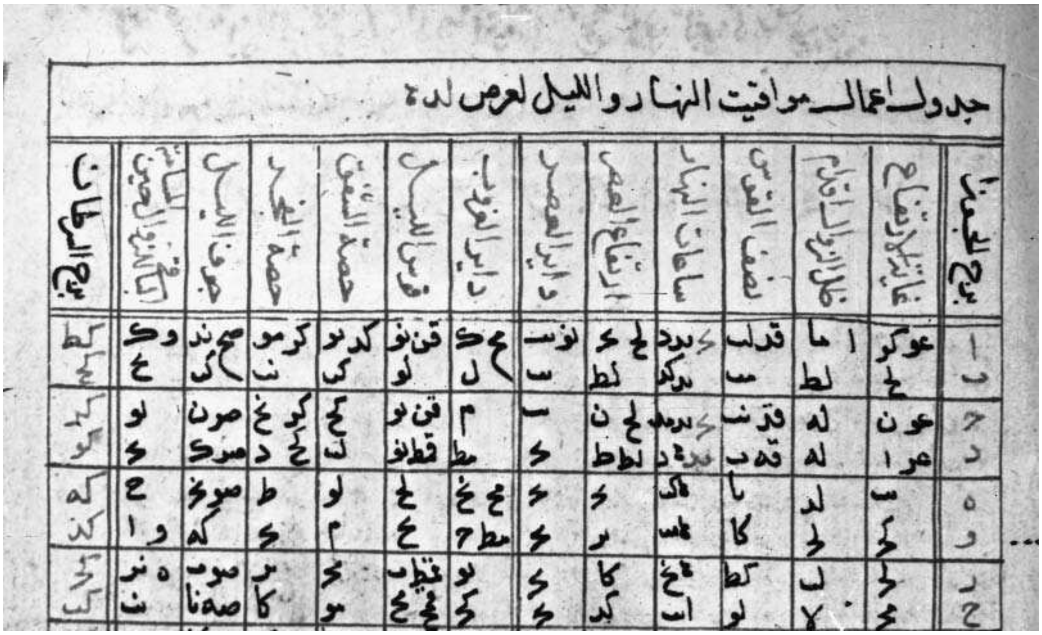


Fig. 9.3a: An extract serving solar longitudes in Taurus and Cancer from a set of prayer-tables for latitude 34° probably due to Ibn al-Shāṭir. [From MS Cairo DM 1170,2, courtesy of the Egyptian National Library.]

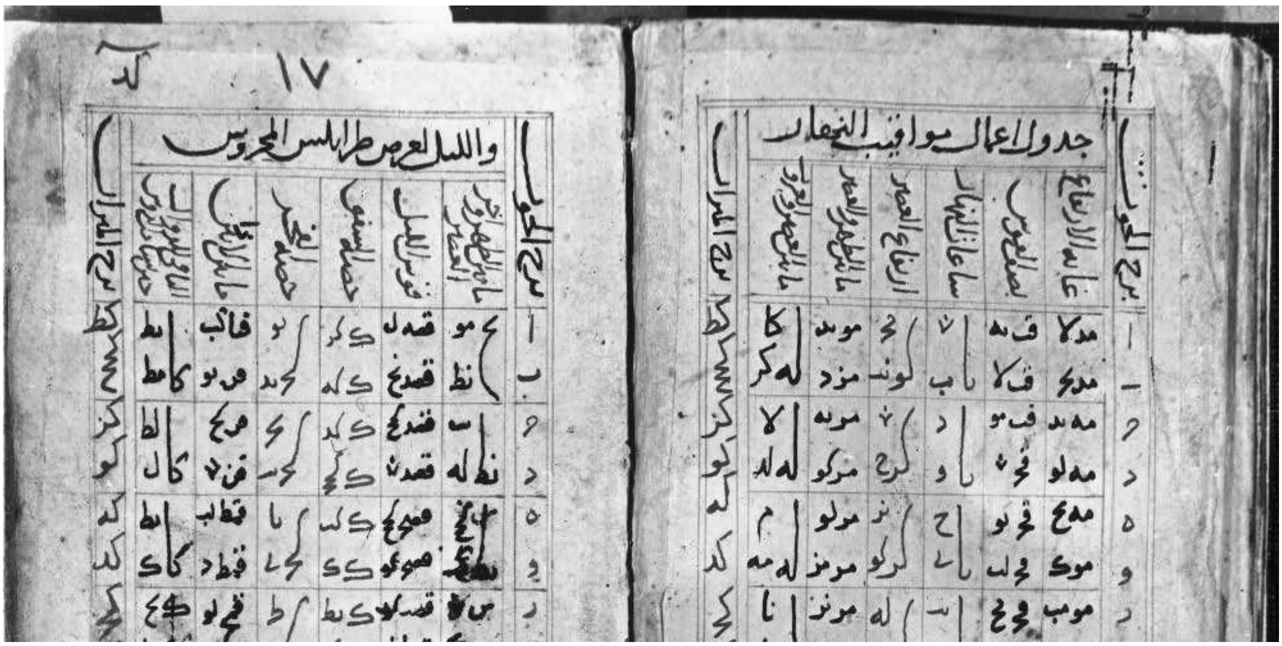


Fig. 9.3b: An extract for solar longitudes in Pisces from an an anonymous set of prayer-tables for Tripoli (latitude 34;0°). [From MS Cairo ṬM 228,3, fols. 16v-17r, courtesy of the Egyptian National Library.]

contained in MS Cairo ṬM 228,3, fols. 9r-9v + 16r-21r, copied *ca*. 1500.[27] I cannot compare these now with those in the Cairo DM manuscript. Suffice it here to note that the functions tabulated are:

$$H, D, 2D^h, h_a, t_a, T_a, t_b, 2N, s, r, n, \text{ and } t_q ,$$

and to present an extract in **Fig. 9.3b**. The functions t_b and n are referred to as *mā bayn al-ẓuhr wa-ākhar al-ʿaṣr* and *mā bayn al-ādhānayn*, respectively. Clearly, the two sets, and possibly also those in **11.6**, merit a new investigation.

The treatise in the Leiden manuscript also contains a description of how to compute the solar longitude from the date, the time of day from shadow lengths, the solar altitude at the beginning of the afternoon prayer, and the times of rising and setting of the moon: all these methods are approximate though this is not stated. For example, to find the time elapsed since sunrise when the shadow is z (measured to base 7), the author suggests:

$$T \approx \{ 42 / [(z + 7) - Z_{(7)}] \}^{sdh} .$$

Equivalent versions of this Indian approximate formula were used by various Islamic astronomers from the 8th century onwards,[28] but one is suprised to see an astronomer of the calibre of Ibn al-Shāṭir advocating it. Likewise, the method outlined by him for finding the altitude at the beginning of the afternoon prayer involves the approximation:

$$h_a \approx {}^1/_2 [H + {}^1/_2 \cdot {}^1/_6 \bar{H}] ,$$

and the reader is warned against using:

$$h_a \approx {}^1/_2 H + {}^1/_2 \cdot {}^1/_6 \bar{H} .$$

Approximate methods of this kind are also given in Ibn al-Shāṭir's treatise on the complete quadrant (MS Damascus Ẓāhiriyya 3098, *e.g.* Chs. 114 and 173). For similar approximations in other sources, see the references in **1.4**.

In his treatise on planetary theory entitled *Nihāyat al-suʾl* (MS Oxford Marsh 139, fol. 7r), Ibn al-Shāṭir mentions that the person who stated that the visible portion of the heaven is 0;4,26 more than the invisible portion was in error. (Shams al-Dīn al-Minūfī (**7.1**) associates that value with the illustrious Ibn al-Haytham: see also **14.9**.) He adds that he had observed the half arcs of visibility of the sun and stars and found that they were greater than the arcs found by calculation by an amount exceeding $^2/_3°$. In what purports to be a set of notes due to the Egyptian astronomer ʿIzz al-Dīn al-Wafāʾī the values of the "difference minutes" for Cairo (4.11) are attributed to Ibn al-Shāṭir (5.7, also 8.3), but I feel that this attribution is somewhat suspect.

In a study of Ibn al-Shāṭir's contributions in astronomical timekeeping it would be amiss not to mention the magnificent sundial which he constructed for the main minaret of the Umayyad Mosque. The remains of this instrument are today preserved in the garden of the Archeological Museum in Damascus, but in its original position there is a faithful replica made by the late-19th-century *muwaqqit* al-Ṭanṭāwī (**11.13**). The sundial serves to regulate time with respect to each of the five times of prayer.[29]

---

[27] *Cairo ENL Survey*, no. C142, *sub* 3.1.17.

[28] See **1.4** and also Arabic **Text 2.9** to **IV**.

[29] See Janin, "Cadran solaire de Damas"; King in *Paris IMA 1993-94 Exhibition Catalogue*, p. 439, no. 334, and also **X-7.2**.

9.4 al-Karakī's timekeeping tables for Jerusalem

MS Leipzig UB 808, fols. 3r-93r, penned in 805 H [= 1402], is an apparently unique copy of a set of tables for timekeeping compiled by Zayn al-Dīn Abū Bakr al-Karakī[30] for the latitude of Jerusalem. He appears to have been a student of al-Mizzī (**9.2**) and can thus be dated to the mid 14th century. The introduction to his tables on fol. 2v is of considerable relevance to the present study and I present a translation:

> "In the Name of God, the Merciful and Compassionate. The *shaykh* and *imām* Zayn al-Dīn Abū Bakr ibn Muḥammad ibn Ayyūb al-Tamīmī known as al-Karakī, *muwaqqit* in Sacred Jerusalem – may God have mercy on him – said the following in the first section (of his book), which concerned the compilation of tables of time since sunrise and the hour-angle for latitude 32° north. When I saw that the virtuous *shaykh*, scholar, and calculator Jamāl al-Din Abu 'l-ʿAbbās Aḥmad ibn ʿUmar ibn Ismāʿīl ibn Muḥammad ibn Abī Bakr al-Ṣūfī al-Maqsī – may God have mercy on him – had compiled tables of the time since sunrise for latitude 30°, which require further calculation to find the hour-angle, and that our teacher the virtuous *shaykh* and meticulous scholar Shams al-Dīn Abū ʿAbdallāh Muḥammad ibn Aḥmad ibn ʿAbd al-Rahīm al-Mizzī – may God Almighty have mercy on him – had compiled tables of the hour-angle for latitude 33;27°, which also require further calculation to find the time since sunrise, and that the *shaykh* Shams al-Dīn Ibn al-Rashīdī – may God have mercy on him – had put the altitude at the head of each table and the hour-angle opposite the solar longitude, I wanted to participate with them in such compilations. So I put the significant (functions) together to facilitate the work of the observer, and tabulated the time since sunrise and the hour-angle opposite the altitude with the solar longitude at the head of the tables, beginning at the first point of the ascending zodiacal signs (*i.e.*, the winter solstice). The same tables can be used for the descending signs in the opposite direction (since the last table is for the summer solstice). I did this, asking help from God and placing my trust in Him ... ."

al-Karakī's tables do indeed display the functions:

$$T(\lambda,h) \text{ and } t(\lambda,h)$$

for each degree of both arguments. Note that values are given for both solstices and the equinoxes. For each value of λ, $D(\lambda)$ is also tabulated. The underlying parameters are:

$$\phi = 32;0° \text{ and } \varepsilon = 23;35° \ .$$

The entries, which are given to two digits, are rather accurately computed. Note that al-Karakī does not actually say that he computed both of the functions himself. Indeed, it may be that he only computed the time since sunrise from someone else's hour-angle tables. On fols. 94v-123v of the Leipzig manuscript, in a different hand, there is another set of hour-angle tables arranged in the same way as al-Karakī's tables, and also showing $D(\lambda)$. The title and colophon indicate that they are for latitude 31;40°. In fact they are for 32;0° and the entries are the same as those in al-Karakī's set. A note in the incomplete introduction to this second set on fol. 94r

[30] The Leipzig manuscript of al-Karakī's tables is mentioned in Brockelmann, *GAL*, SII, p. 156, after *Leipzig Catalogue*, p. 261.

mentions the completion of the minaret on a mosque in Ramla in 797 H [= 1395]. The value 31;40° is used for Ramla in several Islamic geographical tables.[31]

al-Karakī was clearly impressed with al-Maqsī's tables of T(h,λ) for Cairo (**5.4**), and al-Mizzī's tables of t(h,λ) for Damascus (**9.2**). It is a pity that he does not state the latitude for which Ibn al-Rashīdī compiled his hour-angle tables. His name is not otherwise associated with any of the various hour-angle tables that I have found in the manuscript sources. It seems to me likely that Ibn al-Rashīdī compiled a set of tables of t(h,λ) for Jerusalem, which al-Karakī then used to compile his tables with a different format. Now fragments of tables of t(h,λ) for latitude 32° exist in MSS Cairo DM 45 and DM 153 (**6.12**) with identical corresponding entries to al-Karakī's tables. If my supposition is correct, al-Karakī's tables for Jerusalem are merely an adaption of Ibn al-Rashīdī's tables, which were probably compiled in Cairo. See also the next section.

9.5 Anonymous prayer-tables for Jerusalem

The tables for latitude 32° in MS Princeton Yahuda 861,1 of the *Natīja al-kubrā* attributed to al-Wafāʾī (**8.1**) differ from those for other latitudes in such a way that it is immediately obvious that they were lifted from various independent sources. The following functions are tabulated for this latitude:

$$D,\ 2D^h,\ 2N,\ H,\ h_a,\ t_a,\ h_q,\ T_q,\ r,\ s,\ n,\ \alpha_\phi,\ \alpha_\sigma \text{ and } \sigma\ .$$

The entries in certain of the tables are reasonably accurately computed and in others are very carelessly computed. My investigation of the tables reveals the following.

(1) The table of D, which is carelessly computed, is the same as the corresponding table in MS Oxford Marsh 676 (Uri 944 = 995) due to Najm al-Dīn al-Miṣrī (**6.5**).

(2) The table of $2D^h$, the length of daylight in equinoctial hours, is a particularly wretched specimen. Some of the entries, which are badly garbled, are in hours and minutes and others are in hours and degrees.

(3) The table of 2N, which is rather accurately computed, is based on values of D other than those in (1). In fact it is based on the values of D used by al-Karakī to compile his tables of T(λ,h) from (Ibn al-Rashīdī's ?) tables of T(h,λ) (**9.4**).

(4-6) The tables of H, h_a and t_a are likewise rather accurately computed.

(7-8) The tables of h_q and T_q are based on a value of q for some particular locality, although neither parameter nor locality is specified for either function. The latitude 32° is the only latitude other than 30° for which h_q and T_q are tabulated in the Princeton manuscript. However, in the title of the first function tabulated for latitude 32°, namely D(λ), the cities of Gaza, Ramla and Jerusalem are specifically mentioned, and by inspection I find the underlying value of q to be 41;30° (measured from the meridian). The two tables are reasonable accurately computed. Now in the geographical tables in MS Princeton Yahuda 861,1 the following entries for localities in Palestine are given:

[31] Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. 280-281.

	L	ϕ	q	q (acc.)	q (approx.)
Gaza	56;30°	32;0°	52;40°	42;59°	43;41°
Ramla	57;20	32;20	45;0	39;37	40;31
Jerusalem	58;35	32;10	45;51	35;58	37;5
Nablus	58;15	32;20	37;0	36;41	37;45
(Mecca	67;0	21;0	-	-	-

In each of several earlier Egyptian and Syrian sources I have consulted,[32] the qibla values have been carelessly computed, and in some cases miscopied to boot. However, in none of these sources do we find the qibla value 41;30°. It was probably derived for Jerusalem. Indeed it was perhaps based on the value 33;0° for the latitude of Jerusalem, as found, for example, in the tables of Ibn al-Dahhān (**9.1**). Notice that if we take

$$L = 56;30° \text{ and } \phi = 33;0 \quad \text{with} \quad L_M = 67;0° \text{ and } \phi_M = 21;30° ,$$

the accurate and approximate values of q are *41;37°* and 42;26°, and if we take:

$$L = 57;0° \text{ and } \phi = 33;0° \quad \text{with} \quad L_M = 67;0° \text{ and } \phi_M = 21;40° ,$$

then the accurate and approximate values of q are 40;31° and *41;28°*. Ibn al-Dahhān himself gives 41;21° for the qibla at Jerusalem.

(9-11) The tables for twilight are based on parameters 20° and 16° for morning and evening and are reasonably accurately computed. The entries are the same as the corresponding ones in al-Karakī's tables of T(λ,h) for h = 20° and 16°. All of the other twilight tables in the Princeton manuscript are based on parameters 19° and 17°.

(12) The table of α_ϕ is carelessly computed, but is based on a different set of values of d than was used to compile the table of D (see (1) above).

(13) The table of α_s is based on a more accurate set of values of α_ϕ than (12), using:

$$\alpha_s(\lambda) = \alpha_\phi(\lambda + 180°) + s(\lambda) .$$

(14) The table of σ(λ), displaying the time from sunset to the *salām*, is ostensibly based on:

$$\sigma(\lambda) = 2N(\lambda) - r(\lambda) ,$$

so that the *salām* was to be performed precisely at daybreak rather than a few minutes before as was the case in Egypt (**4.10**). However, the last four columns of entries are badly garbled.

(15) Finally, amongst the tables for latitude 31° in the Princeton manuscript there is one displaying the functions t(h) and T(h) for each degree of h at the equinoxes. The underlying latitude is 32° not 31° as stated, and the tables originally formed part of the more extensive set of tables compiled by al-Karakī. The entries are badly garbled,

From these investigations it is clear that there existed a set of carefully-computed prayer-tables for Jerusalem displaying at least the functions:

$$D,\ 2N,\ H,\ h_a,\ t_a,\ h_q,\ T_q,\ r,\ s,\ n \text{ and } \alpha_\phi ,$$

thus complementing al-Karakī's extensive set of tables for timekeeping.

[32] Namely, the geographical tables in MSS Paris BNF ar. 2513, fol. 89r, and Paris BNF ar. 2520, fol. 82v, of the *Muṣṭalaḥ Zīj*; MS Oxford Seld. A30, fols. 155r-157v, of the *Zīj* of Ibn al-Shāṭir; MSS Paris BNF ar. 5968, fols. 162v-163r, of the anonymous *Dastūr al-munajjimīn*; and Gotha A1403 of the derivative *zīj* of Ibn Zurayq. See further King, *Mecca-Centred World-Maps*, pp. 76-84, on the unhappy state of such tables.

CHAPTER 10

THE DAMASCUS CORPUS OF AL-KHALĪLĪ

10.0 Introductory remarks

An impressive corpus of astronomical tables was used in Damascus from the 14th to the 19th century. The tables in their entirety contain over 40,000 entries. Their compiler was a Damascene astronomer named Shams al-Dīn al-Khalīlī, a man recognized only in the 1970s for his remarkable competence in computation.[1] al-Khalīlī's corpus is more clearly-defined than the Cairo corpus, by which it was obviously inspired. We know who compiled the tables, and we are even provileged to have an earlier work of his which shows how his thinking had progressed. And the question why we had to wait to the late 20th century to have a clear picture of his achievements is simple to answer: his tables were rehashed and modified over the centuries and numerous of the manuscripts bear the names of later astronomers who simply appropriated them.[2]

Damascus was home to a vigorous tradition of astronomy in the 14th century, centred on the Umayyad Mosque in the heart of the city.[3] This tradition was but one of many colourful aspects of life under the Mamluks, who ruled Egypt, Syria, Palestine and the Hejaz from the mid-13th to the early 16th century. A team of astronomers called *muwaqqit*s (that is, professional timekeepers), which included al-Khalīlī, was employed at the Mosque. Their main duty was to determine the times of prayer so that the muezzins could perform the call to worship from the minarets at the appropriate time. In addition, they specialized in the regulation of the lunar calendar and the determination of the qibla. The interests of some of the *muwaqqit*s extended far beyond these somewhat mundane tasks (**V**).

The chief *muwaqqit* at the Umayyad Mosque during the 1360s and 1370s was ʿAlāʾ al-Dīn ʿAlī ibn Ibrāhīm called Ibn al-Shāṭir, known today for his innovative and successful research on geometric models for the sun, moon and planets (**9.3**). An earlier Damascus astronomer was al-Mizzī, who started the business of compiling prayer-tables for the city (**9.2**). As we shall see, both al-Mizzī and Ibn al-Shāṭir influenced the work of al-Khalīlī. Both had themselves

[1] Before the 1970s al-Khalīlī was known only from the bio-bibliographical references in Suter, *MAA*, no. 418, Brockelmann, *GAL*, II, pp. 156-157, SII, p. 157, and Sarton, *IHS*, III:2, pp. 1526-27. My first studies on his tables were "al-Khalīlī's Auxiliary Tables" (1973) and "al-Khalīlī's Qibla Table" (1975). A summary of his achievements and a critical list of the available manuscripts of his works appeared in *DSB*, XV (Supp. vol. I), pp. 259-261. See also *Cairo ENL Survey*, no. C37.

[2] "Appropriated" is a nicer word than "plagiarized": see n. 4:12 above. The Damascus of the astronomers, like Cairo, was a "small world" and it was not possible for astronomers to pass off as their own tables that were computed by their illustrious predecessor.

[3] This activity is surveyed in King, "Astronomy of the Mamluks", and *idem*, "Astronomie en Syrie". On timekeeping in particular see my "Astronomical Timekeeping in Medieval Syria".

been influenced by developments in Egypt in the 13th century (**6.0**). al-Khalīlī recomputed al-Mizzī's tables for the new parameters (local latitude and obliquity of the ecliptic) derived by Ibn al-Shāṭir in 765 H [= 1363/64], namely:

$$\phi = 33;30° \text{ and } \varepsilon = 23;31°.$$

His corpus of tables for solar timekeeping and the regulation of the times of prayer for Damascus continued to be used in one form or another until the 19th century. In addition, al-Khalīlī prepared a set of auxiliary tables for solving the standard problems of spherical astronomy for all latitudes, as well as a universal qibla table.

The work of the early-14th-century Aleppo astronomer Shihāb al-Dīn Aḥmad ibn Abī Bakr called Ibn al-Sarrāj, which represents the high point of Islamic instrumentation, was also known to al-Khalīlī.[4] Indeed, the tables of al-Khalīlī and the instruments of Ibn al-Sarrāj are singularly important developments in the particularly Islamic tradition of providing solutions to the problems of spherical astronomy for all latitudes, which I have documented elsewhere (**VIa-b**). However, al-Khalīlī makes no reference to any of his predecessors except Abū ʿAlī al-Marrākushī (**10.8**).

In later centuries al-Khalīlī's universal auxiliary tables (**10.7**) were also used in Egypt, Tunisia and Turkey. As far as we know, al-Khalīlī had no influence on the development of astronomy or trigonometry in Europe. There in the 15th and 16th centuries auxiliary trigonometric tables were developed by a series of astronomers of whom the most important were Regiomontanus and Magini (**I-10.1-2**). Yet despite the sophistication of these European tables they are not as useful for practical purposes as those developed by al-Khalīlī. His tables would have been of great interest to European astronomers of the from the 15th century right through to the early 20th century, had they been known to them.

10.1 Shams al-Dīn al-Khalīlī

Biographical information on Shams al-Dīn Abū ʿAbdallāh Muḥammad ibn Muḥammad ibn Muḥammad al-Khalīlī is scant indeed, although the epithet al-Khalīlī indicates that his family came from Hebron. His name is given in the above form in MS Berlin Wetzstein 1138 (Ahlwardt 5754-6), fol. 1r, and is attested in the same form in numerous other manuscripts. On fol. 1v of the same Berlin manuscript he is also called al-Ḥanafī, indicating that he adhered to the predominant Ḥanafī legal school. His minor auxiliary tables were completed in 763 H [= 1361] (MS Dublin CB 4091, fol. 1r) and his main corpus some years later. In the earlier work he is referred to as a *muwaqqit* at the Yılbughā Mosque, a new mosque in Damascus established by the governor Sayf al-Dīn Yılbughā in 747 H [= 1346].[5] But in copies of his main corpus he is referred to as a muezzin (MS Paris BNF ar. 2558, fols. 1r and 64r) or a *muwaqqit* (MS Berlin Wetzstein 1138 (Ahlwardt 5754-6), fol. 1r) at the Umayyad Mosque,[6] which as we have seen had become the main centre of astronomy in the 14th-century Islamic

[4] On Ibn al-Sarrāj see n. 9:12.

[5] On the Yılbughā Mosque see Henri Sauvaire, "Description de Damas: Sur les Grandes-Mosquées", *Journal Asiatique*, mars-avril 1896, pp. 185-285, *etc*., esp. pp. 236-237.

[6] On the Umayyad Mosque see *op. cit*., pp. 185-230.

world. The present author is not aware of any references to al-Khalīlī in the standard Mamluk biographical sources. al-Khalīlī had a nephew who was also an astronomer. Sharaf al-Dīn Abū ʿImrān Mūsā ibn Muḥammad ibn ʿUthmān al-Khalīlī studied under his maternal uncle and is known from several short treatises on timekeeping.[7]

10.2 The manuscript sources

al-Khalīlī's minor auxiliary tables and solar azimuth tables survive in the unique MS Dublin CB 4091, copied in 833 H [= 1429]. Likewise, his auziliary tables for finding the solar azimuth for all latitudes are known only from MS Bursa Haraççioğlu 1177,4, copied *ca*. 1425. The various tables in the main Damascus corpus survive in numerous copies, in most of which they are correctly attributed to al-Khalīlī. MS Paris BNF ar. 2558, the best copy by far, is not only the most complete but also the earliest, having been penned in the year 811 H [= 1411]. See **Fig. 10.2**. It even bears evidence of having been proofread by the copyist. The title folio gives a very good indication of the contents (see **10.4**). A table of contents of this and various other early copies of the Damascus corpus is given below.

Although only one complete manuscript survives, there is little doubt that the entire corpus was frequently copied. Various parts were also copied separately, as the surviving manuscripts attest. The distribution of the tables in the sources is as follows:

Fig. 10.2: The title-folio of the magnificent Paris copy of al-Khalīlī's corpus. [From MS Paris ar. 2558, courtesy of the Bibliothèque Nationale de France.]

[7] On Sharaf al-Dīn al-Khalīlī see Suter, no. 427, and *Cairo ENL Survey*, no. C38.

Location of the various tables of the main corpus in the available manuscripts

I al-Khalīlī's prayer tables
II al-Khalīlī's hour-angle tables
III al-Ḥalabī's azimuth tables
IV al-Khalīlī's universal auxiliary tables
V al-Khalīlī's qibla table

MS	I	II	III	IV	V
Berlin Ahlwardt 5754-6	*	-	-	*	*
Berlin Ahlwardt 5758	*	-	-	-	-
Berlin Ahlwardt 5759/5771	*	-	-	-	-
Berlin Ahlwardt 5760/5772	*	-	-	-	-
Berlin Ahlwardt 5739	-	-	-	*	-
Beirut PC	-	-	-	*	-
Cairo MM 71	-	*	*	-	-
Cairo MM 43	-	-	-	*	-
Cairo DM 184	*	-	-	-	-
Cairo MM 98	-	-	-	*	-
Cairo ṬM 255,7	*	-	-	-	-
Cairo DM 758	-	-	-	*	-
Cairo K 8525	*	*	*	-	-
Cairo ṬM 228,5	*	*	-	*	-
Cairo ṬM 173	*	*	-	-	-
Cairo TR 129	*	*	-	-	-
Cairo DM 1007	*	*	-	-	-
Damascus 3116	-	*	-	-	-
Damascus 10387 (?)	*	-	-	-	-
Damascus 9227	-	-	*	-	-
Damascus 9233	*	*	-	-	-
Damascus 4893	*	-	-	-	-
Escorial ár. 931,8	-	-	-	*	-
Gotha A1412	*	-	-	-	-
Istanbul Ayasofya 2590	-	-	-	*	-
Istanbul Esat Efendi 1990	*	-	-	-	-
Istanbul Hamidiye 1453,3	-	-	-	*	-
Istanbul Serez 1914	-	-	-	*	-
Leipzig 814	*	-	-	-	-
London BL Add. 9599,31	-	-	-	*	-
Mosul al-Muḥammadiyya 129	-	*	-	-	-
Oxford Seld. Supp. 100	*	*	-	-	-
Oxford Marsh 39 (Uri 1042)	-	*	-	-	-
Oxford Marsh 95 (Uri 961)	*	-	-	-	-
Paris BNF ar. 2558	*	*	-	*	*
Paris BNF ar. 2560,11	-	-	-	-	*
Princeton Yahuda 861,2	*	-	-	*	-

Descriptions of the most important manuscripts

As in the case of the manuscripts of the Cairo corpus (**4.2**), it seems worthwhile to present descriptions of the most significant copies of the Damascus corpus.[8]

MS Dublin CB 4091:[9]

Title folio:	(MS 4090): Statement that the copyist was Shihāb al-Dīn al-Biqāʿī al-Shāfiʿī (?) (not identified). Notices of possession by Muḥammad ibn Muḥammad ibn Aḥmad al-..... (illegible); Jalāl al-Dīn Muḥammad ibn Muḥammad al-Ramlī, *muwaqqit* at the Umayyad Mosque; Muḥammad al-*Qwʾf* (?, unidentified); and the Egyptian astronomer ʿAlī al-Khashshāb al-Falakī al-Dimyāṭī (*Cairo ENL Survey*, no. D120).
1r:	(MS 4091): Title (see **10.3a**); introduction to the auxiliary tables; notice of possession by al-Khashshāb (see above and also fol. 17r).
1v:	Table of "universal meridian altitudes", table of Sines, table of solar altitude at the ʿ*aṣr*.
2r-6v:	Tables of the half base.
7r-16v:	Tables for finding the hour-angle. Colophon.
17r:	Blank except for notice of possession by al-Khashshāb.
17v:	Introduction to list of formulae and solar azimuth tables.
17v:	Table of formulae for spherical astronomy.
18r-24r:	Tables of solar azimuth.
24r:	Colophon dated 24 Ṣafar 833 H [= 22 November 1429].

MS Bursa Haraççioğlu 1177,4, fols. 72r-90r:[10]

Notes: Only a part of this manuscript, with folios numbered 72-92 in fairly recent Syro-Egyptian-type Arabic numerals, is available to me for study. The hand is an elegant *naskhī*, dateable between about 1400 and 1450.

72r:	The title reads: *jadwal al-samt li-kulli 'rtifāʿ fī ʿurūḍ al-aqālīm al-sabʿa*, that is, "Table of the Azimuth for all Altitudes in the Latitudes of the Seven Climates".
72v:	Instructions, without indication of the author.
73r-85r:	Tables of the first and second auxiliary functions.
85v-90r:	Tables of ther third auxiliary function.
90v:	A note in a similar, likewise elegant hand: *naẓara fīhi al-ʿabd al-faqīr ila 'llāh taʿālā Muḥammad ibn Muḥammad al-Tīzīnī al-Mīqātī sanat 867 wa-lam yaktub minhu shay* (*sic* for *shayʾan*), that is: "(This manuscript was) inspected by ... Muḥammad ibn Muḥammad al-Tīzīnī al-Mīqātī in the year 867 (Hijra) [= 1462/63], and he did not write anything of it (??)."

---

[8] These were prepared in 1993 in the process of the compilation of my detailed study mentioned in n. 1:23 above. I prefer them to the descriptions presented in **4.2**.

[9] The manuscript (MS 4091) is part of a collection of treatises in the same hand (MSS 4090-92). The Arabic foliation runs from 159 to 181 skipping a folio between 165 and 166. The modern foliation is also incorrect. The notes here follow the correct foliation.

[10] I am most grateful to my colleague Dr. Sonja Brentjes for obtaining a microfilm of this manuscript.

MS Paris BNF ar. 2558:[11]

Notes: The folios are numbered on the verso, and there are two marked '2'. Throughout the manuscript there appear the words *balagha* (*sic*) *muqābalatuhu*, indicating that it was proofread. No changes or corrections have been inserted, however.

1r:	Title folio, illustrated in **Fig. 10.2**. The full title of the corpus is given (see the beginning of **10.4** for a translation). The manuscript was prepared for the treasury (*khizāna*) of the chief chamberlain (*amīr ḥājib*) of Damascus, Qarabughā al-ʿAlāʾī. Reader's notice of Buṭrus ibn Dīb, *turjumān* of the "Sultan" of France (is this how the manuscript came to Paris?). The copyist is named as Aḥmad ibn ʿAlī al-Khabbāz (on fols. 64r and 105v) and the date of copying as Jumādā I 11, 811 H [= October 2, 1411] on fol. 64r, and Jumādā II of the same year on fol. 105v. The handwriting is a most elegant *naskhī*.
1v:	al-Khalīlī's introduction to his calendrical and solar tables (duplicated in mirror image on fol. 2r).
2v-3r:	al-Tīzīnī's introduction to his calendrical and solar tables.
3v-8r:	al-Tīzīnī's calendrical tables.
8v-9r:	al-Tīzīnī's solar tables.
9v-14r:	Prayer-tables for Damascus.
14v-18r:	Tables of ascensions.
18v-20r:	Tables of stellar coordinates.
20v-50r:	Hour-angle tables for Damascus.
50v:	Table for finding solar motion in fractions of a day.
51r:	Tables of Cotangents, and solar rising amplitude and altitude in the prime vertical.
51v:	Tables of terrestrial longitudes, latitudes and qiblas.
52r:	Tables of latitudes of localities on the Syrian pilgrim route.
52v-53r:	al-Khalīlī's introduction to his qibla table.
53r:	Tables for marking the base on sine quadrants.
53v-61r:	al-Khalīlī's qibla table.
61v-64r:	al-Khalīlī's introduction to his universal auxiliary tables.
64r:	Copyist's colophon (see also fol. 105v).
64r:	Table of solar declination and solar altitude at the *ʿaṣr*.
64v-92r:	al-Khalīlī's universal auxiliary tables (first and second functions).
92v-104r:	al-Khalīlī's universal auxiliary tables (third function).
104v-105v:	Table of normed right ascensions.
105v:	Colophon (see also fol. 64r). Notices of possession by Ibrāhīm ibn Aḥmad ibn Yaḥyā ibn ʿAbd al-Ṭāhir (!?) dated 915 H [= 1509/10], and Muḥammad *R-ḥ-ʾ-l-h* (?) al-*Ḥ-m-ʾ-r-y* (?) *al-muʾadhdhin* (the muezzin), both unidentified.

MS Berlin Wetzstein 1138 (Ahlwardt 5754-6):[12]

Notes: The MS is in three different hands: A = 1v-55r + 64r-94r (elegant *naskhī*, *ca.* 1500); B = 56v-61v (later); and C = 62v-64r (very late).

[11] *Paris Catalogue*, p. 460; also *Paris IMA 1993-94 Exhibition Catalogue*, p. 440. no. 335.
[12] *Berlin Catalogue*, pp. 207-208.

No title folio.

1v-4r:	al-Khalīlī's introduction to his universal auxiliary tables.
4r-5v:	Sharaf al-Dīn al-Khalīlī's notes on the use of his uncle's tables.
6r:	Title for al-Khalīlī's universal auxiliary tables (*jadwal faḍl al-dāʾir al-āfāqī*).
6v-46v:	The universal auxiliary tables (including a sub-table for ϕ = 21;30°). For each latitude various localities are named. The tables run to latitude 55° with a page ruled for 56° without entries.
47r:	al-Khalīlī's introduction to his qibla table.
47v-55r:	The qibla table.
55v:	Blank.
56r:	Scribbles.
56v-58v:	al-Khalīlī's introduction to his universal auxiliary tables (incomplete), in hand B.
59r:	Cotangent table to base 10 (argument 1°-90°).
59v:	Sine table.
60r-61r:	Notes on Cotangents.
61v:	Table of solar declination and solar altitude at the *ʿaṣr* (attributed to al-Khalīlī).
62r:	Blank.
62v-94r:	Special tables for the *ʿaṣr* (hand C, hand A resumes on fol. 64r).
94v:	Blank, no colophon.

MS Escorial ár. 931,8, fols. 171r-211v [13]

171r:	Title folio (*Kitāb al-Dāʾir wa-faḍl al-dāʾir wa-'l-samt*) attributed to al-Khalīlī.
171v-209r:	al-Khalīlī's universal auxiliary tables (no sub-table for ϕ = 21;30°, latitude argument only up to 50°).
209v:	Table of solar declination (ε = 23;35° !).

No colophon. Dated 933 H [= 1526].

MS Cairo MM 43:

Notes: The manuscript contains some 50 unnumbered folios and the tables are unattributed. It is of Egyptian provenance and the copyist is recognizable as ʿAlī ibn Muḥammad al-Dalāmī and thus datable to *ca.* 1450. The tables listed here are followed by a variety of other tables for trigonometry, spherical astronomy and instrument construction (some universal, some specifically for the latitude of Cairo). There is also a table of geographical coordinates.

1r:	Miscellaneous scribblings. No title.
1v-5r:	(al-Khalīlī's) introduction to his universal auxiliary tables. Definitions of the functions f and g (fol. 5r).
5v:	Tables of solar declination (ε = 23;31°) and universal *ʿaṣr*.
6r-24r:	(al-Khalīlī's) tables of the first and second functions (ϕ = 1°-50° including 21;30° and 33;30°) and the third function.

[13] *Escorial Catalogue*, pp. 42-43.

MS Cairo ṬM 228,5:

Notes: All of the tables are anonymous. The manuscript is undated, but is copied in a legible *naskhī* script which is estimated *ca*. 1500. The folios have been bound in disorder, but the universal auxiliary tables and the hour-angle tables are complete.

1r-3r:	No title. (al-Khalīlī's) introduction to the universal auxiliary tables.
3r-3v:	Notes on the determination of the qibla and the use of the sine quadrant by an unidentified astronomer Zayn al-Dīn ʿUmar al-Zuhrī (*Cairo ENL Survey,* no. C79).
4r:	Tables of solar declination (ε = 23;31°) and universal ʿ*aṣr*.
4v-7v:	(al-Khalīlī's) tables of the first and second functions (ϕ = 1°-7° only).
8r:	(al-Khalīlī's) hour-angle tables (h = 79°-80° only).
8v:	Notes on the determination of the qibla.
9r-9v:	Prayer-tables for Tripoli (latitude 34°) (incomplete, only 6 functions for Capricorn and 6 for Aquarius): see fols. 16r-21r and **9.3**.
10r-13r:	(al-Tīzīnī's) tables for converting from the Hijra to the Syrian calendar (incomplete).
13v-14r:	(al-Tīzīnī's) table of solar longitude.
14v-15r:	Table of solar mean motion.
15v:	Blank.
16r-21r:	Continuation of prayer-tables for Tripoli (12 functions for the remainder of Aquarius up to Gemini): see contents of fols. 9r-9v above.
21v-25v:	Tables of ascensions (for Damascus).
26v:	Cotangent tables (bases 12 and 7).
27r-28r:	Table of solar altitude at the ʿ*aṣr* (for each 0;15° of H).
28r-60v:	(al-Khalīlī's) hour-angle tables (h = 1°-78° only).
61r-85r:	Remainder of (al-Khalīlī's) first two functions (ϕ = 8°-54° including ϕ = 21;30° and 33;30°).
85v-86r:	Geographical tables as follows: (a) list of localities, mainly in Egypt, with their coordinates and qiblas; (b) list of localities giving latitudes only; and (c) list of localities on the Egyptian pilgrim route with coordinates and qiblas.
86v-98r:	Tables of (al-Khalīlī's) third function (complete).
98v-101r:	Miscellaneous notes on calendars, scribblings, notice of possession by Muḥammad Ṣalāḥ al-Dīn (1)117 H [= 1705/06] (fol. 98v).

10.3 The minor corpus

The set of tables contained in the unique source MS Dublin CB 4091 was compiled before the main Damascus corpus. It consists of three parts, which are considered in turn (see already **I-9.4** and **I-5.3.1** on the tables, here considered in more detail). al-Khalīlī refers to these tables in the introduction to his universal auxiliary tables (**10.7**).

a) Auxiliary tables for solar timekeeping

In MS Dublin CB 4091, fols. 2r-16v, a pair of auxiliary tables by al-Khalīlī for finding t(h,λ,φ) survive. These are quite different from his universal auxiliary table. The principal functions tabulated are the following:

$$B'(\lambda,\phi) = \text{Cos}\ \delta(\lambda)\ \text{Cos}\ \phi\ /\ 2R = \tfrac{1}{2}\ B(\lambda,\phi) \quad (\varepsilon = 23;31°)\ ,$$

computed for the domains:

$$\phi = 1°,\ 2°,\ \ldots\ ,\ 49° \text{ and } 33;30°, \text{ and } \lambda = 1°,\ 2°,\ \ldots,\ 90°,$$

and:

$$V'(x,y) = \text{arc Vers}\ \{\ R\ y\ /\ 2\ x\ \}\ ,$$

computed for the domains:

$$x = 30,\ 29,\ \ldots,\ 19 \quad \text{and} \quad y = 0;10,\ 0;20,\ \ldots\ ,\ 60;0.$$

These two sets of tables contain respectively 4,500 and 4,680 entries, rather accurately computed to two digits. Horizontal differences are shown in the second set. The first function is called *al-aṣl*, "the base" (*cf.* **I-6.0**) but a marginal note at the beginning of the table points out that it is actually *niṣf al-aṣl*, "half the base". The second function is called *faḍl al-dāʾir āfāqī min al-aṣl wa-min faḍl jayb al-ghāya ʿan jayb al-irtifāʿ*, "the hour-angle for all latitudes as a function of the base and the excess of the Sine of the meridian altitude over the Sine of the instantaneous altitude". The instructions translate as follows:

> "The Book entitled "A Compilation on Timekeeping in the Different Regions" (*Mawḍiʿ al-awqāt fī 'l-aqālīm al-muqassamāt*) by the scholar Shams al-Dīn Muḥammad al-Khalīlī *al-muwaqqit*.
>
> In the Name of God the Merciful and Compassionate. I have no success but through God, and upon Him I have placed my trust. The celebrated scholar Shams al-Dīn Muḥammad al-Khalīlī, the *muwaqqit* at the Mosque of al-Sayfī (= Sayf al-Dīn) Yılbughā – may God have mercy upon him – said:
>
> Praise be to God who takes away (our) fears and brings about miracles. May (God) bless and save Muḥammad and his family, the virtuous and honorable, with a blessing by which I beg for what is good and which will erase for me what is bad.
>
> I consider the hour-angle to be one of the most important concepts (*abwāb*) in astronomy because it is the basis for finding the times of the obligatory prayers. So I wanted to prepare a table for finding it for (all) inhabited regions. I ask God – who knows the secrets and what is hidden – that He should protect me from all kinds of mistakes, and that He should make it useful to all those who know about timekeeping (lit., the number of hours and degrees). Verily He responds to prayers and gives what is asked (of Him). I called (this work) "Compilation on Timekeeping in the Different Regions" (*Mawḍiʿ ... ... al-maqsūmāt*). God is sufficient for me and He is the best guardian; (He it is who is) the Creator of the Earth and of the Heavens.
>
> Chapter on the determination of the hour-angle, half the arcs of daylight and night, the time of the *ʿaṣr* and the time remaining until sunset, and the duration of morning and evening twilight, in localities between the equator and latitude 49°.
>
> You find the base from the appropriate table, entering with the longitude of the sun and the local latitude. Then find the meridian altitude from the appropriate table and find its Sine from the Sine table using proportional parts of the first differences (*bi-*

nisbat al-tafāḍul). Next find the difference between the Sines of the meridian altitude and the instantaneous altitude by subtracting the Sine of the altitude from the Sine of the meridian altitude. Enter with the base (as horizontal argument) in the universal hour-angle table until you come opposite the difference between the two Sines (as vertical argument), using linear interpolation if (the difference) is not equal to the argument. The result will be the hour-angle if the base is integral. If it is not integral, round the fraction to unity and determine the hour-angle by interpolating linearly for the difference between the two Sines. Then find the first difference from the appropriate box and recall the fraction which you rounded: if it was one-half or one-third or more or less, add one-half or one-third or more or less of the first difference to the hour-angle.

Example. The base is 24;15. We round (the fraction) to unity and obtain 25. The difference between the two Sines is 29;30. With this we find the hour-angle, which is 65;48. We find that the first difference is 1;32, and the amount we added to the fraction to make it whole is 0;45. We add one-half and one-quarter of the difference, namely 1;9, to the hour-angle and the result is 66;57°, which is the required hour-angle.

If we enter with the base (as horizontal argument) until we come opposite [the Sine of] the meridian altitude or the Sine of the meridian altitude of the opposite degree of the ecliptic, the result will be the half arc of daylight or of night (, respectively). (To determine) the time from midday to the *ʿaṣr*, find the altitude at the *ʿaṣr* from the special table and then find the corresponding hour-angle. The time remaining until sunset is the excess of half the arc of visibility over the time from midday to the *ʿaṣr*. (To determine) the durations of morning and evening twilight (*al-ḥiṣṣatān*), enter with the base (as horizontal argument) until we come opposite the excess of the Sine of the meridian altitude of the opposite point of the ecliptic over the Sine of 17° for evening twilight and 19° for morning twilight. The resulting hour-angles are the times between evening and morning twilight and midnight: subtract either of them from the half nocturnal arc and the remainder will be the duration of twilight. God knows best. The calculation was completed in the Holy Month of Muḥarram in the year 763 H [= November, 1361]."

The main tables are preceded by two small tables, the first of which is for the function:

$$\delta^*(\lambda) = 90° + \delta(\lambda) \quad (\varepsilon = 23;31°)$$

labelled *al-ghāyāt āfāqiyya baʿd isqāṭ al-ʿarḍ minhā*, "meridian altitudes for all latitudes when the latitude is subtracted". The second is for the Sine function, with entries to two digits for each degree of argument. To find $t(h,\lambda,\phi)$ al-Khalīlī proposes first using the table of $\delta^*(\lambda)$ to find $H(\lambda,\phi)$, thus:

$$H(\lambda,\phi) = \delta^*(\lambda) - \phi$$

and then using the Sine table to find Sin H and Sin h to determine their difference:

$$H'(h,\lambda,\phi) = \text{Sin } H(\lambda,\phi) - \text{Sin } h.$$

Using the two main sets of auxiliary tables one should then be able to find $B'(\lambda,\phi)$ and finally:

$$t(h,\lambda,\phi) = V' \{ B'(\lambda,\phi), H'(h,\lambda,\phi) \} .$$

al-Khalīlī does not mention that his tables can also be used to determine the solar azimuth, but there is an obscure reference in the introduction to his major tables which indicates that he was aware of this possibility. The procedure would be to form the quantity:

$$H''(h,\lambda,\phi) = \mathrm{Sin}\ (\bar{\phi} + h) - \mathrm{Sin}\ \delta(\lambda),$$

with which the azimuth (measured from the north point) is given by:

$$a(h,\lambda,\phi) = V' \{ B'(\lambda,\phi),\ H''(h,\lambda,\phi) \} .$$

Although only one copy of these original auxiliary tables has come to light, as opposed to the several extant copies of the universal table, it is clear that their circulation was not limited. They were known even in Tunis and Cairo (see **10.11**).

b) Formulae for timekeeping

MS Dublin CB 4091, fol. 17v, contains a list of 24 formulae for spherical astronomy. They are expressed in terms of a set of four quantities written in words. As explained in the introduction (see the second paragraph of the translation in (c) below), for the tetrad (a,b,c,d) the relation a/b = c/d is valid. See **Table 10.3b**.

Tabulation of formulae in this fashion was not initiated by al-Khalīlī, for a list of 62 formulae appears in the treatise of Abū ʿAlī al-Marrākushī (**6.7**) and a list of 184 (many, of course, redundant because they are trivially equivalent to others) appears in the *Zīj* of Ibn al-Shāṭir.[14] See **Fig. I-1.2a** for a similar list. This method of recording formulae was no doubt considered useful for memorization, for although algebraic notation would facilitate the tabulation, the symbols would still have to be defined in words.

Let us consider first a simple example, the fourth entry in the table. It indicates that:

horizontal shadow ÷ length of gnomon = length of gnomon ÷ vertical shadow

that is:

$$\mathrm{Cot}_n\ h \div n = n \div \mathrm{Tan}_n\ h ,$$

or, in modern terms:

$$\cot h = 1 / \tan h .$$

More complicated examples are nos. 5, 8, 7, and 6. The first defines a quantity called the "base" (*al-aṣl*), here denoted by B (**I-6.0**):

Cosine of latitude reduced ÷ base = unity ÷ Cosine of latitude reduced ,

where the term "reduced" (*munḥaṭṭ*^{an}) means "divided by 60"), so that:

$$B = \mathrm{Cos}\ \delta\ \mathrm{Cos}\ \phi / R^2 .$$

The second presents an equivalent definition of this quantity, namely:

base ÷ Sine of meridian altitude = unity ÷ Versed Sine of semi diurnal arc ,

that is:

$$B = \mathrm{Sin}\ H / \mathrm{Vers}\ D .$$

With this quantity the third relation defines another quantity called the "auxiliary Sine" (*jayb al-tartīb*), here denoted by j:

base ÷ Sine of altitude = unity ÷ auxiliary Sine ,

whence:

$$j = \mathrm{Sin}\ h / B = \mathrm{Sinh} \cdot \mathrm{Vers}\ D / \mathrm{Sin}\ H .$$

---

[14] Sédillot-*père*, *Traité*, I, pp. 351-359, and Kennedy, "*Zīj* Survey", p. 163a, respectively. See now Charette, *Mamluk Instrumentation*, pp. 357-358, on Najm al-Dīn al-Misrī's list of 30 such formulae.

Table 10.3b
List of formulae for spherical astronomy
(MS Dublin CB 4091, fol. 17v)

1.	Sine of rising amplitude	Sine of declination	60	Cosine of latitude
2.	[Cosine][a] of altitude	[Sine][b] of altitude	Length of gnomon	Horizontal shadow
3.	Cosine of altitude	[Sine][c] of altitude	Length of gnomon	Vertical shadow
4.	Horizontal shadow	Length of gnomon	Length of gnomon	Vertical shadow
5.	Cosine of latitude reduced	Base	Unity	Cosine of declination reduced
6.	Base	Difference between Sines of meridian and instantaneous altitudes	Unity	Versed Sine of hour-angle
7.	Base	Sine of altitude	Unity	Auxiliary Sine (*jayb al-tartīb*)
8.	Base	Sine of meridian altitude	Unity	Versed Sine of half diurnal arc
9.	Hypotenuse of horizontal shadow	Length of gnomon	60	Sine of altitude
10.	Hypotenuse of vertical shadow	Length of gnomon	60	Cosine of altitude
11.	Sine of ecliptic distance from equinox	Sine of declination	60	Sine of obliquity
12.	Sine of latitude	Tangent of complement of rising amplitude	60	Tangent of half diurnal arc
13.	Sine of altitude	Component (*ḥiṣṣa*) of the azimuth	Cosine of latitude	Sine of latitude
14.	Cosine of azimuth	Component (*ḥiṣṣa*) of hour-angle	Cosine of declination	Cosine of altitude
15.	Sine of altitude in prime vertical	Sine of declination	Total Sine	Sine of latitude
16.	Sine of altitude in prime vertical	Sine of rising amplitude	Cosine of latitude	Sine of latitude
17.	Tangent of solar declination (base 60)	Sine of half excess	Unity	Tangent of latitude (base 60)
18.	Sine of half excess	60	Tangent of declination	Tangent of complement of latitude
19.	Tangent of half excess	Tangent of latitude	Sine of declination	Cosine of rising amplitude
20.	Sine of declination	Cosine of declination	"The excess"	5
21.	Tangent of declination	12	"The excess"	5
22.	Cotangent of latitude	Sine of half excess	Unity	"The excess"
23.	Sine of ecliptic distance from equinox	Tangent of second declination	60	Tangent of obliquity
24.	Cosine of declination	Cosine of rising amplitude	60	Sine of half diurnal arc

Notes: [a] text Sine (!); [b] text Cosine (!); [c] text Cosine (!)

The fourth relation allows a determination of the hour-angle from the difference between the Sines of the meridian and instantaneous altitudes:

$$\text{base} \cdot \text{Sine of altitude} = \text{unity} \cdot \text{auxiliary Sine} \; ,$$

whence:

$$\text{Vers } t \;=\; [\text{Sin } H - \sin h] \;=\; R^2\,[\text{Sin } H - \text{Sin } h] \,/\, [\text{Cos } \delta \text{ Cos } \phi]$$

$$=\; [\text{Sin } H - \text{Sin } h] \cdot \text{Vers } D \,/\, \text{Sin } H = \text{Vers } D - \text{Sin } h \cdot \text{Vers } D \,/\, \text{Sin } H \; ,$$

or, in the simplest possible terms:

$$\text{Vers } t \;=\; \text{Vers } D - j \; .$$

Rules such as these, and the accompanying technical terminology, were standard knowledge amongst contemporaneous Muslim astronomers.

c) Solar azimuth tables for Damascus

In MS Dublin CB 4091, fols. 18r-24r, there is a table of about 2,700 entries for the solar azimuth at Damascus. The function tabulated is a(H,h) computed to two digits for the domains:

$$H = 33°, 34°, \ldots , 80° \text{ and } h = 1°, 2°, \ldots , H \; .$$

The underlying latitude is 33;30° as stated at the end of the tables, and the values are independent of the obliquity. Horizontal differences are also tabulated and labelled positive or negative by the Arabic letters *d* (for *zā'id*, increasing) or *ṣ* (for *nāqiṣ*, decreasing). The use of the tables is explained in the short introduction (Dublin CB 4091, fol. 17v):

> "In the Name of God the Merciful and Compassionate. O my Lord, make (things) easy and help, O Generous One! The celebrated scholar (*al-shaykh al-imām al-ʿallāma*) Shams al-Dīn Muḥammad al-Khalīlī – may God have mercy upon him – said: Praise be to God, Lord of the Worlds. Blessings and Salvation for Muhammad, the most excellent of God's creation, and his Family and all of his Companions.
>
> These are four tables which contain all (important) astronomical operations. The way to use them is to look for the quantity you want. If it is one of the outer values multiply one of the inner entries by the other and divide the product by the outer entry which is known: the result will be the unknown outer quantity. If the unknown quantity is one of the inner two, multiply one of the outer quantities by the other and divide it by the inner quantity which is known: the result will be the unknown inner quantity. Section on the use of the table of the azimuth. You enter with the instantaneous altitude and the maximum altitude for the solar longitude (text has incorrectly: *ghāyat jadwal juz' al-shams*) and what you find is the azimuth. The direction of the azimuth from meridian altitude 33° to the limit of 57° is always southerly. The direction for the other meridian altitudes is also southerly if the instantaneous altitude is greater than the altitude in the prime vertical, and northerly if it is less.
>
> Section. If the meridian altitude is non-integral take a proportion corresponding to its fractional part of the first difference (*fa-khudh min al-tafāḍul bi-nisbatihi*) and add it to the azimuth of the differences which are marked positive (with the letter *d* for *zā'id*, increasing) and subtract it from the azimuth if they are marked negative (with the letter *ṣ* for *nāqiṣ*, decreasing): the result will be the azimuth. Example. The altitude is 24° and the meridian altitude 40;20°. You find (the entry) 41;53° opposite

(argument) 24° and you subtract one third of the difference from it, namely 0;34°: the remainder is 41;19°, which is the azimuth. If (the difference) had been marked positive we would have added them. Know this!"

The simplest formula for compiling such a table of a(H,h) is:

$$a(H,h) = \text{arc Sin} \{ [\text{Sin } h - \text{Sin } h_0(H)] \cdot \text{Tan } \phi / \text{Cos } h \} ,$$

where:

$$\text{Sin } h_0(H) = R \text{ Sin } (H - \bar{\phi}) / \text{Sin } \phi ,$$

$h_0(H)$ being the altitude of the sun in the prime vertical on a day when it has meridian altitude H.

Although several other solar azimuth tables are known from the Islamic sources (**I-5**), mostly with arguments h and λ; al-Khalīlī's is the only one with arguments H and h. Tables of the time since sunrise were often computed for arguments H and h, the advantage being that only about 3,000 entries have to be computed, as compared with about 10,000 entries in tables with arguments h and λ. al-Khalīlī's azimuth table was soon replaced by Shihāb al-Dīn al-Ḥalabī's solar azimuth table (**11.2**) which had the same format as al-Khalīlī's hour-angle tables and which was occasionally included in copies of the main Damascus corpus.

10.3* The universal auxiliary tables for finding the solar azimuth

The following tables are mentioned by al-Khalīlī in the introduction to his main universal auxiliary tables (**10.4**) but the unique surviving copy come to my attention only in January, 2001.[15] I have mentioned them briefly in **I-8.5.1a** and **I-9.4a**, and show here how they fit into al-Khalīlī's development.

MS Bursa Haraççioğlu 1177,4 (fols. 72r-90r), copied *ca*. 1450, is a unique copy of a set of tables by al-Khalīlī, not identified as the compiler, and entitled simply *Jadwal al-samt li-kulli 'rtifāʿ fī ʿurūḍ al-aqālīm al-sabʿa*. The set contains sub-tables for the following functions:

$$f'_\phi(\lambda) = R \text{ Sin } \delta(\lambda) / \text{Cos } \phi \quad \text{and} \quad g_\phi(h) = \text{Sin } h \text{ Tan } \phi / R$$

for the domains:

$$\lambda \text{ and } h = 1°, 2°, \dots , 90° ,$$
$$\phi = 1°, 2°, \dots , 48°, \text{ as well as } 33;30° \text{ (Damascus)} ,$$

and:

$$K(x,h) = \text{arc Sin} \{ R \cdot x / \text{Cos } h \}$$

for the domains:

$$x = 1, 2, \dots , 59 \text{ and } h = 0°, 1°, \dots , n(x) ,$$

where n(x) is the largest integer such that $x \leq \text{Cos } n(x)$. The functions f'_ϕ and g_ϕ are labelled *jayb saʿat al-mashriq* and *ḥiṣṣat al-samt* by al-Khalīlī, and it is clear that they represent Sin ψ(λ) and k(h) – see **5.0** and **8.5**. The value of ε underlying the first table is about 23;30°, possibly 23;31°.

The instructions (see **Fig. 10.3***) describe how to determine the *taʿdīl al-samt* using the first two tables, and then how to find the azimuth from the third table. These universal auxiliary tables represent the ultimate solution to the problem of determining the solar azimuth a(h,λ,ϕ).

[15] The manuscript is mentioned in İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 805.

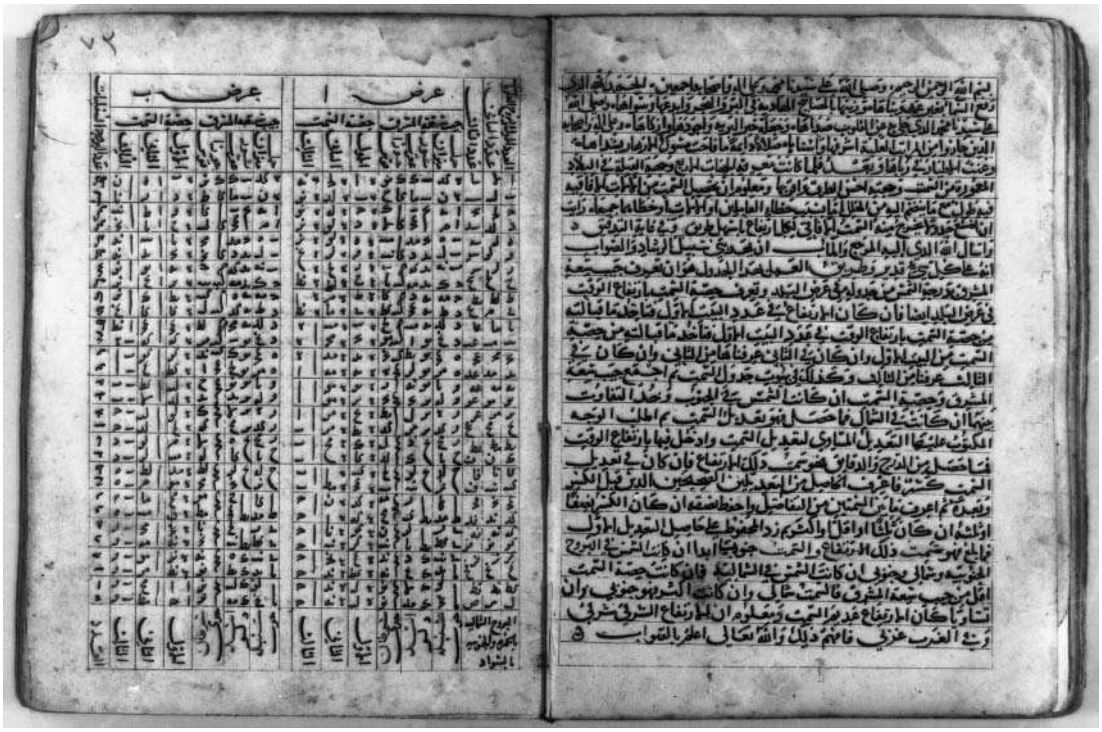


Fig. 10.3*: The introduction to al-Khalīlī's tables and the first two sub-tables of f_ϕ and g_ϕ for latitudes 1° and 2°. See also **Figs. I-9.4*a-b**. [From MS Bursa Haraççioğlu 1177,4, fols. 72v-73r, courtesy of the Genel Kütüphanesi.]

It is clear that all al-Khalīlī had to do to compile his splendid tables described in **10.7** was to replace the table of $f'_\phi(\lambda)$ with a new one of $f_\phi(\theta)$ and rewrite the instructions!

There is a note on fol. 90v of the Bursa manuscript in the hand of (the Damascus astronomer) Muḥammad ibn Muḥammad al-Tīzīnī stating that he had looked at this table in the year 867 H [= 1462/63].[16] al-Tīzīnī himself compiled a set of tables for sundial construction for different latitudes which is extant in MS Vatican ar. 105,3.[17]

10.4 The main Damascus corpus: the introduction and solar tables

The full title of the Damascus corpus of tables for timekeeping is given on MS Paris BNF ar. 2558, fol. 1r. It translates: "Tables of the hour-angle and the operations of night and day for latitude 33;30° computed by *al-Shaykh* Shams al-Dīn Muḥammad ibn Muḥammad ibn Muḥammad al-Khalīlī, muezzin at the Umayyad Mosque, followed by a table for finding the azimuth of the qibla and a universal table which can be used in all localities."

Fol. 1v of MS Paris BNF ar. 2558 contains virtually all that remains of an introduction to

[16] On al-Tīzīnī see Suter, *MAA*, no. 450; Brockelmann, *GAS*, II, p. 160, and SII, p. 484; *Cairo ENL Survey*, no. C95; and also **10.4**.

[17] These are not listed in any reference work from Suter, *MAA*, to Rosenfeld & İhsanoğlu, *MAIS* (no. 903).

some tables for calendar conversion and for finding the solar longitude.[18] The bulk of the introduction and the tables themselves have apparently been removed and replaced by a later set (fols. 2v-9r) in a different hand. At this time fol. 1v was apparently glued to an empty fol. 2r. More recently this introductory fragment was separated from fol. 2r with the result that a mirror image of part of the text may be read. Although the introduction and the missing tables are anonymous they are most probably the work of al-Khalīlī himself; they were already outdated in 1408, when the manuscript was copied. It is clear from this fragment that al-Khalīlī had prepared a table for converting Hijra lunar dates into dates in the Syrian solar calendar and another table displaying the solar longitude for each day of an artificial Syrian year which needed to be corrected from an auxiliary table. This correction provided the necessary adjustment for leap years in the Syrian calendar and was tabulated as a function of the excess Hijra years over al-Khalīlī's epoch (not stated) reckoned modulo 60.

The new introduction and tables in MS Paris BNF ar. 2558, fols. 2v-9r, are the work of al-Tīzīnī (**10.3***). They are similar in conception to those of al-Khalīlī, and tabulate the Syrian dates corresponding to the beginnings of each lunar month from 851 to 1000 Hijra. al-Tīzīnī then presents a table of solar longitude of the *shabaka* variety for an artificial Syrian year to which a correction must be applied. The correction is to be taken from a sub-table copied around the main table; values are given for each year after 850 Hijra from 1 to 150 (corresponding, of course, to years 851 to 1000 Hijra). al-Tīzīnī then goes on to discuss various timekeeping operations which need not concern us here.

Solar tables arranged according to the date in a solar calendar are attested in other Islamic sources both earlier and later, but none of these has been investigated for structure or accuracy.[19] It is likely that al-Khalīlī's solar longitude table would have been computed using the solar tables in the *Zīj* of Ibn al-Shāṭir, and that the entries would have been accurate to within a few minutes.

MS Paris BNF ar. 2558, fol. 50v, contains a simple table for correcting the solar longitude at midday to yield the solar longitude for any other time of day. Corrections $\Delta\lambda$ are given as a double-argument function of the daily solar motion λ (Arabic, *buht*) and the time t before or after midday. Values to one digit (minutes) for each 3° of t from 3° to 180°, and each unit of the daily solar motion from 57 min/day to 62 min/day are given. The underlying formula[20] is simply:

$$\Delta\lambda = \lambda \times t / 360.$$

MS Paris BNF ar. 2558, fol. 64r, contains a table of solar declination $\delta(\lambda)$ with values to degrees and minutes for each degree of solar longitude. The underlying value of the obliquity is 23;31°.

Finally, in MS Paris BNF ar. 2558, fols. 18v-20r, there is a set of equatorial coordinates (α',Δ) for 81 stars. This table is not the work of al-Khalīlī: the entries are, as stated in the heading, those of Ibn al-Shāṭir's star catalogue, with 0;15° added to the normed right ascensions.[21] Of course the correction for precession should be added to the stellar longitudes, so we have here a very crude and rather unsuccessful attempt to update the tables. A precessional

[18] On calendars in Islamic astronomy see n. **I**-1:3.
[19] See nn. 1:43.
[20] See Kennedy, "*Zīj* Survey", p. 162b.
[21] On Ibn al-Shāṭir's star catalogue see Kennedy, "*Zīj* Survey", p. 164a.

correction of 0;15° corresponds to about 15-20 years using the standard medieval values (1° in 66 or 70 solar years) for this motion. Ibn al-Shāṭir's catalogue is for 765 H [=1363/64].

For our present purposes it is important to realize that al-Khalīlī assumed that the user of his tables for timekeeping knew either the longitude or declination of the sun or the stellar declination.

10.5 The hour-angle tables

al-Khalīlī's corpus contains a complete set of tables of the hour-angle $t(h,\lambda)$ for Damascus based upon the parameters:

$$\phi = 33;30° \text{ and } \varepsilon = 23;31°.$$

There is no mention of these tables, or the prayer-tables (**10.6**), in the introduction. Values of the hour-angle are given for arguments:

$$h = 1°, 2°, \ldots, 80° \quad \text{and} \quad \lambda = 271°, 272, \ldots, 359, 0, 1, 2, \ldots, 90°.$$

Note that because of the format of the tables no entry is given for the winter solstice, that is, $\lambda = 270°$. There are about 3,000 entries. Extracts are shown in **Figs. I-2.1.4a-b**.

The standard of al-Khalīlī's computational accuracy is remarkably high. He may have used his first set of auxiliary tables (**10.3**) to compile these hour-angle tables. The procedure would be equivalent to:

$$t(h,\lambda) = \text{arc Vers } \{ [\text{Sin } h(\lambda) - \text{Sin } h] / B'(\lambda) \} .$$

Alternatively he may have tabulated the functions $B(\lambda)$ and $C(\lambda)$ for this latitude and then used the simple relationship:

$$t(h,\lambda) = \text{arc Cos } \{ R \cdot [\text{Sin } h - C(\lambda)] / B(\lambda) \} .$$

The hour-angle tables are contained in MS Paris BNF ar. 2558, fols. 20v-50r, and at least seven later copies (see **10.2**). In MS Cairo MM 71,1, copied *ca*. 1600, triplets of entries (t,T,a) for Damascus are given for each pair of arguments (h,λ). The values of both t and T are attributed to al-Khalīlī, and the values of a are attributed to Shihāb al-Dīn al-Ḥalabī (**11.2**). Some tables for the time remaining until moonset based on al-Khalīlī's tables of $T(h,\lambda)$ for the sun are described below (**10.11**).

MSS Paris BNF ar. 2558, fol. 33r, and Oxford Seld. Supp. 100, fol. 16v, of the corpus contain two tables by al-Khalīlī without instructions for their use. The two tabulated functions are labelled *qaws al-aṣl bi-'l-murī* and *qaws al-aṣl bi-ghayr murī*, "arc of the base with / without a movable marker". These functions are denoted by b_1 and b_2, and inspection reveals that for al-Khalīlī's parameters:

$$b_1(\lambda) = \text{arc Sin } \{ B(\lambda) \} \quad \text{and} \quad b_2(\lambda) = \text{arc Tan } \{ B(\lambda) \}.$$

where:

$$B(\lambda) = \text{Cos } \delta(\lambda) \text{ Cos } \phi / R .$$

These tables, discussed in more detail in **I-6.5.1**, were designed to facilitate the marking of $B(\lambda)$ on an instrument called the sine quadrant (Arabic: *al-rubʿ al-mujayyab*), a computational device which in different forms was popular among Muslim astronomers from the 9th to the 19th century. Complicated trigonometric computations can be made with the sine quadrant without any calculations whatsoever.

10.6 Tables for regulating the times of prayer

The main set of prayer-tables compiled by al-Khalīlī, for which there is likewise no introduction (**10.5**), consists of tables displaying some twelve functions for each degree of solar longitude, beginning with Capricorn 1°. The tables are entitled *jadwal aʿmāl al-layl wa-'l-nahār*, that is, "tables for the operations of night and day", and are based on the same parameters: an extract is shown in **Fig. 10.6**. The functions tabulated are:

H	the solar meridian altitude (*ghāyat al-irtifāʿ*);
D	half the diurnal arc (*niṣf qaws al-nahār*);
$2D^h$	the number of hours of daylight (*sāʿāt al-nahār*);
h_a	the solar altitude at the beginning of the *ʿaṣr* (*irtifāʿ al-ʿaṣr*);
t_a	the hour-angle at the beginning of the *ʿaṣr* (*dā'ir al-ʿaṣr*);
T_a	the time between the beginning of the *ʿaṣr* and sunset (*mā bayn al-ʿaṣr wa-'l-ghurūb*);
t_b	the time between midday and the end of the *ʿaṣr* (*mā bayn al-ẓuhr wa-ākhir waqt al-ʿaṣr*);
2N	the duration of night (*qaws al-layl bi-kamālihi*);
s	the duration of evening twilight (*ḥiṣṣat al-shafaq*);

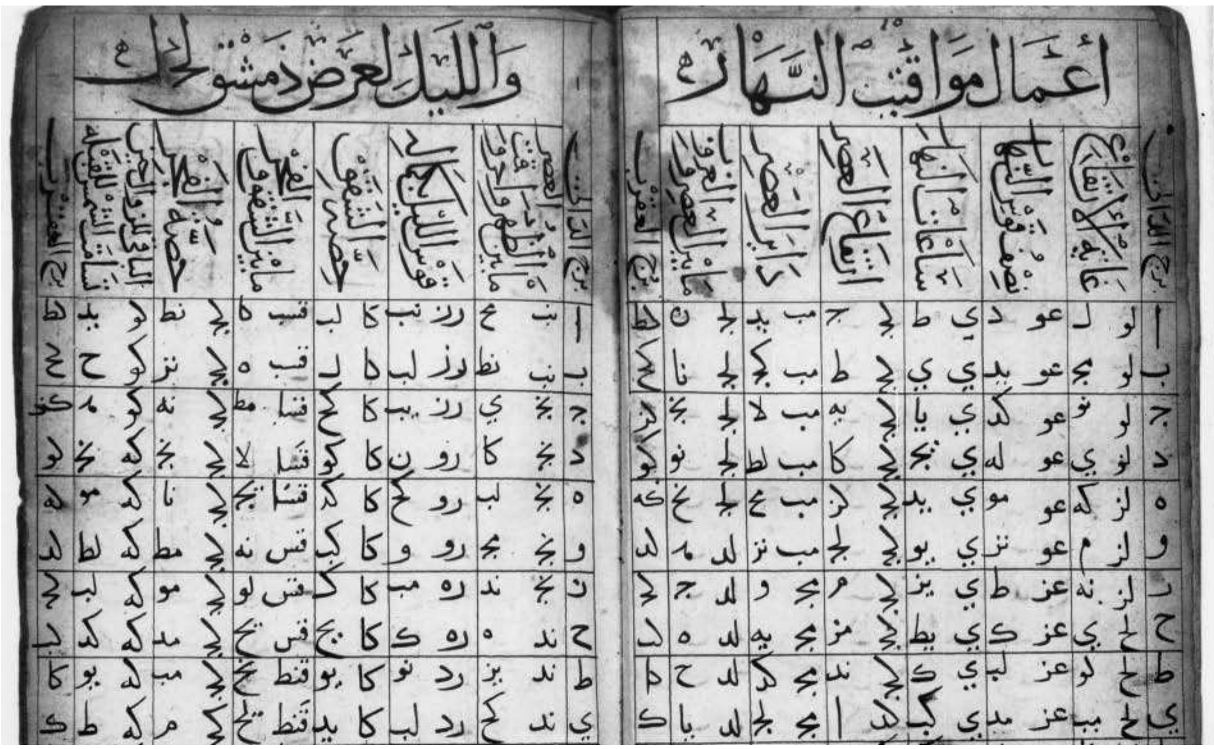


Fig. 10.6: The sub-table for solar longitudes in Aquarius in al-Khalīlī's corpus of prayer-tables. [From MS Paris BNF ar. 2558, courtesy of the Bibliothèque Nationale de France.]

n the duration of darkness (*mā bayn al-shafaq wa-'l-fajr*);
r the duration of morning twilight (*ḥiṣṣat al-fajr*);
t_q the time remaining until midday when the sun is in the azimuth of the qibla (*al-bāqī li-'l-zawāl ḥīna tusāmit al-shams al-qibla* (text: *li-'l-qibla*)

Entries are to two sexagesimal digits and are arranged side by side in columns for each zodiacal sign. Since the vertical arguments run downwards from 1° to 30° for each sign from Capricorn to Gemini and upwards from 0° to 29° in the same tables from Sagittarius to Cancer, there is no entry for the winter solstice. (The different format of the Cairo corpus means that there is no entry for the equinoxes.) The total number of entries in al-Khalīlī's prayer-tables is 2,160.

These tables appear in MS Paris BNF ar. 2558, fols. 9v-14r, and at least 15 later copies (see **10.2**). In MS Princeton Yahuda 861,1 they have the format of the Cairo corpus in a collection of prayer-tables for all latitudes. In MSS Berlin Ahlwardt 5758, 5759/5771 and 5760/5772, they are preceded by the solar longitude tables of ʿAbd al-Laṭīf called Ibn al-Kayyāl (**11.4**). In MS Damascus Ẓāhiriyya 9233 they form part of a set of tables attributed to Muḥammad ibn Muṣṭafā al-Ṭanṭāwī (**11.13**).

al-Khalīlī's tables defining the interval for the afternoon prayer are based on the following formulae for the solar altitudes at the beginning and end of the interval:

$$h_a = \text{arc Cot} \{ \text{Cot } H + R \} \quad \text{and} \quad h_b = \text{arc Cot} \{ \text{Cot } H + 2R \} .$$

The values of $t_a(\lambda)$ are more accurate than those that can be derived by linear interpolation using the argument $h = h_a$ in the tables of $t(h,\lambda)$. Nevertheless, the values of t_a, and also T_a and T_b, are often incorrect by several digits in the second sexagesimal place. Possibly al-Khalīlī used his table of $h_a(H)$ to generate the values of $h_a(\lambda)$ and the resulting values suffered from the interpolation scheme used. (See **11.3** below for some additional, rather extensive tables based on al-Khalīlī's tables for the *ʿaṣr*.)

al-Khalīlī defined the *ʿaṣr* as beginning when the shadow of a vertical object had increased by the length of the gnomon and ending when the shadow increase was double this amount. This is the definition usually associated with the Ḥanafī legal school, to which al-Khalīlī adhered, and presumably it corresponded to the currently-accepted practice at the Umayyad Mosque in his time.

The tables defining the length of twilight and darkness are based on the parameters 19° and 17° for morning and evening twilight. The values of $r(\lambda)$ and $s(\lambda)$ can be derived from the tables of $T(h,\lambda)$ for 19° and 17°, using:

$$r(\lambda) = T(19°, \lambda^*) \quad \text{and} \quad s(\lambda) = T(17°, \lambda^*), \quad \text{where } \lambda^* = \lambda + 180°.$$

The table defining the hour-angle when the sun is in the azimuth of the qibla is, of course, based on a value for the azimuth of Mecca at Damascus. This value is not stated in any of the manuscripts in the corpus, but al-Khalīlī's values of $t_q(\lambda)$ correspond most closely to the parameter q = 28;57° (**10.9**).

In some of the manuscripts there are separate tables of certain additional functions:

(a) Tables of $\delta(\lambda)$ (*al-mayl*) (*e.g.*, MS Paris BNF ar. 2558, fol. 64r), $\psi(\lambda)$ (*saʿat al-mashriq*) and $h_0(\lambda)$ (*al-irtifāʿ al-ʿadīm al-samt*) (*e.g.*, fol. 51v of the Paris copy) to two digits for each degree of λ.

(b) Tables of $\text{Cot}_R\, h$ (*al-ẓill al-mabsūṭ*) to two sexagesimal digits for each degree of h, for the bases R = 20 (only in the Paris copy, fol. 51v) and R = 10 (only in MS Berlin

Wetzstein 1138 (Ahlwardt 5754-6), fol. 59r). The use of these bases is remarkable. No explanation is provided in the manuscript and none is forthcoming from this author.[22]

(c) Tables of the function $h_a(H)$, which is universal as it is independent of terrestrial latitude (*al-ʿaṣr al-āfāqī*) (*e.g.*, fol. 64r of the Paris copy). This is computed to two digits for each degree of H from 1° to 90°, and is based on the standard formula. al-Khalīlī's values of $h_a(\lambda)$ (see above) may have been computed using this table, although they would have been more accurate had they been computed directly.

(d) Tables of two functions which we label $b_1(\lambda)$ and $b_2(\lambda)$ for use in timekeeping calculations with quadrants (only in the Paris copy of the corpus, fol. 33r): see already **10.5**.

(e) Most of the sources (*e.g.*, fols. 104v-105r, 17v-18r, 15v-16r, 16v-17r, of the Paris copy) contain four tables of ascensions, with values in degrees and minutes for each degree of ecliptic longitude. The functions tabulated are:

$$\alpha'(\lambda),\ \ \alpha_\phi(\lambda),\ \ \alpha_s(\lambda)\ \text{ and }\ \alpha_r(\lambda).$$

Each table is arranged in twelve columns of thirty entries, with the name of the zodiacal sign at the head of each column and a vertical argument running from 1 to 30. The actual arrangement of the columns differs for each table, beginning with the columns for the quadrant of the ecliptic at the beginning of which the function tabulated is between 0° and 90°. Thus the first columns in the table are for Capricorn, Aries, Libra, and Cancer, respectively.

The tables of α_r and α_s are based on the relations:

$$\alpha_r(\lambda) = \alpha_\phi(\lambda_H) = \alpha_\phi(\lambda) - r(\lambda) \quad \text{and}$$
$$\alpha_s(\lambda) = \alpha_\phi(\lambda_H) = 360° - [\ \alpha_\phi(180° - \lambda) - s(\lambda)\ ]\ .$$

In the first case H is the ascendant at daybreak, and in the second it is the ascendant at nightfall.

(f) The Paris copy (fols. 18v-20r) contains a star catalogue attributed to Ibn al-Shāṭir listing the equatorial coordinates of 81 stars.

Each of the sub-tables in this set is rather carefully computed, with the exception of the tables for the *ʿaṣr* (noted above).

10.7 Universal auxiliary tables

al-Khalīlī's most original contribution was his second set of auxiliary tables for solving all of the standard problems of spherical astronomy for all latitudes:[22a] see **I-9.5** and **VIa-9.1** and the extracts illustrated there. Over a dozen copies are now known, of which the best is on fols. 61v-104r of the superior Paris manuscript. al-Khalīlī tabulated three auxiliary functions, each to two sexagesimal digits. For the first two, the horizontal argument is terrestrial latitude, and the functions are denoted by f_f and g_f, where f is the local latitude. The vertical argument is

---

[22] The same holds for the use of 10 by Ibn al-Ādamī in the 10th century: see **I-4.2.3**. On the use of base 20 see n. **I**-1:35, and the text to n. **II**-10:39a.

[22a] The first study of these was in King, "al-Khalīlī's Universal Tables" (1973).

defined in the instructions, and the functions are called *al-maḥfūẓ al-awwal* and *al-maḥfūẓ al-thānī*, "first and second functions", respectively. The third function, which is denoted by K, has a quantity called *jayb al-tartīb*, perhaps best rendered as "the auxiliary Sine", as horizontal argument. The vertical argument is again defined in the instructions. The *jayb al-tartīb* used here is not the same as the one defined in the minor auxiliary tables above (**10.3**).

The functions are not explained in the instructions, but are in fact:

$$f_\phi(\theta) = R \text{ Sin } \theta \text{ / Cos } \phi \text{ ,} \quad g_\phi(\theta) = \text{Sin } \theta \text{ Tan } \phi \text{ / R , and}$$
$$K(x,y) = \text{arc Cos } \{ R \cdot x \text{ / Cos } y \},$$

for appropriate domains (see below), and the trigonometric functions are to base R = 60. The functions are defined in a marginal note in MS Cairo MM 43, fol. 5r:

> "The first function is the quotient of the Sine of an arc divided by the Cosine of the latitude. The second (function) is the product (*musaṭṭaḥ*, lit., rectangle) of the Sine of the arc and the Tangent of the latitude, (both expressed) to base sixty (*sittīniyy*^{an}) The function tabulated in the table labelled *jayb al-tartīb* (*ḥāṣil jayb al-tartīb*) is the complement of the arc (*i.e.* inverse Cosine) of the quotient [multiplied by a factor of sixty] of (the *jayb al-tartīb*) and the Cosine of the arc."

The first two functions are tabulated for the domains θ = 1°, 2°, ..., 90°, and ϕ = 1°, 2°, ..., 55°, as well as 21;30° (Mecca) and 33;30° (Damascus). In some sources (*e.g.* MS Paris ar. 2558) there is no table for Mecca, and in others (*e.g.* MSS Escorial ár. 931 and London BL Add. 9599,31) the argument ϕ runs only to 50°. The arguments for the third functions are x = 1, 2, 3, ... , 59 and y = 0°, 1°, ... , n(x), where n(x) is the largest integer such that x ≤ Cos n(x). The values for each function are arranged in columns each of thirty entries. On a given double opening in the first set of tables one finds values of f and g for two consecutive values of ϕ. In the second set one generally finds values of K for four consecutive values of x. However, when the values of K are, in modern terms, non-real, no entry is given in the table. Columns which would be empty are omitted; thus, for example, the values of K for x = 52, 53, ... , 59 fit on a double opening in MS Paris ar. 2558, fols. 103v-104r. If the values of K were displayed in a single table it would be trapezoidal in shape.

The first two tables each contain 5,040 entries and the third about 3,500, all computed with remarkable precision. Most of the entries are accurate, and the error in the remainder is usually only ±1 in the second digit, and occasionally ±2. Errors in the K table are slightly more frequent than in the f_ϕ and g_ϕ tables, and larger isolated errors do occur. Some entries in the table of K(x,y) for small values of x and large values of y are rather inaccurately computed. This author is unable to explain how these errors might have arisen.

However, Glen Van Brummelen, armed with an impressive combination of mathematical and statistical skills and not lacking in historical sensitivity, has investigated these error patterns in an attempt to establish precisely how al-Khalīlī computed his tables; this was a topic beyond the scope of the present study, and I cannot claim to understand all that Van Brummelen has published on this subject.[23] He would be the first to admit that al-Khalīlī's tables still actually defy explanation. He has, however, applied various statistical procedures in his investigation

---

[23] Van Brummelen, "al-Khalīlī's Auxiliary Tables", and *idem* & Butler, "Interdependence of Astronomical Tables", pp. 46-48.

of the structure of the auxiliary tables and – to our joint disappointment, for I do not think it was too much to hope for – was also unable to come up with precisely the procedures that were used by al-Khalīlī. The interested reader should consult Van Brummelen's paper, of which I here cite the conclusion (using the mathematical notation of this study):[24]

> "We have found in al-Khalīlī's auxiliary tables an interesting mix of ingenious insights and rudimentary errors. The $g_\phi(\theta)$ table was computed from two-place sine and tangent values on an interpolation grid of 5° of θ, and some of the remaining entries were filled in using a well chosen variant of linear interpolation. Many of the entries, however, seem to be unexplainable other than by simple guesswork. The $f_\phi(\theta)$ table was computed from the $g_\phi(\theta)$ table by a clever application of prosthaphairesis. The K(x,y) table was calculated by using only two places for the value of Cos y, followed by an extremely careful and precise calculation of the quotient Rx / Cos y and the arc Cosine."

Van Brummelen has shown that the errors in the entries for f_ϕ and g_ϕ in a large part of the tables, if not throughout, are linked by the following prosthaphaeretical relation:

$$f_\phi(\ \bar{\phi} \pm n\ ) = \text{Cos } n \pm g(\phi,n)\ ,$$

and he suggests that al-Khalīlī, having computed the table of $g_\phi(\theta)$ and noted that $f_\phi(\bar{\phi}) = R = 60$, used this relation to generate a major part of the table of $f_\phi(\theta)$. When he published this in 1991, I found the implications surprising. However, we now have independent confirmation of the fact that al-Khalīlī did tabulate $g_\phi(\theta)$ first, because he simply took over the table from his universal auxiliary tables for finding the solar azimuth.

We are on safer ground regarding what al-Khalīlī intended with his tables. The rules for their use which he outlines in words will now be compared with the medieval formulae listed in **I-1.2**. Most of the rules can be explained by transforming standard medieval expressions into a form which is equivalent to al-Khalīlī's rule, given the definitions of the functions f, g, and K. The arguments to be used in the tables are considered positive, and although al-Khalīlī's introduction (MS Paris ar. 2558, fols. 61v-64r) gives detailed instructions to cover all possible cases, modern notation renders these superfluous in the commentary below.

> "In the name of God, the Merciful and Compassionate: Praise be to God, by whose grace good things are achieved, and to whose might the earth and the heavens are obedient, and by whose wisdom the months and the times (of prayer) are known, and to whose order(s) the sun, moon and stars are subservient. May God bless our Lord Muḥammad, master of brilliant miracles, and his family and his companions, (all of whom) possess great virtue and noble qualities, with a blessing such that he who pronounces it be saved from misfortunes in life and in death.
>
> The aspect of astronomy most deserving of investigation is that by which the times of the five prayers and the qibla can be found and which deals with other information useful for guiding oneself on land and sea. The instruments which achieve this are very numerous, and the best of them are those which are easy to use and which are not specific to one latitude, but there are only a few people who can use these properly. Finding (the times of prayer and the qibla) by means of calculation is more precise

[24] Van Brummelen, "al-Khalīlī's Auxiliary Tables", p. 689.

but it involves more work, and only someone skilled in this discipline can do it properly. Thus if a table is prepared by calculation it is better to use such a table than to use instruments, although (??) the table is easier to comprehend than (the instruments).

Previously I had compiled two universal tables, from one of which one could determine the times, and from the other, the azimuth. Now God has enabled me to compile a table for finding all that could be found from those two tables and more besides. In addition, it is simpler than them. Praise and thanks be to God.

Section on the half diurnal or nocturnal arc for the sun or a star. Find the longitude of the sun from an ephemeris or some other tables and use it to find the declination from the declination table, or find the declination of the star corrected for the epoch in which you are working from the table of stellar coordinates. Then look for the page on which is written the local latitude and find the value of the second function corresponding to the solar or stellar declination. If the declination is in the first, second or third column, take the value of the function from the first, second or third column, respectively. Enter the result in the table of the third function (as the horizontal argument) and the declination as the (vertical) argument. What you find in the first, second or third box will be the half diurnal arc if the declination is southerly and the half nocturnal arc if it is northerly. If one of them is subtracted from 180°, the result will be the other one.

Section on the hour-angle. Find the value of the first function on the page corresponding to the terrestrial latitude with the altitude as (vertical) argument and the value of the second function as before in the section on the half arc. If the declination is southerly add the two values, otherwise take the difference. The result is the auxiliary sine. Enter it in the table of the third function with the solar or stellar declination as (vertical) argument, and you will find the hour-angle. If the declination is northerly and the first function is less than the second, subtract the result from 180° and the remainder will be the hour-angle. If the two functions are equal then the hour-angle will be 90°.

Section on the maximum altitude, the local latitude, the altitude of (the sun) and the hour-angle at the *ʿaṣr*, the time remaining from the *ʿaṣr* until sunset, and the duration of evening and morning twilight. Add the northern declination to 90° and subtract the southern declination: the result will be the sum of the maximum altitude and latitude. Then find the local latitude by observation or otherwise and subtract it from this sum: the result will be the maximum altitude. If it is greater than 90° subtract it from 180°: the remainder will be the maximum altitude to the north of the zenith (i.e. measured from the north point). If the maximum altitude has been found by observation and it is subtracted from the sum, the remainder will be the local latitude. If the latitude is added to the maximum altitude and the difference between the sum and 90° is taken, the result will be the solar or stellar declination. If the maximum altitude is northerly with respect to the zenith, subtract it from 180° and subtract the remainder from the sum: the result will be the local latitude, or add the latitude to the remainder, subtract 90° from the sum, and the remainder will be the declination.

Next enter with the maximum altitude of the sun in the universal *ʿaṣr* table and you will find the altitude of the *ʿaṣr*. Find the hour-angle for this altitude: it will be the time from midday to the *ʿaṣr*. Subtract this from the half diurnal arc: the remainder will be the time from the *ʿaṣr* to sunset. Next you find the value of the first function on the page for the latitude corresponding to (vertical argument) 17° for nightfall or 19° for daybreak. Then add to it the previous value of the second function for the half diurnal arc if the declination is northerly, otherwise subtract it from the arc: the result will be the auxiliary sine. Enter this in the table of the third function and the declination as (vertical) argument. Subtract what you find from the half nocturnal arc: the result will be the duration of twilight.

Section on the rising amplitude, the altitude in the prime vertical, and the azimuth corresponding to a particular altitude. Find the value of the second function for latitude 45° with (vertical argument equal to) the declination and enter the result as argument in the table of the third function with the local latitude or its complement as vertical argument, and you will find the complement of the rising amplitude or [complement of] the altitude in the prime vertical. Another method: Find the value of the first function for the solar or stellar declination on the page corresponding to the latitude and enter whatever it is as (vertical) argument in the table of the second function for latitude 45° or in the page corresponding to the latitude. The corresponding argument will be the rising amplitude or the altitude with no azimuth.

Then find the value of the first function for the solar or stellar declination on the page corresponding to the latitude, and find the value of the second function for the altitude. Add the two values (if the declination is) southerly and take the difference between them (if it is) northerly. The result is the auxiliary sine: enter it in the table of the third function with the altitude as the (vertical) argument and you will find the azimuth measured from the meridian. If the declination is southerly the azimuth is southerly. If it is northerly and the value of the first function is greater than that of the second, the azimuth will be northerly; if it is less, then (the azimuth) will be southerly. If the two values are equal then there is no azimuth.

Section on finding the declination by observation given that the latitude is known and finding both the latitude and the declination. Determine the meridian or the prime vertical either by means of an azimuth found from an altitude observation (*samt al-irtifāʿ*) or by using the Indian circle or a qibla-box (*ḥuqq al-qibla*) or by some other well-known means.[25] When one of these two lines is known use it to find the azimuth of a particular altitude by measuring the altitude at that time to obtain (a pair of) altitude and azimuth (values). Then look in the table of the third function for a box such that if the azimuth is entered in it (as one argument) the altitude would be the corresponding (vertical) argument: the auxiliary sine is (the argument) above the box. Then find the value of the second function with the altitude (as argument) on the page for the latitude. Subtract the value of the second function from the auxiliary sine for southern

[25] On the Indian circle see n. 2:23. On qibla-boxes in Egypt *ca*. 1300 see Schmidl, "Earliest Arabic Sources on the Compass", and King, *Mecca-Centred World-Maps*, pp. 113-114.

declinations, or add the two for northern declinations if the azimuth is northerly, otherwise take the difference between them. The result is a value of the first function, so find the corresponding argument: it will be the declination.

If you do not know either the latitude or the declination, make an observation of the azimuth and altitude and find the auxiliary sine as before, then observe another azimuth and altitude and find the auxiliary sine (for this pair) also. If the azimuths are in the same direction, take the difference between the two, otherwise add them: the result will be the *ḥiṣṣa*. Next find the values of the second function for latitude 45° and for each of the altitudes and take the difference between them. Enter with the result in the (table of the) second function for latitude 45° and find the corresponding argument: the result will be the *taʿdīl*. Then look for a page of (the tables of) the second function so that if you enter with the *taʿdīl* as argument, the value is the *ḥiṣṣa*: (you do this) to obtain the latitude at the top of the page. Then find the value of the second function for the larger altitude on that page and add to it the auxiliary sine for the larger altitude if the azimuth is northerly, otherwise take the difference between them: enter the result in (the table of) the first function on the (same) page, and find the corresponding argument to obtain the declination.

Conclusion on operations with stars. Enter the longitude of the sun in the table of (normed) right ascensions and you will find the (normed) right ascensions for that day. Subtract from these the half diurnal arc if possible, otherwise add a complete revolution (of 360° and then subtract): the result will be the oblique ascensions. If you add the half diurnal arc to the (normed) right ascensions, the result will be the (oblique) ascensions of the opposite point of the ecliptic. If the sum is greater than one revolution then the excess will be the ascensions of the opposite point. Subtract the half diurnal arc of a star or its eastern hour-angle from its ascensions – if this is not possible add a revolution to the ascensions: the result will be the ascensions of midheaven when the star is rising or when its altitude was measured. If the half diurnal arc or the hour-angle is added to the ascensions (of the star) the result will be the ascensions of midheaven when (the star) is setting or when its altitude was measured.

If the ascensions of midheaven are found by these operations or by means of a star which is culminating, subtract from them the ascensions of the opposite point of the ecliptic and the remainder will be how much of the night has passed. If the ascensions of the opposite point are added to the ascensions of midheaven, the culmination will be before sunset by the amount of the sum (lit. increase). If you subtract the ascensions of midheaven from the oblique ascensions, the remainder will be the time remaining of the night. If you add the ascensions of midheaven to the oblique ascensions, the culmination is after sunrise by the amount of the sum (lit. increase).

Section on the declination of a star. Look in the table of the third function for a box such that if you enter in it with the longitude of the star measured from the beginning of Capricorn or Cancer the corresponding (vertical) argument would be the latitude (of the star): you obtain the auxiliary sine above that box. Then enter in (the table of) the second function for latitude 49° the altitude of the star, and double the result and add it to the auxiliary sine if the longitude of the star and its latitude are

in the same direction, otherwise take the difference. Enter one half of the result as a value of the first function for latitude 37° and find the corresponding argument: it will be the declination of that star. The direction of the declination is the same as the direction of the longitude and the latitude (when these are) in the same direction. If they are in different directions and if the auxiliary sine is less than the value of the second function, I mean the doubled value, then the direction of the declination is the same as that of the longitude. If they are equal then there is no declination and no direction.

Section on the ascensions of a star. Enter with the latitude of the star in (the table of) the first function for latitude 37°, double what you find and keep it in mind. Next enter the declination of the star in (the table of) the second function, double what you find and keep it in mind. Then add the two quantities which you kept in mind if the latitude of the star and its declination are in different directions and take the difference between them if they are in the same direction: the result will be the auxiliary sine. Enter this in the table of the third function with the declination of the star as the (vertical) argument, and see what you find in the table. If the latitude and the declination are both northerly and the value of the first function is more than that of the second, and the longitude is between Capricorn and Cancer, then the result is the ascensions of the star. If the longitude is between 3 (signs) and 9 subtract the result from 360°: the remainder will be the ascensions. Likewise for (both latitude and declination) southerly. If the value of the first function is less than that of the second and the longitude is between 3 and 9, then subtract the result from 180°, but add it to 180° if the longitude is between 3 and 9 to obtain the ascensions. Likewise for both (latitude and declination) in the south if the value of the first quantity is greater than that of the second. If the latitude is northerly and the declination is southerly and the longitude is greater than Capricorn, then the result is the ascensions of the star, and if (the longitude) is less then subtract the result from 360° to obtain the ascensions. If the latitude is southerly and the declination is northerly and the longitude is less than Cancer, subtract the result from 180°, and if (the longitude) is more, add the result to 180° to obtain the ascensions of the star. God knows best."

Commentary:

(i) To find the half arc of daylight, D, or night, N, of a celestial body with declination δ, given the terrestrial latitude ϕ, use:

$$D \text{ or } N = K\{\ g_\phi(\delta),\ \delta\ \}$$

for $\delta \gtrsim$% 0, respectively. From **F7** in **I-1.2** for all δ:

$$D = 180° - N = 90° + \text{arc Sin } \{\ \text{Tan } \delta \text{ Tan } \phi\ /\ R\ \}$$

so that:

$$D \text{ or } N = \text{arc Cos } \{\ \text{Sin } \delta \text{ Tan } \phi\ /\ \text{Cos } \delta\ \} \text{ for } \delta < \text{ or } > 0, \text{ respectively.}$$

(ii) To find the hour-angle, t, given the solar altitude, h, for any declination and terrestrial latitude, use:

$$t = K\ \{\ [\ f_\phi(h) - g_\phi(\delta)\ ],\ \delta\ \}.$$

To illustrate the correctness of this procedure, we transform **F12** in **I-1.2** into:

$$t = \text{arc Cos } \{\ R\ [\ R \text{ Sin } h\ /\ \text{Cos } \phi - \text{Sin } \delta \text{ Tan } \phi\ /\ R\ ]\ /\ \text{Cos } \delta\ \}.$$

(iii) To find the lengths of evening and morning twilight, use:

$$r \text{ or } s = N - K\{\ [\ f_\phi(x) + g_\phi(\delta)\ ],\ \delta\ \}$$

for $x = 19°$ and $17°$, respectively. These are the parameters for the angle of depression at twilight used in al-Khalīlī's prayer-tables.

(iv) To find the solar rising amplitude, ψ, use:

$$\psi = 90° - K\{\ g_{45}(\delta),\ \phi\ \}\ ,$$

or solve:

$$g_{45}(\psi) = f_\phi(\delta)\ .$$

Now **F14** in **I-1.2** can be expressed in the form:

$$\psi = 90° - \text{arc Cos}\ \{\ \text{Sin}\ \delta\ \text{Tan}\ 45° / \text{Cos}\ \phi\ \}$$

and also:

$$\text{Sin}\ \psi\ \text{Tan}\ 45° / R = R\ \text{Sin}\ \delta / \text{Cos}\ \phi\ ,$$

so the procedure is valid. Note that $g_{45}(x)$ is the sine function to base 60.

(v) To find the altitude of a celestial body in the prime vertical, h_0, use:

$$h_0 = 90° - K\ \{\ g_{45}(\delta),\ \bar{\phi}\ \}$$

(the Paris copy incorrectly states that $h_0 = K\ \{\ g_{45}(\delta),\ \bar{\phi}\ \}$, where $\bar{\phi}$ is the complement of ϕ) ,

or solve:

$$g_{45}(h_0) = f_\phi(\delta)\ .$$

Now **F13** in **I-1.2** yields:

$$h_0 = \text{arc Sin}\ \{\ \text{Sin}\ \delta\ \text{Tan}\ 45° / \text{Cos}\ \bar{\phi}\ \}\ ,$$

and also:

$$\text{Sin}\ h_0\ \text{Tan}\ \phi / R = R\ \text{Sin}\ \delta / \text{Cos}\ \phi\ ,$$

so that again al-Khalīlī's procedure is correct.

(vi) To find the solar azimuth, a, measured from the meridian, use:

$$a = K\ \{\ [\ g_\phi(h) - f_\phi(\delta)\ ]\ ,\ h\ \}.$$

The standard medieval formula **F15** in **I-1.2** can be rendered:

$$a = \text{arc Cos}\ \{\ R\ [\ \text{Sin}\ h\ \text{Tan}\ \phi / R - R\ \text{Sin}\ \delta / \text{Cos}\ \phi\ ] / \text{Cos}\ h\ \}$$

which is mathematically equivalent.

(vii) To find the solar declination from an observed pair of values (h,a), when the local latitude is known, first solve for x in:

$$a = K(x,h)$$

and then solve for δ in:

$$f_\phi(\delta) = g_\phi(h) - x\ .$$

This procedure is the reverse of that in (vi) above.

(viii) To find both the solar declination and the local latitude from two such pairs, subscripted 1 and 2, observed on the same day, first solve $a = K(x,h)$ for x_1 and x_2. Then define the quantity:

$$\Delta x = x_2 - x_1$$

called the argument (*ḥiṣṣa*), and the quantity:

$$\Delta g = g_{45}(h_2) - g_{45}(h_1)\ .$$

Next find the quantity ξ, called the correction (*taʿdīl*), defined by:

$$g_{45}(\xi) = \Delta g\ .$$

Then solve for ϕ in:

$$g_\phi(\xi) = \Delta x\ .$$

The declination may then be found from $f_\phi(\delta) = g_\phi(h) - x$, as in (vii) above. These procedures are easily verified by projecting the celestial sphere onto the meridian plane.

(ix) To find the declination of a star, Δ, from its ecliptic coordinates (λ,β), first find λ'', the longitude difference between the star and the nearest solstice, and then solve for x in:

$$\lambda'' = K(x,\beta) .$$

The declination can then be found by solving:

$$f_{37}(\Delta) = {}^1/_2 \{ 2g_{49}(\beta) + x \} .$$

To explain this procedure, let ε denote the obliquity of the ecliptic. The modern formula[26] can be transformed thus:

$$\sin \Delta / \sin \varepsilon = \sin \beta \cos \varepsilon / \sin \varepsilon + \cos \beta \sin \lambda .$$

In order to adapt this formula to serve al-Khalīlī's tables, write:

$$R \operatorname{Sin} \Delta / \operatorname{Cos} \phi_1 = {}^1/_2 \{ 2 \operatorname{Sin} \beta \operatorname{Tan} \phi_2 / R + \operatorname{Cos} \beta \operatorname{Sin} \lambda / R \}.$$

It follows that the values ϕ_1 and ϕ_2 are defined by:

$${}^1/_2 \cos \phi_1 = \sin \varepsilon \quad \text{and} \quad 2 \tan \phi_2 = \cot \varepsilon .$$

For al-Khalīlī's parameter $\varepsilon = 23;31°$, the accurate values of ϕ_1 and ϕ_2 are 37;4° and 48;58°: he used the approximations 37° and 49°.

(x) To find the normed right ascension α' of a star, use:

$$\alpha'' = K \{ [2f_{37}(\beta) - 2g_{49}(\Delta)], \Delta \}$$

where α'' is the normed right ascension measured from $\alpha' = 0°$ or $\alpha' = 180°$. To justify this, we write the modern formula for the right ascension as:

$$\alpha = \operatorname{arc Sin} \{ R [\operatorname{Sin} \Delta \operatorname{Cot} \varepsilon / R - R \operatorname{Sin} \beta / \operatorname{Sin} \varepsilon] / \operatorname{Cos} \Delta \},$$

so that, with ϕ_1 and ϕ_2 as defined in (ix) above:

$$\alpha' = \operatorname{arc Cos} \{ - R [2 \operatorname{Sin} \Delta \operatorname{Tan} \phi_2 / R - 2 R \operatorname{Sin} \beta / \operatorname{Cos} \phi_1] / \operatorname{Cos} \Delta \}$$
$$= \operatorname{arc Cos} \{ R [2 R \operatorname{Sin} \beta / \operatorname{Cos} \phi_1 - 2 \operatorname{Sin} \Delta \operatorname{Tan} \phi_2 / R] / \operatorname{Cos} \Delta \},$$

al-Khalīlī's handling of these problems can, by any criteria, only be described as brilliant!

10.8 The universal qibla table

Three copies of al-Khalīlī's corpus contain a qibla table, namely, MSS Paris ar. 2558, fols. 53v-61r, copied 811 H [= 1411]; Berlin Ahlwardt 5754 (Wetzstein 1138, fols. 47v-55r), *ca.* 1700; and Paris BNF ar. 2560,11, fols. 148v-159v, *ca.* 1750.[27] The tables in the first two of these sources are preceded by a short introduction but in the third source there is no introduction and indeed no indication of the nature of the table.

The function tabulated by al-Khalīlī is here denoted by q, although in fact it defines the *acute* angle which the arc joining a given locality to Mecca makes with the local meridian. (See **Fig. VIa-3.1**.) Values of the function $q(\phi,L)$ are given in two sexagesimal digits for the domains:

$$\phi = 10°, 11°, \ldots , 56° \text{ and } 33;30°, \text{ and } \Delta L = 1°, 2°, \ldots, 60°.$$

26 Smart, *Spherical Astronomy*, p. 40.

27 My detailed study listed as King, "al-Khalīlī's Qibla Table" (1975) contains a critical edition of the table based on the first Paris manuscript and the Berlin one (pp. 87-94). The second Paris manuscript came to my attention too late to be used for the edition (see *ibid*., p. 84, n. 10).

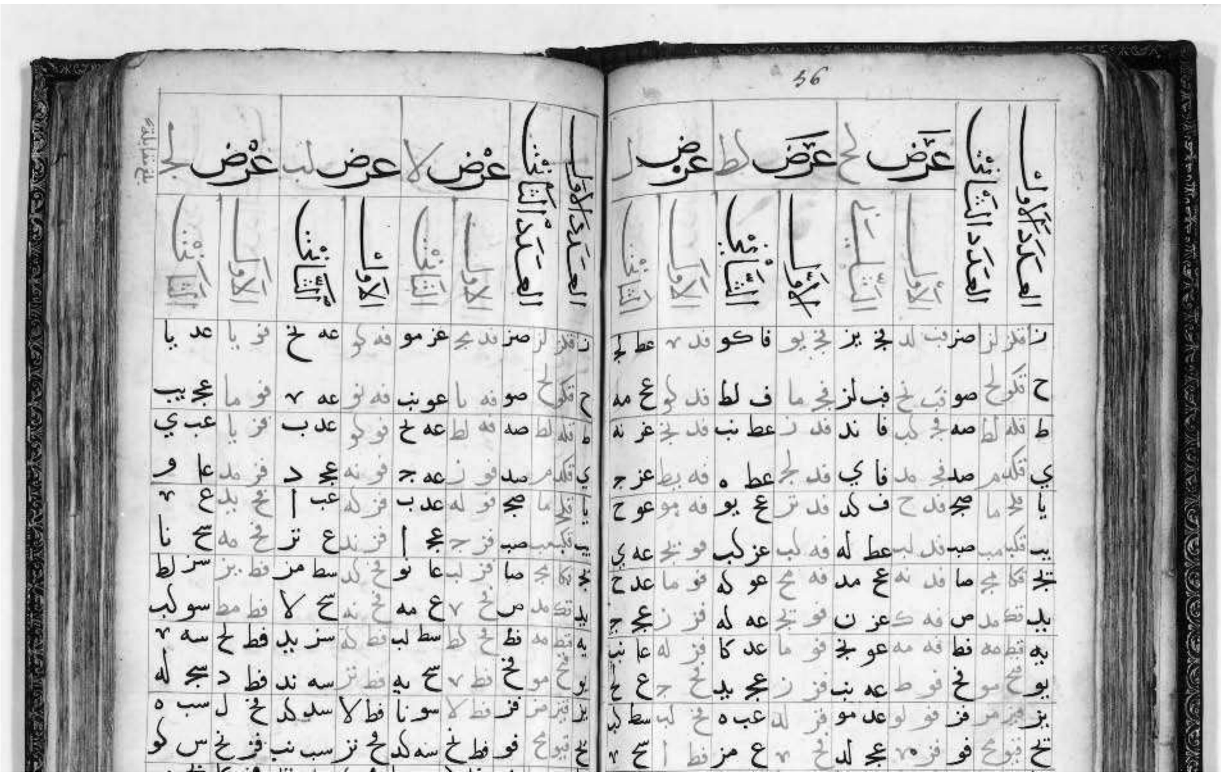


Fig. 10.8: An extract from al-Khalīlī's universal qibla table serving latitudes 28°-33°. The entries in light (actually red) ink indicate that the qibla is northerly. [From MS Paris BNF ar. 2558, courtesy of the Bibliothèque Nationale de France.]

For each value of ϕ there is a sub-table in which L is entered vertically – see **Fig. 10.8** for an extract. The double vertical argument runs from 7° to 66° (longitudes west of Mecca, for which the qibla is easterly) and from 127° to 68° (longitudes east of Mecca, for which the qibla is westerly). These two sets of longitudes correspond to a change in ΔL from 60° to 1°, since the longitude of Mecca is taken as 67;0°, and the underlying latitude of Mecca is found by inspection (see below and also **10.9**).

The entries for each latitude are arranged in two columns of thirty entries, and two facing pages of the manuscripts serve six consecutive latitudes. There are 2,880 entries in the entire table.

In the introduction (see below) he mentions that the qibla is southerly if the entry in the table is in red ink and northerly if it is in black ink. In the first sub-tables all entries are northerly, and in the remaining five sub-tables all entries above the broken line are also northerly; all other entries are southerly.

al-Khalīlī's qibla values are remarkably accurate: the errors in most of the table are less than ±0;2°, although larger errors do occur in certain sections. It is reasonable to assume that al-Khalīlī would have computed a matrix of values for, say, each degree of ϕ and each 5° of ΔL, and that he would have then used an interpolation scheme to compute intermediate values.

The error patterns in the table confirm this, and the interpolation scheme used, which was more sophisticated than a second order scheme, generally works rather well. Note, however, the relatively large errors in the entries for small values of ΔL and values of ϕ close to ϕ_M. For these arguments q is very sensitive to small changes in ΔL. Another minor point to be noted is that for $\Delta L = 1°$ (L = 66°, 68°) the entries for values of ϕ from 21° down to 17° are identical to the entries for ϕ from 22° up to 26°, and the qibla differs only in direction for these values close to ϕ_M. It is clear from the errors in these values that the entries for ϕ from 21° to 17° were copied from those for ϕ from 22° through 26°, which were computed.

It is of interest that al-Khalīlī has tabulated q(ϕ,L) for the latitude of Damascus, 33;30°. The latitude of Baghdad is also close to this, but the only reason for computing such a table would appear to be that al-Khalīlī had already tabulated his auxiliary functions for this latitude and he used them to compile his qibla table. On the other hand, he also tabulated q(ϕ,L) for latitude 56°, but as far as we know, he computed the auxiliary functions only up to latitude 55°.

The introduction to the qibla table (fols. 52v-53r of the first Paris copy) reads:

> "In the Name of God the Merciful and Compassionate. Praise be to God who made us pray towards His Sacred House (*i.e.*, the Kaʿba). May God bless our Lord Muhammad, who was sent (as a Prophet to institute) the religion of Islam, His Family, His Companions, and His Wives – noble persons (all) – with a blessing that will heal whosoever utters it of sickness and make him want to be in the Abode of Peace (*i.e.* Paradise).
>
> I have not seen a better method for (determining) the azimuth of the qibla than the one mentioned by Abū ʿAlī al-Marrākushī in the sixty-seventh chapter of his "Treatise on the Sine (Quadrant)" (*Risālat al-Jayb*). If the moon is at some instant (directly) above the Kaʿba beginning to be eclipsed then people looking at it from (different) parts of the earth will see it in different parts of the sky relative (to the meridian of) the (particular) location. One person will see before it culminates, another will see it culminating in the south or the north, another will see it after it culminates. Anyone who faces the moon at that time will be facing the qibla. If the declination of the moon at that time is known, and it is equal to the latitude of Mecca, and the hour-angle corresponding to its position (relative to the meridian) in any locality is known, and it is equal the difference in longitude (between the locality and Mecca), the altitude of the moon will be known, and similarly its azimuth, that is, the azimuth of the qibla. If the azimuth of the qibla is subtracted from 90°, the remainder will be the azimuth of the qibla measured from the local meridian (*al-inḥirāf*).
>
> Almighty God has enabled me to compile a table for finding the meridian azimuth of the qibla from the longitude and latitude of the locality on the basis that the longitude of Mecca – may God honour it – is 67° and the latitude is 21;30°. The way to use this table is to look for the page on which is written the latitude of the locality and to enter in the first or second argument with its longitude. Then move vertically and horizontally until the column and line meet and you will find the azimuth of the qibla for that locality. If the longitude is in the first longitude argument column then take the first entry for the azimuth, and similarly for longitudes in the second column. The

azimuth is southerly if the entry is written in black and northerly if it is written in red. It is easterly if the longitude of the locality is less than 67°, otherwise it is westerly.

Example for a locality with longitude 25° and latitude 35°. We find its qibla azimuth from the table to be 81;12° east of south. I performed (the calculations for) this example with a sine quadrant using the method of al-Marrākushī and I found the altitude (of the zenith of Mecca above the local horizon) to be 51° and the qibla to be 81;10°. I performed the operation using another method involving the differences in longitude and latitude and I found (the qibla) to be 68;50°, which is an atrocious error.

I ask God for success (in achieving a goal) which He will like and with which He will be pleased. Verily, He is the Excellent and the Noble. Praise be to God alone. May God bless and save our Lord Muḥammad, his Family and his Companions."

To illustrate how the arguments are to be entered in the table, al-Khalīlī considers the case $\phi = 35°$ and $L = 25°$. The value given in the table for these coordinates (which do not correspond to those of any specific locality) is 81;12° east of south, and is, in fact, accurate. al-Khalīlī also mentions that he used a sine quadrant (*rubʿ mujayyab*), to perform the calculation and that he found the results h = 51;0° and q = 81;10°. Actually this value of h is in error by only -0;2°, as is, fortuitously, the final value of q. al-Khalīlī does not describe the procedure he used to arrive at these results, nor does he tell us the size of the instrument he used, although it is known that the quadrants of his colleague al-Mizzī are between 15 and 20 cm. in radius.[28]

al-Khalīlī goes on to state that when he used the "well-known" approximate method involving latitude and longitude differences he found the "terribly inaccurate" value 68;50° for q. Now there are five known approximate methods involving ΔL and $\Delta\phi$,[29] four of which can be represented in modern notation by:

$$q = \arc\tan(x/y),$$

where:

(1)	$x = \Delta L$	$y = \Delta\phi$
(2)	$x = \Delta L \cos \phi_M$	$y = \Delta\phi$
(3)	$x = \sin \Delta L$	$y = \sin \Delta\phi$
(4)	$x = \sin \Delta L \cos \phi_M$	$y = \sin \Delta\phi$

The fifth method involves a more complicated formula, nowhere explicitly stated. Recomputation yields:

72;11° 70;56° 70;46° 69;27° 70;1°

instead of al-Khalīlī's 68;50°. However, if we assume $\phi_M = 21;0°$ instead of 21;30° the corresponding values are:

71;34° 70;21° 70;7° *68;50°* 69;24°

Thus al-Khalīlī's result was probably computed with formula (4), but it seems that he either made the computation using this other well-attested value for the latitude of Mecca, or he simply overlooked the fraction in the value of ϕ which he used to find $\Delta\phi$.

---

[28] On al-Mizzī see n. 9:21 above. On his quadrants see n. 9:22 and **X-6**.

[29] See King, "Earliest Qibla Methods", p. 90, and *idem*, *Mecca-Centred World-Maps*, pp. 57-59 and 335-336, on these procedures.

Unfortunately al-Khalīlī does not describe the method by which he compiled his table. However, he does remark that he knows of no better method for finding the qibla than the one advocated by Abū ʿAlī al-Marrākushī (**6.7**).[30] This involves first finding the altitude h of the zenith of Mecca above the local horizon using:

$$\mathrm{Sin}\ h(\phi,L) = \mathrm{Sin}\ (\ \bar{\phi} + \phi_M\ ) - \mathrm{Vers}\ \Delta L\ \mathrm{Cos}\ \phi_M\ \mathrm{Cos}\ \phi\ /\ R^2\ ,$$

and then finding the corresponding azimuth q using:

$$q(\phi,L) = \mathrm{arc\ Cos}\ \{\ R\ [\ \mathrm{Sin}\ h\ \mathrm{Tan}\ \phi\ /\ R\ -\ R\ \mathrm{Sin}\ \phi_M\ /\ \mathrm{Cos}\ \phi\ ]\ /\ \mathrm{Cos}\ h\ \}.$$

al-Khalīlī's universal auxiliary tables can also readily generate qibla values for a particular value of ϕ. First calculate $h(\phi,L)$, an operation simplified by the fact that:

$$\mathrm{Sin}\ h = c_1 + c_2\ \mathrm{Cos}\ \Delta L,$$

where c_1 and c_2 are constants. Then compute the quantity:

$$x(\phi,L) = g_\phi(h) - f_\phi(\phi_M)$$

using the first two auxiliary tables and enter this as horizontal argument in the third table since:

$$q(\phi,L) = K\ \{\ x(\phi,L)\ ,\ h\ \}.$$

Although it cannot be established that this procedure was actually followed by al-Khalīlī, by using his auxiliary tables it is possible to generate qibla values which are less accurate than, the same as, or more accurate than his own.[31] Van Brummelen has noted that the errors in al-Khalīlī's table of $f_\phi(\theta)$ for $\phi = \phi_M$ are not reflected in his qibla table, and he agrees that al-Khalīlī did not use his auxiliary tables to compute the qibla table.[32]

10.9 List of qibla values

Some copies of the Damascus corpus, for example, MS Paris ar. 2558, fol. 51v, contain a table of geographical coordinates "copied from the handwriting of ... al-Mizzī" with associated qibla values "computed by Shams al-Dīn al-Khalīlī": see **Fig. 10.9**.[33] In this copy the first four entries (Mecca, Medina, Cairo and Alexandria) are doubtless later additions, firstly because the title in the Egyptian copy MS Cairo MM 167, fol. 204r (dated 989 H [= 1581/82]), indicates that the localities belong to "the climate of Syria to the (further) edge of al-ʿIrāq", but secondly because the qiblas are carelessly computed. The entries in the table as found in the Paris and Cairo copies are presented below. Clearly the final qibla values offer some measure of control over copyists' mistakes.

The final column in the edited table displays values computed using linear interpolation in al-Khalīlī's qibla table. It is clear that these are slightly more accurate than those in the list of qiblas, which indicates that al-Khalīlī did not use his qibla table to compile them.

---

30 For details of al-Marrākushī's procedure and an attempt to recompute al-Khalīlī's qibla values see King, "al-Khalīlī's Qibla Table", pp. 99-108.

31 See the examples listed *ibid*., pp. 106-108.

32 Van Brummelen, "al-Khalīlī's Auxiliary Tables", pp. 688-689, and *idem* & Butler, "Interdependence of Astronomical Tables", pp. 46-48.

33 Kennedy & Kennedy, *Islamic Geographical Coordinates* (pp. xxii *sub* KHL and xxv *sub* MIZ) have determined that the list is related to an early set of geographical tables recorded by Abu 'l-Fidāʾ (*fl*. Hama, *ca*. 1310 – see n. 2:33). This table is now published in King, *Mecca-Centred World-Maps*, App. F7.

Fig. 10.9: The geographical tables in al-Khalīlī's Damascus corpus. [From MS Paris BNF ar. 2558, fols. 51v-52r, courtesy of the Bibliothèque Nationale de France.]

The geographical table of al-Mizzī
with qibla-values computed by al-Khalīlī
A: MS Paris BNF ar. 2558; B: MS Cairo MM 167

Notes: The entries Δq are the errors compared with computation using the accurate formula. The entries q* are those derived by linear interpolation in al-Khalīlī's qibla-table and Δq* the corresponding errors.

	Locality	L	ϕ	q	Δq	q*	Δq*
1	Shawbak	56; 0	31; 0	48;42	[+3]	48;42	[+3]
2	Gaza	57; 0	32; 0	42;46	[+1]	42;46	[+1]
3	Ramla	56;50	32;10	42;56[a]	[+8]	42;50	[+2]
4	Hebron	56;30	31;35	45;21	[-6]	45;29	[+2]
5	Jerusalem	56;30	32; 0	44;14	[-2]	44;16	[0]
6	Nablus	57;30	32;10	40;38	[-6]	40;44	[0]
7	Baysan	58; 0	32;50	37;[2]5[b]	[0]	37;26	[+1]
8	Tiberias	58;15	32; 5	38;34	[+4]	38;30	[0]
9	Ajlun	58;10	32;10	38;35	[+1]	38;34	[0]
10	Salt	58;10	32; 0	39; 2	[+2]	39; 0	[0]
11	Adhraat	60; 0	31;55	32;40	[+3]	32;39	[+2]
12	Bosra	60; 0	32;15[c]	31;52	[+4]	31;51	[+3]
13	Sarkhad	60;20	32;15[d]	30;31	[-1]	30;33	[+1]
14	Damascus	60; 0	33;30[e]	29; 4	[+1]	29; 7	[+4]
15	Safad	57;35	32;30[f]	39;35	[0]	39;36	[+1]
16	Shaqif Tayrun	57;40	33; 5	37;53	[+1]	37;51	[-1]
17	Baniyas	59; 0	33; 0	33;40	[0]	33;40	[0]
18	Sidon	58;15	33; 0	36;13	[+2]	36;10	[-1]
19	Beirut	59;15	33;20	31;59	[-4]	32; 4	[+1]
20	Baalbek	60; 0	33;50	28;24[g]	[0]	28;27	[+3]
21	Tripoli Sh	59;40	34; 0	29;13[h]	[-1]	29;14	[0]
22	Buluniyas	60; 0	34;45	26;47	[+4]	26;46	[+3]
23	Homs	61; 0	34;20	23;53[i]	[-2]	23;59	[+4]
24	Salamiyya	61;20	34; 0	23;14	[-1]	23;15	[0]
25	Barin	60;45	34;20	24;49	[0]	24;52	[+3]
26	Ḥisn al-Akrād	60;30	34;10	25;58	[-2]	26; 2	[+2]
27	Hama	61;45	34;45	20;32	[-2]	20;35	[+1]
28	Maarra	61;45	35; 0	20;12	[-1]	20;13	[0]
29	Sarmin	61;50	35;45	18;50	[-7]	19; 0	[+3]
30	Lattakia	60;40	35;25	23;21	[-3]	23;26	[+2]
31	Sahyun	60;10	35;25	25; 3	[-1]	25; 6	[+2]
32	Antioch	60;35	35;50	23; 4	[0]	23; 6	[+2]
33	Aleppo	62;10[j]	35;50	17;42	[0]	17;44	[+2]
34	al-Bāb / Buzaa	62;15	36;10	17; 1	[-2]	17; 4	[+1]
35	Bira	62;30	36;50	15;31	[-1]	15;33	[+1]
36	al-Rahba	64;30	36; 0	9;14	[0]	9;13	[-1]
37	Mardin	64; 0	37;55	9;48	[0]	9;48	[0]
38	Sinjar	66; 0	36;20	3;38	[0]	3;41	[+3]
39	Baghdad	70; 0	33;25	13;19	[0]	13;19	[0]
40	Anbar	69;30	33;15	11;19	[+1]	11;18	[0]
41	Kufa	69;30	31;30	13;12	[+1]	13;12	[+1]

42	Basra	74; 0	30; 0	38;11	[+2]	38; 7	[-2]
43	Tustar	74;30	31;30	35;41	[+4]	35;37	[0]
44	Shiraz	78; 0	29;36	53; 8	[-8]	53;20	[+4]
Additional entries in A only:							
45	Mecca	67; 0	21;30	-			
46	Medina	65;20	24;40	??;40	[?]	26;23	[+13]
47	Cairo	54;40	30; 0	55;58[k]	[+34]	55;24	[0]
48	Alexandria	51;54	31; 0	58;35	[+5]	58;30	[0]

Notes: [a] B: 42;36°; [b] A/B: 37;35°; [c] B: 32;55°; [d] B: 32;55°; [e] B: 32;55° (!), a value not attested in other known source; [f] B: 33;30°; [g] B: 28;30°; [h] A: 29;53°; [i] B: 26;47°; [j] B: 60;10°; [k] The standard approximate formula yields q*: 55;19°, and the 55;58° given here may be a copyist's error for 55;18°.

Notice that the qibla value 29;4° for Damascus (in error by only +1′) is at variance with the value 28;57° which underlies the table of solar altitude in the azimuth of the qibla in al-Khalīlī's prayer-tables (**10.6**). Different approximate formulae as well as the exact formula have been applied to a number of sets of medieval geographical coordinates for Damascus and Mecca to no avail. Two pairs of values, however, which yield precisely 28;57° using the exact formula are:

$$L = 60;\,0^\circ \text{ and } \phi = 33;33^\circ \qquad L_M = 67;\,0^\circ \text{ and } \phi_M = 21;30^\circ$$
$$L = 60;\,0^\circ \text{ and } \phi = 33;24^\circ \qquad L_M = 67;\,0^\circ \text{ and } \phi_M = 21;20^\circ$$

The value 33;33° is not attested in any known geographical tables, but the value 33;24° is found in an early anonymous *zīj* which unfortunately has no coordinates for Mecca,[34] and is also used by Ibn al-Mushrif (**I-7.3.1** and **9.8** and **II-9.10**). The value 21;20° was used by Ibn al-Shāṭir for Mecca. Perhaps al-Khalīlī used the second set of parameters to derive 28;57° for the qibla at Damascus; perhaps he did not.

Opposite this geographical table in the first Paris manuscript (and also in MS Paris BNF ar. 2560,13, fol. 164v, *ca*. 1750) is a list of stations on the pilgrim road from Damascus to Mecca (*manāzil al-Ḥijāz al-Sharīf ʿala ʾl-darb al-Shāmī*) with their respective latitudes: see also **Fig. 10.9**.[35] It is unlikely that al-Khalīlī made these measurements himself, and the list should be compared with similar ones in other sources (*e.g*. MS Cairo MM 167, fol. 203v). In some (*e.g*. Cairo MM 167, fol. 203r), we find also a list of pilgrim stations on the road from Cairo to Mecca. See also **8.4**.

10.10 The arithmetical and trigonometric tables available to al-Khalīlī

There can be no doubt that for his calculations al-Khalīlī used a sexagesimal multiplication table displaying products:

$$m \times n \quad \text{for} \quad m, n = 1, 2, 3, \ldots, 59 \,.$$

Such tables had been in use amongst Muslim astronomers since at least the 10th century. He may also have made use of a table of sexagesimal quotients m/n for the same arguments. The

[34] Kennedy & Kennedy, *op. cit*., p. 473 (*sub* UTT). See also n. **I**-7:28.

[35] On the pilgrim routes see EI_2, III, p. 34 (in the article "Ḥadjdj" by Bernard Lewis), and Brice *et al*., eds., *Atlas*, p. 22.

more extensive Islamic tables for arithmetical operations with sexagesimals were probably compiled after his time. See **I-1.2**.

The contemporary *Zīj* of Ibn al-Shāṭir contains tables of the Sine function (base 60) to four sexagesimal places for each degree of argument, and the Tangent function (base 60) to three places for each half degree. Both tables also display first differences. In addition Ibn al-Shāṭir tabulated the Cotangent to bases 7 and 12 to three places for each degree of argument and the inverse Cotangent function.[36]

Tables of the Sine and Cotangent function with values for each minute of arc computed to four or five sexagesimal places had been available since the 10th century. The *muwaqqit*s of Cairo also used tables of the Sine and Cotangent for each minute of argument to three places.[37] Such tables would have been available to al-Khalīlī. Yet the only table of the Sine function in the available manuscripts gives values to only two sexagesimal digits for each degree of argument (MS Dublin CB 4091, fol. 1v), and, rather surprisingly, is not without mistakes. This may be compared with the values of al-Khalīlī's function $g_\phi(\theta)$ (**10.7**) for $\phi = 45°$, which is identical to the Sine function. Van Brummelen has determined that al-Khalīlī used Sine and Cotangent tables with values to two sexagesimal places in the computation of his auxiliary functions.[38]

It is also possible that al-Khalīlī used tables of the Secant function and the inverse Sine / Cosine function since these functions are fundamental to his tables. But while such tables are attested in the Islamic sources,[39] they do not occur in any known copies of the Damascus corpus.

Only two other trigonometric tables are found in the various manuscripts of al-Khalīlī's tables, both relating to the Cotangent function. In the first (MS Paris ar. 2558, fol. 51r) the base used is 20. Values are given to two digits for each 1° of altitude from 1° to 80° (this being the maximum integral solar altitude at Damascus). It is likely that al-Khalīlī had a gnomon divided into 20 units for which such a table could have been useful. I know of only one other attestation of the use of base 20 in the medieval sources.[39a]

In the second trigonometric table (MS Berlin Wetzstein 1138 (Ahlwardt 5754-6), fol. 59r) the base used is 10. Values are given to two digits for each 1° of altitude from 1° to 90°. No reason is given for the choice of 10 as base, not even in the instructions (fols. 60r-61r) on the use of the shadow tables. The base 10 was also used in the trigonometric tables of the 16th-century Istanbul astronomer Taqi 'l-Dīn ibn Maʿrūf.[40] In the tables in the Damascus corpus the entries are expressed sexagesimally, whereas in Taqi 'l-Dīn's tables they are expressed decimally.

Possibly al-Khalīlī used some kind of interpolation scheme to compile his tables, which contain some 40,000 entries. Several descriptions of second-order interpolation schemes are

---

[36] Kennedy, "*Zīj* Survey", pp. 162b-163a.
[37] For copies see *Cairo ENL Survey*, no. C137.
[38] See the text to n. 10:24.
[39] For examples of Islamic tables of the Secant function see **I-6.9** and **8.4**.
[39a] The 15th-century Egyptian astronomer al-Wafāʾī, in his treatise on the universal astrolabe of Ibn al-Sarrāj (see n. 9:12), proposes 20 for the length of the gnomon: see Charette & King, *Universal Astrolabe of Ibn al-Sarrāj* (forthcoming).
[40] On these tables by Taqi 'l-Dīn (**I-2.3.6** and **II-14.9**) see King, "Islamic Multiplication Tables", B, pp. 413-414.

known from the Islamic sources,[41] including one advocated by the 10th-century Cairo astronomer Ibn Yūnus and ideally suited for generating tables of trigonometric functions.[42] Nevertheless Glen Van Brummelen's detailed analysis of the error patterns in al-Khalīlī's tables has not cast any light on the procedures that he used.[43]

10.11 The influence of al-Khalīlī's tables

Since al-Khalīlī's auxiliary tables for solar timekeeping survive in their original form only in a single manuscript (MS Dublin CB 4091), it is probable that they were not widely used in Syria. However it is known that the tables travelled as far as Tunis, Tlemcen, and Cairo, where they were either extended or modified. In each case the attribution to al-Khalīlī was suppressed (or at least is not found in the surviving copies).

Tunisian redaction

MS Cairo DM 689 contains an extensive set of auxiliary tables copied in an elegant Maghribi hand *ca*. 1600.[44] They conclude with a star catalogue dated 801 H [= 1398], and appear to have been compiled in Tunis. The title folio, instructions, and first few tables are missing from the manuscript, which begins with the last page of a set of tables displaying the solar longitude for each day of a period of four Syrian years. The tables for solar timekeeping follow.

The main functions tabulated are al-Khalīlī's $B'(\lambda,\phi)$ and $V'(x,y)$. In fact these Tunisian tables are merely an extension of al-Khalīlī's tables, and the corresponding entries in both sets are the same. The Tunisian tables of B' are computed for each degree of ϕ from 1° to 48° and also 21;40° (Mecca) and various other non-integral latitudes between 30° and 40° intended to serve localities in Ifdrīqiyya and the Maghrib and perhaps also Sicily. al-Khalīlī had a separate table for latitude 33;30° (Damascus), and the Tunisian set has separate tables for latitudes:

30;30°, 31;30, 32;30, 33;30, 33;40, 34;30, 35;30,
36;30, 36;40, 37;10, 37;30, 38;30 and 39;30.

The Tunisian tables of $V'(x,y)$ are simply those of al-Khalīlī rearranged so that the horizontal argument is increasing.

The tables of B' are preceded by tables of the function Sin $H(\lambda)$ (called *jayb al-ghāya*) computed to two digits for each degree of λ and the latitudes between 30° and 38° for which B' is tabulated. These were probably used by the anonymous Tunisian astronomer to compile his tables of $B'(\lambda)$, since:

$$B'(\lambda,\phi) \;=\; {}^{1}\!/_{2}\ [\ \sin H(\lambda,\phi) + \sin H(\lambda*,\phi)\ ],$$

where $\lambda* = 180° + \lambda$. The underlying value of ε is 23;35°, whereas al-Khalīlī used 23;31°.

However, the change in ε hardly affects the values of $B'(\lambda)$ given to two digits. The Tunisian tables also contain a table of Sin $H(\lambda)$ for $\phi = 0$, which is simply Cos $\delta(\lambda)$. See further **13.4**.

[41] See n. **I**-1:21. MS Cairo Zakiyya 917,4 (pp. 50-54, copied 1163 H) contains some notes on interpolation attributed to al-Khalīlī but I have been unable to obtain a microfilm of it.

[42] King, "Astronomical Timekeeping in Medieval Cairo", pp. 354-357, and Hamadanizadeh, "Islamic Interpolation Schemes", p. 146.

[43] See the text to nn. 10:23-24.

[44] Listed in *Cairo ENL Survey*, no. F30.

Tlemcen extract

MS London BL Or. 411,2 contains the only known complete copy of a commentary on the astronomical poem of the late-14th-century Maghribi writer ʿAbd al-Raḥmān ibn Muḥammad al-Jādarī.[45] This anonymous commentary was written in Tlemcen and contains several spherical astronomical tables computed for parameters:

$$\phi = 35;0° \text{ (Tlemcen) and } \varepsilon = 23;35° .$$

The same manuscript also contains interesting historical accounts of trepidation, twilight, and the obliquity of the ecliptic. Among the spherical astronomical tables two display the functions:

$$\text{Sin } H(\lambda) \text{ and } {}^1/_2 \; B(\lambda),$$

and another displays the function:

$$t \; \{ \; {}^1/_2 \; B, \; (\text{Sin } H - \text{Sin } h) \; \}$$

for the same latitude. Unfortunately it has not been possible to compare these tables with the Tunisian corpus of auxiliary tables (see above), from which they may have been lifted. See further **13.6**.

Egyptian version

Two Egyptian manuscripts (Istanbul S. Esad Efendi Medresesi 119,2 and Cairo DM 644,1) contain an anonymous set of auxiliary tables entitled *Fatḥ al-Karīm al-Bāqī fī maʿrifat al-dā'ir wa-faḍlihi āfāqī* ("The Victory of God the Noble and Eternal for Finding the Time since Sunrise and Hour-Angle for all Latitudes").[46] The main functions tabulated are:

$$B'(\lambda,\phi) \;=\; {}^1/_2 \; B(\lambda,\phi) \;=\; {}^1/_2 \; \text{Cos } \delta(\lambda) \text{ Cos } \phi \;/\; R$$

for the domains:

$$\lambda = 1°, 2°, \ldots , 90°; \; \phi = 3°, 6, \ldots , 27, 28, 30, 32, 33;30, 36, 38, \ldots , 48°$$

and:

$$V'(x,y) \;=\; \text{arc Vers } \{ \; R \cdot y \;/\; 2x \; \}$$

for the domains:

$$x = 19, 20, \ldots , 30 \text{ and } y = 0;5, 0;10, \ldots , 60;0.$$

The first function is labelled *al-aṣl*, "(half) the base" and the arguments in the table of V′ are called *al-aṣl al-muʿaddal*, "the modified base", and *faḍl al-jaybayn*, "the difference between the two Sines". The entries in both tables were simply coped from al-Khalīlī, in whose set B′ is tabulated for some 50 values of ϕ and the increment in the argument y for the tables of V′ is 0;10 rather than 0;5. Also, this later Egyptian set gives no horizontal differences in the tables of V′, but the simplified format makes them slightly easier to use. See further **I-9.11**.

al-Khaṭā'ī's auxiliary tables

The unique source MS Vatican Borg. ar. 217,2 (fols. 6r-7r) contains part of a set of auxiliary tables intended for use in Cairo, attributed to Muḥammad ibn al-Amīr Fakhr al-Dīn ʿUthmān al-Khaṭāʾī (*ca.* 1475). Some minor tables display:

$$\text{Sin } \theta, \text{ Sin } H(\lambda), \text{ and } B(\lambda),$$

[45] On al-Jādarī see **I-6.15.2** and **II-13.6**.
[46] Listed in *Cairo ENL Survey*, no. C144.

each to three digits for each degree of argument. The second and third functions are computed for parameters:

$$\phi = 30;0° \text{ (Cairo) and } \varepsilon = 23;35° .$$

The main set, incomplete in the Vatican manuscript, displays the function:

$$V(x,y) = \text{arc Vers } \{ R \cdot y / x \} ,$$

with values to two digits for the domains:

$$x = 47;37,\ 47;57,\ ...\ ,\ 51;57 \text{ and } y = 1,\ 2,\ ...\ ,\ Y(x),$$

where Y(x) is a certain maximum defined below (see an extract in **Fig. I-9.9b**). The instructions for finding t(h,λ) indicate that one should first find B(λ) and z(h,λ) (*i.e.*, Sin H(λ) - Sin h) and enter these arguments in the main table. This is valid since:

$$t(h,\lambda) = V \{ B(\lambda),\ z(h,\lambda) \} .$$

The argument x runs in intervals of 0;20 between the limits of B(λ) for Cairo, and the argument y runs in unit intervals up to Y(x), the greatest integer less than the value of Sin H(x). The table originally contained about 825 entries, but in the Vatican manuscript only the page for y > 31 remains. The entries are fairly accurately computed.

Being devised for latitude 30° al-Khaṭāʾī's tables are not as useful as al-Khalīlī's minor auxiliary tables for solar timekeeping. It is probable that al-Khaṭāʾī knew of al-Khalīlī's tables, but the former's tables were redundant anyway because by the 13th century there existed a complete set of tables of t(h,λ) for Cairo. See further **1-6.15.1** and **II-6.8**.

Prayer-tables

With the passage of time only al-Khalīlī's calendrical tables and the solar longitude table needed to be updated. The decreasing obliquity (23;30° observed in *ca.* 1440 by Ulugh Beg[47] and 23;29° used by later Ottoman astronomers[48]) was not considered sufficient cause to recompute the corpus, or even the prayer-tables.

After the time of al-Tīzīnī, the Damascus *muwaqqit* Ibn al-Kayyāl provided a new set of calendrical and solar tables to precede al-Khalīlī's tables for timekeeping.[49] These are extant in numerous copies including MSS Berlin Ahlwardt 5758 (Sprenger 1858), 5759/5771 (Wetzstein 1146), and 5760/5772 (Wetzstein 1148), none of which credit al-Khalīlī as the author of the main tables. The same is true of the 19th-century Damascus *muwaqqit* Muḥammad ibn Muṣṭafā al-Ṭanṭāwī,[50] whose recension is contained in several copies including MSS Damascus Ẓāhiriyya 9233, Cairo ṬM 173, Cairo TR 129, and Cairo DM 1007. al-Ṭanṭāwī, however, modified the tables so that the entries displayed actual times in hours and minutes according to the Ottoman convention that sunset is 12 o'clock. Thus al-Khalīlī's tables survive in three versions: the original, and the two recensions of Ibn al-Kayyāl and al-Ṭanṭāwī. See further **11.4** and **11.13**.

Besides these modifications to al-Khalīlī's corpus, various new sets of prayer-tables were compiled for a variety of localities after the 14th century (see further **11**). Mention may be made

---

47 Kennedy, "*Zīj* Survey", p. 166b.
48 See, for example, King, "Astronomical Timekeeping in Ottoman Turkey", p. 251.
49 On Ibn al-Kayyāl see n. 11:8.
50 On al-Ṭanṭāwī see n. **I**-2:27.

of the anonymous prayer-tables for latitude 34;20° (Tripoli ?) in MS Cairo ṬM 228,3; for latitudes 41;15° (Istanbul) and 36° (Aleppo) in MS Cairo ṬM 255,6; for latitude 33;45° (Tripoli) in MS Damascus Ẓāhiriyya 4893; and for latitude 41;0° (Istanbul) and 21;30° (Mecca) in MS Cairo DM 184. All of these are based on obliquity 23;31°, which betrays the influence of Ibn al-Shāṭir and al-Khalīlī. On the other hand, ʿAbd al-Fattāḥ al-Dīsṭī's tables for latitude 34;30° (Lattakia) in MSS Aleppo Awqāf 911 and Leiden Or. 2808,2 are based on obliquity 23;35°, which betrays Egyptian influence, and have the format of the Cairo corpus. The later tables in MS Aleppo Awqāf 943 for latitude 35;50° (Aleppo) are based on obliquity 23;30°, which is the parameter of Ulugh Beg (rounded to minutes).

In MSS Oxford Seld. Supp. 100 and Paris BNF ar. 2521 there are separate tables of various supplementary functions in the same format as the Cairo corpus. These functions are:

$$(r+D),\ (D+2N),\ (2N-s),\ (2N-s+D),\ (D+s),\ \text{and}\ (D+2n-r)\ .$$

Although not attributed to al-Khalīlī, the entries are based on those in his main set of prayer-tables.

Similarly, MS Leipzig 814, fols. 31r-31v, is one remaining folio of a set of prayer-tables for Damascus which was drawn up using al-Khalīlī's tables. The functions displayed are:

$$Z_{(12)},\ H,\ D,\ z_{a(7)},\ \tilde{h},\ \delta,\ d,\ h_a,\ t_a,\ T_a,\ h_b,\ t_b,\ 2N,\ N,\ \psi,\ s,\ r,\ n,\ t_q,\ \text{and}\ 2N^h\ .$$

No other examples of these prayer-tables are known. The function t_b is labelled *ākhir waqt al-ikhtiyār*, "the last time for choosing (to begin the afternoon prayer)" and h_b is called *irtifāʿ al-ʿaṣr al-thānī*, "the altitude at the second afternoon prayer". In al-Khalīlī's main set t_b is called *mā bayn al-ẓuhr wa-ākhir waqt al-ʿaṣr*, "the interval between midday and the last time for the afternoon prayer".

Hour-angle tables

al-Khalīlī's tables of t(h,λ) are supplemented in various copies by a set of tables for T(h,λ). Although it is not certain that these are the work of al-Khalīlī himself (they are not contained in MS Paris ar. 2558), evidence that they might be is contained in MS Cairo MM 71, where both the tables of t(h,λ) and T(h,λ) are attributed to al-Khalīlī while the table of a(h,λ) is attributed to Shihāb al-Dīn al-Ḥalabī. al-Ṭanṭāwī, besides converting the prayer-tables as mentioned above, also converted al-Khalīlī's hour-angle tables so that the entries (given for solar altitudes in both the east and west) were expressed in hours and minutes according to the Ottoman convention, as, for example, in MSS Cairo ṬM 173 and DM 1007. See further **11.13**.

Finally, in MS Damascus Ẓāhiriyya 7387, fols. 57v-60r, of an abridgement of the *Zīj* of Ibn al-Shāṭir by ʿAbd al-Raḥīm al-Qazwīnī of Damascus (*fl. ca.* 1610),[51] there is a set of tables entitled *jadwal al-dā'ir li-rtifāʿ al-shams wa-yuʿraf minhu qaws al-ruʾya min al-makth idhā dakhalta bi-naẓīr al-burj* ..., displaying the time remaining till moonset for lunar altitudes:

$$h = 7°,\ 8°,\ \dots\ ,\ 16°,$$

and each degree of ecliptic longitude (symmetrically arranged). The entries are from al-Khalīlī's tables of T(h,λ), and, as stated in the title, are to facilitate calculations relating to lunar

[51] On al-Qazwīnī see n. 11:5.

crescent visibility. The tables can be used to find the arc of visibility (*qaws al-ru'ya*), that is, the altitude of the moon above the horizon at sunset, from the difference in setting times of the sun and the moon (*qaws al-makth*). The longitude of the point of the ecliptic which sets with the moon, λ', is computed using the longitude and latitude of the moon and the latitude of the locality, and then the difference in setting times can be determined using tables of oblique ascensions.[52] The tables can then be used to find the lunar altitude h corresponding to this time s, with the modified longitude λ' entered as argument.

Miscellaneous related tables

Shihāb al-Dīn al-Ḥalabī's solar azimuth tables

al-Khalīlī's tables of $t(h,\lambda)$ and $T(h,\lambda)$ for Damascus were supplemented in *ca.* 1425 by Shihāb al-Dīn Aḥmad ibn Ibrāhīm al-Ḥalabī[53] with a complete set of tables of the function:

$$a(h,\lambda)$$

based on al-Khalīlī's parameters. These are contained in MS Cairo MM 71, where values are tabulated side by side with al-Khalīlī's functions, and in MSS Damascus Ẓāhiriyya 9227 and Cairo K 8525 where they are tabulated separately. al-Ḥalabī clearly felt the need to compile a table of $a(h,\lambda)$ to supplement the Damascus corpus even though al-Khalīlī had already tabulated $a(H,h)$ (**10.3c**). Possibly al-Ḥalabī was aware that the tables of $a(h,\lambda)$ for Cairo computed by Ibn Yūnus had been incorporated into some copies of the Cairo corpus, with values of the functions t/T/a tabulated together.[54] See further **11.2**.

Ibn Barakāt's tables for the afternoon prayer

MS Damascus Ẓāhiriyya 9227 of al-Ḥalabī's solar azimuth tables also contains a set of tables attributed to Ibn Barakāt, who is otherwise unknown to us. These tables display the time τ_a remaining to the beginning of the interval for the afternoon prayer as a function of solar longitude and solar altitude in the east, and are computed for the latitude of Damascus. The function tabulated is simply:

$$\tau_\alpha(\lambda,h) = t_a(\lambda) - t(h,\lambda) \ .$$

For each integral degree of solar longitude the arguments for solar altitude are the integral values such that:

$$h_a < h < H.$$

Similar tables were computed for Cairo.

In MS Berlin Wetzstein 1138 (Ahlwardt 5754-6), fols. 62v-94r, appended to al-Khalīlī's tables, there are also some anonymous tables of a function described as *al-maktūb bi-'l-sawād al-bāqī li-'l-ghurūb wa-bi-'l-ḥamra li-'l-ʿaṣr*, "time remaining until sunset written in black, or time remaining until the beginning of the afternoon prayer, written in red". These tables have the same format and argument domains as al-Khalīlī's hour-angle tables, and it is understood that the altitudes are eastern. The tabulated function is simply:

$$f(h,\lambda) = T(h,\lambda) \quad \text{for} \quad h < h_a(\lambda) \quad \text{or} \quad \tau_\alpha(h,\lambda) \quad \text{for} \quad h \geqslant h_a(\lambda).$$

---

52 For the procedures see King, "Tables for Lunar Crescent Visibility", pp. 186-189.
53 On al-Ḥalabī see n. **I**-2:42.
54 On this see **5.6** (illustrated).

(Note the different format of these tables in MSS Damascus Ẓāhiriyya 9227 and Berlin Wetzstein 1138 (Ahlwardt 5754-6).) See further **11.3**.

Universal auxiliary tables
al-Khalīlī's main auxiliary tables were used for several centuries not only in Syria but also in Egypt, the Maghrib, and Turkey.

Egyptian copies
MSS Cairo MM 43, MM 98, DM 758, and Princeton Yahuda 861,2 are Egyptian copies of these tables. Evidence exists that the leading Egyptian astronomers in the 15th century, men such as ʿIzz al-Dīn ʿAbd al-ʿAzīz ibn Muḥammad al-Wafāʾī[55] and Shams al-Dīn Muḥammad called Ibn Abi 'l-Fatḥ al-Ṣūfī,[56] had seen the tables. From a passing remark by al-Wafāʾī in his treatise *Khulāṣat al-durar* (MS Manchester Rylands 361, fol. 45r) it can be inferred that the expressions "first and second functions" (*al-maḥfūẓ al-awwal* and *al-thānī*) were well known to contemporary Egyptian astronomers. al-Wafāʾī advocates the use of these functions without even defining them. See further **6.14**.

The Egyptian astronomer Muḥammad ibn Aḥmad al-Bilbaysī wrote a commentary on the auxiliary tables,[57] stating that he found al-Khalīlī's instructions to be long-winded. This commentary, extant in the unique MS Cairo DM 442, copied *ca*. 1700, includes Sharaf al-Dīn al-Khalīlī's instructions about the use of his uncle's tables to compute functions for predicting lunar crescent visibility (also found in MS Berlin Wetzstein 1138 (Ahlwardt 5754-6).

MS Cairo MM 167,8, copied in 989 H [= 1581/82], contains a fragment of an anonymous Egyptian treatise on the sine quadrant with many numerical examples, mostly for latitude 30° (*i.e*. Cairo) and obliquity 23;35°. The author deals at length with the functions f and g and the determination of the qibla ($\phi_M = 21°$). He mentions various earlier Mamluk astronomers such as ʿAlāʾ al-Dīn Ṭībughā al-Baklamīshī, (Taqi 'l-Dīn) Abū Ṭāhir, and Jamāl al-Dīn ʿAbdallāh ibn Khalīl al-Māridīnī,[58] but not al-Khalīlī. MSS Cairo MM 177,2, and Cairo DM 624,2, both copied *ca.* 1700-1750, are two other treatises dealing with al-Khalīlī's functions f and g, but neither of them mentions his name.[59]

Maghribi copy
MS London BL Add. 9599,31 is a late Maghribi copy of al-Khalīlī's universal auxiliary tables. This could be taken as evidence that it also travelled to the Islamic West, but the possibility of a Maghribi copyist working in, say, Cairo, should not be excluded. See further **13.4**.

Turkish copies
MS Istanbul Hamidiye 1453/3 is a copy of the tables prepared about 1465 in Edirne, while MS Istanbul Ayasofya 2590 contains another copy of the tables compiled in 1491 by the

---

55 On al-Wafāʾī (Suter, no. 437, and *Cairo ENL Survey*, no. C61) see n. **I**-9:29.
56 On al-Ṣūfī (Suter, nos. 447/460, and *Cairo ENL Survey*, no. C98) see n. **I**-9:30.
57 On al-Bilbaysī see *Cairo ENL Survey*, no. C80.
58 On these see *Cairo ENL Survey*, nos. C53, C56, and C47 (also Suter, *MAA*, no. 421).
59 *Ibid*., nos. C145 and D232.

Istanbul *muwaqqit* Muḥammad ibn Kātib Sinān,[60] preceded by a Turkish translation of al-Khalīlī's instructions. An unnumbered manuscript formerly (*ca*. 1970) in the private collection of the late Professor Buhairi, American University of Beirut contains tables of f and g for $\phi = 41°$ (Istanbul) and also of K for x = 41. The hapless individual who put these three tables together was unaware that the argument x is in no way related to the latitude. See further **14.6**.

Qibla table and list of qibla values

al-Khalīlī's qibla table was not widely used after his time, and no references to it have been found in treatises by any contemporary or later Muslim astronomers. On the other hand, his list of qiblas appears in several later Syrian and Egyptian manuscripts. Even so, the mathematically computed qibla values for various Syrian cities were not always taken seriously by those who built mosques.

In Syria early mosques faced the traditional qibla direction of due south, while later mosques are often oriented at variance to the directions which can be computed from medieval geographical coordinates. This is particularly evident in the new Mamluk city of Tripoli, where the religious architecture faces several different directions.[61] The modern qibla value for Damascus (based on the correct longitude difference from Mecca) is some 15° closer to south than al-Khalīlī's value, but this is irrelevant to any discussion of the orientation of medieval mosques.

No new tables of any real consequence were compiled in Syria after the time of al-Khalīlī (see **11**), just as no new Syrian *zīj*es of consequence were compiled after the time of Ibn al-Shāṭir.

---

[60] On Muḥammad ibn Kātib Sinān see nn. **I-**2:43 and **II-**14:28.
[61] On the whole problem of mosque orientation see now **VIIa**.

CHAPTER 11

LATER SYRIAN TABLES FOR TIMEKEEPING

11.0 Introductory remarks

After the destruction of Damascus in 1401 there was little original in Syrian astronomy, but the activity certainly continued. In the early 15th century Shihāb al-Dīn al-Ḥalabī prepared a redaction for Damascus of the *Īlkhānī Zīj* (Maragha, mid 13th century);[1] at the end of that century ʿAbd al-Raḥmān al-Ṣāliḥī prepared a redaction for Damascus of the *Zīj* of Ulugh Beg (Samarqand, *ca*. 1430);[2] and in the mid 16th century Ibn al-Kayyāl prepared a redaction for Damascus of al-Kāshī's *Khāqānī Zīj* (Samarqand, *ca*. 1438).[3] Recensions of the *Zīj* of Ibn al-Shāṭir (Damascus, *ca*. 1350) were prepared in the mid 16thcentury by Ibn Zurayq,[4] around 1600 by al-Qazwīnī, and even in the 18th century by al-Mukhallālātī.[5] There was also some activity in astronomical timekeeping. Shihāb al-Dīn al-Ḥalabī contributed some solar azimuth tables to the main Damascus corpus (**11.2**), and Ibn al-Kayyāl compiled some calendrical and solar longitude tables which were used with al-Khalīlī's prayer-tables until the 19th century (**11.4**). Various sets of prayer-tables prepared in order to serve other cities in Syria such as Aleppo, Tripoli, Nablus and Lattakia (**11.5-9**). In the 19th century al-Ṭanṭāwī made an attempt to revive traditional astronomy in Damascus, and brought up to date the calendrical and solar tables, which were used alongside al-Khalīlī's prayer-tables (**11.13**).

11.1 Anonymous prayer-tables for Damascus

In MS Paris BNF ar. 2521, fols. 69v-75r, penned in the 16th century, there is an anonymous set of prayer-tables which the title states are for latitude 33;6°. Elsewhere in the same manuscript (fols. 14v-16r), there are prayer-tables based on those of al-Khalīlī (discussed in **10.11**), for parameters ϕ = 33;30° and ε = 23;31°. The anonymous set gives values of the following functions for each degree of λ, a page of tables serving each zodiacal sign:

$$2D^h, \delta, H, D, t_a, h_a, T_a, n, 2N, s \text{ and } r .$$

[1] On al-Ḥalabī (**5.2**) see n. 11:6. His *Zīj* is preserved in MS Cairo M 226,1 (see *Cairo ENL Survey*, no. C69/2.1.12).

[2] On al-Ṣāliḥī see Suter, *MAA*, no. 454; *Cairo ENL Survey*, no. C87; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 204, 205, and 225.

[3] On Ibn al-Kayyāl (**11.4**) and n. 11:8. This work of his is extant in MS Dublin CB 4677.

[4] On Ibn Zurayq see Suter, *MAA*, no. 426; Mayer, *Islamic Astrolabists*, suppl., p. 296; *Cairo ENL Survey*, no. C116; and İhsanoğlu *et al*., *op. cit*., I, pp. 155-158, no. 79. On various recensions of Ibn al-Shāṭir's *Zīj* see my article "Ibn al-Shāṭir" in *DSB*.

[5] On al-Qazwinī see *Cairo ENL Survey*, no. D38; and İhsanoğlu *et al*., *op. cit*., I, pp. 301-302, no. 160. On al-Mukhallalātī see *Cairo ENL Survey*, no. D90, and İhsanoğlu *et al*., *op. cit*., II, pp. 544-545.

It is immediately apparent that the tables are very corrupt, and analyzing them is made difficult by the fact that the individual who compiled them did not understand the argument system used in the tables from which he was copying. Thus the entries in his tables do not display the symmetry which the tabulated functions naturally enjoy.

The table of δ is based on ε = 23;35°, but the values of H are not consistent with the corresponding values of δ or any particular value of ϕ. It seems, however, that the compiler wished his tables to work for ϕ = 33;30°, because most of his values are lifted from al-Khalīlī's prayer-tables. It is also curious that the second digit in these entries for D assumes values only between 0 and 5, which suggests that the units used are sixths of a degree.

An analysis of the entries for a particular longitude will suffice to illustrate their deficiency. We consider λ = 45°, and first note that the corresponding entries for λ = 135°, which should be the same, differ in several instances. The entries 13;23° (λ = 45°) and 13;25° (λ = 135°) for $2D^h$ differ from al-Khalīlī's value 13;30°. The entry 16;26° for δ is accurate for ε = 23;35°, and the entry 72;56° for H corresponds to ϕ = 33;30°. For D our table has 100;2° (λ = 45°), which seems to mean 100;20°, and 100;3° (λ = 135°), that is 100,30°: the accurate value is about 101;15°. The entries 55;38° and 45;48° for t_a and T_a are al-Khalīlī's values for λ = *46*°, but the entry 37;25° for h_a is his for λ = 45°. The entries 108;59° and 157;34° for n and 2N are likewise al-Khalīlī's for λ = 45°. The 23;4° (λ = 45°) and 22;56° (λ = 135°) for s differ from al-Khalīlī's 22;50° (λ = 45°) and 22;54° (λ = 46°), but the 25;50° (λ = 45° and 135°) for r is al-Khalīlī's for λ = *46*°.

The tables are thus full of inconsistencies, and it was doubtless from tables of this kind originally computed for Cairo that those described in **7.6** were copied out in words. Two sets of these corrupt prayer-tables now preserved in Syria, MSS Aleppo Aḥmadiyya 1310 and Damascus 9227, appear to be based on tables computed for Cairo.

11.2 Shihāb al-Dīn al-Ḥalabī's solar azimuth and twilight tables

The Damascus astronomer Shihāb al-Dīn Aḥmad ibn Ibrāhīm al-Ḥalabī (d. 1455)[6] computed a set of tables of the function a(h,λ), measured from the prime vertical, for each integral degree of h and each degree of λ. The underlying parameters, that is, the values used for ϕ and ε, are the same as those used by al-Khalīlī (see **10.5-6**). These tables are preserved in MSS Cairo K 8525, copied in al-Ḥalabī's own hand and thus datable *ca*. 1425, Cairo MM 71,1, copied *ca*. 1600, and Damascus 9227. The first manuscript contains al-Khalīlī's prayer-tables, and al-Khalīlī's values of t(h,λ) are tabulated side by side with al-Ḥalabī's value of a(h,λ). In the second the function a(h,λ) is tabulated separately, and in the third, triplets of values (t,T,a) are given, the first pair of functions being attributed to al-Khalīlī.

al-Ḥalabī's azimuth tables, which are fairly accurately computed, are similar in conception to those of Ibn Yūnus for Cairo, computed over four centuries previously (**4.4** and **5.2**), and to the later azimuth tables computed for Alexandria (**8.5**), but they have al-Khalīlī's format. al-Khalīlī had also compiled azimuth tables for the latitude of Damascus, but having arguments H and h rather than h and λ (**10.3**).

---

[6] On al-Ḥalabī see Suter, *MAA*, no. 434; Brockelmann, *GAL*, II, p. 159; and *Cairo ENL Survey*, no. C69.

In MS Cairo K 8525, copied by al-Ḥalabī himself, he also reproduces al-Khalīlī's tables of α', and α_ϕ and α_s for Damascus. His table of $\alpha_r(\lambda)$ is presented as his own calculation, and differs from that of al-Khalīlī in that it is based on the parameter 20°. al-Ḥalabī further includes (fols. 14r-20r) a set of tables displaying the time remaining until twilight in equatorial degrees and minutes as a function of solar longitude when any of seven prominent stars (which vary from sign to sign) are culminating (*cf.* **I-2.7.1**). In his brief introduction to these tables he states that daybreak (*al-fajr*) is the time of the prayer-call (*al-ādhān*) in Ramaḍān and of the *salām* (written *s-l-m*) in the other months of the year (see further **4.10** on the institutions associated with twilight in medieval Cairo).

11.3 Ibn Barakāt's tables for the afternoon prayer

MS Damascus Ẓāhiriyya 9227 of Shihāb al-Dīn al-Ḥalabī's solar azimuth tables (**11.2**) also contains a set of tables attributed to Muḥammad ibn Barakāt ibn al-Buṣrawī, on whom I have no further information.[7] These display $\tau_a(\lambda,h)$, the time to the beginning of the interval for the afternoon prayer as a function of solar longitude and solar altitude in the eastern sky over Damascus. The entries are based on the simple relation:

$$\tau_a(\lambda,h) \quad = \quad t_a(\lambda) - t(h,\lambda) \ ,$$

and for each degree of λ' the arguments for h are the integral values such that $h_a(\lambda) \leq h \leq H(\lambda)$. A marginal note by al-Ṭanṭāwī (**11.13**) explains the method of computing the entries in the table from those of $t_a(\lambda)$ and $t(h,\lambda)$. See also **7.3** on some similar tables for Cairo.

In MS Berlin Ahlwardt 5754/5/6, fols. 62v-94r, there are some anonymous tables appended to others by al-Khalīlī (**10.5**) of a function which is described as *al-maktūb bi-'l-sawād al-bāqī li-'l-ghurūb wa-bi-'l-ḥamra li-'l-ʿaṣr*, "time remaining until sunset if written in black ink or time remaining until the beginning of the afternoon prayer if written in red ink". The tables have the same format and the same argument domains as al-Khalīlī's hour-angle tables, and it is understood that the altitudes are eastern. The tabulated function is simply:

$$f(h,\lambda) \quad = \quad T(h,\lambda) \text{ for } h < h_a(\lambda) \quad \text{and} \quad \tau_a(h,\lambda) \text{ for } h \geq h_a(\lambda) \ .$$

Note that the tables of τ_a in the Damascus and Berlin manuscipts have different formats.

11.4 Ibn al-Kayyāl's prayer-tables

MSS Damascus Ẓāhiriyya 4893, Istanbul Esat Efendi 1990, Berlin Ahlwardt 5759/5771 (Wetzstein 1146), *ca*. 1700, and 5760/5772 (Wetzstein 1148), *ca*. 1800, are copies of al-Khalīlī's prayer-tables (**10.6**) in which the main tables are preceded by an introduction and some calendrical tables by the mid-16th-century Damascus astronomer Ibn al-Kayyāl.[8] There is no mention of al-Khalīlī.

---

[7] Ibn Barakāt, whose full name is given in MS Damascus Ẓāhiriyya 9227, is not listed in the modern sources.
[8] On Ibn al-Kayyāl see Suter, *MAA*, no. 474; Brockelmann, *GAL*, II, p. 469; *Cairo ENL Survey*, no. D60; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, p. 127, no. 59.

11.5 Anonymous prayer-tables for Aleppo and Istanbul

MS Cairo ṬM 255,6, copied 1060 H [= *ca*. 1650], contains al-Khalīlī's introduction to his prayer-tables followed by some anonymous prayer-tables for latitudes:

ϕ = 36° (Aleppo) and 41;15° (Istanbul).[9]

Mention of these was omitted from the first version of this text, and all that is available to me now is a photo of an extract from the tables for Aleppo (**Fig. 11.5**). This shows tables of the same kind as those of al-Khalīlī but for ε = 23;35° rather than his more up-to-date value 23;31°. The functions tabulated are:

$H, Z, D, h_a, t_a, T_a, N, n, r, s, \tilde{h}, d$ and t_q .

The quantities t_a and T_a are labelled *ḥiṣṣat al-ʿaṣr* and *ḥiṣṣat al-ghurūb*, expressions which I do not recall having seen elsewhere (though see below). I have not investigated the parameters used for r, s and q. See also **14.8**.

Fig. 11.5: An extract from an anonymous set of prayer-tables for the latitudes of Aleppo and Istanbul. [From MS Cairo ṬM 255,6, fols. 78v-79r, courtesy of the Egyptian National Library.

MS Aleppo ʿUthmāniyya 14, copied *ca*. 1750, contains another anonymous set of prayer-tables for Aleppo. Values of the following functions are displayed in equatorial degrees and minutes for each degree of λ from Capricorn 1°:

D, t_a, T_a, s, n and r

and I have not investigated the underlying parameters. The entries are written in Arabic num-

[9] *Cairo ENL Survey*, no. C142.

erical notation. The five functions are labelled *ḥiṣṣat al-zuhr*, *ḥiṣṣat al-ʿaṣr*, *ḥiṣṣat al-maghrib*, *ḥiṣṣat al-ʿishāʾ*, and *ḥiṣṣat al-fajr*.

11.6 Anonymous prayer-tables for Tripoli

MS Damascus Ẓāhiriyya 4893 contains calendrical and solar tables due to Ibn al-Kayyāl (**11.4**), followed by some anonymous prayer-tables for the latitude of Tripoli, stated to be 34°. (See also **9.3** on two earlier sets of prayer-tables for the latitude of Tripoli, whioch may ber related to each other and to some of these.) The functions tabulated are:

$$D,\ t_a,\ T_a,\ s,\ n,\ r \text{ and } z_{a(7)}\ .$$

The entries are arranged in six columns of thirty, beginning with the column for Capricorn/Aquarius, although the entries for D(λ) begin with the column for Aries/Virgo. Analysis reveals firstly that the table of D(λ) is based on the parameters:

$$\phi = 33;45° \text{ and } \varepsilon = 23;31°.$$

Some of the entries are garbled in such a way as to give the impression that they were copied from another set with a different format: the original table appears to have been rather accurately computed. Secondly, certain of the other tables, including those of t_a, r and s, are the same as those for latitude 34° in MS Princeton Yahuda 861,1 of the *Natīja* attributed to al-Wafāʾī (**8.1**). Thus they are supposedly based on a value of 23;35° for ε, but the entries were so carelessly computed that the value of ε is of little significance. Finally, the table of n(λ) is based on the above-mentioned table of D(λ) (computed for ϕ = 33;45° and ε = 23;31°) and the anonymous values for r(λ) and s(λ) in the Princeton manuscript (computed for ϕ = 34;0° and ε = 23;35°)!

11.7 Anonymous prayer-tables for Nablus

MS Berlin Ahlwardt 5765 [= Wetzstein 1149, fols. 85v-88v], copied *ca*. 1850, consists of anonymous prayer-tables compiled for the latitude of Nablus, taken as 32°.[10] The tables are preceded by a short introduction in which the title *Natījat al-afkār fī aʿmāl mawāqīt al-layl wa-'l-nahār* (compare the title of al-Lādhiqī's tables for Cairo in **7.8**) is mentioned. The functions tabulated for each degree of solar longitude are:

$$D,\ t_a,\ T_a,\ s,\ n \text{ and } r\ ,$$

expressed in equinoctial hours and minutes. Values are displayed side by side in columns, and are written in modern Arabic numerals in an untidy hand – see **Fig. 11.7** for an extract. They are generally inconsistent for a given solar longitude. Consider for example, the entries for Gemini 1° (λ = 61°), which are:

$$D:\ 7;4^h \quad t_a:\ 3;56^h \quad T_a:\ 3;3^h \quad s:\ 1;49^h \quad n:\ 8;40^h \quad r:\ 1;33^h\ .$$

[10] The only medieval source which gives 32;0° for Nablus (32;10° is a common value) is labelled "*qiyās*" ("measurement") by Abu 'l-Fidāʾ: see Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. xxx and 235. Given the crude nature of the tables, this may not be significant: possibly 32° was simply rounded from 32;10°.

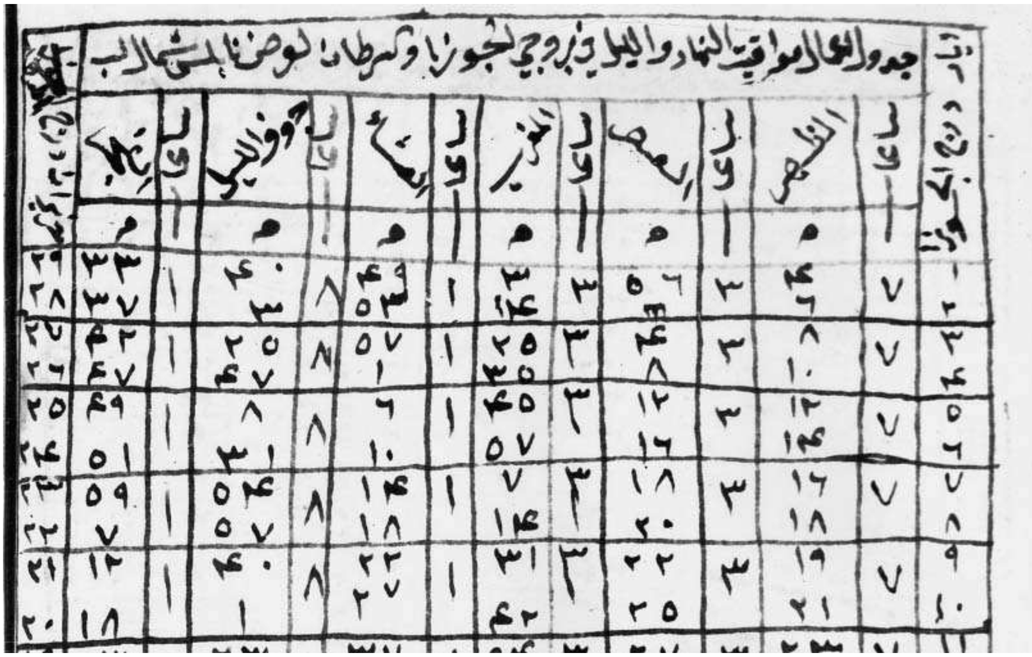


Fig. 11.7: An extract from some messy prayer-tables for Nablus. [From MS Berlin Ahlwardt 5765 [< Wetzstein 1149], fol. 87v, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

Notice that the sum of these values, which should correspond to a complete day, is over 26 hours.

11.8 al-Dīsṭī's prayer-tables for Lattakia

MSS Aleppo Awqāf 911 and Leiden Or. 2808(2) are copies of a work entitled *Asna 'l-ghāyāt fī ʿilm al-mīqāt,* "The Best Ends in Timekeeping", compiled by ʿAbd al-Fattāḥ ibn Ibrāhīm al-Mālikī al-Azharī al-Dīsṭī,[11] an Egyptian astronomer who was a student of Riḍwān Efendī (**7.10**). After a brief introduction in 3 *bāb*s al-Dīsṭī presents a set of prayer-tables for parameters:

$$\phi = 34;30° \text{ (Lattakia) and } \varepsilon = 23;35°.$$

The format of the tables is the same as that of the main Cairo corpus. The functions tabulated are:

$$\delta,\ d,\ H,\ D,\ h_a,\ t_a,\ T_a,\ s,\ r,\ \Delta D,\ B,\ C,\ h_0,\ 2N,\ \alpha \text{ and } \alpha_\phi\ .$$

The values of ΔD are those for Cairo found, for example, in MS Cairo DM 108 of the main Cairo corpus (**4.11**). The work concludes with a table of the solar motion between the prayer-times as in the tables of Shams al-Dīn al-Lādhiqī (**7.8**), and a star catalogue for 1160 H [= 1747].

[11] On al-Dīsṭī (**I-4.8.7**, *etc.*) see Brockelmann, *GAL*, SII, pp. 1017, where the manuscript of his prayer-tables which is now in Leiden is listed; *Cairo ENL Survey*, no. D62 (there incorrectly as al-Daystī without a dot); and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, p. 514, no. 352 (now as al-Daysaṭī).

11.9 al-Ḥakīm al-Lādhiqī's tables for Lattakia

Cairo ṬM 92, copied *ca*. 1750, contains a small *zīj* by an individual named Muḥammad ibn ʿAbd al-Maḥmūd al-Lādhiqī compiled in Lattakia in 1170 H [= 1756/57].[12] This contains tables of three spherical astronomical functions, namely, δ, α', and α_ϕ, computed to two digits for each degree of λ and based on the parameters:

$$\phi = 34;30° \text{ (Lattakia) and } \varepsilon = 23;35°.$$

MS Istanbul Bağdatli Vehbi Efendi 887 contains a selection of anonymous planetary tables as well as others for α' and α_ϕ attributed to "al-Ḥakīm al-Lādhiqī".

The tables are of interest in that the values in the table of $\alpha_\phi(\lambda)$ have been derived by linear interpolation between the entries for λ = 0°, 30°, 60°, and 90°, which are rather accurately computed.[13] I know of no other Islamic ascension tables as crude as this, although see **8.6** on some late Egyptian tables also based on a simplistic interpolation scheme. The table of $d(\lambda)$, on the other hand, is not based on such a scheme, and so al-Lādhiqī had tables of $\alpha'(\lambda)$ and $d(\lambda)$ at his disposal with which he could have compiled a proper table of $\alpha_\phi(\lambda)$, but he preferred his own curious technique.

11.10 al-Manāshīrī's tables for Damascus, Cairo, Mecca, and Istanbul

MS Cairo DM 184, penned in 1171 H [= 1757/58], is the only copy known to me of a treatise on astronomy compiled in the early 17th century by Muḥammad ibn Maḥmūd al-Manāshīrī, otherwise known for his work in the Qurʾānic sciences.[14] al-Manāshīrī presents various calendrical and planetary tables and also four sets of prayer-tables, for Damascus, Cairo, Mecca, and Istanbul. The tables for Damascus are those of al-Khalīlī (**10.6**), based on parameters:

$$\phi = 33;30° \text{ and } \varepsilon = 23;31° ,$$

and are attributed to him. The functions tabulated include various ones not generally displayed in the main Damascus corpus (**10.6**):

$$\text{Sin } H, \text{ Sin } \delta, \text{ and Sin } \psi .$$

al-Manāshīrī tabulates some twenty six functions side by side for each degree of λ starting at Capricorn 1°, namely:

$$H, \text{ Sin } H, Z_{(12)}, Z_{(7)}, D, 2D^h, \tilde{h}, d, \delta, \text{ Sin } \delta, h_a, t_a, T_a, h_b,$$
$$2N, N, 2N^h, \tilde{h}', s, r, \psi, \text{ Sin } \psi, h_0, n \text{ and } t_q .$$

In the column for Sin ψ ($\delta > 0$) al-Manāshīrī tabulates al-Khalīlī's values. However in the column for Sin ψ ($\delta < 0$) a completely different function appears, which I am unable to identify. Sample values from this garbled table are:

1°	89;3	31°	76;31	61°	165;25°
15	89;1	45	69;43	75	60;29°
30	88;56	60	62;31	91	57;36°

[12] On this al-Lādhiqī see *Cairo ENL Survey*, no. D66.
[13] See n. **I**-1:21.
[14] On al-Manāshīrī (**I-6.1.2**) see Brockelmann, *GAL*, I, p. 427; *Cairo ENL Survey*, no. D8; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. pp. 274-275, no. 138.

The prayer-tables for Cairo are taken from the main corpus, although this is not stated, and are thus based on the parameters:

$$\phi = 30;0° \text{ and } \varepsilon = 23;35° .$$

The columns are ruled for functions:

$$H, D, h_a, t_a, T_a, s, r, 2N \text{ and } n ,$$

but only the values for D, t_a, T_a, s and r have been filled in.

The prayer-tables for Mecca and Istanbul are not attested in any other source known to me. The functions tabulated are:

$$H, D, h_a, t_a, T_a, s, r, \text{ and } 2N .$$

It is stated that the underlying latitudes are 21;30° and 41;15° but in fact the tables for Istanbul are based on latitude 41;0°. The obliquity is 23;31° in both cases, and the parameters for morning and evening twilight are 19° and 17°. It seems unlikely that al-Manāshīrī compiled the prayer-tables for Mecca and Istanbul himself. However, in view of the fact that both sets are based on obliquity 23;31°, a value used in *mīqāt* tables only – as far as we know – by al-Khalīlī and Ibn al-Shāṭir, it seems probable that these tables were compiled in Damascus.

al-Manāshīrī states in his introduction that his calendrical tables are based on the techniques of ʿAbd al-Laṭīf ibn Ibrāhīm al-Kayyāl (**11.4**), which were in turn taken over from "al-Suyūṭī al-Jalāl ibn al-Kamāl" (?).[15] In his discussion of the determination of the prayer-times he refers to the statement of Sharaf al-Dīn al-Khalīlī concerning the observations made on Jabal Qāsiyūn with his uncle Shams al-Dīn al-Khalīlī.

11.11 Ottoman-type prayer-tables for Aleppo

MS Damascus Ẓāhiriyya 7564, fols. 53r-55v, contains an anonymous set of prayer-tables of the same type as those of Darendelī (**14.4**), computed for latitude 36°. These are followed (fol. 56r) by a set of geographical coordinates in which no locality is listed with latitude 36°. I presume the tables were intended for use in Aleppo, for which the geographical tables in the Damascus manuscript give the widely-accepted latitude 35;50°. The functions tabulated are:

$$2D, m', a', s', r' \text{ and } R' ,$$

with values in equinoctial hours and minutes, rather accurately computed. The entries are written in standard Arabic numerals, with some unusual abbreviations. The parameters for morning and evening twilight are 19° and 18°.

In the top margin at the beginning of these tables it is stated that:

(a) the interval between the appearance of the zodiacal light[16] and the time to extinguish the candles (*bayn ṭulūʿ al-raʾs wa-ṭafy al-qanādīl*) is 0 45 minutes;

[15] This can hardly be a reference to the famous 16th-century polymath Jalāl al-Dīn al-Suyūṭī (Suter, *MAA*, no. 449; *Cairo ENL Survey*, no. 103). His treatise on the references to astronomy in the *Qurʾān* and *ḥadīth* has been published in Heinen, *Islamic Cosmology*. However, al-Suyuṭī is not known to have written anything of consequence on the calendar. On his poem on the midday shadows for each month of the Coptic year see **III-9.7a**.

[16] See the article "Shafaḳ" on twilight in EI_2.

(b) the interval between the time to extinguish the candles and the time to start fasting in Ramaḍān (*bayn al-ṭafy wa-'l-imsāk*) is 0 25 minutes; and

(c) the interval between the time to start fasting and daybreak (*bayn al-imsāk wa-'l-fajr*) is 0 45 minutes.

I am not sure what these intervals mean, as the time between the appearance of the zodiacal light and the true dawn can hardly be 0;45 + 0;25 + 0;45 = 1;55 hours. See **4.10** on these institutions as cultivated in Cairo.

11.12 Anonymous timekeeping tables for Jerusalem and Nablus

MS Cairo ṬM 81,1 (fols. 1r-84v), copied *ca*. 1900, contains a set of tables of the function $T(\lambda,h)$ for altitudes in the east and west, and with entries in hours and minutes for each degree of both arguments. The underlying latitude is found by inspection to be $\phi = 32°$ (Jerusalem), and values of $D(\lambda)$ are given on each sub-table of entries: see **Fig. I-2.2.6**. These tables are followed by a set of tables of the function $t(h,\lambda)$ with values in degrees and minutes, for parameters:

$$\phi = 32;0° \text{ (Jerusalem) and } \varepsilon = 23;35° .$$

These tables are copied without the solar longitude arguments, and without the degrees of the entries, except at the head of each column. These are probably the tables computed by Ibn al-Rashīdī (**I-2.1.5** and **II-6.12**). The manuscript concludes with tables of the three functions:

$$N, \ d \text{ and } h_0$$

for latitude $\phi = 32;10°$ (Nablus).

I suspect that all of these tables in the Cairo manuscript were copied from an original in the Khālidiyya Library in Jerusalem.

11.13 al-Ṭanṭāwī's prayer-tables for Damascus

Muḥammad ibn Muṣṭafā ibn Yūsuf al-Ṭantāwī al-Ṭanda'tā'ī al-Azharī al-Dimashqī was a *muwaqqit* in the Umayyad Mosque at Damascus who died in 1889, and thus represents the last period of "medieval" Islamic astronomy in Syria.[17] Numerous of the astronomical manuscripts in the Ẓāhiriyya Library in Damascus are written in his hand: in particular, two manuscripts are actually attributed to him. The first, MS 9353, is a collection of tables for constructing sundials, in which he used on the parameters (ϕ and ε) of Ibn al-Shāṭir. It was al-Ṭanṭāwī who by accident broke the sundial of his illustrious predecessor which embellished the main minaret (*al-ʿarūs*) of the Umayyad Mosque in Damascus: whilst trying to realign it: he constructed a new one with identical markings which is still in position on a balcony of the minaret.[18] The second, MS 9233, contains a brief introduction explaining the calendrical and solar tables which form a small part of the manuscript (pp. 1-21). These are followed by a complete set of al-Khalīlī's hour-angle tables, prayer-tables and tables of ascensions (pp. 22-156), but the only reference to al-Khalīlī is a statement that it was he who computed the tables

[17] He is not mentioned in the modern sources.
[18] See n. 9:29.

of Cot_n $H(\lambda)$ and Cot_n $h_a(\lambda)$ (n = 7 and 12). These are not contained in MS Paris ar. 2558 of al-Khalīlī's prayer-tables (**10.2**). MS Damascus 9233 concludes with some tables (pp. 157-161) for the lunar mansions, computed for the years 1292 and 1294 H [= 1875/77]. Finally, MS 7387 contains a set of tables attributed to al-Ṭanṭāwī displaying the function T(λ,h) expressed in equinoctial hours and minutes for each integral degree of both arguments. I have little doubt that these were computed from the corresponding values of T(h,λ) in equatorial degrees and minutes in the tables of al-Khalīlī (*cf.* **10.5**) by a simple division by 15. Note that the format of the two sets of tables is different.

MS Cairo ṬM 173, copied *ca*. 1900, is the only one of the manuscripts associated with al-Ṭanṭāwī that I have located which contains a set of prayer-tables for Damascus with times expressed according to the Ottoman convention (**14.0**). Here:

$$s', i', r', R', m' \text{ and } a'$$

are displayed side by side for each degree of λ starting with Capricorn 1°. The time of the *imsāk*, i′, is always 15^m before daybreak, r′.

Several of the manuscripts containing tables associated with al-Ṭanṭāwī, such as MSS Damascus Ẓāhiriyya 3116, fols. 1v-2v, 9233, p. 37, 10076, fol. 16r, and 7388, fol. 16v, contain a table computed by him entitled *daqāʾiq al-ikhtilāf wa-niṣf quṭr al-shams li-ʿarḍ lām-jīm lām*, "the difference minutes and the radius of the sun for latitude 33;30°", which purports to represent the interval in equatorial minutes between the time when the centre of the sun is on the true astronomical horizon, and the time when the solar disc has disappeared below the visible horizon at Damascus. Values are given for each degree of λ to one or two digits and are derived by linear interpolation from three significant entries, namely:

EQ: 67 SS: 84 WS: 51 .

In two of the sources, MSS Damascus Ẓāhiriyya 9233, p. 9, and 7388, fol. 16v, it is stated that the value used for the radius of the sun is 0;15°, and if this is subtracted from the entries in the table the resulting values are the time taken for the centre of the sun to move between the two horizons, that is, the ΔD(λ) of earlier Egyptian sources (**4.11**, **5.7**, **6.10**, **7.1**, **8.2**, and **8.8**). Indeed, if we subtract 15 minutes from the above values we obtain:

EQ: 52 SS: 69 WS: 36 .

If we multiply the corresponding values for Cairo, namely, 47, 62, and 32 minutes, by the ratio of the latitude of Damascus to that of Cairo, namely, 33;30°/30°, according to the method of ʿAbd al-Qādir al-Minūfī (**8.2**), we obtain:

EQ: 52;29 SS: 69: 14 WS: 35;44 .

which, when rounded, are the same as those underlying al-Ṭanṭāwī's tables. (If we assume that the distance between the true horizon and the circle 0;15° below the visible horizon is 0;40° + 0;15° = 0;55°, then the time in equatorial minutes taken by the sun to move between these two positions is actually 66 at the equinoxes and 74 at both solstices.)

In MS Damascus 9233, p. 9, there are some notes in the hand of al-Ṭanṭāwī which explain the use of this table to modify the various functions of timekeeping which relate to horizon phenomena. The procedures he outlines are along the same lines as those of Muḥammad al-Minūfī (**7.1**). al-Ṭanṭāwī concludes with the statement that all the *muwaqqit*s use a fixed amount to adjust for horizon phenomena. They call this quantity, which does not vary throughout the

year, *tamkīn*. al-Ṭanṭāwī complains that the *muwaqqit*s do not know why they do this and they fall into error. What surprises him more, he says, is that they do not know the truth and will not accept it from one who does. al-Ṭanṭāwī was very much in control of his subject, but his hapless colleagues represent the ultimate decline of Syrian astronomy, and the end of the brilliant tradition associated with al-Mizzī, Ibn al-Shāṭir, and al-Khalīlī.

In 1971 the Director of the Umayyad Mosque in Damascus told me that he had never heard of al-Ṭanṭāwī, the last traditional *muwaqqit* at the Mosque not dead a century, let alone al-Khalīlī. The manuscripts in the nearby Ẓāhiriyya Library had just been catalogued for the first time, but the name of al-Khalīlī does not appear there: the tables attributed to Ibn al-Kayyāl and al-Ṭanṭāwī were listed but had not inspired any interest locally.

11.14 ʿAbdallāh al-Ḥalabī's prayer-tables for Aleppo

MS Aleppo Awqāf 943, copied *ca*. 1800, contains an anonymous set of anonymous auxiliary tables for computing the hour-angle (**I-6.14.4**), and a set of prayer-tables for parameters:

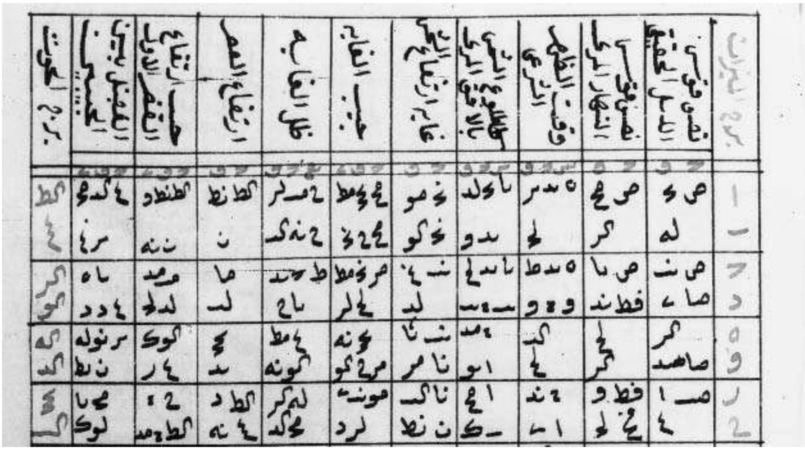
$$\phi = 35;50° \text{ (Aleppo) and } \varepsilon = 23;30°$$

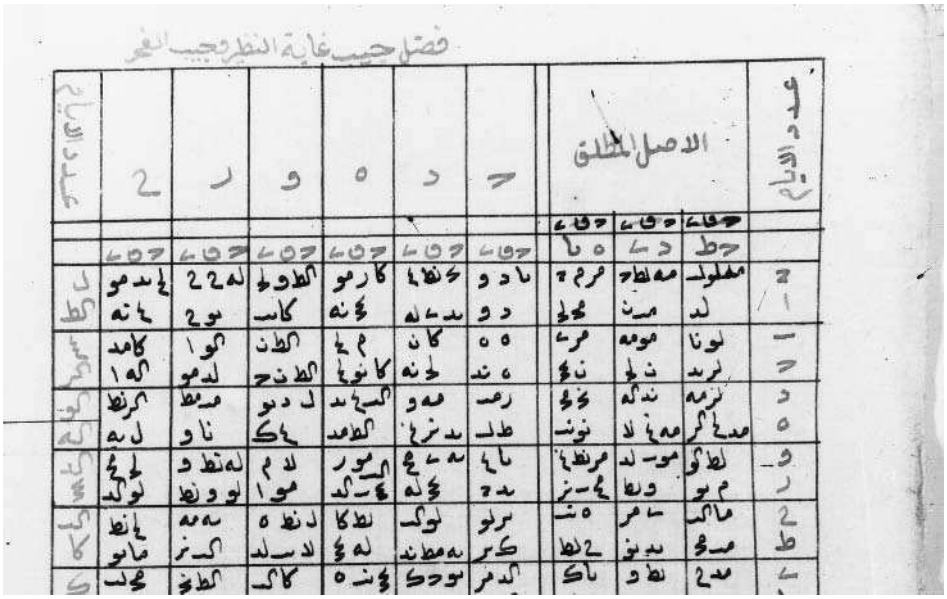
attributed to ʿAbdallāh al-Ḥalabī, *muwaqqit* at the Umayyad Mosque in Aleppo *ca*. 1750.[19] The same prayer-tables occur in MS Aleppo Awqāf 962. The functions tabulated are the following:

N	*niṣf qaws al-layl al-ḥaqīqī*
D	*niṣf qaws al-nahār al-marʾī*
m'	*waqt al-ẓuhr al-sharʿī*
R'	*ṭulūʿ al-shams bi-ʾl-ufuq al-marʾī*
H	*ghāyat irtifāʿ al-shams*
Sin H	*jayb al-ghāya*
Z = Cot H	*ẓill al-ghāya*
h_a	*irtifāʿ al-ʿaṣr al-awwal*
Sin h_a	*jayb al-ʿaṣr al-awwal*
Sin H - Sin h_a	*al-faḍl bayn al-jaybayn*
Vers t_a	*sahm dāʾir al-ʿaṣr*
t_a	*faḍl dāʾir al-ʿaṣr*
a'	*waqt al-ʿaṣr al-awwal*
h_b, ... , b'	ditto (6 functions) for *al-ʿaṣr al-thānī*
Vers (D + r)	*sahm faḍl dāʾir al-fajr*
D + r	*faḍl dāʾir al-fajr*
r'	*waqt al-fajr*

Values are given for each degree of λ beginning with Cancer 0°. The times are given according to the Ottoman convention, and are expressed in hours, minutes and seconds, and values of other functions are given to two digits. Separate tables display values of:

[19] On ʿAbdallāh al-Ḥalabī (**I-6.2.7**) see İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 560-561, no. 398.





Figs. 11.14a-b: An extract from ʿAbdallāh al-Ḥalabī's prayer-tables, showing the first half of the entries (a), and his tables of the absolute base (90 values) and the difference between the Sines of the solar meridian altitude for 180° plus the solar longitude and the Sine of 19° (b). [From MS Aleppo Awqāf 943, courtesy of the Aḥmadiyya Library.]

Sin d	*jayb taʿdīl niṣf qaws al-nahār al-ḥaqīqī*
d	*taʿdīl ...*
Cos δ	*jayb tamām al-mayl al-awwal*
B	*al-aṣl al-muṭlaq*
Sin H(λ*) - Sin 19°	*faḍl jayb ghāyat al-naẓīr wa-jayb al-fajr*

Extracts are shown in **Figs. 11.14a-b**. ʿAbdallāh al-Ḥalabī's tables are of interest because they display the various stages of his calculations of the prayer-times. See **8.3** on the related tables for twilight of Ibn Abi 'l-Khayr al-Ḥusnī.

11.15 Anonymous auxiliary tables for computing prayer-times

The late Syrian treatise on the 15 times of day with religious significance preserved in MS Aleppo Awqāf 970 (see already **2.11**) concludes with a set of auxiliary tables for timekeeping. The functions are labelled *mayl*, *sahm al-mayl* (= *sahm al-aṣl al-muṭlaq*), and *nisba sahmiyya*, and I have not investigated their character. Values appear to be given to *six decimal places* (!), and the underlying parameters are stated to be:

$$\phi = 35;45° \text{ (Aleppo) and } \varepsilon = 23;28°.$$

Some numerical examples follow, as well as a computation of the qibla at Jerusalem as 45;27°.

The anonymous author states that the scholars of astronomy and geometry have established that the visible horizon is half a degree below the true horizon. At the solstices the legal day is 2° longer than the day derived mathematically, the difference being twice the sum of the solar radius and the time taken by the sun to move between the two horizons, both $^1/_2$°. For the summer and winter solstices he advocates respectively differences of 3° and 4°. This means that he is assuming the following corrections for the "difference minutes":

EQ: $^1/_2$° SS: 1° WS: $1^1/_2$°

For $\phi = 36°$ and $\Delta h = {}^1/_2°$, the accurate values are:

EQ: 0;37,5° SS: 0;42,41° WS: 0;42,31° .

Thus the underlying theory is the (inappropriate) one used by earlier Egyptian astronomers (see, for example, **4.11** and **7.1**) and some earlier, but still late, Syrian astronomers (**11.8** and **11.13**).

CHAPTER 12

YEMENI AND HEJAZI TABLES FOR TIMEKEEPING

12.0 Introductory remarks

In the Yemen there was considerable activity in mathematical astronomy between 900 and 1700.[1] More than 15 *zīj*es were compiled and several sets of tables for timekeeping were prepared during this period. This colourful Yemeni tradition is of interest to the history of science not least because it was based mainly on Egyptian, ʿIrāqī, and Tunisian sources, some of which are no longer extant.in their original form. The earliest Yemeni tables which survive are those on the *Muẓaffarī Zīj* of Muḥammad ibn Abī Bakr al-Fārisī, based on Iranian sources.[2] About 700 H [≈ 1300] the Yemeni astronomer Abu ʾl-ʿUqūl compiled an enormous corpus of tables for timekeeping for the latitude of Taiz (**12.1**); he was obviously more influenced by the Mamluk Eyptian tradition. There are tables for instrument construction and timekeeping in the two treatises on instruments and mathematical astrology by the late-13th-century Rasulid Sultan al-Ashraf (**12.2**). The late-14th-century Rasulid Sultan al-Afḍal has left us a disordered pile of tables in his astronomical miscellany (**12.4**). Some simpler tables are found in a remarkable Yemeni ephemeris from 1405 (**12.5**). An early-14th-century Yemeni astronomer apparently named Ibn al-Mushrif, who cannot be identical with the early-15th-century Egyptian astronomer of the same name (**I-9.8,** *etc*.), may have compiled a complete set of tables for the latitude of Zabid (**12.1**), incorporating some of Abu ʾl-ʿUqūl's tables for Taiz. Another Yemeni astronomer named ʿAbdallāh ibn Ṣalāḥ Dāʿir (**12.6**) compiled a set of tables for timekeeping for Sanaa, of which only a fragment survives. The prayer-tables of al-Thābitī (**12.7**), compiled for the Yemen (latitude of Sanaa), are for timekeeping by lunar mansions. Those of al-Wāsiʿī *ca*. 1940 (**12.12**) give no hint of any awareness of the achievements of his predecessors in Rasulid times.

As far as I know, no astronomical works of scientific consequence were compiled in Arabia outside the Yemen. (See, however, the highly-significant treatise on folk astronomy by Ibn Raḥīq (**2.3**).) Some prayer-tables for Mecca (**6.10** and **11.10**) were probably compiled in Cairo and Damascus, respectively. Some late prayer-tables for Mecca and Medina have been located (**12.8-10**).

[1] A survey of Yemeni astronomers and their surviving works, based on about 100 manuscripts, is in King, *Astronomy in Yemen*. New information is contained in my review of the facsimile edition of al-Afḍal's miscellany cited in n. 12:9.

[2] On al-Fārisī (**I-7.1.7** and **II-2.2**) see King, *Astronomy in the Yemen,* no. 9.

12.1 Abu 'l-ʿUqūl's corpus of tables for Taiz

The extensive corpus of tables for timekeeping entitled *Mirʾāt al-zamān*, "The Mirror of Time", was compiled for the latitude of Taiz by the Yemeni astronomer Muḥammad ibn Aḥmad al-Ṭabarī, known as Abu 'l-ʿUqūl, *ca*. 1300.[3] Most but not all of the tables in this corpus are extant in MSS Berlin Ahlwardt 5720 (Mq. 733,3), copied in Mukhkha in 1209 H [= 1795]. MS Milan Ambrosiana C84 is a late anonymous abridgement of the tables, and a manuscript in a private collection in Sanaa of an astronomical miscellany by the Yemeni Sulṭān al-Afḍal compiled *ca*. 1375 (**12.4**) contains a few isolated tables from the corpus. Abu 'l-ʿUqūl also compiled two *zīj*es. The first, called the *Mukhtār Zīj*, is extant in MS London BL Or. 3624, copied in 1008 H [= 1599], and is based mainly on an early *zīj* of Ibn Yūnus. The second is no longer extant, but a few tables "from the *Zīj* of Abu 'l-ʿUqūl" which are not from the *Mukhtār Zīj* are found in the Sanaa manuscript.

The title folio and beginning of the introduction are missing from the Berlin manuscript, and the tables in it are in fact anonymous. The attribution to Abu 'l-ʿUqūl is secured from references in the Sanaa manuscript. The introduction (fols. 16r-23v) is in two *maqāla*s of nine and seven *bāb*s dealing with the various operations of timekeeping by day and night, respectively. All the tables are based on the parameters:

$$\phi = 13;40° \text{ (Taiz) and } \varepsilon = 23;35° \ .$$

The tables in the Berlin manuscript are bound in disorder, but we can distinguish the following main sets:

(a) Tables of $t(h,\lambda)$ and $T(h,\lambda)$, the hour-angle and time since sunrise as a function of solar altitude and solar longitude (fols. 128v-166r). See further **I-2.1.2**, with extract.

(b) Tables of $\lambda_H(h,\lambda)$, the longitude of the horoscopus as a function of solar altitude (fols. 26r-31v, 33r-10v, 122r-123v). See further **I-3.1.1**and the extract illustrated there.

(c) Tables of coordinates of fixed stars and lunar mansions, and tables of $h(\lambda_H)$, the altitudes of various fixed stars at daybreak as a function of the longitude of the horoscopus (fols. 32r-32v, 24r-25v, 108v-120v). See further below and also **I-4.5.1** (illustrated).

(d) Some spurious tables for Sanaa, computed by Ibn Dāʿir (**12.6**).

Various minor tables by Abu 'l-ʿUqūl relating to timekeeping are contained in the various manuscripts mentioned above. Firstly, the Sanaa manuscript contains an anonymous set of spherical astronomical tables displaying the functions:

$$\delta,\ 2D,\ 2D^h,\ H,\ 2N \text{ and } \tilde{h}' \ ,$$

side by side for each degree of λ. Values are given to two digits and the underlying parameters are:

$$\phi = 13;40° \text{ (Taiz) and } \varepsilon = 23;35° \ .$$

Details of the rising, culmination and setting of certain fixed stars, as well as agricultural information, are also given for each 1° of λ, roughly corresponding to each day of the year. It is stated that this information is based on the positions of the stars in 631 H [= 1233/34]. The tables are in fact based on those of the *Mukhtār Zīj* (MS London BL Or. 3624, fols. 176v-184r), in which each of the functions:

[3] On Abu 'l-ʿUqūl (**I-2.1.2**, *etc*.) see King, *Astronomy in Yemen*, no. 9.


القول على الدائر من الفلك عند طلوع الفجر لكل يوم من البروج

Fig. 12.1: A table displaying the oblique ascension of the horoscopus at daybreak, most probably by Abu ’l-ʿUqūl. [From a manuscript in a private collection in Sanaa; taken from Varisco & Smith, eds., *al-Afḍal’s Anthology*, p. 317, with permission of Professor Daniel Varisco.]

$$\tilde{h},\ 2D^h,\ H,\ d \text{ and } \alpha_\phi$$

is tabulated to two digits for each degree of λ for latitudes 13;0° (Aden), 13;40° (Taiz), 14;0° (Zabid), and 14;30° (Sanaa). Tables of the functions:

$$H,\ D^h \text{ and } d$$

computed for ϕ = 13;40° and ε = 23;35° also occur on fol. 107r of the Berlin copy of the *Mirʾāt al-zamān*. I conclude that the tables in the Sanaa manuscript were also compiled by Abu ’l-ʿUqūl. The additional information on the stars and on cultivation of crops formed the basis of an almanac, extant in several copies, of which I have examined MS Milan Ambrosiana C46 (fols. 52-58).[4]

Elsewhere in the Sanaa volume there is a table of the functions:

$$Z_{(7)} \text{ and } h_a(H)$$

with entries to two digits for each degree of H from 1° to 90°. It is stated that this table is taken from the *Mirʾāt al-zamān* by Abu ’l-ʿUqūl, and the same table occurs on fol. 108r of the Berlin copy.

A table for twilight which is probably by Abu ’l-ʿUqūl is extant only in the Sanaa manuscript (see **Fig. 12.1**), where it is not actually attributed to him. It is, however, rather accurately computed for his parameters and is based on other tables compiled by him. The function tabulated is simply the oblique ascension of the horoscopus at daybreak, $\alpha_r(\lambda)$, as in the Cairo and Damascus corpuses (**4.10** and **10.6**), and the underlying parameter for morning twilight

[4] See now Varisco, *Yemeni Almanac*.

is 20°. The function is called *al-dāʾir min al-falak ʿinda ṭulūʿ al-fajr*, literally, "the amount the celestial sphere has revolved at daybreak", and is defined by:

$$\alpha_r(\lambda) \;=\; \alpha_\phi(\lambda) - r(\lambda) \;=\; \alpha_\phi(\lambda) - T(20°,\lambda^*)\ .$$

Values of $\alpha_\phi(\lambda)$ for the above parameters are given in the second twilight tables described below, and a separate table of $\alpha_\phi(\lambda)$ with the same entries occurs in MS Paris BNF ar. 2523, fol. 95v, of an anonymous late-14th-century Yemeni *Zīj* (**12.3**). The table of $\alpha_\phi(\lambda)$ for Taiz in MS London BL Or. 3624 of Abu ʾl-ʿUqūl's *Mukhtār Zīj* is based on $\phi = 13;40°$. Abu ʾl-ʿUqūl tabulated $t(h,\lambda)$ and $T(h,\lambda)$ for each degree of both arguments (see above), and his table of $T(h,\lambda)$ for $h = 20°$ is found on fol. 138r of the Berlin manuscript. The values of $\alpha_r(\lambda)$ in the Sanaa manuscript can be derived from the appropriate values of $\alpha_\phi(\lambda)$ in the same Berlin manuscript and Paris BNF ar. 2523, and of $T(h,\lambda)$ for $h = 20°$ again in the Berlin manuscript.

The other twilight tables of Abu ʾl-ʿUqūl are found on fols. 24v-25v, 32r-32v, *etc*., of the Berlin copy. They show the altitudes at daybreak of about eight prominent stars in the eastern and western sky as a function of the longitude of the horoscopus. For each degree of λ_H, values to two digits are given for the stellar altitudes, and also for $\alpha_\phi(\lambda_H)$ and the duration of morning twilight $r(\lambda_H)$, and for the corresponding solar longitude. I have not investigated the parameters used for the various stars, but there is a table of right ascensions of thirty stars on fol. 24r of the Berlin manuscript which Abu ʾl-ʿUqūl states he used for the twilight tables. On fol. 108v there is another table giving the ecliptic coordinates and declination, and, for the latitude of Taiz, the meridian altitude, half arc of visibility, and the co-ascendants and co-descendants of the stars. No doubt these are the coordinates which underlie all of Abu ʾl-ʿUqūl's tables for timekeeping by the stars. The star catalogue was originally dated, but the year has been deliberately erased. However, the stellar positions correspond to about 1300. In the Sanaa volume there is a similar star catalogue compiled for a later date, with an additional set of entries displaying the meridian altitudes at Zabid: this table is stated to have been taken from a *Zīj* by Ibn al-Mushrif, otherwise unknown to the literature for he cannot be identical with the early-15th-century Egyptian astronomer with this name (**I-9.8**). In the Sanaa manuscript there are also various astrological tables by Abu ʾl-ʿUqūl, some of which originally belonged to the *Mirʾāt al-zamān,* such as tables of the longitudes of the astrological houses.[5]

In all, the Taiz corpus contains over 100,000 entries, and is the largest single set of tables for a specific locality known to have been compiled by a medieval astronomer. Abu ʾl-ʿUqūl was well named as *al-ḥāsib*, "the calculator", although this actually meant simply "the astronomer" in some circles.

12.2 The Sulṭān al-Ashraf's tables for timekeeping

The major astronomical works of the Rasulid Sultan al-Ashraf (*reg*. 1295-96),[6] compiled whilst he was still a prince, were his treatise on the construction of astrolabe and sundials, as well as the qibla-compass,[7] extant in the autograph MSS Cairo TR 105 (149 fols.) and a late copy

5 On the houses see n. **I**-3:2.
6 On al-Ashraf (**I-3.3.1**, *etc*.) see Suter, *MAA*, no. 394, and King, *Astronomy in Yemen*, no. 8.
7 See now Schmidl, "Early Arabic Sources on the Compass".

thereof in a library in Tehran (unidentified), and his astrological work extant in MS Oxford Bodleian Hunt. 233, copied in the 14th century (?). See **I-3.3.1**, **4.1.4**, **4.2.5**, and **5.4.6**, on various rather unusual tables contained in these.

In the former treatise the qibla for Taiz, Zabid, and central Yemen is said to be 27°-29° W. of N. (p. 145 of the Cairo copy). The modern value for Taiz is 26;47°! This good news is somewhat vitiated by the fact that the diagram of the qibla-compass in the same manuscript shows the qibla of Taiz at 20° West of North and the qibla at Aden at 20° East of North: see **Fig. X-9.2.2**.

12.3 A *ṭaylasān* table for all latitudes in a Yemeni *zīj*

The anonymous Yemeni *Zīj* preserved in MS Paris BNF ar. 2523 (late 14th century), which was compiled in Taiz in 775 H [= 1374], contains a table of T(H,h) (fols. 97v-100v). Entries are computed to two digits for the domains:

$$H = 46°, 47°, \ldots, 90°, \text{ and } h = 1°, 2°, \ldots, H\ .$$

In the same work there is a table (fol. 86r) of the function h(T,H) computed for the domains T = 1, 2, ... , 11^{sdh} and H = 51°, 52°, ... , 90°. Entries are given to two digits. Note that both tables are adequate for use in the Yemen, where the minimum value of the solar altitude is about 50°. On these tables see also **I-2.5.3** and **I-4.3.3**.

12.4 The astronomical tables in Sulṭān al-Afḍal's astronomical miscellany

A precious manuscript in a private collection in Sanaa contains an enormous collection of treatises and notes on a whole range of subjects ranging from lexicography, grammar, poetry, Rasulid history, genealogy, biography, medicine and pharmaceutics, and far more besides, to astronomy and astrology, copied in the hand of the Rasulid Sulṭān al-Afḍal *ca*. 1375.[8] This precious manuscript is now available in a facsimile edition, and a survey of the astronomical contents would be a valuable contribution but would merit a doctoral dissertation. See **I-2.1.2**, **3.3.2**, **4.2.6**, **4.5.1**, **6.16**, **7.1.8**, **II-9.1**, and **III-4.7** on various tables relating to timekeeping in this work.[9] There is also an agricultural almanac by al-Afḍal for the latitude of the Yemen (see **Fig. 12.4**),[10] containing values of the following functions to sexagesimal minutes for each degree of solar longitude:

$$\delta,\ 2D,\ 2D/15,\ H,\ 2N \text{ and } 2N/15\ .$$

12.5 An anonymous almanac for Taiz, 808 H

MS Cairo TR 274 (152 pp.) is an anonymous Yemeni almanac for the year 808 H [= 1405/

---

[8] On al-Afḍal and his astronomical miscellany (**I-2.1.2**) see King, *Astronomy in Yemen*, no. 18.

[9] See Varisco & Smith, eds., *al-Afḍal's Anthology*, and also my review in *Yemeni Update* (www.aiys.org/webdate/kngrev.html), which includes of a survey of research over the past 20 years on Yemeni astronomy.

[10] This has been studied in Varisco, "Yemeni Almanac II". The entire text of the almanac is reproduced in Varisco & Smith, eds., *al-Afḍal's Anthology*, pp. 97-109.

Fig. 12.4: An extract for the beginning of the sign of Libra of the almanac in al-Afḍal's collection of treatises. [From a manuscript in a private collection in Sanaa, photo courtesy of Professor Daniel Varisco.]

06].[11] It contains considerable historical information,[12] as well as material of an astrological nature and an ephemeris (pp. 102-125) giving the positions of the sun, moon, and planets for each day of the year.[13] By the side of the ephemeris values of the functions:

$$2D^h,\ H \text{ and } h_a$$

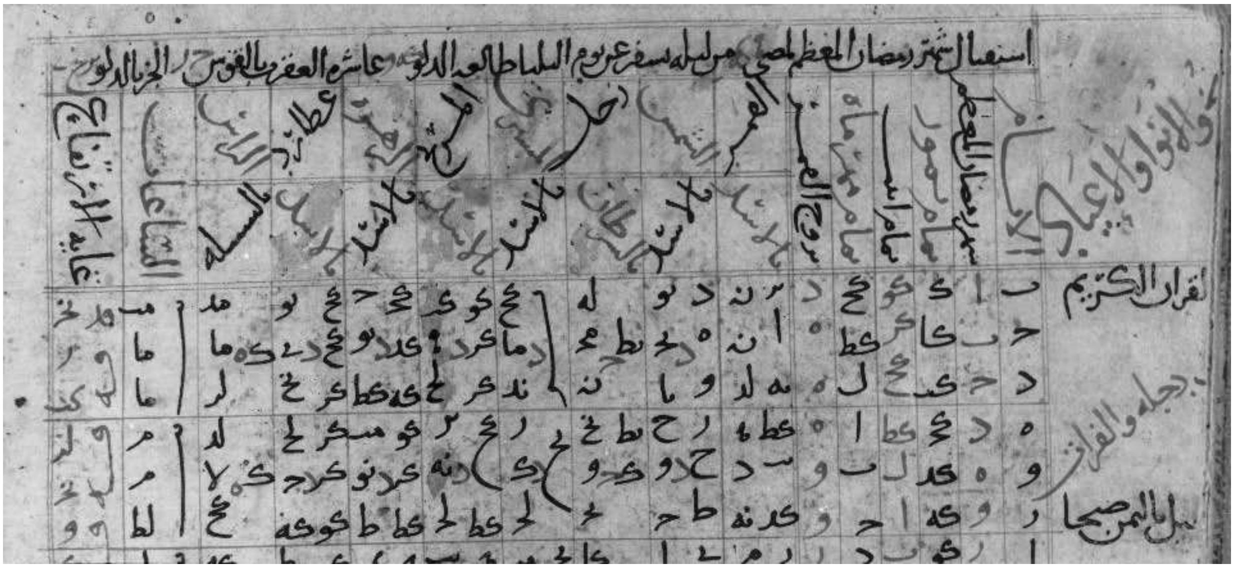
are displayed to two digits also for each day of the year: see the extract in **Figs. 12.5a-b**. The underlying parameters are:

$$\phi = 13;40° \text{ (Taiz) and } \varepsilon = 23;35°.$$

[11] On this almanac see further *Cairo ENL Survey*, no. E11, and King, *Astronomy in Yemen*, no. 22.

[12] See Ayman Fuad al-Sayyid, *Sources de l'histoire du Yémen à l'époque musulmane*, Cairo: Institut Français d'Archéologie Orientale du Caire, 1974, pp. 159-160.

[13] This almanac, together with another Yemeni work of the same genre for the year 727 H [= 1326/27] (King, *Astronomy in Yemen*, no. 11), is currently being investigated by Michael Hofelich of Frankfurt. See his article "Taḳwīm. i" in EI_2.



Figs. 12.5a-b: An extract from the ephemeris (a) and the associated astrological consequences (b) for the first few days of Ramaḍān, 808 H. [From MS Cairo TR 274, courtesy of the Egyptian National Library.]

I know no other tables of $h_a(\lambda)$ for Taiz.

In the only other table specifically for timekeeping (p. 96), the following linar mansions are shown for each 13 days of the Coptic year starting with Barmūda 23:

manzilat al-fajr, the lunar mansion rising at dawn

mutawassiṭ rubʿ al-layl, *etc*., the lunar mansions culminating at one-quarter, one-third, one-half, two-thirds, and three-quarters of the night

mutawassiṭ al-fajr, the lunar mansion culminating at dawn

ghārib al-fajr, the lunar mansion setting at dawn

manzilat al-shams, the lunar mansion of the sun.

Only the first half of this simple table, containing 14 rows of entries, is found in the manuscript now.

12.6 Ibn Dāʿir's timekeeping tables for Sanaa

MS Berlin Ahlwardt 5720 of the corpus of tables for Taiz (**12.1**) contains some additional tables of a less sophisticated type for the latitude of Sanaa (fols. 121v, 124r-127r). These are all for timekeeping by the stars, and I suspect that they were originally part of a larger corpus of such tables for Sanaa. The functions tabulated are: (a) the longitude of the horoscopus as a function of the altitude of 16 stars; (b) the longitude of the horoscopus as a function of each hour of visibility of 18 stars; and (c) the altitudes of 18 stars at each hour of visibility. The hours of visibility represent twelfths of the arc of visibility for each star and hence differ for each star. In the heading of the first table it is stated that the stellar coordinates were based on the observations of Ibn Yūnus, but the name of the compiler and the date for which he computed his stellar coordinates have been deliberately erased. However, it is just possible to read the name ʿAbdallāh ibn Ṣalāḥ Dāʿir,[14] which also appears on fol. 121r in reference to some instructions on the use of the third table. On these tables and their author see further **I-3.2.3**, and also **I-3.4.1** and **I-4.6.1**.

12.7 al-Thābitī's prayer-tables for the Yemen

MSS Berlin Ahlwardt 5769 (Mq. 733,2), fols. 8v-15v, copied in Mukhkha in 1209 [= 1795], Vatican ar. 962, fols. 13r-19r, and Sanaa GML *majāmīʿ* 27,1, contain a set of prayer-tables compiled for the latitude of the Yemen by Muḥammad ibn ʿAbd al-Laṭīf al-Thābitī. MS Algiers Fagnan 1485,3, which I have not examined, contains the same tables.[15] al-Thābitī was a Syrian who lived in Zabid, and he compiled these tables in 1047 H [= 1637/38].[16]

For each day of the Syrian year al-Thābitī tabulates the solar longitude and which 13th part of the appropriate lunar mansions is:

(a) culminating at sunset,
(b) rising at daybreak,
(c) culminating at midnight, and
(d) culminating at daybreak.

Entries are given as $M_1, M_2, \dots, M_{13}$, where M is the name of a given mansion. A rearrangement of al-Thābitī's column of entries for the lunar mansions rising at daybreak is found in an almanac called *Ḥisāb al-Shibāmī*, which was still in use in the Ḥaḍramawt in the mid 20th century.[17]

---

[14] On Ibn Dāʿir (**I-3.2.3**, *etc.*) see King, *Astronomy in Yemen*, no. 31, and now Ihsanoğlu *et al.*, *Ottoman Geographical Literature*, I, pp. 76-78, no. 39.

[15] *Algiers BN Catalogue*, p. 409.

[16] On al-Thābitī see King, *Astronomy in Yemen*, no. 33.

[17] This table is published on p. 435 of Serjeant, "S.W. Arabian Almanac", where it is implied that it displays the lunar mansions corresponding to the position of the sun. See King, *Astronomy in Yemen*, no. 44.

al-Thābitī further tabulates the hours of day and night, giving entries to two digits, where the second represents *thirtieths of an hour*, as well as the shadows to base $6^1/_2$ corresponding to the solar altitude at midday and at the beginning of the afternoon prayer, giving entries to two digits, where the second now represents *twelfths of a shadow digit.* The midday shadows are computed according to a linear zigzag scheme,[18] with a maximum of $5^5/_{12}$ at the winter solstice and a minimum of $2^2/_{12}$ (south) at the summer solstice. Note that these extremal values are equivalent to 10 and 4 digits for a gnomon of 12 digits, which suggests that al-Thābitī converted these from base 12 to base $6^1/_2$. In fact, for $\phi = 15°$ the extremal values are closer to 10 and 2 digits, al-Thābitī's afternoon shadow lengths are always $2^2/_{12}$ feet more than (the absolute value of) his midday shadow lengths. See further **III-4.7**.

12.8 al-Hattārī's prayer-tables for Medina

MS Cairo Sh 89,11-15 (fols. 157r-322v), copied 1070 H [= 1659/60], is the only known copy of some scientific treatises by Ḥusayn ibn Shāmī al-Hattārī, otherwise known only for his writings on religious topics.[19] Some of the former, which deal with the standard topics of *zīj*es, the astrolabe, and trigonometric and astrolabic quadrants, as well as arithmetic, are here copied in al-Hattārī's own hand. Scattered throughout all of his treatises on astronomy are some spherical astronomical tables computed for $\phi = 25;0°$ (Medina).

Some of the tables are based on $\varepsilon = 23;35°$ and others are based on $\varepsilon = 23;30(,17?)°$, and I have not systematically investigated their accuracy. The functions tabulated include:

$$\delta,\ \delta_2,\ \alpha,\ \alpha',\ \alpha_\phi,\ \psi,\ \ H,\ d,\ D,\ N,\ t_a,\ T_a,\ s,\ r,\ n \text{ and } \alpha_\sigma\ ,$$

and al-Hattārī also presents some simple trigonometric tables, and some tables for trepidation (fols. 203r and 218v-220r).[20] An extract from the prayer-tables is shown in **Fig. 12.8**. MS

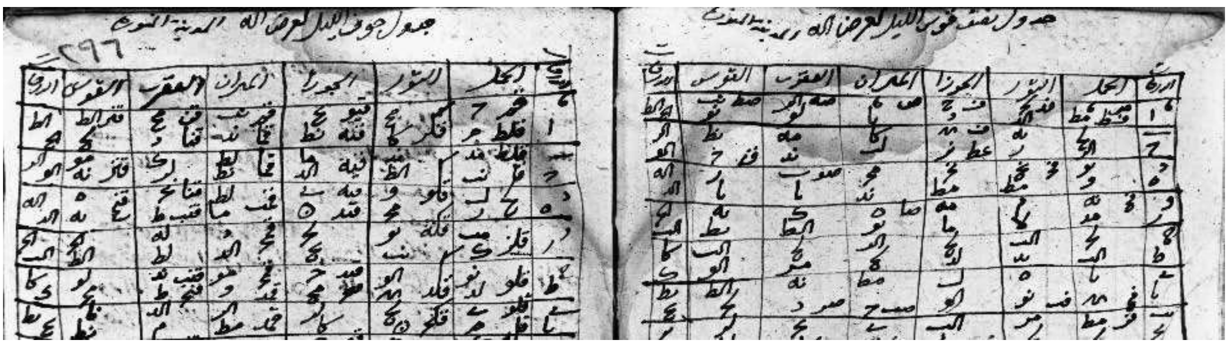


Fig. 12.8: This extract from al-Hattārī's prayer-tables for Medina shows the sub-tables for the functions N(λ) and n(λ), that is, the length of night from sunset to sunrise and the length of darkness of night from nightfall to daybreak. [From MS Cairo Sh 89, fols. 295v-296r, courtesy of the Egyptian National Library.]

---

[18] Such a crude procedure is virtually unknown in medieval Islamic astronomy. For one example see the study by Kennedy and Ukashah cited in n. 1:9.

[19] On al-Hattārī see Brockelmann, *GAL*, II, p. 523, and SII, pp. 543 and 1039; *Cairo ENL Survey*, no. D42; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 388-389, no. 252.

[20] See n. **I**-6:22.

Leiden Or. 2538 is another copy of one of al-Hattārī's treatises entitled *al-Sirāj al-wahhāj* which also contains some of the tables listed above.

12.9 Ḥusayn Ḥusnī's timekeeping tables for Mecca

MS Cairo K 4002, 40 fols., penned in 1249 H [= 1833/34], is the only known copy of a set of tables for timekeeping compiled for the latitude of Mecca, taken as 21;45°, by the Ottoman astronomer Ḥusayn Ḥusnī ibn Aḥmad Ṣabīḥ.[21] The main set of tables displays the time of day according to the Ottoman convention as a function of solar longitude, with entries for solar altitudes in the east and west given in hours, minutes and seconds. See further **I-2.2.4** (illustrated). These are followed (fols. 35v-38r) by a set of prayer-tables for the same latitude. For each degree of solar longitude the following functions are displayed with entries to minutes:

2D, 2N, m′, a′, b′, s′ and r′

I have not investigated the parameters used for twilight.

12.10 Tāj al-Dīn's Ottoman-type prayer-tables for Mecca

MS Damascus Ẓāhiriyya 27 contains a set of prayer-tables for latitude 25° (Medina) similar to those for Aleppo in MS Damascus 7564 (**11.11**). The entries are expressed in hours and minutes and are written in standard Arabic numerals, without the unusual numerical abbreviations found in the Aleppo tables. Values of α' and α_r are also given, expressed in equinoctial hours and minutes. The parameters used for morning and evening twilight are 19° and 17°.

These tables are attributed to Tāj al-Dīn, a *muwaqqit* at the Prophet's Mosque in Medina,[22] and were apparently compiled in 1256 H [= 1840]. A star catalogue giving equatorial coordinates for the year 1100 H [= 1688/89] is also contained in the manuscript and perhaps the prayer-tables were adapted from an earlier source. Tāj al-Dīn also wrote a commentary on the *Ruznāme* of Shaykh Vefa, extant in Cairo.

12.11 Anonymous Ottoman-type prayer-tables for Sanaa

An unnumbered manuscript in the Grand Mosque Library in Sanaa is a comparatively recent copy of an older set of prayer-tables for the latitude of Sanaa, stated as $\phi = 15;0°$. The following times are given in hours and minutes according to the Ottoman convention (**14.0**), written in modern Arabic numerals, for each day of the Syrian year:

R′, z′, a′, b′, s′, r′, m′, and also 2N and 2D .

There are separate tables of the functions:

$z_{z(7)}$ (h_z = H), $z_{a(7)}$, $z_{b(7)}$, H and δ

---

[21] On Ḥusayn Ḥusnī see n. **I**-2:25.

[22] Tāj al-Dīn is mentioned in Brockelmann, *GAL*, SII, p. 538, but the work attributed to him there is apparently by al-Hattārī, on whom see **12.8**. He wrote a commentary to the prayer-tables of Shaykh Vefā (**14.3**): see *Cairo ENL Survey*, nos. D239 and H2; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, p. 53. See also **III-3.4** on his shadow-scheme for timereckoning.

for each degree of solar longitude. The first three of these five tables are computed to two digits, but the remaining two are computed to four digits (ε = 23;35°) and were clearly lifted from some earlier source.

I have not investigated parameters used for morning and evening twilight. The entries for z′ (*ẓuhr*) are in fact simply (12-D^h). The entries for m′ (*niṣf al-nahār*, midday) have been made earlier than z′ by a few minutes (about 8^m at the equinoxes), and the entries for R′, sunrise, have been made earlier by double this correction (about 18^m at the equinoxes). These corrections represent a generous allowance for horizon phenomena (see also **14.15**). The entries for all functions but R′ and m′ correspond quite closely to recomputed values.

12.12 al-Wāsiʿī's prayer-tables for the Yemen

The work *Kanz al-thiqāt fī ʿilm al-mīqāt*, "The Treasure of Reliable Scholars for the Science of Timekeeping", prepared by Shaykh ʿAbd al-Wāsiʿ ibn Yaḥyā al-Wāsiʿī[23] for the Imām Yaḥyā was published in Cairo (Maṭbaʿat Ḥijāzī) in 1358 H [= 1939/40]. It contains two sets of tables, the first for finding the *feria* of a date in the Muslim calendar (from 1358 H to 1596 H) as well as the corresponding day number of the solar year, and the second for reckoning the prayer times, apparently based on ϕ = 15° (Sanaa) although the actual latitude of Sanaa is 15;23°.

In this second set of tables the following information is given for each day of the solar year:

manzilat al-shams, the 13th division of the lunar mansion of the sun
ṭāliʿ awwal al-layl, the ascendant at the beginning of night (in signs and degrees)
maʿālim al-zirāʿa, information about the rising or setting of a prominent star or constellation
daraj al-shams fī 'l-burūj, the solar longitude (to the nearest degree)
al-māḍī min al-shuhūr al-rūmiyya, the date in the Syrian calendar
aqdām al-ẓuhr / al-ʿaṣr wa-banānuhu wa-sāʿatuhu wa-daqāʾiquhu, the shadow length in feet and fingers at the midday / afternoon prayer and the corresponding times in hours and minutes
ṭulūʿ al-fajr, daybreak
shurūq al-shams, sunrise.

The times of daybreak, sunrise, midday and the afternoon prayer are given in hours and minutes according to the Ottoman convention. Daybreak is made to precede sunrise by 1^h10^m throughout the year, which corresponds to a parameter of about 17° at the equinox. The shadow lengths are to base 7 and the fractional twelfths are called *banān*, fingers, as in al-Thābitī's prayer-tables (**12.7**). The shadow lengths are computed according to a very primitive scheme. al-Wāsiʿī's shadow has the constant value $^4/_{12}$ feet from Taurus 8° to Cancer 18°. Since the midday shadow in the Yemen actually becomes southerly in the summer, al-Wāsiʿī's values represent a gross distortion. The equinoctial and solstitial values are given as:

VE: 2   SS: $^4/_{12}$ AE: $1^4/_{12}$ WS: 5 ,

[23] On al-Wāsiʿī see King, *Astronomy in Yemen*, no. 47. A more detailed investigation of his shadow-schemes is in **III-4.10**.

whereas for $\phi = 15°$ they should be:

VE: 1;53 SS: 1;03S AE: 1;53 WS: 5;35 .

Note that al-Wāsiʿī even gives different values for the two equinoxes. The afternoon shadow lengths are always 7 feet more than the midday shadows, so that the summer shadows are not considered negative.

al-Wāsiʿī's times for sunrise, midday and the afternoon prayer were adopted in the prayer-tables currently in use in the Yemen when I visited there in 1974. These also showed nightfall at 1 o'clock (Ottoman convention!) throughout the year.

In the introduction to the tables al-Wāsiʿī mentions a method for timekeeping by night (p. 16):

> "To find the hours by the stars. If you want to know how many hours of the night have passed, first find the ascendant at the beginning of the night (from the tables), and then the ascendant at the time in question (by observation), and then see how many lunar mansions there are between the two ascendants. Multiply this by six and then divide the product into sevenths, taking each seven sevenths as one hour and the remainder less than seven as a fraction of an hour."

This crude method ignores the fact that the mansions are measured along the ecliptic and time is measured along the celestial equator. The factor 6/7 results from the fact that 14 mansions rise in the 12 seasonal night-hours between sunset and sunrise.

al-Wāsiʿī concludes his work (pp. 69-70) with a brief discussion of the qibla, which for the Yemen is, he says, due north. Thus one can use either the pole star or a compass to find it. Elsewhere, he says, the compass is useless for finding the qibla. Compare **12.2**!

On the cover of the work it is stated that the author worked for years to prepare the two tables, and consulted several *zīj*es. Now that we know that the Yemen was an important centre of astronomy from the 10th to the 17th century we can judge al-Wāsiʿī in the light of this early tradition and can only regard his scientific output as rather pathetic.

CHAPTER 13

MAGHRIBI TABLES FOR TIMEKEEPING

13.0 Introductory remarks

I know of no tables relating to astronomical timekeeping from al-Andalus, other than the simple shadow-schemes of the kind found in the 10th-century *Calendar of Cordova*, which I have discussed elsewhere (**III-5.1**). We can, however, be sure that some tables for *mīqāt* were compiled there, not least because the earliest European tables of this kind, some from medieval Chrisitan Spain, show some Islamic influence (**I-10.1**). It is therefore to the Maghrib that we turn next, a region whose distinctive astronomical tradition was influenced successively over the centuries by Baghdad, then by al-Andalus and then by Egypt and Syria, whence came also the *Zīj* of Ulugh Beg.

Only recently has the history of astronomy in the Maghrib been surveyed.[1] According to the late 14th-century Tunisian historian Ibn Khaldūn,[2] the *zīj* that was commonly used in his time (in Tunis or in Morocco or both?) was that of Ibn Isḥāq, who worked in Tunis in the early 13th century. A manuscript of this *zīj* was discovered only in 1978 and has been subjected to detailed study in Barcelona.[3] An abridgement of it was made by Ibn al-Bannāʾ of Marrakesh *ca*. 1300, and this survives in a number of copies.[4] Four 14th-century Maghribi *zīj*es have survived, namely three *zīj*es by Ibn al-Raqqām, compiled in Tunis,[5] and a small *zīj* by al-Qusanṭīnī, compiled in Fez.[6] This last is of particular interest because it contains planetary equation tables based on Indian rather than Ptolemaic theory. The later Tunisian *zīj*es of Ḥusayn Quṣʿa and Muḥammad Sanjaqdār are based on the 15th-century *Zīj* of Ulugh Beg of Samarqand.[7] Also, a recension of the 14th-century *zīj* of Ibn al-Shāṭir was prepared for Algiers.[8] The three Tunisian sets of tables for timekeeping described here (**13.2-4**) were probably also compiled in the 14th century, and reflect the influence of the Syrian tradition in astronomical timekeeping. Certain later Maghribi sources are full of surprises (**13.5-8**).

In the past hundred or so years Gäetan Delphin, Henri-Paul-Joseph Renaud, and Georges Colin conducted a series of studies on astronomy in medieval times in what is now Morocco.[9]

---

1 See King, "Astronomy in the Maghrib", already to some extent superseded by various works of the Barcelona school, such as Samsó, "Maghribi *zīj*es", and *idem*, "Astronomical Observations in the Maghrib". See also King & Samsó, "Islamic Astronomical Handbooks and Tables", pp. 60-64.

2 Franz Rosenthal, *Ibn Khaldûn: The Muqaddimah – An Introduction to History*, 2nd edn., 3 vols., Princeton, N.J.: Princeton University Press, 1967, III, pp. 133-137.

3 On Ibn Isḥāq see n. **I**-6.17, and especially the new studies of Àngel Mestres there cited.

4 On Ibn al-Bannāʾ see n. **I**-6:18.

5 On Ibn Raqqām see n. **I**-6:19.

6 On al-Qusanṭīnī (Suter, *MAA*, no. 371) and his tables see Kennedy & King, "Indian Astronomy in Fourteenth-Century Fez".

7 See n. **I**-4:20.

8 *Cairo ENL Survey*, nos C30/2.1.19 and F66.

9 *Cf*. Delphin, "L'astronomie au Maroc", and Colin & Renaud, "Abū Miqraʿ".

Their investigations suggested that the most celebrated *muwaqqit*s there such as the 13th-century scholar Abū Miqraʿ (**2.0**) were innocent of the developments in astronomical timekeeping further East (which the French scholars did not know about anyway). However, the numerous sets of prayer-tables mentioned below, mostly unknown to modern scholarship before the 1970s, show that there was more activity in *mīqāt* in Morocco than was previously thought and that some of it was rather sophisticated.

The *mīqāt* specialists of the Maghrib were involved with three times of day, two of which were not generally used by their colleagues further East. These three times are graphically displayed on an elegant 14th-century sundial from Tunis.[10] The first of these was the *ḍuḥā* prayer, beginning around mid-morning, or, more precisely, at a time before midday equal to the time after midday of the beginning of the time for the *ʿaṣr*.[11] The *ḍuḥā* was also considered important in Ottoman Turkey (**Ch. 14**). The second was the *taʾhīb*, a "preparation" for the communal prayers on Friday, which took place one equinoctial hour or 15 equinoctial degrees before midday.[12] The third time is the beginning of the *ẓuhr*, which was taken not as immediately after midday as it was in the East but rather as the moment when the shadow of any object z has increased by one-quarter of its length over its midday minimum Z. This definition of the *ẓuhr* is attested in some early Eastern Islamic sources (see, *e.g.*, **2.4**), but remained standard in Maghribi and Andalusī practice.[13] The corresponding solar altitude h_z is defined by

$$h_z = \text{arc Cot}_n \ \{ Z_{(n)} + {}^1/_4\, n \} \ .$$

In this chapter I use the subscripts ∂, t and z to refer to the *ḍuḥā*, *taʾhīb* and *ẓuhr*.

13.1 The spherical astronomical tables for Tunis in the *Zīj* of Ibn Isḥāq

MS Hyderabad Āṣafiyya 298, copied *ca*. 1400, is a unique copy of a voluminous (*ca*. 400 pp.) Syrian recension of the *Zīj* of Ibn Isḥāq.[14] During my brief encounter with this manuscript in Hyderabad in 1978, I was able to record the existence of some auxiliary tables for calculations in timekeeping and computing ascensions also found in earlier works of Ibn al-Zarqālluh and the later works of Ibn Isḥāq, Ibn al-Bannāʾ and Ibn al-Raqqām: see further **I-6.9.1*** and **7.1.5***. The only other tables specifically relating to timekeeping in this manuscript are tables of the functions:

$$2D^h,\ \tilde{h},\ H \text{ and } \delta^*,$$

with entries in degrees and minutes for each degree of λ′ based on parameters:

$$\phi = 36;40° \text{ (Tunis) and } \varepsilon = 23;33°.$$

Two tables whose titles indicate that they were to display r(λ) and s(λ) in hours and minutes have no entries (see further **XI-5.3**). Another table, unique of its kind, displays the half-excess

---

[10] See King, "Tunisian Sundial", for an analysis of this instrument and (pp. 193-196) a first interpretation of the institutions represented by the markings (including the *ḍuḥā*, *taʾhīb*, *ẓuhr* and *ʿaṣr*), accomplished with the help of contemporaneous Tunis tables for timekeeping. More information is in **IV-4-6**.

[11] On the *ḍuḥā* see *ibid*., pp. 189 and 193-194, and **IV-5.**

[12] On the *taʾhīb* see *ibid*., p. 190, and **IV-6.2**.

[13] *Cf*. Wiedemann & Frank, "Gebetszeiten". On the origin of this definition see **IV-4.4**.

[14] On Ibn Isḥāq see n. **I**-6:17.

of the arc of visibility of the moon for each of the seven climates and for each degree of its declination from 1° to 30° (see further **Fig. VIa-7.2**).

Whilst this manuscript may be somewhat disappointing for the history of astronomical timekeeping it provides exciting new material for the history of most other aspects of mathematical astronomy and astrology.

13.2 Anonymous corpus of timekeeping tables for Tunis

MS Berlin Ahlwardt 5724 (Wetzstein 1150), fols. 1r-41r, copied *ca*. 1700, contains an introduction and a set of tables for timekeeping computed for the latitude of Tunis. The work is anonymous but is entitled *ʿUmdat al-nuẓẓār fī mawāqīt al-layl wa-'l-nahār*, roughly "The Support of those who look closely of the Times of Night and Day". The author dedicated his work to the Ḥafṣid ruler Abū Fāris (al-Mutawakkil) ʿAbd al-ʿAzīz ibn Abi 'l-ʿAbbās Aḥmad (*reg*. 1394-1434).

The first tables (fols. 11r-13r) are for calendar conversion and for finding the solar mean longitudes (from 600 H to 900 H) and equations. These are followed (fols. 13v-14r) by Sine and Cotangent (base 12) tables, and (fols. 14v-15v) by tables of:

$$\delta,\ H \text{ and } D$$

for parameters:

$$\phi = 37° \text{ (Tunis) and } \varepsilon = 23;35° ,$$

which also underlie the remaining tables. There follow (fols. 16r-28v) tables displaying the hour-angle t(H,h) for each degree of both arguments ($h \leq H$) (**I-2.3.5**, illustrated). A full set of prayer-tables follows (fols. 29r-41r), in which the following functions are displayed:

$$h_0,\ \psi,\ \text{Sin}\ \psi,\ \tilde{h},\ D^h,\ 2D^h,\ h_t,\ h_z,\ h_a,\ (D{+}s),\ (D{+}2N{-}r),\ \alpha' \text{ and } \alpha\ ,$$

as well as k(h) and sec h for azimuth calculations (**I-8.1.3** and **8.4.1**). There is also a separate table (fols. 3v-38r – see **Fig. 13.2**) with entries to one digit for the functions:

$$D,\ (D{+}s),\ (D{+}2N{-}r) \text{ and } (D{-}r)\ .$$

Those tables with entries for $\delta \geq 0$ begin with Capricorn 1°. The tables for the solar altitude at the time of the *ẓuhr* and *taʾhīb* are based on the definitions noted in **13.0**. The tables for twilight are based on parameters 20° and 18° for morning and evening, respectively. The entries in all of the tables are rather accurately computed to two digits. The remainder of the tables in the Berlin manuscript are based on latitude 36;40° for Tunis and are the same as those in the *Zīj* of Ḥusayn Quṣʿa (see the next section).

13.3 Anonymous prayer-tables for Tunis

A second set of prayer-tables for the latitude of Tunis, based on parameter 36;40°, is attested in a number of sources. The prayer-tables for latitude 37;0° in MS Berlin Ahlwardt 5724 (**13.2**) are followed (fols. 42r-53r) by this second set, as well as a star catalogue for the year 773 H [= 1371/72] attributed to Ibn al-Shaṭīr (fols. 53v-54v), and a table for reckoning time from stellar altitudes (fols. 55r-56r), on which see **I-2.4.1**. The prayer-tables for latitude 36;40° also occur in MS Princeton Yahuda 147c, pp. 113-132, of the later *Zīj* of Ḥusayn Quṣʿa, based

Fig. 13.2: An extract from the prayer-tables in the main Tunis corpus. [From MS Berlin Ahlwardt 5724, fol. 36r, courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

Fig. 13.3: An extract from anonymous prayer-tables for Tunis serving Aquarius / Leo. [From MS Berlin Ahlwardt 5724, fol. 56v. [Courtesy of the Deutsche Staatsbibliothek (Preußischer Kulturbesitz).]

mainly on the 15th-century *Zīj* of Ulugh Beg, and the prayer-tables are likewise not original.

The tables for latitude 36;40° include some for the *taʾhīb* and *ẓuhr* like those for latitude 37;0°. The functions t_z and τ_{za} denote the time from midday to the *ẓuhr* and from the *ẓuhr* to the *ʿaṣr*. Altogether these prayer-tables display the following functions, reasonably accurately computed to two digits for each degree of solar longitude:

(Berlin and Princeton): h_t, H, Sin H, $Z_{(12)}$, $z_{z(12)}$, $z_{a(12)}$, h_z, t_z, τ_{za},
h_a, t_a, T_a, s, (n-45), n, (2N-r), r
(Princeton only): D, $\tilde{h}$, D^h, (D+2n-r), and (D+s) .

The entries begin with those for either Capricorn 1° or Cancer 1°. An extract from part of the tables in the Berlin manuscript is shown in **Fig. 13.3**. The function (2N-r), representing the time from sunset to daybreak, is referred to as *qaws al-layl al-sharʿī,* "length of night as defined by religious law", an expression also used in the tables for latitude 37;0°. The function (n-45) is referred to as *mā bayn al-shafaq wa-'btidāʾ al-ādhān*, "the time between nightfall and the beginning of the prayer-call". This means that the muezzin would perform a prayer-call three hours before daybreak. Probably this particular table was intended for use only in the festive nights of Ramaḍān, to announce the time for the *suḥūr*. A similar table for Damascus, allowing more time for sleep, is contained in MS Berlin Ahlwardt 5758, fol. 49v, of the Damascus corpus (**10.2**).

A manuscript in the Topkapı Library, whose number is not available to me, is another anonymous copy of some of these tables without introduction or title. The following functions are represented:

h_a, t_a, T_a, $Z_{(12)}$, $z_{z(12)}$, $z_{a(12)}$, ψ, Sin ψ, s, (n-45°), n, (2N-r) and r .

It is noted that the tables of ψ and Sin ψ are based on ϕ = 37° rather than 36;40°.

13.4 Anonymous auxiliary tables for timekeeping by the sun

MS Cairo DM 689 contains an extensive set of auxiliary tables copied in an elegant Maghribi hand *ca*. 1600.[15] The tables conclude with a star catalogue dated 801 H [= 1398], and they appear to have been compiled in Tunis. The title folio and instructions are missing from the manuscript, which begins with the last page of a set of tables displaying the solar longitude for each day of a period of four Syrian years (**Fig. I-1.2d**). The auxiliary tables are simply those of al-Khalīlī for timekeeping by the sun (**10.3**), with a few additional tables of B′ for the latitude of Mecca, taken as 21;40°, and various unspecified localities in the Maghrib, al-Andalus and Sicily (?) with latitudes:

30;30°, 31;30°, 32;30°, 33;40°, 34;30°, 36;30°, 36;40°, 37;10°, 37;30°, 38;30° and 39;30°

and also tables of Sin H(λ) for these latitudes, from which those of B′(λ) are easily derived. See further **I-6.2.5**, **I-6.3.3**, and **I-9.7** (illustrated).

13.5 al-Jannād's prayer-tables for the Western Maghrib

MS Cairo TR 338,2 (pp. 204-287), penned *ca*. 1850**,** is the only known copy of a set of prayer-tables for various latitudes in what is now Morocco.[16] The tables are preceded by a short introduction in which the author is named as Abū ʿAbdallāh Muḥammad ibn Muḥammad ... al-Jannād al-Anṣārī al-Andalusī, and the work is dedicated to the ruler (called *imām* and *sulṭān*) Nāṣir al-Dīn Abū ʿAbdallāh Muḥammad ibn ʿAbdallāh ibn Ismāʿīl, surely the ʿAlawī *sharīf* Muḥammad III ibn ʿAbdallāh ibn Ismāʿīl who ruled Morocco during 1171-1204 H = 1757-1790].[17]

al-Jannād's tables begin with a table of solar longitude for each day of the Christian year. According to this table the sun enters Aries 0° on March 8-9, and in the title of this table and of all those which follow it is stated that the operations are all *li-ḥarakat yḥ*, which shows that al-Jannād accepted that the equinox had moved 18° from the sidereally-fixed Aries 0°, as a result of the (false) notion known as the trepidation of the equinoxes.[18] A table of geographical coordinates promised in the introduction is no longer contained in the manuscript. The remaining tables consist of three sets for latitudes:

34° (Meknes, Zarhun, Fez, and Sakka), 31° (Marrakesh, Kairouan (!), Alexandria, *etc*.), and 30° (Sijilmasa, Cairo) ,

with underlying obliquity $\varepsilon = 23;30°$. An extract is shown in **Fig. 13.5**. The functions displayed are in each case:

$$\delta,\ H,\ h_a,\ \mathrm{Sin}\ h_a,\ C,\ B,\ d,\ t_a{}^h,\ 2D^h,\ 2N^h,\ D^h,\ (D+s)^h,\ (N\text{-}r)^h \text{ and } N^h\ .$$

Values of the first seven functions are given to two sexagesimal digits, and all times are given in hours, degrees and sexagesimal minutes thereof, except the lengths of day and night, which are tabulated to one extra digit.

---

[15] *Cairo ENL Survey*, no. F30.
[16] *Ibid*., F35.
[17] Boswell, *Islamic Dynasties*, p. 53. See also the article "ʿAlawīs" in *EI*$_2$ by the Editors, especially p. 356.
[18] See n. **I**-6:22, and also n. 13:20 below.

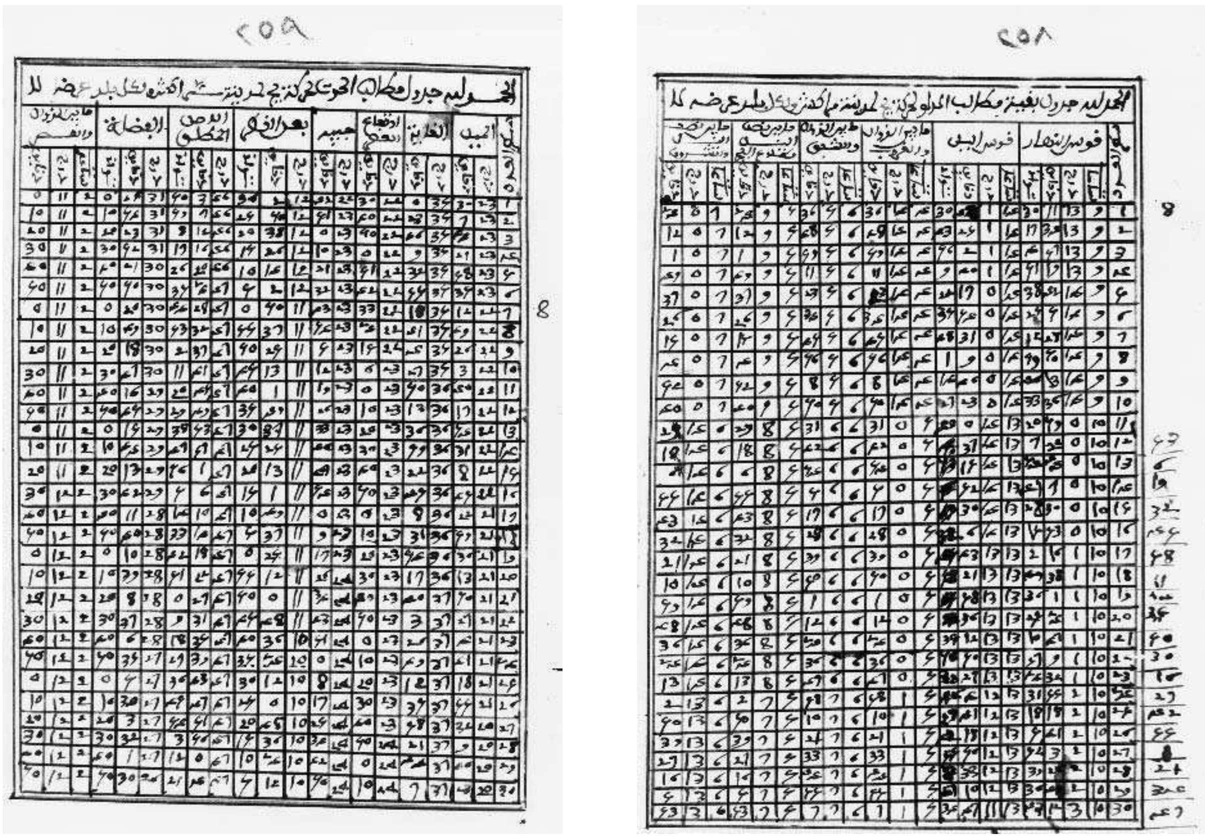


Fig. 13.5: An extract from al-Jannād's prayer-tables for the Maghrib. [From MS Cairo TR 338,2, pp. 258-259, courtesy of the Egyptian National Library.]

13.6 Anonymous prayer-tables for Tlemcen

One of the numerous commentaries on al-Jādarī's poem on timekeeping *Rawḍat al-azhār* (**2.0**, also **I-6.15.2**)[19] is an anonymous work entitled *Natāʾij al-afkār*. This commentary is of interest for the historical information it contains about the medieval Islamic tradition of obliquity measurements, twilight parameters, and theories of trepidation.[20] It is extant in two copies, MSS London BL Or. 411,2, and Cairo K 4311, dated 1183 H [= 1769/70]. The first of these contains a set of tables for timekeeping scattered in the text of the commentary; the second contains no tables. All of the tables based on local latitude are computed for:

$$\phi = 35;0° \text{ (Tlemcen)} .$$

The tables include some for the solar mean motion and the solar equation, simple trigonometric functions, and a star catalogue. The tables relating specifically to timekeeping display the functions:

$$H,\ \alpha_{\phi},\ \alpha,\ D,\ D^{h},\ h_{z},\ h_{a}\ t_{a} \text{ and } T_{a}\ ,$$

---

[19] On al-Jādarī see n. **I**-6:21.

with values given in degrees and minutes for each degree of solar longitude. There are also tables of various auxiliary functions for use in determinations of the solar azimuth and hour-angle – see **I-6.15.2** and **I-8.1.4**.

13.7 Anonymous prayer-tables for Algiers

MS Algiers Fagnan 1472 contains six pages of prayer-tables for the latitude of Algiers. I have not been able to inspect the manuscript, but according to the catalogue,[21] the prayer-times are arranged in eight columns, values are given for each day of the year, and the manuscript itself dates from the end of the 18th century.

MS Cairo ṬFT 9,1, penned *ca*. 1900, is the only copy known to me of an extensive anonymous almanac and set of prayer-tables for the latitude of Algiers, stated to be 36;40°.[22] The introduction to the work is in Turkish, and the author pays his respects to Shaykh Vefā (**14.3**) and ʿAlī Efendī (the latter presumably made some alterations to Shaykh Vefā's almanac),[23] and then proceeds to mention Ulugh Beg, ʿAlī al-Maghribī (perhaps Abū ʿAlī al-Marrākushī of **6.7** is intended), Abū Miqrāʿ (**2.0**) and Ibn al-Bannāʾ (**13.0**). His almanac contains extensive calendrical tables, and a table for the solar declination in a four-year cycle, as well as three sets of tables for timekeeping. The first of these displays for each day of each month in the Syrian calendar the duration of day (*al-nahār*) and night (*al-layl*). The second displays likewise – see **Fig. 13.7** – the following ten functions:

$$2D,\ 2N,\ (2D{+}r),\ (2N{-}r),\ r,\ D,\ t_z,\ t_a,\ T_a,\ s\ .$$

The third and fourth functions are labelled *al-nahār al-sharʿī* and *al-layl al-sharʿī*, as opposed to the first and second, labelled ... *al-ʿurfī*. In both of these sets values are given in hours and minutes. The third set of tables – see **Fig. IV-5.5** – displays the time in hours and minutes, according to the Turkish convention, of the three institutions, the *ṣanjāq*, the *ʿaṣr*, and the *imsāk*. These tables are not based directly on those in the second set. The term *ṣanjāq*, which here refers to a time shortly before midday, is new to me. These tables merit more detailed investigation: see **IV-5.4**.

13.8 Anonymous tables for timekeeping for Sfax

MS Cairo K 7584,1, copied in 1257 H [= 1841/42] in an elegant Maghribi hand, contains a substantial fragment of an anonymous Tunisian treatise on sexagesimal arithmetic and timekeeping compiled in Sfax.[24] The treatise, which probably dates from the 17th or 18th centuries, contains various auxiliary functions for timekeeping (**I-6.4.16**, **I-6.10.13, I-7.2.3** and **I-8.4.2**), as well as trigonometric tables, and tables of the functions:

$$H \text{ and } 2D^h$$

---

[20] See nn. 2:12 and 13:18.

[21] *Algiers BN Catalogue*, p. 406.

[22] *Cairo ENL Survey*, no. F68.

[23] He is apparently too early to be identified with ʿAlī Efendī, the *muwaqqit* at the Laleli Mosque in Istanbul in the 19th century (İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, p. 632).

[24] *Cairo ENL Survey*, no. F69.

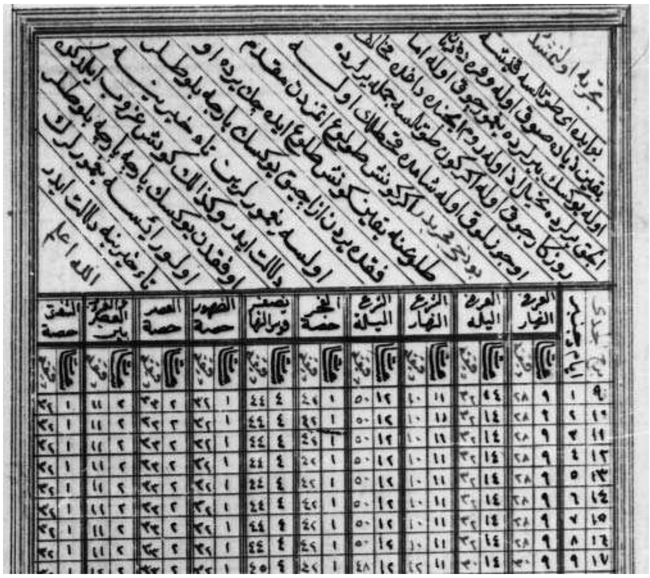


Fig. 13.7: An extract from the anonymous prayer-tables for Algiers. See also **Fig. IV-5.5**. [From MS Cairo ṬFT 9,1, fols. 34v-35r, courtesy of the Egyptian National Library.]

with entries to four digits for each degree of λ, and based on the parameters:

$$\phi = 34;48° \text{ (Sfax) and } \varepsilon = 23;30,17° \ .$$

13.8* Miscellaneous tables for timekeeping from the Maghrib

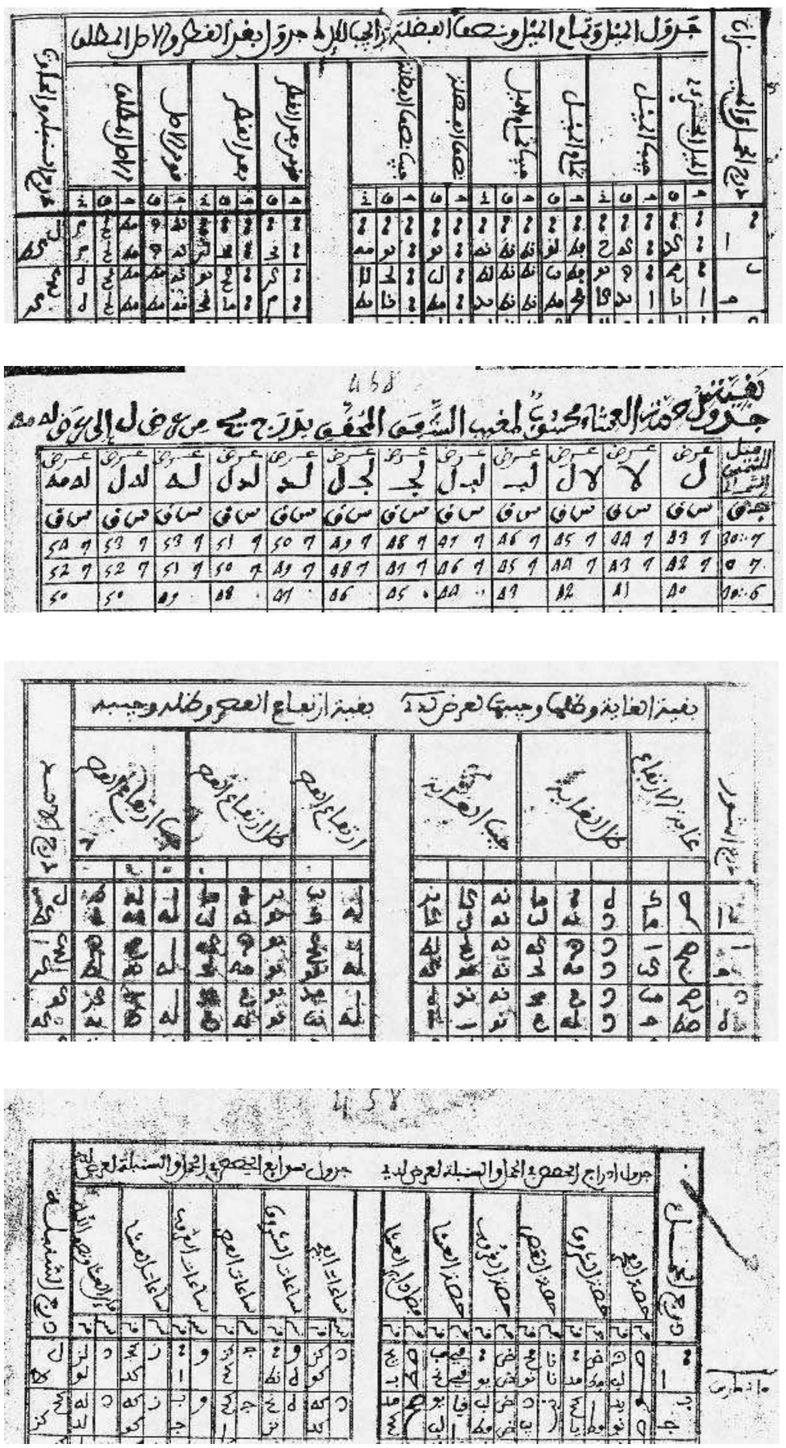
As this book was going to press I came across some photocopies of photos from a Maghribi manuscript labelled "Ms. 7, v. 3. Zijes. Manuscript", with the pages numbered 446 to 474. The original document dates from about 1800. I have no recollection of the source of these documents. There are some interesting tables for timekeeping, which I can only briefly list here: see **Figs. 13.8*a-d**.

These tables may have been taken from a larger set; certainly, there is no indication of a compiler or a date. The tables are stated to serve latitude 34;10°, which is not an attested value for any location in the Maghrib, and may well be the result of a new set of observations for Fez, whose latitude is accurately 34°05′, but was traditionally taken as 33° or 33;40°, so that was certainly need for a new measurement.[25] The underlying obliquity is 23;29°, which may also have been derived by observation, although it is an attested Ottoman (and European) value. See **I-6.4.20** and **6.10.17** for related tables.

The first group of tables displays the following functions side-by-side for each degree of λ' (1° to 90°):

$$\delta, \text{ Sin } \delta, \bar{\delta}, \text{ Sin } \delta, \text{ d, Sin d},$$
$$\text{arc Sin C, C, arc Sin B, and B,}$$

[25] Kennedy & Kennedy, *Islamic Geographical Coordinates*, pp. 117-118.



Figs. 13.8*a-d: Four extracts from the remarkable anonymous prayer-tables for Fez that survive in an apparently unique copy, whose location is a mystery to me. Possibly the compiler is al-Warjāni – see **I-6.4.20**. [From a set of photocopies of a manuscript labelled “Ms. 7, v. 3, Zijes, Manuscript”, provenance unknown.]

with values to two digits. The functions arc Sin C and arc Sin B are called *qaws buʿd al-quṭr* and *qaws al-aṣl*, respectively; on their purpose see **I-6.5**.

The second group displays in a similar fashion, but now for λ from 270° to 90°, the functions:

$$H,\ \mathrm{Cot}_{12}\ H,\ \mathrm{Sin}\ H,\ h_a,\ \mathrm{Cot}_{12}\ h_a,\ \text{and}\ \mathrm{Sin}\ h_a\ .$$

The third set displays two sets of functions labelled *ḥiṣaṣ* – and *sāʿāt* – (sometimes *sawāyiʿ*), also for λ from 270° to 90°. The *ḥiṣaṣ* are for daybreak (*al-fajr*) and sunrise (*al-shurūq*), both measured from midnight, and for the afternoon prayer (*al-ʿaṣr*), sunset (*al-ghurūb*), and nightfall (*al-ʿishā*), all three measured from midnight, followed by the hour-angle at nightfall (*faḍl dāʾir al-ʿishā*). The *sāʿāt* are for the same five times, now expressed in hours and minutes, followed by the time between nightfall and midnight (*mā bayna ʻl-ʿishā wa-niṣf al-layl*). The *sāʿāt* are written in Maghribi "Hindu-Arabic" notation, whereas the *ḥiṣaṣ* are in *abjad* notation.

Apart from a single table showing the solar longitude to the nearest degree from the date in the Western calendar, the remaining tables are of the same format, and are of a kind not known from any other source. A series of functions:

h_a (*irtifāʿ al-aṣr*), t_a^h (*ḥiṣṣat al-ʿaṣr*), $(D+s)^h$ (*ḥiṣṣat al-ʿishā*),
$(N+r)^h$ (*ḥiṣṣat al-fajr*), and d^h (*niṣf al-faḍla*)

are tabulated for each half degree of solar declination (entered vertically) for each half degree of latitude (entered horizontally) from 30° to 35;30°, then 35;45°. The parameters underlying the entries for evening and morning twilight are stated to be 18° and 19°, respectively. Again the entries are in Maghribi "Hindu-Arabic" numerals. Since only the first of three pages for the last function is present in the photocopy at my disposal it may be that the set originally served yet other functions.

In the light of the existence of this late set of sophisticated tables for timekeeping from the Maghrib, and no less the set described in **13.5**, it is perhaps all the more remarkable that H.-J.-P. Renaud, who wrote the first serious historical essays on Maghribi astronomy (see **13.0**), was not aware of the existence of any tables of this kind.

13.9 Miscellaneous notes

A motley collection of corrupt approximate rules for the prayer-times is contained in a late Maghribi source that I have consulted, but forgot where. The rules outlined are as follows:

$$t_a = 50° + \tfrac{1}{4}\,d \quad \text{or} \quad (45° + \tfrac{1}{4}\,\varepsilon - 1°) + \tfrac{1}{4}\,d \ (\text{where } \varepsilon = 24°!)$$
$$\text{or} \quad 51° + \tfrac{1}{4}\,d \quad \text{or} \quad \tfrac{4}{7}\,[90° - \tfrac{1}{2}\,d]$$
$$r = s + 3°\ .$$

The anonymous author also explains how to find the time of the *ḍuḥā* using the fact that $t_\partial = t_a$.

CHAPTER 14

TURKISH TABLES FOR TIMEKEEPING

14.0 Introductory remarks

The only astronomical activity amongst the Muslim community in Anatolia before the rise of the Ottomans is restricted to some legendary accounts of observations in specific *madrasa*s of Kirshehir and Kütayha documented by Aydın Sayılı,[1] and the sojourn of the well-known astronomer Quṭb al-Dīn al-Shīrāzī, who compiled some of his astronomical works in Konya, Sivas and Malatya in the middle of the second half of the 13th century.[2] If my assumption that the two sets of tables discussed below (**14.1-2**) are of Seljuk Anatolian provenance is correct, then clearly further research in this field would be worthwhile.

The later Ottoman tradition in astronomy has until recently been generally neglected by historians, despite the vast amount of source material. The popular *Ruznāme* of Shaykh Vefā (**14.3**) has never been properly studied, in contrast with the less widely-used *Rūznāme* (almanac) of Darendelī (**14.4**), on which three descriptive studies were published in Europe between 1676 and 1804.[3] Some brief notes on Ottoman astronomical works were published by J.-B. Toderini in 1789,[4] and an eloquent study by Adnan Adıvar, *La science chez les Turcs Ottomans*, alas devoid of any serious scientific content, was published in Paris in 1939. Otherwise, apart from a series of articles on specific topics that were published over many decades by Aydın Sayılı and his former student, Sevim Tekeli,[5] the notes which follow represented in the 1970s the first serious attempt to investigate the activities of the astronomers of Ottoman Turkey.[6] Now, however, we have a monumental survey of the available literature prepared by Ekmeleddin İhsanoğlu and his colleagues in Istanbul, which will serve as a starting-point for all future research.[7]

In the sequel (**14.3-14**), I describe several Ottoman sets of tables for timekeeping in general and prayer-tables in particular. These are of two kinds: in the first, time intervals are expressed in equatorial degrees and/or equinoctial hours, and in the second the times of day are expressed in equinoctial hours according to the convention that sunset is 12 o'clock. This latter convention, which in the 1970s was still used in the Levant, and more especially in Arabia,

---

[1] Sayılı, *The Observatory in Islam*, pp. 253-255.
[2] *Ibid*., pp. 215-217, *etc*., on his association with the Observatory at Maragha. See also the article by Seyyed Hossein Nasr in *DSB*. MS Paris BNF ar. 2516 of his *Tuḥfa shāhiyya* on mathematical cosmology is in his own hand and dated Sivas, 684 H [= 1285/86]: see *Cairo ENL Survey*, no. G27.
[3] See nn. 14:20-21.
[4] Jean-Baptiste Toderini, *De la littérature des Turcs, traduit de l'Italien en Français par Cournant*, 3 vols., Paris, 1789, I, pp. 147-148 and 406.
[5] A list of Sayılı's publications is in *Sayılı Memorial Volumes*, I, pp. 22-29. Many of Tekeli's publications are listed in İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 956-957.
[6] See King, "Astronomical Timekeeping in Ottoman Turkey".
[7] İhsanoğlu *et al*., *Ottoman Astronomical Literature*, and also *Ottoman Mathematical Literature*.

including the Yemen, seems to be of Ottoman Turkish origin, although, of course, the Islamic day has always been considered to begin at sunset.[8] It is the reckoning of time from sunset in equinoctial hours rather than seasonal hours that characterizes Ottoman practice. An obvious disadvantage of this convention in modern usage is that clocks must (in theory) be reset every evening at sunset.[9] In the sequel I use prime notation to indicate those times which are expressed in this convention (see also **1.3** above).

The most popular prayer-tables prepared for Istanbul were the *rūznāme*s of Shaykh Vefā and Darendelī, already mentioned above. I show (**14.3** and **14.4**) that they are based on different values for the latitude of Istanbul, and that the entries are expressed in different units. Both of these almanacs also contain calendrical tables, information on religious festivals, the seasons, and astrology, none of which con-cerns the present study. More sophisticated prayer-tables were prepared for Istanbul and Edirne, probably in the 16th and 17th centuries (**14.7**, **14.8**, and **14.11**). Particularly impressive are the contributions to our subject by Muḥammad ibn Kātib Sinān and Taqi 'l-Dīn (**14.5** and **14.9**). The extensive corpus of tables for timekeeping by the sun compiled by Ṣāliḥ Efendī (**14.12**) in the late 18th century, by which time Ottoman astronomers had come into contact with European astrology and had "translated" the *Zīj*es of Cassini and Lalande into Turkish and Arabic and adapted the tables to their own meridians. Various Ottoman-type prayer-tables for other localities in the Ottoman Empire, such as Cairo, Aleppo, Sanaa, Algiers, Mecca, Crete and Yarqand, have been located in the manuscript sources (see **7.11**, **11.11**, **12.11**, **13.7**, **12.10**, **14.14** and **3.17**, respectively). Doubtless other Ottoman sets of tables of this type for yet other localities are available amongst the several thousand mainly-uncatalogued astronomical manuscripts preserved in Turkey.

14.1 Anonymous Seljuk prayer-tables

MS Istanbul Süleymaniye 1037/32 (fols. 282v-285v) is an apparently unique copy of an anonymous set of prayer-tables, probably of Seljuk origin, computed for some unspecified latitude in Anatolia. The tables are bound amidst a collection of later astronomical works for Istanbul. The following functions are tabulated side by side for each degree of λ from 271° to 90°:

$$H,\ D,\ \tilde{h},\ h_a,\ t_a,\ T_a,\ h_b,\ t_b,\ T_b,\ s,\ 2N,\ n \text{ and } r\ .$$

Values are given to two digits and the underlying parameters are:

$$\phi = 38;30° \text{ and } \varepsilon = 23;35°.$$

This main set of prayer-tables is followed by a smaller single table displaying values of the functions:

$$\delta,\ C,\ B \text{ and } d$$

to two digits for each degree of λ and the same parameters.

The twilight tables are of particular interest. Whilst the values of s correspond quite closely to recomputed values using the accurate formula and parameter 17°, the values of r are based

[8] See Charles Pellat's EI_2 article "Layl and Nahār", esp. p. 708b.
[9] See Kurz, *European Clocks*, pp. 83-84.

on an approximate formula different from the standard approximate one (see **1.4** and, for example, **3.3**), and they correspond most closely to values recomputed with parameter 20°. I have not been able to determine the underlying formula. Whereas the table has values:

26;30°	25;39°	32;56°

for arguments λ = 271°, 1°, and 90°, recomputation with the accurate formula and parameter 20° gives:

27;46°	25;57°	35;11°

and recomputation with the standard approximate formula and parameter 20° gives:

25;24°	25;59°	36;30° .

This anomalous set of entries of course affects the values of n (=2n-r-s). Otherwise the tables are rather accurately computed.

14.2 Anonymous Seljuk prayer-tables (II)

MS Istanbul Nuruosmaniye 2782, copied in Sivas in 773 H [= 1371/72] by Zayn al-Munajjim "the astronomer" ibn Sulaymān al-Qūnawī, consists mainly of calendrical and astronomical tables of a non-numerical character. The accompanying text, which is in Persian, makes mention of al-Khwārizmī, Kūshyār and al-Bīrūnī. Amongst the tables is one displaying the solar longitude for each day of the Syrian year (fol. 39v, the entry for Ādhār I is 11^s 20°), followed by a set of tables for timekeeping (fols. 53v-54r), here labelled in Persian *jadwal-i ṭāliʿ-i awqāt*, which means "table showing the horoscopus at different times of day". It is stated that the entries were derived using a sextile astrolabe (*asṭurlāb sudsī*).[10] Another copy of this table came to light after this study had been completed (the first time round). It is contained in the unique MS Cambridge Browne O.1, fol. 179r, of the Persian *Zīj-i Mufrad* of the 11th(?)-century astronomer Muḥammad ibn Ayyūb al-Ṭabarī.[11] However, this table – see **Fig. IV-5.3** – is not necessarily original to the *Zīj,* since it is appended to the main tables in a different hand. It is not certain whether the Persian or the Arabic terminology is original, and the precise significance of the *chāsht* or *ḍaḥwa* is uncertain. The names of the functions tabulated, that is, the ten times of day, are given in the two sources as follows:

1	*ṣubḥ-i kādhib*	*al-fajr al-kādhib*	false dawn
2	*ṣubḥ-i-ṣādiq*	*al-fajr al-ṣādiq*	true dawn
3	*chāsht-i kūchak*	*al-ḍaḥwa 'l-ṣughrā*	the small (= first) *chāsht* or *ḍaḥwa*
4	*chāsht-i-miyān*	*al-ḍaḥwa 'l-wusṭā*	the middle *chāsht* or *ḍaḥwa*
5	*chāsht-i-buzurg*	*al-ḍaḥwa 'l-kubrā*	the large (= last) *chāsht* or *ḍaḥwa*
6	*nīm-i-rūz*	*anṣāf al-nahār*	midday
7	*miyān-i-dū-namāz*	*mā bayna 'l-ṣalātayn*	(the time) between two prayers
8	*awwal-i-namāz-i-dīgar*	*awwal al-ʿaṣr*	beginning of the other (= afternoon) prayer

[10] See Hartner, "Astrolabe", and also **X-4.1**.
[11] On al-Ṭabarī see Kennedy, "*Zīj* Survey", no. 65; and Storey, *PL*, II:1, pp. 3-4 and 43-44.

9	*ākhir-i-namāz-i-dīgar*	*ākhir al-ʿaṣr*	end of the afternoon prayer
10	*mughīb-i-shafaq*	*maghīb al-shafaq*	end of evening twilight

The underlying latitude is found by inspection to be about $\phi = 38°$, and it is not possible to determine the value used for ε. This latitude corresponds to Konya, although the possibility that the table was lifted from an earlier Iranian source cannot be excluded (see below). The three sub-tables for twilight (1, 2, and 10) are based on parameters 23°, 18°, and 18°, respectively. The remaining seven sub-tables correspond to the division of the interval from sunrise to sunset into eight equal parts, *i.e.*:

$^1/_8$, $^1/_4$, $^3/_8$, $^1/_2$, $^5/_8$, $^3/_4$ and $^7/_8$.

This is the only source in which such a division of the day is attested; on the significance of this division see **IV-5.3**.

14.3 The *Rūznāme* of Shaykh Vefā

The *Rūznāme-yi Vefāʾī* exists in dozens of manuscript sources, of which I have examined MSS Istanbul Hamidiye 842, Istanbul Nuruosmaniye 2914,1, Paris BNF turque 186, 187, 188, 194, turque supp. 537, Vienna 1426 and 1427, and several copies in Cairo, including Cairo KhMT 3: see **Fig. 14.3a-b**. See The identity of the compiler of this almanac has not previously been established with any certainty: the work has previously been ascribed to Shaykh Vefā, a celebrated saint who lived in the time of Muḥammad II and Bāyezīd II and died in the year 1491,[12] and to Shaykh Vefāʾī Muḥammad, who wrote a history of Murād III *ca*. 1585. A new source, MS Cairo K 4037, copied *ca*. 1700,[13] contains an anonymous commentary on the *Rūznāme* and confirms the former attribution, providing the following biographical information (fols. 1v-2r):

> "Actually his name is Muṣṭafā ibn Aḥmad ibn al-Ṣīrawi (?) al-ʿĪsawī, known as Vefā. This is as found in his handwriting on some of his copies (of the *Ruznāme*), according to the statement of the author of the (book entitled) *al-Sh[aq]āʾiq*.[14] [Note: the rest of these notes are probably taken from that work.] Vefā was the name of his father, but (the son) was also known by (this name). He studied mysticism under Shaykh ʿAbd al-Laṭīf al-Qudsī and with him completed the stages of the order.[15] (al-Qudsī) authorized him (to give) instruction (himself). (Shaykh Vefā) – may God Almighty have mercy on him – was a Ḥanafī, versed in all of the phenomenological and esoteric sciences and also in the science of timekeeping. He had noble behaviour because of his blessing. He also had a complete knowledge of music and rhetoric in poetry and prose. He was contemporary with the Sulṭān Muḥammad al-Fātiḥ [reg. 1444-1446] and he died in the time of the Sulṭān Bāyezīd (II) [reg. 1481-1512] in Constantinople.

[12] See *Cairo ENL Survey*, no. H2; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, I, pp. 51-54, no. 19 (with references to many more manuscripts).

[13] For the Arabic text see *Cairo ENL Catalogue*, II, pp. 234-235.

[14] This is the biographical encyclopedia *al-Shaqāʾiq al-nuʿmāniyya fī ʿulamāʾ al-dawlat al-ʿUthmāniyya* by Ṭashköprüzāde (d. 1561 – see the EI_2 article by F. Babinger updated by Christine Woodhead).

[15] See the article "Taṣawwuf" in EI_2.

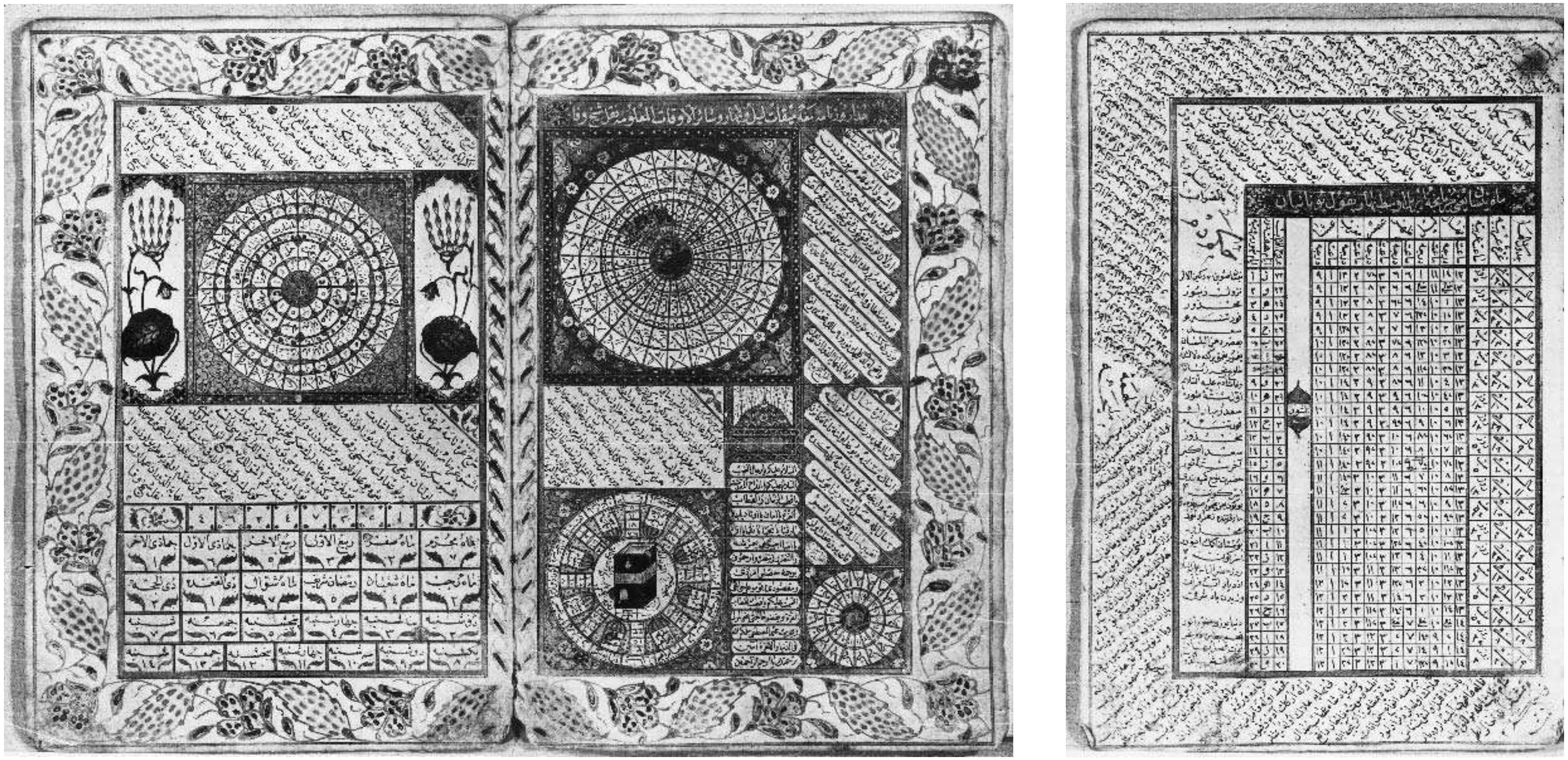


Fig. 14.3a-b: More extracts from another copy of the *Rūznāme* of Shaykh Vefā: calendrical tables and mystical interpretations of the Kaʿba, mentioning the *rijāl al-ghayb*, intermediaries between God and Man (b), and the first two of twelve pages of tables for the times of prayer (c). [From MS Cairo KhMT 3, fols. 1v-2r and 2v-3r, courtesy of the Egyptian National Library.]

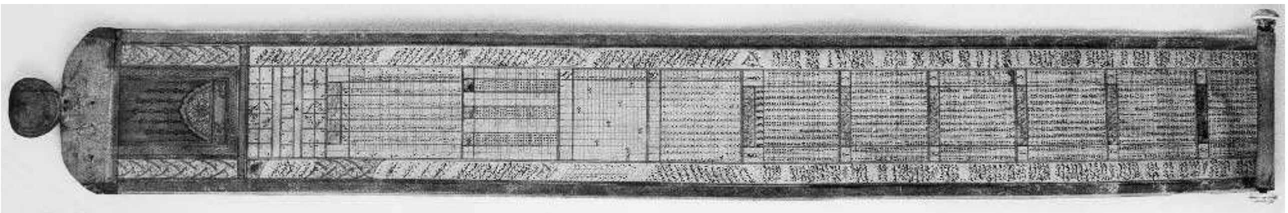


Fig. 14.4b-c: The Oxford copy of Darendelī's prayer-tables, apparently dated 1795. [From MS Oxford MHS 84-33, courtesy of the Museum of the History of Science.]

(This was) after he had journeyed on the pilgrimage by sea. The Christians took him prisoner and incarcerated him in the Citadel of Rhodes.[16] He compiled these tables in prison. The *amīr* Ibrāhīm Bey ibn Qarāmān[17] bought him from (the Christians), then he went home to Constantinople and died there. He has a prayer room and mosque which are known, and his tomb is in front of the mosque. It is famous and is visited (by the people) to obtain blessing from him.[18] The date of his death is the year eight hundred and ninety six of the Hijra of the Prophet [= 1490/91] ... ."

[16] On the history of Rhodes under the Ottomans see the EI_2 article "Rodos" by S. Soucek.

[17] Perhaps the grandson of the better known Ibrāhīm Beg (d. 1464): see the EI_2 article "Ḳarāmān-Og͟hullari (Ḳarāmānids)" by F. Sümer, esp. pp. 619 and 622.

[18] İhsanoğlu, *Ottoman Astronomical Literature*, I, p. 51.

All copies of the *Rūznāme* contain calendrical tables. However, MSS Istanbul Hamidiye 842 and Nuruosmaniye 2914,1, Paris BNF turque supp. 537, and Vienna 1426 contain in addition a set of prayer-tables. In MS Vienna 1426 there are twelve sub-tables, one for each month of the Syrian year. In this particular Paris manuscript the tables for each month have been copied on both sides of a single folio. The solar longitude corresponding to each day of the year is given in the argument column in MS Vienna 1426, and in a separate table in the Paris manuscript, which is described below. The functions tabulated are the following, and the values are expressed in *equinoctial hours* (*sāʿāt*) *with fractions thereof in equatorial degrees* (*daraja*):

2D	*nahār*, length of daylight
2N	*layl*, length of night
D	*ẓuhr*, time from sunrise to midday or midday to sunset
t_a	*ʿaṣr*, time from midday to the afternoon prayer
T_a	*maghrib*, time from the afternoon prayer to sunset
s	*ʿishāʾ*, duration of evening twilight.

The entries are written in Eastern Arabic numerical characters. Pretty designs (MS Vienna 1426) or the Arabic word *khālī* meaning "empty" (MS Paris BNF turque supp. 537) are used for units which are zero. Half degrees are indicated by the symbol ι. The parameters underlying these prayer-tables are:

$$\phi = 41;30° \text{ and } \varepsilon \approx 23;29° \ ,$$

with 19° and 17° for morning and evening twilight. In other Ottoman tables the latitude 41;30° is used for Edirne (**14.7**), but it seems certain that the *Rūznāme-ye Vefāʾī* was intended for use in Istanbul, where the latitude is in fact 41°.[19]

In MS Paris BNF turque supp. 537 there is a separate table of two functions for regulating the *ifṭār* and *imsāk* in Ramaḍān, that is, the time of the evening meal after the day's fast and the beginning of fasting the next day. The functions are labelled *ḥiṣṣat-i ghurūb* and *ḥiṣṣat-i ṣuḥūr* (*sic* for *suḥūr*) and values are given to one digit for each day of the Syrian year. The solar longitude is also given for each day. In fact the main functions are simply T_a and r, expressed in equatorial degrees.

MS Istanbul Esat Efendi 1973, fol. 148v, contains a table entitled *jadwal mīqāt imsākiyya,* which means "table for the time of fasting in Ramaḍān". It is stated that the *tamkīn*, the time before daybreak when one should start fasting, is 12^m. The entries in the table are given in hours and minutes according to the Ottoman convention, for each degree of λ from Capricorn 1° to Gemini 30°. The entries are symmetrically arranged, but both vertical arguments run from 1° to 30°, whereas the upward arguments should run from 0° to 29°. The entries are 10^m (not 12^m) less than the times for daybreak given in the *Rūznāme-yi Vefāʾī*. The remaining tables in this manuscript consist of planetary tables for Istanbul based on the parameters of Ulugh Beg.

[19] See n. 14:25.

14.4 The *Rūznāme* of Darendelī

The *Rūznāme* of Darendelī has been published three times, by Velschius (1676), d'Ohsson (1787), and Navoni (1804).[20] From an examination of the material published by d'Ohsson and Navoni, as well as of MSS Istanbul Aşir Efendi 470, Istanbul Kandilli 440, Oxford MHS 84-33, and an unnumbered manuscript in the Yale Medical School Historical Library, New Haven, Conn., it is clear that at least these copies are recensions of the same work. The relatively small number of surviving manuscripts suggests that this *rūznāme* was less popular than that of Shaykh Vefa (**14.3**). Navoni and d'Ohsson attributed the almanac to an individual named Darendelī (Arabic, al-Darandawī), as did Toderini, who stated in 1789 that the almanac had been in use in Turkey for a century.[21] Muḥammad ibn ʿUmar ibn ʿUthmān Darendelī[22] is also known as the author of a treatise on the astrolabic quadrant,[23] and he died in 1739: his name indicates that he or his family was associated with the town of Darende in central Anatolia.[24] Judging by the number of extant copies his almanac was less popular than that of Shaykh Vefā (**14.3**).

The lithograph copy of Darendelī's almanac published by d'Ohsson and also the Yale and Oxford manuscripts are in the form of scrolls, about 10 cm wide and 1 m long: see **Fig. 14.4a-b**. The non-numerical material is written around the tables in Osmanli Turkish, and the entries in the tables are written in Eastern Arabic numerals as in the *Rūznāme-ye Vefāʾī*. An entry such as 2512 (in Arabic numerals) indicates 25 minutes past 12 o'clock. Zero is represented by a dot in the number 10, but by three dots ∴ when it is one of the units of an entry. One-half is represented by a tick or by /. Time is reckoned in equinoctial hours and minutes, according to the Ottoman convention.

The prayer-tables of Darendelī consist of six sub-tables for each pair of zodiacal signs: see **Fig. IV-5.4**. The vertical argument in each generally runs from 1° to 30°, but extra entries enable the user to find the prayer times for each day of the solar year using a second argument column. Nine functions are tabulated side by side in each sub-table. These are:

2D	*nahār*, the length of daylight
2N	*layl*, the length of night
M′	*ẓuhr*, the time of midday
a′	*ʿaṣr-i awwal*, the earliest time for the first afternoon prayer
b′	*ʿaṣr-i thānī*, the earliest time for the second afternoon prayer
s′	*ʿishāʾ*, the time of nightfall
r′	*imsāk*, the time of daybreak
t_q'	*qibla*, the time when the sun is in the azimuth of Mecca
∂′	*ḍaḥwa*, mid-morning

[20] G. H. Velschius, *Commentarius in ruzname naurus*, Augsburg, 1676; M. d'Ohsson, *Tableau général de l'Empire Othoman*, vol. I, Paris, 1787; and J. B. Navoni, "Rouz-name ou calendrier perpetuel des Turcs", *Fundgruben des Orients (Mines de l'Orient)* 4 (Vienna, 1814), pp. 38-67, 127-153, 253-277, and appendix.

[21] See n. 14:4.

[22] İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 406-410, no. 271, also p. 464.

[23] In *Cairo ENL Survey*, no. D176, this treatise is incorrectly attributed to "Muḥammad Efendī Darandar", listed under Egyptian (and Syrian) authors.

[24] See the EI_2 article "Lāranda", and also King, *Mecca-Centred World-Maps*, p. 75 and n. 65.

The entries in the tables are based on the parameters:

$$\phi = 41;0°, \varepsilon \approx 23;29°, q = 42;0°$$

with 19° and 17° for morning and evening twilight. The value for the latitude of Istanbul is very accurate, and much better than that used by Shaykh Vefā (**14.3**) or the first value derived by Taqi 'l-Dīn (**14.9**), let alone the earlier (and absurd) Islamic and Byzantine value 45°.[25] The value of q indicates that the longitude difference between Istanbul and Mecca was taken as about 17°: in fact, it is only about 11°, and the modern value of q is approximately $28^1/_2$°.

The time called *imsāk* is of particular significance during Ramaḍān. The word means "abstinence" and denotes the beginning of the fast which lasts until sunset. In these tables the *imsāk* is identified with daybreak (*fajr*). However, at the foot of each of the three copies of Darendelī's almanac which I have consulted, it is stated that the *imsāk* should be 15^m before the time given in the table. In al-Gedūsī's treatise on the sine quadrant, a late Ottoman source (**14.15**), it is stated that the *imsāk* should be 16^m before dawn.[26] On the time called *ḍaḥwa*, Navoni (1814) remarked:[27]

> "[Le] Zahve(?), vulgairement dit *Kouschlouk,* marque un autre temps entre le lever du soleil et le midi, mais sans une prière particulière et d'obligation."

In fact, the *ḍaḥwa* divides the interval between daybreak and sunset in half. I do not know whether any religious activity was associated with the *ḍaḥwa* in Ottoman Turkey. On the *ḍuḥā* or *ḍaḥwa* in late Islamic astronomical sources see further **IV-5.4**.

14.5 Muḥammad ibn Kātib Sinān's tables for timekeeping by the stars

Muḥammad ibn Kātib Sinān was a *muwaqqit* in Istanbul in the late 15th century.[28] One of his works is an enormous table for timekeeping by the stars, preserved in MSS Istanbul Ayasofya 2710 and Istanbul Topkapı T 3046 (Ahmet III 3515). The table occupies some 500 pages and contains about 240,000 entries, and is arranged so that one can feed in both the normed right ascension of a star which is culminating and the solar longitude, and read off:

(1) the time since sunset,
(2) the time remaining until sunrise,
(3) the time remaining until daybreak, and
(4) the time remaining until midday .

The tables are computed for latitude $\phi = 41;0°$ (Istanbul): see further **I-2.7.2**. On a much smaller Turkish table for timekeeping by the stars, or rather, one particular star, see **I-2.8.1**.

---

[25] On this see my notes in "Byzantine Astronomy", pp. 117-118; and King, "Geography of Astrolabes", p. 12.

[26] See n. 14:40.

[27] Navoni, "Rouz-name des Turcs" (cited in n. 14:20), p. 58.

[28] On Muḥammad ibn Kātib Sinān (**I-2.7.2**) see Suter, *MAA*, no. 455; *Cairo ENL Survey*, no. H8; and İhsanoğlu *et al*., *Ottoman Astronomical Literature*, II, pp. 84-90, no. 46.

14.6 Three Ottoman copies of al-Khalīlī's universal auxiliary tables

MS Istanbul Hamidiye 1453,3 (fols. 232v-266v) contains al-Khalīlī's universal auxiliary tables (**10.7**) and was copied in Edirne in 869 H [= 1464/65] by ʿUmar ibn ʿUthmān al-Ḥusaynī. The tables follow a number of treatises on timekeeping and quadrants (see, for example, **2.5** and **2.6**) and are preceded by al-Khalīlī's instructions written in Arabic (see below). The functions f_ϕ and g_ϕ are tabulated only for arguments:

$$\phi = 20°, 21°, \ldots, 49°, \text{ as well as } 21;30° \text{ (Mecca) and } 33;30° \text{ (Damascus)} .$$

The main tables are preceded by a small table (fols. 233v-234r) displaying the qibla as a function of local latitude and longitude, $q(\Delta\phi,\Delta L)$, which, as I have shown elsewhere, is ultimately of Abbasid origin;[29] a table of $\delta(\lambda)$ to two digits, which, although for $\lambda = 90°$ it has entry 23;32°, a value of ε nowhere else attested in the known Islamic sources, is apparently based on $\varepsilon = 23;31°$; and tables for finding the solar longitude from the date in the Muslim and Syrian calendars (the solar longitude for Ādhār 1 is given as Pisces 20°;23°). The main tables are followed by some simple trigonometric tables of the functions:

$$\text{arc Sin } (x), \text{ arc Vers } (x) , \text{Cot}_{12} \text{ h and Tan}_{60} \text{ h} ,$$

as well as a table of max $d(\phi)$ based on $\varepsilon = 23;35°$.

MS Istanbul Ayasofya 2590 is a second Ottoman copy of al-Khalīlī's universal auxiliary tables in a recension prepared by the *muwaqqit* Muḥammad ibn Kātib Sinān (**14.5**) for the Sulṭān Bāyazīd, and dated 897 H [= 1491]. Ibn Kātib Sinān translated al-Khalīlī's introduction into Turkish and then copied the tables in their entirety. He also copied al-Khalīlī's parameters 19° and 17° for twilight in the introduction, although elsewhere (as in his treatise on the astrolabe preserved in MS Istanbul Ayasofya 2708) he advocated 20° and 16°. The main set is preceded by others of $\delta(\lambda)$ and $\delta_2(\lambda)$ computed to three digits for $\varepsilon = 23;35°$ and followed by tables of $h_a(H)$ and $h_b(H)$.

In a late Ottoman miscellany of astronomical treatises preserved in an unnumbered manuscript formerly (*ca*. 1970) in the private collection of the late Professor Buhairi of the American University of Beirut, there are tables of $f_\phi(\theta)$ and $g_\phi(\theta)$ for $\phi = 41°$ (Istanbul) and a single table of $K(x,y)$ for $x = 41$. There are no accompanying instructions, and the hapless individual who thought fit to copy these three tables together was unaware that the argument x is unrelated to the local latitude.

14.7 Anonymous timekeeping tables for Edirne

MS Oxford arab. e. 93, fols. 3v-25r, contains an undated set of anonymous tables of the functions $t(\lambda,h)$ and $T(\lambda,h)$ expressed in equatorial degrees: see **I-2.2.2** (illustrated). Values are given for each degree of λ starting at the winter solstice and each degree of h up to $[H(\lambda)]$, and the city of Edirne is specifically mentioned at the head of some of the tables. The underlying parameters are:

$$\phi = 41;30° \text{ and } \varepsilon \approx 23;30° .$$

---

[29] See King, "Earliest Qibla Methods", p. 118.

There are no prayer-tables accompanying this set, but notice that the *Rūznāme-ye Vefāʾī* for Istanbul (**14.3**) is based on the same parameters. See also **14.11** on some other tables for Edirne.

14.8 Anonymous prayer-tables for Istanbul

MS Cairo ṬM 255,6, copied 1060 H [= *ca*. 1650], contains al-Khalīlī's introduction to his prayer-tables and some anonymous Syrian prayer-tables for latitudes:

ϕ = 36° (Aleppo) and 41;15° (Istanbul).[30]

Mention of these was omitted from the first version of this text, and all that is available to me now is a photo of an extract from the tables for Aleppo (**Fig. 11.5**). This shows tables of the same kind as those of al-Khalīlī but for ε = 23;35° rather than his more up-to-date value 23;31°. The functions tabulated for Aleppo are:

$H, Z, D, h_a, t_a, T_a, N, n, r, s, \tilde{h}, d$ and t_q ,

MS Istanbul UL T1824,1 (fols. 3r-9r) contains another set of prayer-tables for Istanbul, based on the parameters:

ϕ = 41;15° and ε = 23;30° .

No compiler is mentioned. Notice that the value of the latitude is the same as that used by Taqi 'l-Dīn in his early work (**14.9**), and that the value of the obliquity is the one usually associated with astronomers who relied on Ulugh Beg (see, for example, **7.1**). The set begins with a table of $\delta(\lambda)$ computed to two digits for each degree of λ. The main prayer-tables display some 22 functions tabulated side by side for each degree of λ from Capricorn 1° to Gemini 30°, as in the tables of al-Khalīlī (**10.6**). Values are given to two digits. The functions tabulated are the following:

$D^h, C, H, D, 2D^h, h_a, h_b, t_a, t_b, T_a, T_b, 2N^h,$
$s, (2N\text{-}r), r, (r+D), (D+t_a), h_q, t_q, \tilde{h}, B$ and b_1.

MSS Paris BNF ar. 2544,14 and Cairo DM 36 contain an extract from these tables displaying only the functions:

$D, 2D^h, t_a, T_a, s, (2N\text{-}r), r$ and $\tilde{h}$.

No compiler is mentioned in either source.

These tables are rather accurately computed. The parameters used for twilight are 19° and 17°, and I have not investigated the value of the qibla underlying the tables of h_q and t_q. Note that in the geographical tables of Taqi 'l-Dīn (**14.9**), we find the entries:

Istanbul	ϕ: 41;15°	L: 60;0°
Mecca	21;30	77;0°

so that the qibla at Istanbul, computed according to the accurate formula, would be 41;10° E of S. On the tables of B, b_1 and C see **I-6.4.9**, **I-6.5.2** and **I-6.10.7**. The fact that a table of b_1 was included in the set I take to be further evidence that its compiler was inspired by the tables of al-Khalīlī.

MS Istanbul Hamidiye 842 contains another set of less extensive prayer-tables based on the same parameters, but with slightly different entries. Values of the following functions are given

[30] *Cairo ENL Survey*, no. C142.

to two digits in modern Arabic numerical notation for each degree of λ from Capricorn 0° to Gemini 29°:

$$H, Z_{(12)}, D, h_a, t_a, T_a, s, r, \tilde{h} \text{ and } 2N^h .$$

MS Cairo MM 100 (13 fols., *ca*. 1750) contains some anonymous prayer-tables for Istanbul arranged according to the Byzantine months, which I have not investigated.

14.9 Taqi 'l-Dīn on some aspects of timekeeping

Taqi 'l-Dīn (*ca*. 1525-1585)[31] was the principal astronomer associated with the founding of the short-lived observatory in Istanbul.[32] He wrote a number of works of considerable interest, which have not yet received the attention which they deserve. MS Istanbul Kandilli 208 is a copy of some of Taqi 'l-Dīn's works in his own hand, and contains a *ṭaylasān* table displaying the hour-angle and time since sunrise as functions of the meridian altitude and the instantaneous altitude for parameters:

$$\phi = 41° \text{ (Istanbul) and } \varepsilon = 23;30° .$$

I see no reason yet to doubt that this was computed by Taqi 'l-Dīn himself (see further **I-2.3.6**). It may be that Taqi 'l-Dīn also compiled the prayer-tables discussed in **14.8** since he used the latitude 41;15° for Istanbul in his earlier works, such as his treatise on sundial construction. In his later work, however, he used latitude 40;58° for Istanbul and obliquity 23;28,54°: these are the results of his "new Murād Khān observations" (see, for example, **I-6.4.8**).

In his *Zīj* entitled *Jarīdat al-durar wa-kharīdat al-fikar*, of which I have examined MS Istanbul Esat Efendi 1976, Taqi 'l-Dīn advocates the parameters 19° and 17° for twilight and then mentions certain corrections (*taʿdīl*) which should be applied to the length of twilight or the diurnal arc. The following is a translation of the relevant section in Ch. 98 of the *Zīj*:

> "For a day of fasting it is necessary to modify the duration of twilight and the length of daylight by the quantities which I determined by observation and calculation. This is explained in detail in my treatise called *al-Qawl al-marʾī fī 'l-nahār al-sharʿī*. In brief, I say that if Ramaḍān occurs when the sun is at the beginning of Aries then the amount which must be added to the semi diurnal arc in the west is 1;3°, likewise, Taurus 1;20°, Gemini 1;43°, Cancer 1;39°, Leo 1;23°, Virgo 0;59°, Libra 0;54°, Scorpio 0;54°, Sagittarius 0;39°, Capricorn 0;29°, Aquarius 0;59°, and Pisces 1;3°. To the semi diurnal arc in the east should be likewise added, in the order of signs: 0;56°, 0;58°, 1,8°, 1;15°, 1;16°, 1;8°, 1;6°, 1;7°, 1;6°, 1;5°, 0;59°, and 0;58°. In the middle of the signs one should use linear interpolation. Modifying the length of daylight obviates the need to modify the duration of morning twilight, because the latter is defined in terms of the apparent horizon and the light of the upper edge of the disk of the sun (*ḥājib al-shams*). The size (of the sun) never exceeds the sum of the two corrections, that is, a maximum 2;51° in Gemini, which is about 3°, and a minimum 1;34° in Capricorn, which is approximately $1^1/_2$°, an amount that is not at all small (*i.e.*, negligible)."

---

[31] On Taqi 'l-Dīn see n. **I**-2:35. See also the next note.
[32] See Sayılı, *The Observatory in Islam*, pp. 289-305, and also Ünver, *Istanbul Observatory* (in Turkish).

Unfortunately I do not know of any copies of Taqi 'l-Dīn's treatise *al-Qawl al-marʾī ...* . The title means something like "A Statement as Clear and Visible as Light of Day concerning the Length of Daylight as Defined by the Religious Law".[33] It would be interesting to know how he derived these two sets of values. Suffice it to say for the time being that the mean of the two sets is suspiciously close to the values given by ʿAbd al-Qādir al-Minūfī (**8.2**) for $\phi = 41°$. These mean values are:

1;0 1;19 1;26 1;27 1;20 1;4 1;0 1;1 0;53 0;47 0;59 1;1 ,

and al-Minūfī has:

1;4 1;11 1;18 1;25 1;18 1;11 1;4 0;57 0;51 0;44 0;51 0;57 .

Note that al-Minūfī Jr.'s tables for refraction were compiled in Cairo in 975 H [= 1567] and Taqi 'l-Dīn's *Zīj* was compiled in Istanbul about 990 H [= 1582].

In a short treatise preserved in MSS Istanbul Kandilli 208 (fols. 89v-90r) and Istanbul Kandilli 176, of which the former is in his own hand, Taqi 'l-Dīn claims to discuss the difference of opinion between the Cairo *muwaqqit*s concerning the precise length of daylight, nightime, and twilight. He makes no mention of the nature of the difference of opinion, but does state that the visible horizon is 0;2,13° below the true horizon. This number is related to the numerical tradition attributed to Ibn al-Haytham (see **7.1**, also **9.3**). Otherwise the treatise contains no information of historical or scientific consequence.

MS Istanbul Kandilli 441 of the shorter version of Ṣāliḥ Efendī's tables for timekeeping (**14.13**), copied in 1266 H [= 1849/50], contains a simple table for evening twilight (p. 62) and a set of geographical coordinates (p. 73), both of which are attributed to Taqi 'l-Dīn. The following values are given for the duration of twilight in equatorial degrees for each zodiacal sign:

23 25 29 *32* 29 25 23 23 24 25 24 23

If we round the values given in MS Istanbul UL T1824,1 (**14.8**), which are based on Taqi 'l-Dīn's first value 41;15° for the latitude of Istanbul and parameter 17°, we obtain:

23 25 29 *31* 29 25 23 23 24 25 24 23 .

All but the fourth value correspond precisely to Taqi 'l-Dīn's values.

Since there is a dearth of Ottoman geographical tables that have been published,[34] and since the Kennedys did not use any in their survey of Islamic geographical tables, I present the table attributed to Taqi 'l-Dīn in full:

Locality		ϕ	L
1	Istanbul	41;15°	60; 0°
2	Gallipoli	41;15	58; 0
3	Mitilene	39; 0	55; 0
4	Salonica	39;30	50; 0
5	Kavalla	40;30	46;30
6	Cos	38; 0	55; 0
7	Izmir	38; 0	56;20

[33] No copies of this work are mentioned in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, I, pp. 199-217, but Taqi 'l-Dīn's treatises listed as nos. 3, 12 and 16 (pp. 204, 209-210 and 211) relate to the same subject and would repay investigation.

[34] This table is now featured in Sezgin, *GAS*, X, pp. 182-184. See also King, *Mecca-Centred World-Maps*, p. 86-87, on another Otoman table giving only the qiblas of 90 localities.

8	Rhodes	36;50	54;30
9	Crete	35; 0	54; 5
10	Cyprus	35;30	68;30
11	Alexandria	31; 0	61;55
12	Cairo	30;10	64;55
13	Jerusalem	32;10	66;30
14	Damascus	33;35	70; 0
15	Aleppo	36; 0	[3]6;10 [a]
16	Medina	24;30	75;20
17	Mecca	21;30	77; 0
18	Kayseri	39; 0	69; 0
19	Kastamonu	41;30	65; 0
20	Eskişehir	39;30	62;50
21	Konya	38; 0	65;10
22	Bursa	40; 0	58;10
23	? (illegible)	41; 0	60;15
24	Dile	41;30	61; 0
25	Eregli	41;30	67; 0
26	Sinop	40;30	69;30
27	*B-ʾ-r-ṭ-y-z* (?)	41;30	64; 0
28	Trabzon	40;50	71;50
29	Edirne	41;30	58;30
30	*Q-r-q k-l-y-ʾ* (?)	41;30	58;30

[a] text: 26;10°

14.10 Anonymous prayer-tables for Anatolia

MS Cairo KhMT 2, copied about 1750, consists mainly of a set of planetary tables based on the *Zīj* of Ulugh Beg and a few spherical astronomical tables specifically for $\phi = 41°$, that is, Istanbul. Included amongst these (fols. 55v-58r) is a set of prayer-tables for latitude 38°, locality unspecified. The functions tabulated are:

$$H, 2D, D, t_a, t_{a/b}, t_b, T_b, s, 2N, n \text{ and } r ,$$

and the values are given in Eastern Arabic numerical notation to the nearest half degree, one-half being indicated by the symbol ι. I have not investigated the parameters underlying the values of s and r. The function $t_{a/b}$ is refered to as *bayn al-ʿaṣrayn*, and displays the hour-angle at a certain time about halfway between the two *ʿaṣr* prayers. The tables display some discrepancies: for example, the values of t_b and T_b do not total D, and the values of s and n and r do not total 2N.

MS Istanbul Hüsnü Paşa 1286, copied around 1700, contains an anonymous set of tables (fols. 66r-74r) of the functions:

$$2N, N, B, C, H, \delta, d, \psi, h_0, D^h, D, 2D, \alpha_\phi \text{ and } \alpha' ,$$

computed for parameters:

$$\phi = 42° \text{ (Edirne) and } \varepsilon = 23;31° \text{ (?)}.$$

On other tables for Edirne see **14.7**.

14.11 An anonymous *ṭaylasān* table for Edirne (?)

MS Cairo KhMT 2,1, copied about 1750 (**14.10**), contains a flyleaf (fol. 1r) on which part of a *ṭaylasān* table has been copied. The table is entitled *jadwal faḍl al-dāʾir* and displays the hour-angle t(H,h) for the argument domains:

$$H = 71;29°,\ 71°,70°,\ \dots\ ,\ 48° \text{ and } h = 0°,\ 1°,\ \dots\ ,\ H\ .$$

Values are given to two sexagesimal digits. Since the entry for H = 48° and h = 0° is 90;0°, the underlying parameter is:

$$\phi = 42;0° \quad (\text{with } \varepsilon = 23;29°)\ .$$

Not all of the entries have been filled in, and the page of tables for $\delta < 0$ is not contained in this manuscript. There is no mention of the locality for which the table was prepared; this may have been Edirne. See also **I-2.3.7**.

14.12 Aḥmad Efendī's timekeeping tables for Istanbul

MS Istanbul Kandilli 196 is the only copy known to me of a set of hour-angle tables for Istanbul, computed in 1095 H [= 1684] by "Aḥmad Efendī, known as Miṣrī al-Islāmbūlī", "the Egyptian in Istanbul".[35] The function t(h,λ) is tabulated for the domains:

$$h = 1°,\ 2°,\ \dots\ ,\ 72° \quad \text{and} \quad \lambda = 1°,\ 2°,\ \dots\ ,\ 90°\ (\delta \gtrsim 0)$$

and the underlying parameters are:

$$\phi = 41;0° \text{ (Istanbul) and } \varepsilon = 23;30°\ .$$

The tables have the same format as those in the main Cairo corpus (**4.0**) and are quite accurately computed: see already **I-2.1.7**. Aḥmad Efendī prefered to use obliquity 23;30°, as in the late Egyptian tradition (**7.1**), rather than Taqi 'l-Dīn's value 23;28,54° (**14.9**). A century later Ṣāliḥ Efendī recomputed t(λ,h) for latitude 41° using Taqi 'l-Dīn's parameter (**14.13**).

14.12* Prayer-tables for Tillo by Ismāʿīl Fahīm

In February, 2003, as I was preparing the final illustrations for this book, I came across a negative of some prayer-tables in a Cairo manuscript that were not yet described in the text. Checking my 1986 *Survey* of the Cairo manuscripts, I found that I had promised a description in *SATMI*. The following must suffice.

MS Cairo DMT 2,1 (fols. 1r-26v, 1202 H [= 1787/88]) is a copy of a set of prayer-tables by Ismāʿīl Fahīm, son of the better-known Ibrāhīm Ḥaqqī who was the author of a well-known encyclopaedia entitle *Maʿrifat-nāme*.[36] Five other manuscripts are preserved in Turkey. The tables are preceded by an introduction in Turkish and the work is dated 1193 H [= 1779] and dedicated to the father. The underlying latitude is stated to be 38;30°, and apparently no locality is specified. However, in the colophon of some of the copies mention is made of the village

[35] I did not find mention of Aḥmad Efendī or these tables in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*.
[36] On the author see *Cairo ENL Survey*, no. H39; and İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, 627-628 (no. 462), where the other manuscripts of these tables are listed. On his father see *Cairo ENL Survey*, no. H38.

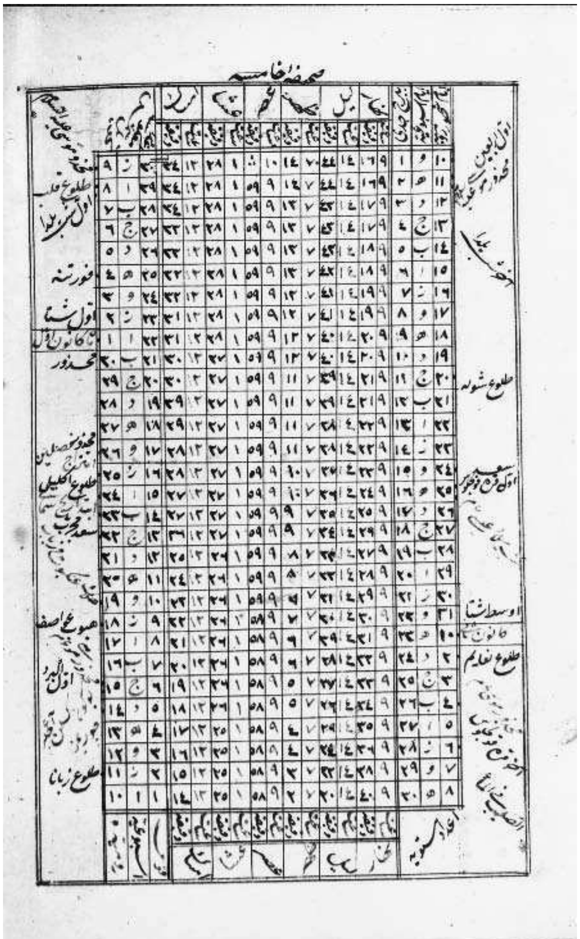


Fig. 14.12*: An extract from the prayer-tables of Ismāʿīl Fahīm. [From MS Cairo DMT 2,1, courtesy of the Egyptian National Library.

of Tillo, where the father, originally from Erzerum, was living at the time. Ekmeleddin İhsanoğlu kindly informs me that Tillo is in the district of Aydınlar in S. E. Turkey, directing me to www.tillo.net. The tables display for each degree of each sign the functions:

$$2D^h,\ 2N^h,\ z',\ a',\ s' \text{ and } i' .$$

I have not investigated the underlying twilight parameters.

14.13 Ṣāliḥ Efendī's corpus of timekeeping tables for Istanbul

Ṣāliḥ Efendī Miʿmārī "the architect" (*fl. ca.* 1700) compiled an extensive corpus of tables for timekeeping by the sun, extant in numerous copies.[37] MSS Princeton Yahuda 353, Istanbul Aşir Efendi 224, Istanbul Kandilli 219, Vienna 2379 (Mixt. 989), Cairo ṬM 151 and 215, and

[37] On Ṣāliḥ Efendī see n. **I**-2:23.

Cairo K 18199, are copies of this corpus; all but the last are complete. The title is simply *al-jadwal al-kabīr*, "the large table", which is appropriate since the corpus contains over 80,000 entries. Ṣāliḥ Efendī's tables, and simpler versions thereof, were widely used in Istanbul in the 19th century.

The main tables are for timekeeping by the sun; the remainder is for regulating the times of prayer. All are based on the parameters:

$$\phi = 41;0° \text{ (Istanbul) and } \varepsilon = 23;28,54° .$$

This distinctive value of the obliquity is the value derived by Taqi 'l-Dīn (**14.9**), who worked in Istanbul two centuries prior to Ṣāliḥ Efendī.

In the main tables of the corpus Ṣāliḥ Efendī tabulated the hour-angle and time since sunrise $T(\lambda,h)$ and $t(\lambda,h)$ for each degree of l° starting at the winter solstice and for each degree of h up to $[H(\lambda)]$. A given sub-table displays entries for one particular solar longitude, with the times for solar altitudes in the east and west expressed in equatorial degrees (*al-dāʾir*, *faḍl al-dāʾir*), equinoctial hours (*al-mustawiya*), seasonal hours (*al-zamāniyya*), and in equinoctial hours according to the Ottoman convention (*al-muwāfiqa*). Entries are given to three sexagesimal digits in each case and are very carefully computed. The way in which the tables were compiled is clear from the nature of certain auxiliary functions also tabulated by Ṣāliḥ Efendī. See further **I-2.2.3**. Various functions are displayed around each of these sub-tables, often in different coloured inks. These are the following:

$$H,\ 2D,\ 2N,\ a',\ b',\ r' \text{ and } s' .$$

The parameters used for twilight are 19° and 17°, and the entries, which but for the first function H are in hours, are expressed to three sexagesimal digits.

At the end of the main set of tables the functions:

2D	*al-nahār*	length of daylight
r'	*al-fajr*	time of daybreak
$a' = T_a'$	*al-ʿaṣr al-awwal*	the first afternoon prayer
$b' = T_b'$	*al-ʿaṣr al-thānī*	the second afternoon prayer
2N	*al-layl*	length of daylight
s'	*al-shafaq*	time of nightfall

are tabulated separately (the functions T being labelled *al-bāqī ila 'l-ghurūb*, "time remaining until sunset", and the times *al-muwāfiqa*, meaning "time according to the Ottoman convention"), with entries expressed in equinoctial hours to three digits. A given page of tables serves a pair of zodiacal signs symmetrical with respect to the solstices.

MS Istanbul Husrev Paşa 232 contains a set of tables attributed to Muḥammad Ṣādiq Jihāngīrī, *muwaqqit* in the Mosque called Wālidat al-Sulṭān (Valide Sultan Camii) in Istanbul.[38] This displays the two times when the sun has a particular altitude on the assumption that *midday is 12 o'clock*. Here we see European influence at work. Values are given in equinoctial hours to three digits and are derived from those of Ṣāliḥ Efendī. Anonymous copies of these same tables are contained in MSS Istanbul Bağdatli Vehbi Efendi 990, Istanbul Lala Ismail 287, Istanbul Kandilli 220, 440 and 441, Cairo ṬM 120, and anonymous copies of similar tables

[38] *Ibid*., II, pp. 561-562, no. 399, and also p. 456 and 684. See the illustration in King, "Astronomical Timekeeping in Ottoman Turkey", pl. 9.

with entries to two digits rather than three are contained in MSS Istanbul Esat Efendi 1979, Istanbul UL T1963 and T1964.

MS Baghdad Awqāf 325/12248 contains some 50-odd pages of tables attributed to Jihāngīrī. I have not been able to inspect this manuscript, but it is catalogued under the title *ikhtilāf mā bayna ʾl-ufuq al-ḥaqīqī wa-ʾl-marʾī*, which refers to corrections for refraction at the horizon. This title probably relates to something written on the first page of the manuscript rather than the main tables.

On the flyleaf of MS Istanbul Lala Ismail 287, copied in 1195 H (= 1781), is a set of numbers for each of the zodiacal signs labelled *daqāʾiq ḥiṣṣat al-qurṣ li-ʿarḍ mīm-alif,* "minutes of the correction for the solar disc at latitude 41°". The numbers are as follows, in the order of signs:

3 2 2 2 2 2 3 4 5 5 4 3 .

14.14 Anonymous Ottoman-type prayer-tables for Crete

MS Istanbul Kandilli 522 contains a set of prayer-tables for the year 1290 H [= 1873/74] computed for the latitude of Khania in Crete (see already **8.8** on some other tables for Crete).[39] The following functions are tabulated for each day of the year, expressed in equinoctial hours and minutes:

$$R', m', a', s' \text{ and } r' .$$

The underlying parameters are found by inspection to be:

$$\phi = 35;20° \text{ and } \varepsilon \approx 23;29° ,$$

and the parameters used for morning and evening twilight are 19° and 17°. The entries for m′, a′ and s′ correspond very closely to recomputation without taking horizon phenomena into consideration. However both the times r′ and R′, daybreak and sunset, are earlier than the recomputed times by an amount which varies between 16^m at the equinoxes and 20^m at the solstices. This appears to be a generous correction for refraction and the size of the solar disc (see also **12.11**), half of which the anonymous calculator should also have applied to the times m′ and a′.

14.15 Gedūsī's twilight tables for different latitudes

At the end of the treatise on the trigonometric quadrant by the 19th-century Ottoman astronomer al-Gedūsī,[40] printed in Istanbul in 1311 H [= 1893/94], there is a set of *imsāk* tables for different latitudes. The time of the *imsāk* in hours and minutes, Turkish time, is displayed for each 2° of solar longitude and for latitudes:

21° (Mecca), 24° (Medina) and 35°, 36°, ... , 45°.

I have not investigated the underlying value of the solar depression and the definition used

---

39 This source is mentioned in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, p. 807.

40 On Sulaymān Murād ibn ʿUmar ibn Aḥmad Saʿdī al-Gedūsī see İhsanoğlu *et al.*, *Ottoman Astronomical Literature*, II, pp. 601-602, no. 436. A translation of this treatise and a commentary were published in Würschmidt, "Gedosi über den Quadranten". Würschmidt mentions the tables (p. 154) and apologizes for not reproducing them. Copies of the tables were kindly prepared for me by the late M. Alain Brieux of Paris.

for the *imsāk*. In Gedūsī's treatise the parameters used for morning and evening twilight are 19° and 17°, and the *imsāk* is taken as being 16^m before daybreak.[41]

14.16 Saʿīd Beg Zāde's auxiliary tables for computing the prayer-times for all latitudes

It is appropriate that we should conclude the present study with a few remarks on a set of tables compiled in Istanbul in the 19th century for computing the prayer-times for all latitudes. The compiler of these tables, which are extant in MS Istanbul Kandilli 226, was one Saʿīd (or possibly Sayyid) Beg Zāde Mīr Ibrāhīm, on whom I have no further information.[42] The tables are preceded by an introduction in Turkish and followed by some worked examples. The notation used by Saʿīd Beg Zāde is rather unfortunate: most of his functions are called *taʿdīl*, "equation" or "correction". Also *he uses logarithms to base sixty expressed sexagesimally*. Nevertheless his tables are of interest because they are really still part of the medieval tradition, and indeed bear some resemblance to the auxiliary tables of al-Wafāʾī compiled several centuries previously (**6.15**). One of the two known copies of al-Wafāʾī's tables (MS Nuruosmaniye 2921,2) is preserved in Istanbul, and I consider it likely that Saʿīd Beg Zāde had seen such a copy. He advocates the use of parameters 20° and 17°. He also gives tables for finding the time of prayers at the two major religious holidays in Islam (*ṣalāt-i ʿīdayn*), based on the assumption that the prayer should take place when the solar altitude is 6° in the east. I know of no other tables for this prayer-time, or reference to this particular definition in the Islamic sources. I have outlined the principle underlying these tables in **I-9.13**.

The first set of tables displays three functions of λ, with values for each degree of argument expressed to two sexagesimal digits in the case of the first and to three digits in the case of the second and third. These functions are:

$\delta(\lambda)$ (ε = 23;28°)	*mayl awwal*	first declination
$L(H) = \log\{\mathrm{Sin}\ H\}$	*taʿdīl-i ẓuhr*	equation of midday
??	*taʿdīl-i mayl*	equation of the declination

The first main table displays values of the following functions for each degree of argument (*ʿadad*) H, or H* in the case of the table for twilight, from 1° to 90°. Values are given to three sexagesimal digits, except for the last function whose values are given to two digits.

$L_a(H) = \log\{\mathrm{Sin}\ H - \mathrm{Sin}\ h_a(H)\}$	*taʿdīl-i ʿaṣr-i awwal*	equation of the first afternoon prayer
$L_b(H) = \log\{\mathrm{Sin}\ H - \mathrm{Sin}\ h_b(H)\}$	*taʿdīl-i ʿaṣr-i thānī*	equation of the second afternoon prayer
$L_r(H) = \log\{\mathrm{Sin}\ H^* - \mathrm{Sin}\ 16°\}$	*taʿdīl-i shafaq*	equation of nightfall
$L_s(H) = \log\{\mathrm{Sin}\ H^* - \mathrm{Sin}\ 20°\}$	*taʿdīl-i fajr*	equation of daybreak
$L_\iota(H) = \log\{\mathrm{Sin}\ H - \mathrm{Sin}\ 6°\}$	*taʿdīl-i ṣalāt al-ʿīdayn*	equation of the prayers at the two religious festivals
$S(h) = \mathrm{Sin}\ h$	*taʿdīl-i irtifāʿ*	equation of the altitude

(No values are given for $L_r(H)$ for $H^* < 16°$, *etc.*)

[41] *Ibid.*, pp. 152-153.
[42] I did not find him in İhsanoğlu *et al.*, *Ottoman Astronomical Literature*.

The second main table displays the following functions for each 0;15 of argument *x* (*taʿdīl-i awwal*) up to 120 in the case of the second and third functions, and up to 60 in the case of the remainder. Values are given to two sexagesimal digits except for the second function whose values are given to three:

$S^{-1}(s)$ = arc Sin (s)	*taʿdīl-i thānī*	second equation
L(x) = log x	*taʿdīl-i muʿaddal*	modified equation
V(x) = $^{1}/_{15}$ arc Vers (x)	*sāʿāt-i dāʾir*	hours of time
C′(x) = $^{1}/_{15}$ arc Cos (x)	*ẓuhr-i shamālī*	midday for northern latitudes
12^{h} - C′(x)	*ẓuhr-i janūbī*	midday for southern latitudes

A major disadvantage of these tables, as we shall see, is that one must find V from L in a table where both functions are tabulated with x as argument. I introduce the function

$$V^*(x) = {}^{1}/_{15} \text{ arc Vers (alog x)} ,$$

which Saʿīd Beg Zāde would have done well to tabulate. The third main table displays the function:

$$G^*(\lambda,\phi) = \log G(\lambda,\phi) ,$$

called *taʿdīl-i ʿarḍ,* "the equation of the latitude". Values are given to three significant sexagesimal digits for each 5° of λ and each 1° of ϕ up to 90°.

The prayer-times expressed according to the Ottoman convention can be found using these functions in the following way:

ẓuhr:	$m' = V^* \{ L(H) + G^*(\lambda,\phi) \}$
ʿaṣr 1 and 2:	$t_{a/b}' = m' + V^* \{ L_{a/b}(H) + G^*(\lambda,\phi) \}$
shafaq:	$s' = m' - V^* \{ L_s(H^*) + G^*(\lambda, \phi) \}$
fajr:	$r' = m' + V^* \{ L_r(H^*) + G^*(\lambda,\phi) \}$
ṣalāt-i ʿīdayn:	$\iota' = m' + 12^{h} - V^* \{ L_\iota(H) + G^*(\lambda,\phi) \}$

To find the hour-angle corresponding to a given solar altitude h, form:

$$L \{ H'(H,h) \} = L \{ S(H) - S(h) \}$$

and then:

$$t(h,H,\lambda,\phi) = V^* \{ L \{ H'(H,h) \} + G(\lambda,\phi) \} .$$

Saʿīd Beg Zāde gives numerical examples for ϕ = 41° and λ = 60°. The prayer-times agree with the corresponding entries in the prayer-tables of Ṣāliḥ Efendī (**14.13**) to within a minute of time. He also demonstrates how to compute the qibla, and using coordinates:

Istanbul	L: 56;33°	ϕ: 41;0°
Mecca	68;30	21;15

he somehow derives the value 66;31° (measured from the prime vertical). Accurately computed for these coordinates the qibla is 59;16°. As noted above (**14.4**) the qibla at Istanbul is in fact closer to $61^{1}/_{2}$° S of E.

Postscript: When I first started writing **Part II** of this book some 30 years ago, tables were available from the local authorities in each region of the Islamic world displaying the prayer times. Some examples are presented in **Fig. V-13.1** and in the article "Mīḳāt ii. Astronomical Aspects" in EI_2. Nowadays one can download tables for any locality from the Internet: see, for example, http://prayer/al-islam.com (2003).

Part III

A survey of arithmetical shadow-schemes for time-reckoning

Dedicated to Otto Neugebauer in 1990
on the occasion of his 90^{th} birthday

This page intentionally left blank

ACKNOWLEDGEMENTS AND NOTES ON THIS VERSION

In numerous medieval Islamic treatises on traditional folk astronomy and legal texts various simple arithmetical shadow-schemes are found for regulating the times of the *ẓuhr* and *ʿaṣr* prayers. Less frequently, there occur schemes for the shadows at each of the seasonal hours of daylight. This material represents a development virtually independent of the highly sophisticated astronomical timekeeping (*ʿilm al-mīqāt*) practiced by the astronomers of Islamic world, and it is here investigated for the first time.

As we shall see, some of this material derives from Hellenistic folk astronomy, notably, from the shadow schemes which have been studied by Otto Neugebauer. Also an approximate Indian rule for reckoning time of day from shadow lengths had considerable influence in Islamic folk astronomy, to the extent that it even underlies the definitions for the *ẓuhr* and the *ʿaṣr* which became standard in later practice. The Yemeni sources provide the most original materials, but all of the material discussed here has a distinct Islamic flavour. In passing I mention the shadow-schemes in medieval European folk astronomy, which were also studied by Neugebauer.

Otto Neugebauer (1899-1990) was the leading historian of the exact sciences in the 20th century. His monumental contributions to ancient Babylonian and Greek astronomy and mathematics are well known, his contributions to Islamic, Byzantine and medieval European astronomy perhaps less so. The interested reader should consult Noel Swerdlow, "Otto E. Neugebauer", *Proceedings of the American Philosophical Society* 137:1 (March 1993), pp. 137-165, or a summary available on the Internet under www.nap.edu/html/biomems/oneugebauer.html, as well as the bibliography compiled by Janet Sachs and Gerald Toomer in *Centaurus* 22 (1979), pp. 257-280.

But Neugebauer should also be remembered for his contributions to folk astronomy, Hellenistic, Ethiopic and medieval European. Most historians of science have no conception of folk science, but the fact that it flourished alongside the exact sciences wherever these were cultivated, not least in two of the most significant civilisations in human history, that is, the Hellenistic and the Islamic, suggests that perhaps we historians of science should take it seriously. For an introduction, the interested reader may consult any of Neugebauer's publications listed below:

- "Astronomische Papyri aus Wiener Sammlungen: II. Über griechische Wetterzeichen und Schattentafeln", *Sitzungsberichte der philosophisch-historischen Klasse der Österreichischen Akademie der Wissenschaften* 240:2 (1962), pp. 29-44.
- "On Some Aspects of Early Greek Astronomy", *Journal of the American Philosophical Society* 116 (1972), pp. 243-251, repr. in *idem*, *Essays*, pp. 361-369.
- *Ethiopic Astronomy and Computus*, Vienna: Österreichische Akademie der Wissenschaften, 1979.
- *Abu Shaker's "Chronography" – A Treatise of the 13th Century on Chronological, Calendrical, and Astronomical Matters, written by a Christian Arab, preserved in Ethiopic*, Vienna: Österreichische Akademie der Wissenschaften, 1988.

- "Astronomical and Calendrical Data in the Très Riches Heures," (originally published 1974), repr. in *idem*, *Astronomy and History – Selected Essays*, New York, *etc*.: Springer, 1983, pp. 507-520.

(For writings by other scholars on Islamic folk astronomy see the notes to **1.1** below.)

Most people could only stand in awe of Neugebauer's academic achievements, but those who knew the man will remember him most for his warmth and his inimitable humour. He for one knew that eagles fly along the Bosphorus, could identify those who liked a good "Spiel und Tanz", and recognized that Kindi played a "Doppelhorn". I was happy that he could see the first published version of this study, presented to him on the occasion of his 90th birthday. This appeared as "A Survey of Medieval Islamic Shadow Schemes for Simple Timereckoning", *Oriens* 32 (1990), pp. 191-249.

Most of the sources for this study were gathered during my survey of the scientific manuscripts in the Egyptian National Library conducted during the years 1974-76. My activities at the American Research Center in Egypt during 1972-79 were made possible mainly by the Smithsonian Institution and (U.S.) National Science Foundation, and my research on Islamic folk astronomy during 1983-85 by the (U.S.) National Endowment for the Humanities. This support is gratefully acknowledged. It is also a pleasure to record my appreciation to the Egyptian National Library for the privilege of working with their vast manuscript holdings, and my thanks are also due to the other libraries where the sources are preserved.

To my friend Daniel Varisco I owe much of my enthusiasm for folk astronomy and most of the references to modern practices relating to irrigation. Dr. Abdallah Nasif (University of Riyadh) kindly provided me with copies of relevant sections of his thesis. Dr. Tom Zuidema (University of Illinois) drew my attention to the basic documentation on the Gresik sundial. For remarks on the penultimate draft of this study – and for much more – I am indebted to Ted Kennedy.

Since the original study was published, two new studies have appeared that deal with timekeeping by night using the lunar mansions. These are Miquel Forcada, "*Mīqāt* en los calendarios andalusíes", *al-Qanṭara* 11 (1990), pp. 59-69, and Petra Schmidl, *Volksastronomische Abhandlungen aus dem mittelalterlichen arabisch-islamischen Kulturraum*, doctoral dissertation, Institute for History of Science, Johann Wolfgang Goethe University, Frankfurt am Main, 2003.

In this new version I have simplified the footnotes by using the bibliographical abbreviations listed at the beginning of this book and inserted cross-references to **I-II** and **IV**.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Islamic folk astronomy and mathematical astronomy

In the medieval Islamic world there were two separate traditions of astronomical knowledge. The first was a simple, non-technical, essentially practical tradition, devoid of theories. The second was a mathematical tradition in which theories, tables and computation featured prominently. Since the distinction between the traditions is not generally recognized we shall begin by considering each of the traditions separately.

First, the folk tradition.[1] The Arabs of the Peninsula in the time before Islam possessed an intimate knowledge of the apparent motions of the sun, moon, and stars across the heavens, the months and seasons, the changing night sky throughout the year, and associated meteorological phenomena. A distinctive feature of this tradition was the division of the year into 13-day periods (*anwāʾ*) defined by the (acronychal) settings and (heliacal) risings of the 28 lunar mansions (*manāzil*).[2] Another was the custom of describing the time of day with reference to the length of a person's shadow.[3] The Arabs also had names for various divisions of the day and night, including names for the seasonal hours, that is, the twelve hours of daylight and the twelve hours of night.[4]

With the advent of Islam in the early 7th century, the Muslims expanded out of the Peninsula to establish a commonwealth from Spain to Central Asia. At various stages they incorporated some of the simple and non-technical procedures that they found in use in these regions. The resulting Islamic tradition of folk astronomy was widely practiced until the 19th century, and vestiges of it can be detected to this day in rural areas from Mauritania to Indonesia.

Besides the obvious fact that any man in the street could master the fundamentals of folk astronomy, there were two main reasons for its appeal. First, the sun, moon and stars were mentioned in the *Qurʾān*, and a knowledge of the basics of their apparent courses constituted

---

[1] All Arabic sources dealing with folk astronomy *per se* and compiled prior to *ca*. 1050, as well as all previous studies thereon, are surveyed in Sezgin, *GAS*, VII, pp. 336-370. Some of the sources were seriously investigated for the first time in Nallino, *Scritti*, V, pp. 152-197. A contribution of a more philological kind on the subject is the article "Layl and nahār" by Charles Pellat in *EI*$_2$. A new overview is Varisco, "Islamic Folk Astronomy". Other recent contributions, particularly by Paul Kunitzsch and Miquel Forcada, are mentioned below. On certain distinctive features see also nn. 1:2-4 and on various practical applications see n. 1:9.

[2] On the lunar mansions in Islamic astronomy see the introductory articles "Anwāʾ" by Charles Pellat and "Manāzil" by Paul Kunitzsch in *EI*$_2$, repr. in *idem*, *Studies*, XX, and the overview in Varisco, "Islamic Folk Astronomy". On the Andalusī tradition in particular see now Forcada, "*Anwāʾ* Books in al-Andalus".

[3] Arent J. Wensinck *et al*., *Concordance et indices de la tradition musulmane*, 8 vols., Leiden: E. J. Brill, 1936-1988, (repr. in 4 vols., Leiden: E. J. Brill, 1992), IV, pp. 78-79, and V, pp. 211-213, lists references to the terms *ẓill* and *fayʾ* for "shadow" in the *ḥadīth* literature. See also **2.1** below.

[4] On the seasonal hours in general see, for example, Neugebauer, *HAMA*, III, p. 1069; *idem*, *Ethiopic Astronomy and Computus*, pp. 167-170; Kennedy, "*Zīj* Survey", p. 141; and **IV-2.1**. On these names (which are not mentioned in Pellat's article cited in n. 1:1) see **IV-8**.

part of a standard religious training. Even a distinctively Islamic cosmology developed over the centuries.[5] Second, the *Qurʾān* and Islamic tradition advocate three duties incumbent on every Muslim which are connected with astronomy, at least with the moon, the sun, and the earth, namely:

- Observance of a sacred month of fasting and of other religious festivals regulated by a strictly lunar calendar in which the beginnings of the months are determined by the first visibility of the lunar crescent;[6]
- Performance of five ritual prayers at times determined by the sun's position relative to the local horizon, defined in terms of shadows during the day, by sunset and sunrise, and by twilight phenomena;[7] and
- Performance of various ritual obligations, including prayer, but also recitation of the *Qurʾān*, the call to prayer, slaughter of animals, and burial of the dead, in a sacred direction towards the Kaʿba in Mecca (called in Arabic, *qibla*).[8]

The basic stipulations of each of these duties – correct timing and correct orientation – could be achieved by applications of the techniques of folk astronomy, with virtually no computation beyond simple arithmetic.[9] These procedures were advocated by the scholars of the sacred law and are to be found recorded not only in legal texts, but also in works on folk astronomy and popular works such as lexicons and encyclopaedias. They involved respectively:

- Actual sighting of the lunar crescent and, in the case of bad weather, counting months of roughly $29^1/_2$ days each in pairs;[10]
- Timekeeping by day using rough estimates of shadow lengths (as described in the present study) and by night using the risings and settings of the lunar mansions; direct observation of sunset, nightfall, daybreak, and sunrise; and
- Adopting as indicators of the local sacred direction the north or south points or the risings and settings of various prominent stars or of the sun at the equinoxes or solstices.[11]

The early Muslims must have been particularly happy to come across arithmetical rules for determining the shadows at midday and at the seasonal hours: these, as we shall see, they used not only to formulate precise definitions for the times of the daylight prayers prescribed in Islam, but also – in certain circles – for timekeeping in general.

Second, mathematical astronomy. In the 8th and 9th centuries the Muslims enjoyed an encounter with a very different approach to astronomy.[12] This they found documented mainly

---

5 On this Islamic tradition see Heinen, *Islamic Cosmology*. On the corresponding "scientific" tradition see Jachimowicz, "Islamic Cosmology".

6 See, for example, the articles "Ramaḍān" by M. Plessner and "Ṣawm" by C. C. Berg in EI_1, updated by the editors in EI_2.

7 See the article "Mīḳāt, i: ritual and legal aspects" by A. J. Wensinck in EI_2.

8 See the article "Ḳibla, i: ritual and legal aspects" by A. J. Wensinck in EI_2.

9 See King, "Islamic Folk Astronomy", A-B, for overviews of the folk astronomical tradition and the ways in which it was applied to regulating the calendar, timekeeping, and determining the sacred direction towards Mecca.

10 The best source of information on this subject is still Renaud, "Sur les lunes du Ramadan" (mainly on the practice in the Maghrib).

11 See my forthcoming monograph *Sacred Geography of Islam*, summarized in the article "Makka. As Centre of the World" in EI_2, repr. in King, *Studies*, C-X, for a survey of these procedures, and **VIIa** on the consequences for the orientation of Islamic religious architecture.

12 For an overview of the mathematical tradition in Islamic astronomy see King, "Islamic Astronomy". On

in texts of Indian and Greek provenance. Here the emphasis was on careful observation, preferably with instruments; geometrical models for the sun, moon and planets; and on computation. Greek and Indian astronomers, like their Babylonian predecessors, had a passion for tables and no fear of extensive calculations. Muslim astronomers adopted this earlier Greek and Indian material and modified it for their own use. They conducted new observations, compiled new tables, devised new instruments, and made progress in all aspects of mathematical astronomy within its ancient and medieval framework. Only since the middle of this century has it become clear to us to what extent they too had a passion for tables and instruments, and the basic documentation of their achievements is still in progress.

Muslim astronomers working in the mathematical tradition also treated the three problems mentioned above. Their solutions included:

- Developing limiting conditions on such quantities as the apparent elongation of the moon from the sun, the difference in setting times of the moon and sun, or the height of the moon above the local horizon at sunset, and preparing tables based on such conditions for predicting lunar crescent visibility in specific latitudes;[13]
- Applying trigonometric formulae, either complicated accurate ones or handy approximations, for reckoning time from solar and stellar altitudes and compiling tables for timekeeping by the sun and stars for different latitudes, sometimes containing tens of thousands of entries, based upon such formulae (**I-II**); also developing instruments such as the astrolabe, quadrant and sundial for the same purpose (**X**).
- Developing cartographic, geometric and trigonometric solutions to the problem of determining the direction of one point on a sphere to another; tabulating the direction of Mecca as a function of terrestrial longitude and latitude; listing the qiblas of cities throughout the Islamic world; and devising cartographic techniques for reading the qibla directly from maps.[14]

These approaches are a far cry from the methods advocated by the legal scholars. Only when we realize how different they were do we find ourselves in a position to assess Islamic science on its own terms.

After the formative period of Islamic astronomy in the 8th to 10th centuries there developed regional schools with their own specific interests and authorities.[15] It was in the Yemen in particular that the two traditions of astronomy flourished side by side. Likewise in the Maghrib there were several widely-respected authorities in folk astronomy, this in spite of the

---

the beginnings of this tradition see Pingree, "Indian Influence" and "Greek Influence", and more recently Saliba, "Arabic Science and the Greek Legacy". Pingree's article "ʿIlm al-hayʾa" in *EI*$_2$ provides an overview of the activities of the astronomers in all fields except those related to Islamic ritual (on which see the overview in King, "Islamic Folk Astronomy", A-B). For the manuscript sources from the early period see Sezgin, *GAS*, VI.

[13] See King, "Tables for Lunar Crescent Visibility", on the earliest tables, and the article "Ruʾyat al-hilāl" in *EI*$_2$ for an overview of Muslim activity in this field.

[14] An overview is in the article "Ḳibla (astronomical aspects)" in *EI*$_2$, repr. in King, *Studies*, C-IX. More information is in *idem*, *Mecca-Centred World-Maps*.

[15] On the schools in the Islamic East see Kennedy, "Exact Sciences in Iran", A-C; on those in Egypt and Syria, the Yemen, and the Maghrib see King, "Astronomy of the Mamluks"; *idem*, *Astronomy in Yemen*, and *idem*, "Astronomy in the Maghrib", respectively. On astronomy in al-Andalus see Samsó, *Ciencias en al-Andalus*, and *idem*, *Studies*, and many other publications of the Barcelona school.

availability there of sophisticated tables relating to mathematical astronomy. In Egypt and Syria there were apparently no such authorities.

Now what concerns us here is the second obligation to pray at the right times, particularly in the case of the two prayers performed during the day. It seems that in the first centuries of Islam the times of prayer were regulated by the muezzins who performed the call to prayer five times a day. They were chosen for the excellence of their voices and their good character; they then had to learn the basics of folk astronomy, which was all that they needed in addition to perform their task adequately. However, the 13th century witnessed the rise of the institution of the *muwaqqit*, that is, the astronomer associated with a mosque for the purposes of regulating the times of prayer, as well as the prediction of lunar crescent visibility and the determination of the qibla (**V**). Some impressive scientific achievements were made by certain *muwaqqit*s, particularly in Egypt and Syria. Tables displaying the times of prayer for each day of the year for specific localities became commonplace. Instruments – mainly sundials, astrolabes and quadrants – also bore markings for the times of prayer. We do not know how the *muwaqqit*s functioned in such proximity to legal scholars who were advocating simple techniques of folk astronomy; our sources are silent on this point. Certainly I am not aware of any legal text in which it is suggested that one should consult an astronomer on the prayer-times or use any of the astronomical tables or instruments that were available for this purpose. It would be naïve to suppose that there was any reason why a legal scholar should have consulted an astronomer.

In this study I present a survey of all known Islamic shadow-schemes for regulating the time of day in general and the times of the midday and mid-afternoon prayers in particular. This material belongs almost exclusively to the folk tradition in Islamic science, and most of it has never been studied previously. It has been found mainly in books dealing with the regulation of the prayer-times by non-technical means (*kutub al-mawāqīt*), in popular almanacs, and in books on the sacred law of Islam. Not a few of the schemes are recorded as curiosities in works belonging to the mathematical tradition, or simply as marginalia or notes on flyleaves.

I make no claim to have exhausted the available material (especially legal texts), but I have included all the relevant material currently known to me. I have not come across any Turkish or Persian sources, so the Ottoman heartland and the Islamic East are not well represented in my survey.

1.2 The seasonal hours and the times of the *ẓuhr* and *ʿaṣr* prayers

A minority of our sources displays shadow lengths for each seasonal hour of the day during each month of the year, but most of the Islamic shadow-schemes that I shall discuss were intended to be used primarily for regulating the times of the midday (*ẓuhr*) and afternoon (*ʿaṣr*) prayers. It is not clear how these were regulated during the first few decades of Islam: the *Qurʾān* and the *ḥadīth* do not between them provide precise definitions (**IV-1-3**). But from the 8th century onwards, the times of these two prayers were defined in terms of shadows, or – more specifically – shadow increases: see **Fig. 1.2a**.[16] The *ẓuhr* began shortly after midday

[16] On the definitions of the prayers in the astronomical sources see also Wiedemann & Frank, "Gebetszeiten"; Kennedy, "al-Bīrūnī on Prayer-Times"; and now **IV-4** and **7**.

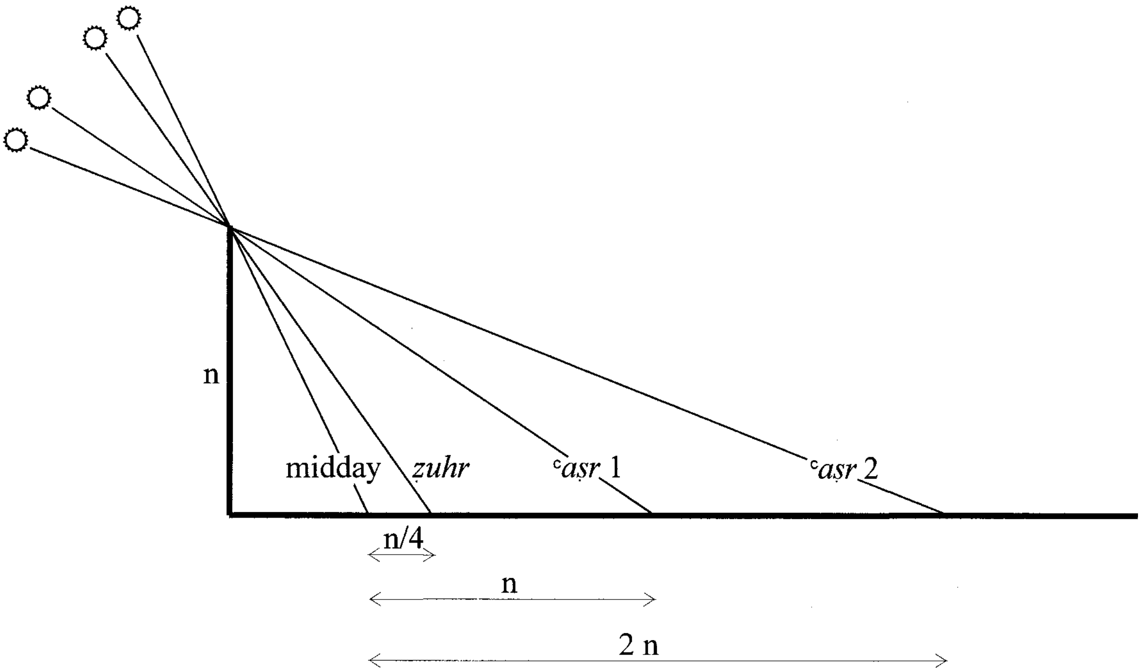


Fig. 1.2a: The standard definitions of the times of the beginning of the *ẓuhr* and the beginning and end of the ᶜ*aṣr*.

when the shadow was observed to increase, and the ᶜ*aṣr* began when the shadow had increased over its midday minimum by the length of the gnomon. Variant definitions for the *ẓuhr* in terms of shadow increases are also attested. Likewise, the end of the interval permitted for the ᶜ*aṣr* is often defined as being when the shadow has increased by twice the length of the gnomon. As I have shown in detail elsewhere (**IV**), these definitions represent practical means of linking the prayers to the seasonal hours, the link being provided by an approximate Indian formula for timekeeping first attested in the Islamic sources of the 8th century – see below and also **2.1**, **2.3** and **3.2**.

Underlying most of the schemes is the implicit notion that one should measure one's own shadow in terms of the length of one's feet. The most common Islamic base for the gnomon length was 7 (*qadam*s or feet) although sometimes 6, $6^1/_2$ and $6^2/_3$ feet were used. These values between 6 and 7 were thought to correspond to the ratio of the height of a man (*qāma*) to the length of his feet.[17]

A gnomon the height of a man is still *in situ* in the courtyard of the 7th-century Mosque of Janad in the Yemen – see **Fig. 1.2b**. It is known locally as the stick (ᶜ*aṣā*) of Muᶜādh ibn Jabal, who was appointed by the Prophet Muḥammad as the first governor of the Yemen.[18]

[17] Kennedy, *al-Bīrūnī's* Shadows, I, pp. 68-79, and II, pp. 25-31, *ad* al-Bīrūnī, *Shadows*, Ch. 7. In the mathematical tradition two other bases were used, namely 12 (*iṣba*ᶜs or digits) and – less frequently – 60: see Schoy, *Schattentafeln*, and Kennedy, "*Zīj* Survey", p. 140; and also the last paragraphs of **I-1.1** and **II.1.3**.

[18] See also King, *Astronomy in Yemen*, p. 8.

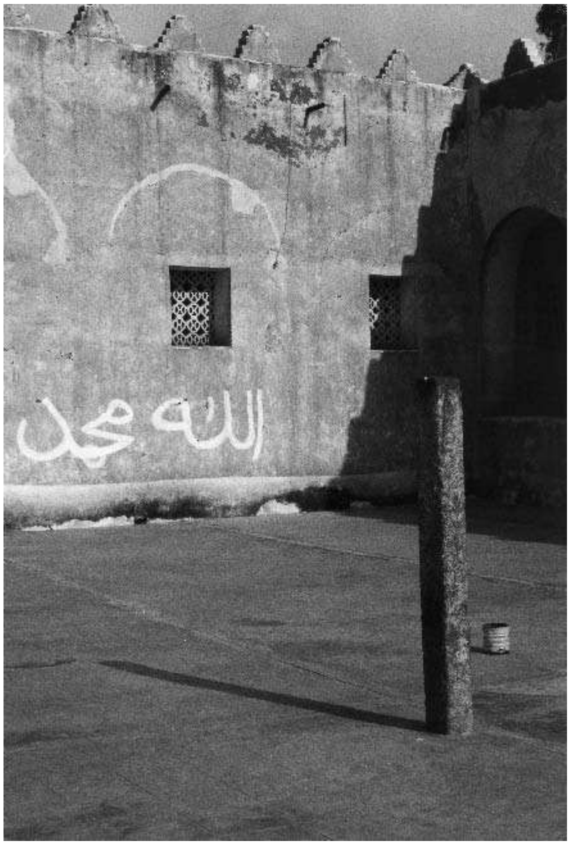


Fig. 1.2b: The gnomon in the courtyard of the Mosque of Janad. [Photo courtesy Professor Daniel M. Varisco.]

The 13th-century legal scholar al-Aṣbaḥī (see **4.1**) states explicitly that he measured the midday shadows in Janad for every 13 days of the Syrian year, that is, for each of the *anwāʾ*. I do not know the age of the present gnomon, but there has clearly been a gnomon there for centuries.

Usually the schemes are presented in words or mnemonics in standard medieval Arabic alphanumerical (*abjad*) notation.[19] The schemes for the hours were sometimes represented in tabular form, but underlying them we also find trivial arithmetical schemes. Indeed, virtually all of the material is arithmetical, devoid of any influence of trigonometry. I have included a few shadow-lists that were observed or derived by calculation; these stem from the murky and unclearly-defined border between the two traditions.

Only rarely do competent astronomers mention these primitive schemes. The celebrated scientist al-Bīrūnī (see **2.4**) devoted a whole book to the topic of shadows,[20] and in this he discussed two Indian schemes, but only because he found them interesting, not because he

[19] On this convention see Irani, "Arabic Numeral Notation", and also Berggren, *Islamic Mathematics*, pp. 41-42.

[20] An uncritical edition of the *Shadows* is listed under al-Bīrūnī, *Shadows*. For translation and commentary see Kennedy, *al-Bīrūnī's* Shadows.

would have approved of their use. On the other hand, two masters of spherical astronomy, Najm al-Dīn al-Miṣrī and Ibn al-Shāṭir (see **9.2** and **8.2**), do advocate using the simple formula of Indian provenance mentioned above as a handy approximation, and they can be forgiven for doing this because in spite of its simplicity the formula is remarkably accurate for most practical purposes (see **1.3**). Before we consider the Islamic material in detail we should briefly mention the earlier traditions (**1.4**).

1.3 A simple arithmetical rule for timekeeping

The approximate method for timekeeping advocated by al-Fazārī (see **2.3**) and the equivalent shadow-schemes for the hours in several later sources investigated below (see **2.1**, **3.2**, *etc.*) are actually of Indian origin[21] although this is not explicitly stated (see, however, **2.4**). It is equivalent to the formula:

$$T \approx 6\,n / (\Delta z + n)\ .$$

Note that, by definition, for $T = 0$, $\Delta z \to \infty$, and for $T = 6$, $\Delta z = 0$. The formula is the simplest means of assuring that these two boundary conditions are satisfied. For the general situation $0 < T < 6$ the formula provides a reasonable approximation.[22] This was the formula mentioned in **1.1** and **1.2** that was used to derive the standard definitions of the times of the daylight prayers in Islam.

1.4 Shadow-schemes from Antiquity

Simple arithmetical schemes representing the shadow lengths at midday for each month of the year, and tables displaying the shadows at each seasonal hour for each month of the year or each zodiacal sign also based on a simple arithmetical scheme, are attested already in early Greek astronomy, and their use continued in the Byzantine, Coptic, and Ethiopic traditions, as well as in the medieval Islamic world and Europe. It is to Otto Neugebauer that we owe the first detailed investigation of this material.[23]

---

[21] On this formula in the already known Islamic sources, see Pingree, "al-Fazārī", pp. 121-122 (al-Fazārī), Kennedy, *al-Bīrūnī's* Shadows, II, pp. 116-121 (al-Bīrūnī), and **IV-2.4**. In this study I refer to this formula as the "standard" Indian formula although the Muslim astronomers also used an accurate formula for timekeeping also of Indian provenance – see Davidian, "al-Bīrūnī on the Length of Day", for details. There is a substantial secondary literature on timekeeping by shadows in Babylonian and Indian astronomy, much of which seems to confuse shadows with shadow increases.

In **XI**, "the standard approximate formula" is the trigonometric formula, also apparently of Indian origin, referred to in n. 4:5 below.

[22] An investigation of its accuracy is in Davidian, *op. cit.*, p. 334. See also **XI-A2**.

[23] See Neugebauer, "Astronomische Papyri"; *idem*, "Early Greek Astronomy", pp. 243-246; and esp. *idem*, *HAMA*, II, pp. 737-746.

For the Ethiopic tradition, based on Hellenistic schemes, with numerous absurdities (*e.g.*, 70 instead of 0 for the twelfth hour, resulting from the misreading of the Greek zero-sign meaning "no shadow" as an *omicron*, that is, 70), see *idem*, *Ethiopic Astronomy and Computus*, pp. 209-215.

See also Kennedy, "Overview of the History of Trigonometry", esp. pp. 5-6, for the first comparison of some of the approximate rules discussed in the present study.

The absurdity of attempting to analyze these purely arithmetical schemes from a modern astronomical point of view is well illustrated in Bremner, "The Shadow Table in Mul.Apin". In this 7th-century B.C.E. source,

The midday shadows are usually symmetrically arranged so that seven values suffice to define them for the whole year, thus for the months of the Julian calendar (type M):

XII XI/I X/II IX/III VIII/IV VII/V VI

or for the zodiacal signs between the winter and summer solstices (type Z):

♑ ♒/♐ ♓/♏ ♈/♎ ♉/♍ ♊/♌ ♋

The midday shadow-scheme:

2 3 4 5 6 7 8

appears to be of early (5th or 4th century B.C.) Greek provenance. Later modifications include *inter alia*:

2 3 4 6 8 10 13 and 1 2 3 4 5 7 8 ,

as well as:

0 $1^1/_2$ 3 $4^1/_2$ 6 $7^1/_2$ 9 .

Also attested are pairings of all the months or signs to yield six rather than seven significant values. The sources usually do not specify the length of the gnomon casting the shadows or the locality for which the values are intended. For the hours it is assumed that the shadows increase at the same rate throughout the year, thus:

hours of daylight:	5/7	4/8	3/9	2/10	1/11
increase each hour:	1	2	3	4	10 (=1+2+3+4)
cumulative increase:	1	3	6	10	20

Each of the classical and medieval traditions was remarkably prone to errors by those who conceived of the tables and yet more by those who copied them, but this feature by no means lessens the interest of the material for the history of science. In his monumental *History of Ancient Mathematical Astronomy* Neugebauer was able to cite just one example of a shadow-scheme from a published Islamic source,[24] and, as might be expected, the unpublished sources offer many more.

1.5 A classification of the Islamic material

I have arranged the sources chronologically according to geographical provenance, as far as this is possible. The monthly schemes are for the Syrian calendar, the Julian (al-Andalus and the Maghrib), or the Coptic (Egypt); I have not found any schemes based on the Persian calendar.[25] We can distinguish seven different kinds of schemes and tables:

a) midday shadow-schemes for the hours based on the approximate Indian formula relating the seasonal hours to the increase of the shadow over its midday minimum (**2.3**, **2.4**, **3.2**, *etc.*);

---

a time function T in degrees (or units of 4 minutes) is tabulated for each cubit of the shadow z from 1 to 10. The values of T(z) are simply T_1/z where T_1 are the times for z=1 at the equinoxes (75) and the solstices (60 in summer and 90 in winter).

[24] Neugebauer, *HAMA*, II, p. 743 (on the scheme listed in **6.3** below).

[25] The various calendars in use in the medieval Islamic world are discussed, for example, in Adolf G. Grohmann, *Arabische Chronologie*, Leiden: E. J. Brill, 1966; the article "Zamān" by Willy Hartner in EI_1; Kennedy, "*Zīj* Survey", p. 139; and the article "Ta'rikh iv" by Benno van Dalen in EI_2.

b) midday shadow-schemes for the months based on Greek schemes, also attested in Coptic sources, and modifications thereof (**7.2**, *etc*.);
c) shadow tables for the hours and for each month based on Greek models (**6.2**, **6.7**, **9.7a**, *etc*. – see also **4.9a** on a possible Himyaritic scheme);
d) midday shadow-schemes for each 13-day period of the year (*anwāʾ*) (**4.1**, *etc*.);
e) simple lists of midday shadows for a given locality, derived by observation (**3.2**, **4.1**, *etc*.);
f) simple lists of midday shadows derived by calculation (for given latitude and obliquity) (**5.1**, *etc*.); and
g) simple shadow tables for the hours and for each zodiacal sign or for each few days of the year based on calculation with an approximate trigonometric formula (only in **4.2**).[26]

1.6 Some technicalities

In the analysis, I use the following notation freely:

AE	autumnal equinox
EQ	equinoxes
M	lunar mansion (numbered with subscripts 1-28)
N	north
S	south
SS	summer solstice
VE	vernal equinox
WS	winter solstice
– – – – – – – – – –	– – – – – – – – – –
h	solar altitude
H	solar meridian altitude
n	length of the gnomon
sdh	as a superscript, seasonal day hours
T	seasonal hours since sunrise or remaining until sunset
z	shadow length
Z	midday shadow length
Δz	increase in shadow length over midday minimum
ε	obliquity of the ecliptic
ϕ	local latitude
+	"n+" indicates a number "a little more than" n (Arabic *rājiḥ*)
-	"n-" indicates a number "a little less than" n (Arabic *shāḥḥ*)
m/n	for seasonal hours the slash indicates two hours for which the sun has the same altitude (n = 12-m), and for zodiacal signs two signs for which the sun has the same declination

The astronomical theory underlying the schemes is simple indeed. The altitude of the celestial pole is a measure of the local latitude ϕ. The meridian altitude of the sun at the

[26] See **I-2.5** and **4.3**, for examples of more extensive tables based on this formula, and also **XI-3** and **4**.

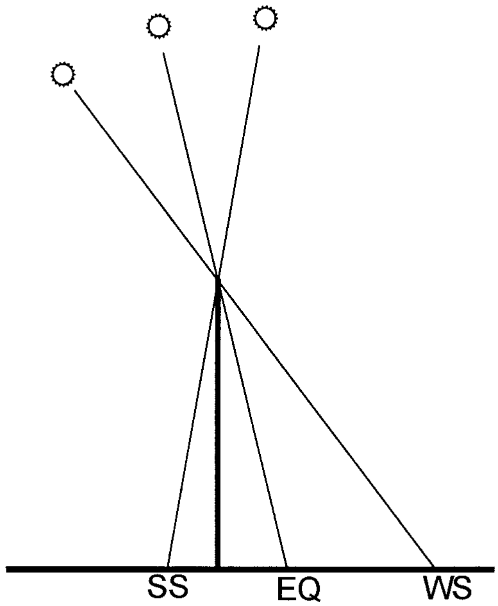


Fig. 1.4a: Solstitial and equinoctial shadows for a tropical locality such as the Yemen (here, specifically Taiz).

equinoxes is $\bar{\phi} = 90° - \phi$. Thus the shadow lengths at the winter solstice, equinoxes, and summer solstice are respectively:

$$n \cot(\bar{\phi} - \varepsilon), \quad n \cot \bar{\phi}, \quad \text{and} \quad n \cot(\bar{\phi} + \varepsilon).$$

Fig. 1.4a shows the solstitial shadows for a tropical locality ($\phi < \varepsilon$) such as the Yemen or Mecca: these vary between almost a full gnomon length north at midwinter to a small fraction of the gnomon length south at midsummer. For the convenience of the reader, I present in **Table 1.5** the approximate shadow lengths at the solstices and equinoxes for the standard gnomon lengths 7 and 12 for latitudes corresponding to the Yemen, Mecca, Cairo, Aleppo, Cordova and Istanbul (assuming $\varepsilon = 23\frac{1}{2}°$). Values are rounded to the nearest digit, as in most of our sources. It is not always sensible to try to establish the latitude for which a given scheme might have been computed.[27]

Table 1.5

Latitude	Midday altitudes			Midday shadows n = 7			Midday shadows n = 12		
	WS	EQ	SS	WS	EQ	SS	WS	EQ	SS
15°	$51\frac{1}{2}°$	75°	$98\frac{1}{2}°$	6	2	1(S)	10	3	2(S)
$21\frac{1}{2}$	45	$68\frac{1}{2}$	92	7	3	0(S)	12	5	0(S)
30	$36\frac{1}{2}$	60	$83\frac{1}{2}$	9	4	1	16	7	1
36	$30\frac{1}{2}$	54	$77\frac{1}{2}$	12	5	2	20	9	3
38	$28\frac{1}{2}$	52	$75\frac{1}{2}$	13	5	2	22	9	3
41	$25\frac{1}{2}$	49	$72\frac{1}{2}$	15	6	2	25	10	4

27 See Neugebauer, *HAMA*, II, p. 739, on the problems.

In our sources fractions are invariably given as combinations of unit fractions of the form $^1/_n$ ($n < 9$) so that, for example (see **Tables 4.1** and **4.3**), $^1/_{12}$ is expressed as *niṣf suds*, that is, $^1/_2$ of $^1/_6$, and $^5/_6$ is expressed as *niṣf wa-thulth*, that is, $^1/_2$ plus $^1/_3$. The one exception to the unit fractions is $^2/_3$ (Arabic has a dual form). This is a feature of simple arithmetic in medieval Islam, reminiscent of the practice in ancient Egypt, and it persisted not least because of the fact that Arabic has no names for unit fractions with denominator greater than ten.[28] Occasionally numbers with fractions are written sexagesimally in the standard manner of the astronomers; a number rendered in the form "a b" in the texts is rendered "a;b" in the commentary, and this means "$a + {}^b/_{60}$".

---

[28] See Neugebauer, *ESA*, pp. 74-77, and the article "ʿIlm al-ḥisāb" by Abdelhamid I. Sabra in EI_2, esp. III, pp. 1140-1141.

CHAPTER 2

EARLY SOURCES

2.1 The Prophetic *ḥadīth*

There are various references to shadow lengths in the statements attributed to the Prophet Muḥammad (known as *ḥadīth*). I shall not attempt to give a survey of these; rather, I shall restrict my attention to just two examples.[1]

First, the Prophet is reported to have said that the angel Gabriel led him in prayer on two consecutive days. On the first day they prayed the *ẓuhr* when the shadow was equal to the width of a thong of a sandal and the *ʿaṣr* when the shadow of every object was the same as its length. These situations correspond to $z = \delta$ ($\delta << n$) and $z = n$. On the second day they prayed the *ẓuhr* when the shadow of every object was the same as its length and the *ʿaṣr* when the shadow of every object was twice its length. These correspond to $z = n$ and $z = 2n$.

As pointed out by al-Bīrūnī, these definitions in terms of shadow lengths could not be applied universally.[2] Only in tropical latitudes can the midday shadow be relatively small, and in non-tropical latitudes the solar altitude does not always attain the 45° necessary to ensure that sometime in the afternoon the condition $z = n$ will be satisfied. The definitions for the *ẓuhr* in terms of shadow increases that were adopted in later practice, namely:

$$\Delta z = {}^1/_{12}\, n\ ,\ {}^1/_5\, n \quad \text{or} \quad {}^1/_4\, n\ ,$$

ensure that the shadow has increased by an observable amount. The definition for the *ʿaṣr* in terms of shadow increases which was adopted by three of the four major legal schools, namely, $\Delta z = n$, and that adopted by the fourth (the Ḥanafīs), namely, $\Delta z = 2n$, can be applied in any habitable latitude. The *ḥadīth* in a sense justifies both of these amounts – n and 2n – for the *ʿaṣr*. Yet other evidence indicates that the reason for adopting these definitions was to regulate the prayers by the seasonal hours. The formula:

$$T \quad = \quad 6\ n\ /\ (\Delta z + n)$$

yields the following shadow increases at the seasonal hours:

T	0	1	2	3	4	5	6
Δz	∞	5n	2n	n	${}^1/_2$ n	${}^1/_5$ n	0

so that the definitions $\Delta z = n$ for the *ʿaṣr* (and occasionally for a prayer at mid-morning called the *ḍuḥā*) represent practical means of linking the *ʿaṣr* to the beginning of the tenth hour (and the *ḍuḥā* to the beginning of the fourth). Likewise, the definitions:

$$\Delta z \quad = \quad {}^1/_{12}\, n\ ,\ {}^1/_5\, n \quad \text{or} \quad {}^1/_4\, n$$

are practical means of relating the *ẓuhr* to one-half or one hour after midday.

---

[1] See **IV-1** and **4.1-2**, for more information on these two *ḥadīth*. For references to shadows in the *ḥadīth* literature see n. 1:3.

[2] al-Bīrūnī, *Shadows*, p. 168, and Kennedy, *al-Bīrūnī's* Shadows, I, p. 218, and II, pp. 137-138.

The second report about the Prophet states that he prayed the *ẓuhr* when $5 < z < 7$ in winter and when $3 < z < 5$ in summer. Since the winter midday shadow in Mecca and Medina is about $7^1/_2$ (n = 7), this cannot be a description of a daily routine.

2.2 ʿAlī ibn Abī Ṭālib

al-Aṣbaḥī (see **4.1**) relates a tradition (MS Cairo DM 948, fol. 41r) on the authority of Yaḥyā ibn Saʿīd[3] about the fourth Caliph ʿAlī ibn Abī Ṭālib.[4] The latter – to whom numerous achievements of a mildly scientific nature are attributed in Islamic legend – is reported to have said that the day on which the midday shadow at Basra was $6^1/_2$ feet (the shadow of any object being then equal to its length) marked the beginning of winter. I do not know of any other sources in which winter is thus defined[5] (see also **4.1** on a specific date for the winter solstice – as opposed to the middle of winter – attributed to ʿAlī). The report continues to list the shadow lengths at each eighth of the year; these are shown in the following table:

Table 2.2

Season	Days elapsed	Midday shadow
Beginning of winter	0	$6^1/_2$
midwinter	45	$9 - {}^1/_8$
end of winter	46	$6^1/_2$
middle of spring	45	$3^1/_2$
beginning of summer	46	$1^1/_2$+ *
midsummer	45	$^1/_2$+
end of summer	47 (leap year: 48)	$1^1/_2$+
midautumn	45	$3^1/_2$+
end of autumn	46	$6^1/_2$

Since for Basra, $\phi \approx 30°$, the shadow (base $6^1/_2$) actually varies between

$$8^3/_4 \text{ (WS)}, \quad 3^3/_4 \text{ (EQ)}, \quad \text{and} \quad {}^3/_4 \text{ (SS)},$$

and so the values given in the text are reasonable. Note that at the latitude of Mecca the midday shadow at the winter solstice – rather than the middle of winter – is equal to the length of the gnomon (see **Table 1.5** and **Section 3.3**).

Ibn Raḥīq reports that, according to ʿAlī, (the end of) the ninth hour of daylight marks the beginning of the *ʿaṣr* prayer – see **3.2**.

---

[3] On possible candidates with this name, both having a connection with al-ʿIrāq, see Sezgin, *GAS*, I, p. 407 (on Abū Saʾīd Y. b. S. al-Anṣārī, a *tābiʿī* from Medina, d. 760 in Basra (!)), and p. 293 (on Abū Ayyūb Y. b. S. al-Umawī, b. *ca.* 732 in Kufa, d. 809 in Baghdad, author of a work on the campaigns of the Prophet).

[4] On ʿAlī ibn Abī Ṭālib see the article by L. Vecchia Vaglieri in EI_2. On various achievements in the sciences attributed to him see Sezgin, *GAS*, VII, pp. 8, 10, 11, 26, and 184; *Cairo ENL Survey*, no. B1; King, "A Medieval Arabic Report on Algebra before al-Khwārizmī" and n. **IV**-2:8.

[5] See, however, Lane, *Lexicon*, V, p. 1916a, where the word *ẓill*, "shadow", is associated with the beginning of winter.

2.3 al-Fazārī

The extant fragments of the writings of the mid-8th-century astronomer al-Fazārī, now gathered and interpreted by David Pingree, display the eclectic nature of early Islamic astronomy.[6] For timekeeping al-Fazārī advocated:

$$T \approx 6n / (\Delta z + n) \qquad (n = 12),$$

a formula which was to be extremely influential in later Islamic folk astronomy and in the formulation of the practical definitions for the times of the daylight prayers which are still in use to this day (see **2.1**, **3.2**, **4.1**, **6.4**, **6.7**, **8.2**, **9.2**, *etc.*).

2.4 al-Bīrūnī

Two numerical shadow-schemes are presented by al-Bīrūnī in his treatise *On Shadows*.[7] The account of the first is garbled, and it was interpreted by E. S. Kennedy thus:

T	1	2	3	4	5	6
[Δz-12]	60	24	12	2;50	0	?
[Δz	72	36	24	14;50	12	0]

However, it seems more likely that the text should be restored to yield a scheme as follows:

T	1	2	3	4	5	6
[Δz]	60	24	12	[6]	[$2^2/_5$]	0

This emendation requires only inserting a value 6 for the fourth hour and reading *wa-khumsayn* (= $^2/_5$) rather than *wa-khamsūn* (= 50) after the 2 for the fifth hour. Now the scheme corresponds precisely to the Indian formula for n = 12.

The second shadow-scheme mentioned by al-Bīrūnī is also Indian. It relates the shadow increase to the passage of time in *muhurta*s, one-fifteenth divisions of the time between sunrise and sunset. The values given are as follows:

T	1/14	2/13	3/12	4/11	5/10	6/9	7/8	$7^1/_2$
Δz	90	60	12	6	5	3	2	0

The same scheme is described in al-Bīrūnī's *India*, with 96 for the first entry.[8]

---

[6] On al-Fazārī see the article by David Pingree in *DSB*. For his use of this formula see Pingree, "al-Fazārī", pp. 121-122.

[7] On al-Bīrūnī see the article by E. S. Kennedy in *DSB*. On this passage see al-Bīrūnī, *Shadows*, pp. 158-159, and E. S. Kennedy's translation and commentary, I, pp. 195-196, and II, pp. 121-122, as well as *idem*, "Overview of the History of Trigonometry", esp. pp. 5-6.

[8] Sachau, *Alberuni's India*, pp. 338-339.

CHAPTER 3

HEJAZI SOURCES

3.1 Aḥmad al-Rāzī

See **Section 3.2**.

3.2 Ibn Raḥīq

Abū ʿAbdallāh Muḥammad ibn Raḥīq ibn ʿAbd al-Karīm wrote a treatise on the calendar, lunar mansions, and simple time-reckoning, which is extant in the unique MS Berlin Ahlwardt 5664 (Landberg 108), 71 fols., copied *ca*. 1350. The work appears to have been compiled in Mecca in the 11th century.[1] Of particular relevance to the present study are, first, a set of midday shadows at Mecca for each of the Coptic months and, second, a scheme for reckoning the seasonal hours by shadow increases.

Ibn Raḥīq quotes Abū ʿAlī ʿAraqa, a muezzin at the Mosque of ʿAmr in Fustat, as having stated that Aḥmad al-Rāzī, a muezzin at the Sacred Mosque in Mecca, had observed the sun over a period of seven years and had determined the shadow lengths for the Coptic months starting with Bāʾūna (X), expressed in terms of the height of a man (*qāma*). His results (fols. 12r-12v) are as follows:

Table 3.2a

Month	Shadow
X	$^1/_3 \bullet {}^1/_6$ (S)
XI	0
XII	$^1/_6$
I	$^1/_3 + (^1/_2 \bullet {}^1/_6)$ [a]
II	$^1/_2$
III	$^2/_3 + {}^1/_4$
IV	$1 + (^1/_6 \bullet {}^1/_8)$
V	$1 - (^1/_2 \bullet {}^1/_8)$
VI	$^1/_3 + {}^1/_4$
VII	$^1/_4 + {}^1/_8$
VIII	$^1/_6$
IX	0

[a] text has $^1/_3 + {}^1/_2 + {}^1/_6$

Since the latitude of Mecca is about $21^1/_2°$, the meridian altitudes at the winter and summer

[1] On Ibn Raḥīq see King, *Astronomy in Yemen*, no. 2, also **II-2.3** and **IV-1.5**. His treatise, as well as those of al-Aṣbaḥī and al-Fārisī (nn. 4:1 and 4:4), are currently being investigated in Schmidl, *Islamische volksastronomische Abhandlungen*.

solstices, measured from the south point, are 45° and 92°, and one would expect maximum shadows in the north of about 1 *qāma* and in the south of about $^1/_{30}$ *qāma*. I have no information on the date of al-Rāzī.

Elsewhere in his treatise (fols. 28v-29r – see **Fig. IV-2.1**) Ibn Raḥīq presents the scheme:

T	1	2	3	4	5
Δz	39	$19^1/_2$	$6^1/_2$	$3^1/_4$	1+ ,

and he uses gnomon length $6^1/_2$. (The value for T = 5 is given as *qadam rājiḥ*.) If we use the standard Indian formula for T = 5, 4, and 3 seasonal hours, we obtain for this value of n:

$$\Delta z \;=\; 1^3/_{10}\;,\;\; 3^1/_4\;,\;\; \text{and}\;\; 6^1/_2\;,$$

as in the text. When T = 2 and 1, Δz is large compared with n, and if we apply the very crude approximation:

$$T \;\approx\; 6n \;/\; \Delta z\;,$$

we obtain for these values of T and n:

$$\Delta z \;=\; 19^1/_2 \quad \text{and} \quad 39\;.$$

Each of Ibn Raḥīq's values is thus explained. The above approximation is nowhere attested, explicitly or implicitly, in any of the other Islamic sources known to me.

Ibn Raḥīq adds a remark that (the end of) the ninth hour marks the beginning of the ʿ*aṣr* prayer according to ʿAlī ibn Abī Ṭālib (see **2.1**). It was this remark that first led me to investigate the standard definitions of the times of the daylight prayers in Islam and find that they were practical means of regulating the prayer-times in terms of the seasonal hours.

The last part of Ibn Raḥīq's treatise (fols. 40r-71v) consists of an almanac arranged according to the lunar mansions rising at daybreak. For each 13-day period, various kinds of information are given, including the midday shadow lengths at Mecca. Part of the almanac is missing (some 20 fols. between fols. 43 and 44), and the shadow lengths that I have been able to extract are shown in **Table 3.2b**.

3.3 al-Shillī

MS Cairo DM 130,2 (fols. 14v-15v, copied *ca*. 1800 – see **Fig. 3.3**) contains a short treatise by Muḥammad ibn Abī Bakr ibn Aḥmad al-Shillī Bā ʿAlawī, a scholar from the Hadramawt who worked in Mecca in the 17th century.[2] al-Shillī discusses the disagreements between scholars about the midday shadow lengths throughout the year at Mecca and Medina and then presents his own opinion on the shadows at Mecca, whose latitude he takes as 21°.

In brief, al-Shillī states that the sun is overhead at Mecca on "the *fifth* day of the sign of Gemini" and "the twenty-*sixth* day of the sign of Cancer".[3] The sun is overhead (in tropical localities, *i.e.* where $\phi < \varepsilon$) when its declination equals the local latitude. For solar longitude 65° and ε = 23;35° (the standard value, which I would have expected al-Shillī to have used) I compute the declination as 21;16°; for longitude 64° the corresponding value is 21;4°, which

---

[2] On al-Shillī see Brockelmann, *GAL*, II, p. 502, and SII, p. 516; *Cairo ENL Survey*, no. D47; and King, King, *Astronomy in Yemen*, p. 59.

[3] *Cf.* the discussions in Lee, "Astronomical Tables of Al Farsi" (1822), p. 265 (Sanaa); and King, "al-Bazdawī on the Qibla in Transoxania", p. 19 (Samarqand).

Fig. 3.3: The full text of al-Shillī dealing with shadow-lengths. [From MS Cairo DM 130,2, fols. 14v-15v, courtesy of the Egyptian National Library.]

is close to his value for the latitude of Mecca. However if we compute with longitude 64° and obliquity 23;30° (rounded from the value of Ulugh Beg) the declination is exactly 21°, which is very nice. (Had al-Shillī taken the trouble to measure the latitude he would have no doubt been surprised to find that it is closer to 21;30°.) He goes on to say that the "small increase (in midday shadow)" (*al-ziyāda al-ṣughrā*) occurs between these two days and reaches a maximum of 25 minutes (*i.e.* sixtieths of a foot, where the gnomon is 7 feet long). He states that the daily increase in the shadow is 1 minute. The "large increase" (*al-ziyāda al-kubrā*) occurs at the winter solstice, and no value is given. al-Shillī states that the daily shadow changes in minutes in the signs are as follows:

♉/♌/♐/♑ : 2   ♍/♈/♏/♒ : 3   ♎/♓ : 4 ,

and from this one could infer that the shadow at the winter solstice was about:

$$4 \times 1 + 30 \times 2 + 30 \times 3 + 30 \times 4 + 30 \times 3 + 30 \times 2 = 424 \text{ minutes,}$$

that is, about 7 feet (see **2.2**). This is reasonable because the solar meridian altitude is close to 45° at this time. The author promises a table displaying the midday shadow lengths at Mecca for (solar longitudes in) each degree of each zodiacal sign, but there is no table appended to the treatise in the Cairo manuscript.

3.4 Tāj al-Dīn

MS Damascus Ẓāhiriyya 5588 (16 fols., copied *ca*. 1900) contains some prayer-tables for Medina by Tāj al-Dīn, *muwaqqit* at the Sacred Mosque there. This individual is otherwise known only by a commentary on the almanac of Shaykh Vefā for Istanbul (*ca*. 1475), of which the unique Cairo copy is dateable to *ca*. 1700.[4]

One of Tāj al-Dīn's tables (fol. 14v) displays midday shadows in feet (*aqdām*) and tenths thereof (*aʿshār*) for "some days of the solar year for the latitude of Medina ...". The entries beginning with Kānūn II (= January) are shown in the following table.

Table 3.4

Month Day	I	II	III	IV	V	VI
1	7 0	5 3	3 5	1 9	0 8	0 2
10	6 6	4 6	3 0	1 5	0 5	0 1
20	6 0	4 1	2 5	1 2	0 3	0 2

For $\phi = 25°$ the maximum shadows for bases 7 and $6^1/_2$ are 7.9 and 7.3. For $\phi = 24°$, a more common medieval value for Medina, the shadows are 7.6 and 7.1. The use of decimal fractions is not attested elsewhere in Islamic tables except in the trigonometric tables of the 16th-century Istanbul astronomer Taqi 'l-Dīn.[5]

---

4 *Cairo ENL Survey*, nos. D239 and H2.

5 King, "Islamic Multiplication Tables", B, pp. 413-415.

3.5 al-Amrānī / al-Imrānī (?)

In the unique copy of the prayer-tables for Medina by Tāj al-Dīn (see **3.4**) there is an introduction by an individual named al-Amrānī or al-Imrānī, who is new to me. In this (fol. 8r, also 2r) we find the following shadow-scheme for the *ẓuhr*:

طزه جبا أبجه حي

9	7	5	3	2	1	1	2	3	5	8	10

advocated for the (Syrian ?) months, with 7 to be added for the *ʿaṣr*. Elsewhere (fol. 4r), the following scheme is given for the hours:

1	2	3	4	5	6	7	8	9	10	11
28	18	9	6	3	0	3	6	9	18	28

as in the treatise of Ibn Tūmart (see **6.1**). The units are *aqdām*, but no indication is given whether these are shadow lengths or shadow increases.

CHAPTER 4

YEMENI SOURCES

4.1 al-Aṣbaḥī

Ibrāhīm ibn ʿAlī al-Janadī al-Aṣbaḥī worked in Janad in the Yemen in the mid 13th century and compiled an extensive treatise on folk astronomy.[1] This is extant in several copies, notably MS Baghdad Awqāf 2982/6276, apparently executed in Taiz in 680 H [= 1281/82]. I have used MS Cairo DM 948,1, copied in 1320 H [= 1902/03] but nevertheless reliable.

al-Aṣbaḥī's treatise is a valuable source of information on Islamic folk astronomy. Of particular interest to the present study is the fact that he advocates (fols. 38r-38v) the shadow-scheme:

T	1	2	3	4	5
Δz (in multiples of n)	5	2	1	$^1/_2$	$^1/_5$

which corresponds precisely to the Indian rule. See also **2.1**, **2.2**, **3.2**, and **6.4**.

al-Aṣbaḥī concludes his treatise with the statement (fol. 51v) that it is the custom of those astronomers who author works (on folk astronomy) to conclude their treatises with a statement on the (midday) shadow at the location where they work. He therefore adds to his treatise a list of shadow lengths observed by him at 13-day intervals throughout the Syrian year and he specifically states that he made the observations in the Mosque of Janad in the year 654 H [= 1256]. The values are displayed in **Table 4.1**. al-Aṣbaḥī neglects to mention the length of the gnomon, but for the latitude of Janad (about 13;40°) his solstitial values correspond best to $6^1/_2$. His values are expressed in *qadam*s and occasionally also a smaller unit *qīrāṭ*, not usually used as a unit of length (see also **Table 4.5**).[2] Some of the values (indicated by an asterisk in **Table 4.1**) are labelled *shāḥḥ*, a term I have not encountered elsewhere but which seems to mean "just less than".[3] The first value is for XII 15, which al-Aṣbaḥī says is the date of the winter solstice according to ʿAlī ibn Abī Ṭālib (see **2.2**).

---

[1] On al-Aṣbaḥī see King, *Astronomy in Yemen*, no. 5, and **IV-1.5**, *etc*. His treatise is currently being investigated by Petra Schmidl: see n. 3:1.

[2] On the use of *qīrāṭ* as a unit of weight see Hinz, *Islamische Masse und Gewichte*, p. 27, and the same author's article "*Ḳīrāṭ*" in EI_2. For another attestation of the term as a unit of length, this time in a 16th-century treatise on timekeeping, see Wiedemann, *Schriften*, II, p. 1039. Wiedemann, on what authority it is not clear, states that it is equal to a finger's breadth.

[3] Thus, for example, the value for XII 15 is *khamsat aqdām illā thumn shāḥḥ*, and the value for X 11 is *thalāthat aqdām shāḥḥa*. The verb *shaḥḥa* means, amongst other things, "to become short, to run out, to decrease, to dwindle". The term *shāḥḥ* thus has the opposite meaning of the more common term *rājiḥ*, "just more than" (see, for example, **III-3.2** and **IV-7.5**). Since sometimes the adjective *shāḥḥ* is applied only to a negative fractional part of the entry I have preferred not to use the notation "n-" in **Table 4.1**.

Table 4.1

Date	Mansion	Midday Shadow
XII 15 (WS)	21	$5 - \frac{1}{8}$*
XII 28	22	$5 - \frac{1}{3}$*
I 10	23	$4\frac{1}{3}$
I 23	24	$3 + \frac{1}{2} + \frac{1}{4} + q$
II 5	25	$3\frac{1}{4}$
II 18	26	$2 + \frac{1}{2}q$
III 3	27	$2 - (\frac{1}{2} \cdot \frac{1}{6})$ +
III 16 (VE)	28	$1\frac{1}{3}$ +
III 29	1	$1 - \frac{1}{6}$ +
IV 11	2	0
IV 24	[3]	*deest*
V 7	4	$1 - \frac{1}{6} + q$ [S]
V 20	5	$1 - (\frac{1}{2} \cdot \frac{1}{6})$* [S]
VI 2	6	$1\frac{1}{2}$* [S]
VI 15 (SS)	7	$1 + (\frac{1}{2} \cdot \frac{1}{6})$ + [S]
VI 30	8	$1 + (\frac{1}{2} \cdot \frac{1}{6})$* [S]
VII 12	9	$1 - \frac{1}{6}$ + [S]
VII 25	10	$\frac{1}{3} + (\frac{1}{2} \cdot \frac{1}{6})$* [S]
VIII 7	11	0
VIII 20	12	$\frac{1}{2}$
IX 2	13	$1 + (\frac{1}{2} \cdot \frac{1}{6})$
IX 15 (AE)	14	$1 + \frac{1}{3} - q$
IX 28	15	$2 + \frac{1}{6}$
X 11	16	3*
X 24	17	$3\frac{1}{2}$
XI 6	18	$4 + (\frac{1}{2} \cdot \frac{1}{6})$
XI 19	19	$4 + \frac{2}{3} - (\frac{1}{2} \cdot \frac{1}{8})$
XII 2	20	$5 - \frac{1}{8}$ +

4.2 al-Fārisī

The Yemeni astronomer Muḥammad ibn Abī Bakr al-Fārisī,[4] author of a *zīj* compiled for the Rasulid Sultan al-Muẓaffar *ca*. 1295, also wrote a treatise on folk astronomy entitled *Tuḥfat al-rāghib.* This is extant in the unique complete copy MS Milan Ambrosiana Griffini 37 (unfoliated, copied *ca*. 1500).

In this work, which was compiled in Aden, al-Fārisī presents the following shadow-scheme for the hours, written out in words (see further below):

Table 4.2a

T / Sign	1	2	3	4	5	6
♈ / ♎	26	12	7	4	$2\frac{1}{2}$	1
♏ / ♓	28	13	8	5	3	$2\frac{1}{2}$
♐ / ♒	31	15	9	6	5	4
♑	32	$15\frac{1}{2}$	10	7	$5\frac{1}{2}$	5

[4] On al-Fārisī (Suter, *MAA*, no. 349 and 349N and Kennedy, "*Zīj* Survey", no. 54) and his works see King, *Astronomy in Yemen*, no. 6. Another anonymous, incomplete copy of the *Tuḥfat al-rāghib* is MS Berlin Ahlwardt 5731. This treatise is currently being investigated by Petra Schmidl: see n. 3:1.

He then states that an approximate scheme for the shadows at the hours throughout the year is:

28 17 15 7 3 0 .

As can be seen from comparing these values with those in **Table 4.2a**, this scheme is approximate indeed.

Elsewhere his work al-Fārisī presents the following values of the midday shadows for various solar longitudes, also written out in words; he specifically mentions the town of Aden, for which he uses latitude 13°:

Table 4.2b

Sign	Shadow	Direction
♈ / ♎	$1 + {}^{1}/_{4} + {}^{1}/_{5} + {}^{1}/_{6}$	N
♉ / ♍	${}^{1}/_{6} + {}^{1}/_{6} \bullet {}^{1}/_{10}$	N
♉ 10° / ♌ 25°0	-	
♊ / ♌	${}^{4}/_{5} + {}^{1}/_{2} \bullet {}^{1}/_{6}$	S
♋	$1 + {}^{1}/_{4} + {}^{1}/_{3} \bullet {}^{1}/_{5}$	S
♎ / ♓	$3 + {}^{1}/_{6} + {}^{1}/_{6} \bullet {}^{1}/_{10}$	N
♏ / ♒	$4 + {}^{1}/_{2} + {}^{1}/_{10}$	N
♑ 0°	$5 + {}^{1}/_{5}$	N

In a section on the prayer times al-Fārisī uses $\Delta z = 7$ and 14 to define the beginning and end of the ʿ*aṣr*, so that he is using $n = 7$. Now if we take al-Fārisī's parameters:

$$\phi = 13° \quad \text{and} \quad \varepsilon = 23;35°,$$

the meridian altitudes at the summer solstice, equinox, and winter solstice measured from the south point are approximately

$100^{1}/_{2}°$ 77° $53^{1}/_{2}°$

and the corresponding shadows to base $n = 7$ are:

1;18 S 1;37 N 5;11 N .

These correspond quite closely with al-Fārisī's values in **Table 4.2b**, these being equivalent to:

1;19 S 1;37 N 5;12 N .

To compute the shadows at the seasonal hours s_i al-Fārisī has used a simple approximation to find the corresponding altitudes h_i equivalent to:

$$\sin h_i = \sin H \sin (i \bullet 15°) ,$$

a procedure derived originally from Indian sources and used by certain Muslim astronomers.[5] For the solstices, using $\phi = 13°$ and $n = 7$, I compute the values:

26.6	12.4	7.2	4.3	2.3	1.3
32.9	15.9	10.1	7.2	5.7	5.2

so that al-Fārisī has in general truncated the results of his calculations.

[5] See n. 1:21. The history of this formula, as well as various tables and instruments based on it, in both the Near Eastern and the European traditions, is traced in **XI**. See already Goldstein, *Ibn al-Muthannā on al-Khwārizmī*, pp. 82-83 and 207-209 (on al-Khwārizmī's use of the formula), and King, "al-Khwārizmī", pp. 7, 11.

4.3 al-Sulṭān al-Ashraf

The Rasulid Sultan al-Ashraf (*reg*. 1296-97) compiled two major works on astronomy. The first was a lengthy treatise on the construction of astrolabes and sundials and also on the use of the magnetic compass. The second was an equally lengthy volume on astrology, which also contains tables for timekeeping for Sanaa and an agricultural almanac.[6]

This second work, entitled *Kitāb al-Tabṣira fī ʿilm al-nujūm*, survives in the unique MS Oxford Hunt. 233, copied *ca*. 1400, and in the almanac on fols. 97r-108v, shadow lengths for the *ẓuhr* and the beginning and end of the *ʿaṣr* are given for each half-month of the Syrian year. The values are written out in words at the side of the almanac; they are reproduced in the following:

Table 4.3

Notes: All fractions are additive unless otherwise stated.
An asterisc denotes that an approximation has been made in a particular set of entries.
A double asterisc denotes that there is an error in the calculation of either of the entries for the *ʿaṣr*.

Month	*ẓuhr*	*ʿaṣr* 1	*ʿaṣr* 2	
I	5	$11\ ^2/_3$	$18\ ^2/_3$	
	$4\ ^1/_2$	$11\ ^1/_6$	$18\ ^1/_6$	
II	$3\ ^1/_2\ ^1/_4$	$10\ ^1/_4\ ^1/_6$	$17\ ^1/_4\ ^1/_6$	
	3	$9\ ^2/_3$	$16\ ^2/_3$	
III	$2\ ^1/_4$	$8\ ^2/_3\ ^1/_4$ [a]	$16\ ^2/_3\ ^1/_4$	
	$1\ ^1/_2$	8	$15\ ^1/_5$	**
IV	$^1/_2\ ^1/_4$	$7\ ^1/_3\ ^1/_{10}$	$14\ ^1/_3 \bullet ^1/_{10}$	**
	$^1/_4$	$6\ ^2/_3\ ^1/_4$	$13\ ^2/_3\ ^1/_4$	
V	$^1/_3$	7	14	
	$^1/_2$	$7\ ^1/_4\ ^1/_6$	$14\ ^1/_4\ ^1/_6$	**
VI	$1\ ^1/_2 \bullet ^1/_{10}$	$7\ ^2/_3$	$14\ ^2/_3$	*
	$1\ ^1/_{10}$	$7\ ^1/_2\ ^1/_4$	$14\ ^1/_2\ ^1/_4$	*
VII	1	$7\ ^2/_3$	$14\ ^2/_3$	
	$^1/_2\ ^1/_4$	$7\ ^1/_3$	$14\ ^1/_3$	**
VIII	$^1/_5$	$6\ ^1/_2\ ^1/_3$	$13\ ^1/_2\ ^1/_3$	*
	1	7	14	**
IX	$1\ ^1/_2 \bullet ^1/_{10}$	7	$14\ ^2/_3$ [b]	**
	$1\ ^1/_2\ ^1/_4$	$8\ ^2/_5$	$15\ ^2/_5$	**
X	$2\ ^1/_3\ ^1/_4$	$9\ ^1/_4$ [c]	$16\ ^1/_4$	
	$3\ ^1/_3$	10	17	
XI	$4\ ^1/_6$	$10\ ^1/_2$	$17\ ^1/_2\ ^1/_3$	**
	$4\ ^1/_2\ ^1/_4$	$11\ ^1/_4\ ^1/_6$	$18\ ^1/_4\ ^1/_6$	
XII	$5\ ^1/_6$	$11\ ^1/_2\ ^1/_3$	$18\ ^1/_2\ ^1/_3$	
	$5\ ^1/_4$	$11\ ^2/_3\ ^1/_4$	$18\ ^2/_3\ ^1/_4$	

[a] text: $3\ ^2/_3\ ^1/_4$; [b] text: fraction partially illegible (2 for numerator is clear); [c] text: $6\ ^1/_4$

We notice immediately that, apart from a few calculating errors, the entries for the end of the *ʿaṣr* are larger than those for the beginning of the *ʿaṣr* by 7, and that the latter are larger than those for the *ẓuhr* by $6^2/_3$. This is curious indeed, and I have no explanation to offer. Elsewhere in his treatise al-Ashraf presents a table of cotangents to base $6^2/_3$, and in his timekeeping tables he uses obliquity 24° (14;30° for the latitude of Sanaa). For these parameters the maximum

[6] On al-Ashraf (Suter, *MAA*, no. 394) and his astronomical works see King, *Astronomy in Yemen*, no. 8, and King, "Yemeni Astrolabe". His shadow-scheme was brought to my attention by Daniel Varisco, whose detailed study of al-Ashraf's almanac is now published as Varisco, *Yemeni Almanac*. See also n. 4:12.

midday shadow is 5;18, which corresponds quite well to the maximum value in the table, *viz.* $5^1/_4$. Note that the fact that the midday shadows around the summer solstice are southerly is overlooked in the computation of the shadows at the *ʿaṣr*.

4.4 al-Sulṭān al-Afḍal

A precious manuscript preserved in a private library in Sanaa contains an encyclopaedic work compiled *ca.* 1375 by the Yemeni Sultan al-Afḍal.[7] In some marginalia to the astronomical sections of this work there are two sets of notes, both in an untidy hand, apparently due to al-Afḍal himself. The two notes give the shadows (*ẓill*) for each hour as follows, the values being stated to be in cubits (*dhirāʿ*):

T	1	2	3	4	5	6
z	60	30	15	$7^1/_2$ (?)	$3^1/_4$	$^2/_3$
z	60	30	19	$7^1/_2$ (?)	$3^1/_4$	1(?)

These values are clearly garbled. One would expect that the values would be for Δz and that they should be five in number. The entries for T = 4 (read 3?) should probably be $6^1/_2$.

4.5 Bā Makhrama

A late Ottoman navigational manual (see **10.2**) contains a table of shadow lengths, said to be *ʿalā manāzil al-Shibāmī al-shaykh ʿAbdallāh ibn ʿUmar Bā Makhrama*, that is, "in accordance with the lunar mansions as described by ... Bā Makhrama". This individual was a Yemeni scholar active *ca*. 1500, and is otherwise known by a treatise on folk astronomy entitled *al-Shāmil*, which survives in a unique manuscript.[8] I have not noticed the shadow table in this treatise. The epithet al-Shibāmī indicates his family's provenance from Shibām in the Ḥaḍramawt.[9]

In the table the 28 mansions are divided in four columns between the seasons (*al-Hanʿa* = M_5 = first mansion of spring), and the shadow lengths in *qadam*s at midday and the *ʿaṣr* are written out in words. The difference is invariably 7, which is clearly the value used for n. The entries are displayed in the following table:

Table 4.5

M	Z	M	Z
6	$5 + ^1/_8$	20	$1 - ^1/_{20}$
7	$5 - ^1/_6$	21	$1 - ^1/_5$
8	$4 + ^1/_3$	22	$^1/_2$
9	$4 - ^1/_5$	23	$^1/_8$ [e]
10	$3 + ^1/_8$	24	$^1/_3$
11	$2 + ^1/_2 - ^1/_{30}$ [a]	25	$1 - ^1/_6$
12	$2 - ^1/_6$	26	$1 + ^2/_5$

[7] On the astronomical activity of al-Afḍal see King, *Astronomy in Yemen*, no. 18, and **II-12.4**.
[8] King, *Astronomy in Yemen*, no. 26.
[9] Another Shibāmī of renown in Yemeni folk astronomy is listed *ibid*., no. 43.

13	$1 + {}^1/_5$	27	$2 + {}^1/_{30}$
14	${}^2/_3$	28	$2 + {}^2/_3$
15	${}^1/_8$ [b]	1	$3 + {}^1/_3$
16	${}^1/_4 + q - {}^1/_4 q$ [c]	2	4
17	${}^1/_3 - {}^1/_{20}$ [d]	3	$4 + {}^1/_2 + {}^1/_{30}$
18	$1 - {}^1/_8$	4	$5 - {}^1/_{20}$
19	$1 - {}^1/_{30}$	5	5 [f]

[a] text: *qadamayn wa-niṣf illā thulth ʿushr*; [b] text adds remark: "on the sixth day of this mansion the midday shadow is zero"; [c] here q = *qīrāṭ* (text: *rubʿ qadam wa-qīrāṭ illā rubʿ*) – see **4.1**; [d] for the *ʿaṣr*: $8 - {}^1/_3$ of ${}^1/_2$ of ${}^1/_{10}$; [e] for the *ʿaṣr*: $7 + {}^1/_3$; [f] for the *ʿaṣr*: $11 + {}^1/_6$

Notice that the extremal values are $5 + {}^1/_8$ and $1 - {}^1/_{30}$ (S), which are mutually inconsistent. They correspond most closely to latitude 13°, commonly used for Aden: for this latitude the approximate values are $5 + {}^1/_6$ and $1 + {}^3/_{20}$ (S).

In some notes appended to MS Cairo DM 948 (fols. 101v-102v) there are some values of the midday shadow (base not stated) at Aden, given for each ten days of the Syrian year. These are specifically attributed to Bā Makhrama. They are followed by an incomplete, anonymous set for Taiz (with $n = 6{}^1/_2$), also for each ten days of the year (fols. 102v-103r).

4.6 al-Ṭawāshī

A Yemeni treatise on folk astronomy entitled *Miftāḥ al-asrār fī ʿilm al-falak al-dawwār* is attributed in both of the extant manuscripts, MSS Cairo DJ 709,6, (fols. 29r-53v, copied *ca*. 1750) and Princeton Garrett (Hitti) 1016 (61 B) (copied *ca*. 1875), to Nūr al-Dīn ʿAlī ibn ʿAbdallāh al-Ṭawāshī, an individual otherwise known to the modern literature.[10] The treatise contains a poem on the midday shadows for the lunar mansions (fol. 35r of the Cairo manuscript): the values are a simplified version of those given by Ibn Tūmart (see **6.1**), varying linearly between a maximum of $5{}^1/_2$ feet for *al-Balda* and a minimum of zero between *al-Haqʿa* and *al-Jabha*.

4.7 al-Thābitī

Muḥammad ibn ʿAbd al-Laṭīf al-Thābitī was a Syrian who lived in Zabid.[11] In 1047 H [= 1637/38] he compiled a set of tables for timekeeping by the lunar mansions and by shadow lengths, of which I have examined MS Vatican ar. 962 (fols. 13r-19r). The shadow-tables display the shadow at midday and the *ʿaṣr*: values to base $6{}^1/_2$ are given to two digits, the second called *banān*, fingers, representing *twelfths of a shadow digit*. The midday shadows are computed according to a linear zigzag-scheme, with a maximum of $5{}^5/_{12}$ at the winter solstice and a maximum of $2{}^2/_{12}$ (south) at the summer solstice. These extremal values are equivalent to 10 and 4 digits for a gnomon length of 12 digits, which suggests that al-Thābitī converted a set of values from base 12 to base $6{}^1/_2$. In fact, for $\phi = 15°$ and base 12 the extremal values are

[10] *Ibid*., no. 27.
[11] *ibid*., no. 33, and **II-12.7**.

closer to 10 and 2 digits. al-Thābitī's afternoon shadow lengths are always $6^1/_2$ feet more than the absolute value of his midday shadow lengths.

In MS Cairo DM 899,2 (fols. 9r-9v, copied *ca*. 1700), appended to an abridged version of the treatise of al-Aṣbaḥī (**4.1**) by Abū Ḥamad Maḥfūẓ ibn ʿAbd al-Raḥmān al-Ḥaḍramī, there is a list of midday shadows to base $6^1/_2$ written out in words for days in Syrian calendar, specifically intended for Zabid. The values are the same as those given by al-Thābitī. Values are also given for the *ʿaṣr* and are all correctly $6^1/_2$ feet more than the midday values.

4.8 Abū Shukayl (?)

On fol. 97v of MS Cairo DM 948 (see **4.1**), amidst some notes on timekeeping, there is a reference to a commentary (*sharḥ*) by one Abū Shukayl (?), not otherwise known to me. In this work the midday shadow is given as zero on Ayyār 1; it has a maximum (southerly) value 2 $^1/_6$ on Ḥazīrān 22, and a maximum (northerly) value 5 $^1/_4$ + $^1/_6$ on Kānūn I 22. No locality is specified, but the extremal values are the same as those of al-Thābitī (see **4.7**).

4.9 Miscellaneous anonymi

(a) In MS Oxford Marsh 134, copied 855 H [= 1451/1452], of the treatise of folk astronomy by al-Aṣbaḥī (**4.1**), there is a shadow-scheme amidst some notes at the end of the treatise (fol. 52v). The scheme is presented in the form of a circle and values of the midday shadow lengths are given for each of the Syrian months; the corresponding names of the Himyaritic (ancient South Arabian) months are also given.[12] The scheme is simply:

5 4 3 2 1 0 1 2 3 4 5 6 ,

and is probably very old. One may wonder whether the scheme was known in the Yemen before the advent of Islam. In the view of what has come to light of Graeco-Roman influence in South Arabia,[13] I suspect that it was indeed known there. The only other Islamic source in which I have come across such a scheme is from Ḥiṣn Kayfā (now in Southern Turkey), where one could presume Byzantine influence – see **7.2**.

(b) MS Aleppo Awqāf 968 (16 pp., copied *ca*. 1600) contains an anonymous treatise entitled *Kitāb al-Īḍāḥ al-shāfī bi-'l-itqān fī maʿrifat al-manāzil wa-'l-azmān*. The author mentions the *Qāmūs* of Majd al-Dīn (al-Fīrūzābādī) so that he must postdate *ca*. 1400. He also mentions the *Kitāb al-Yawāqīt* by al-Janadī, otherwise known as al-Aṣbaḥī (**4.1**). Two shadow-schemes are presented for the seasonal hours. The shadows (in feet), not shadow increases, are as follows:

---

[12] A. F. L. Beeston, "New Light on the Himyaritic Calendar", *Arabian Studies* 1 (1974), pp. 1-6; George Saliba, "A Medieval Note on the Himyarite Calendar", *JAOS* 105 (1985), pp. 715-717, based on MS Oxford Hunt. 233 of al-Ashraf's *Tabṣira* (**4.3**); and Varisco, *Yemeni Almanac*, pp. 64-71, *etc*.

[13] See, for example, A. H. Masry, *An Introduction to Saudi Arabian Antiquities*, Riyadh, 1975; A.R. al-Ansary, *Qaryat al-Fau. A Portrait of Pre-Islamic Civilisation in Saudi Arabia*, Riyadh, 1982, and Annie Caubet, *Aux sources du monde Arabe: L'Arabie avant l'Islam – Collections du Musée du Louvre*, Paris: Institut du Monde Arabe & Réunion des musées nationaux, 1990.

38	18	9	6	3	0
35	15	9	5	3	0

The numbers are displayed in the text in Arabic numerical notation.

(c) In an anonymous Egyptian source MS Cairo Dār al-Kutub K 3769, (fols. 10v-11r, copied 1910), appended to a late Yemeni treatise on timekeeping, the scheme is:

1	2	3	4	5	6	7	8	9	10	11
26(!)	13	7	5	3	-	3	5	7	13	24(!)

4.10 al-Wāsiʿī

Shaykh ʿAbd al-Wāsiʿ ibn Yaḥyā al-Wāsiʿī published a set of prayer-tables for the Yemen in the year 1939/40.[14] Amongst the functions tabulated for each day of the year are the shadow lengths at midday and the *ʿaṣr* given to base 7 in feet and fingers (as with al-Thābitī). The shadow lengths are computed according to a very primitive scheme. From Taurus 8° to Cancer 18° the midday shadows have a constant value $^{4}/_{12}$ feet north instead of south, and different values are given for the vernal equinox and autumnal equinox, as follows:

VE: 2 SS: $^{4}/_{12}$ AE: 1 $^{4}/_{12}$ WS: 5 .

For $\phi = 15°$ and $n = 7$ these are actually closer to:

$1^{10.5}/_{12}$ $1^{5}/_{12}$ (north!) $1^{10.5}/_{12}$ $5^{7}/_{12}$.

[14] On al-Wāsiʿī see King, *Astronomy in Yemen*, no. 47. His tables were published in Cairo, Maṭbaʿat Ḥijāzī, 1358 H [= 1939], under the title *Kanz al-thiqāt fī ʿilm al-mīqāt.* For an analysis see **II-12.12**.

CHAPTER 5

ANDALUSĪ SOURCES

5.1 ʿArīb ibn Saʿd

The earliest known almanac for al-Andalus is the *Kitāb al-Anwāʾ* of Abu ʾl-Ḥasan ʿArīb ibn Saʿd al-Kātib compiled in the late 10th century. This work, known as the *Calendar of Cordova*, was translated into Latin and was transmitted to medieval Europe. The Judaeo-Arabic and Latin texts have been published by Reinhart Dozy and translated into French by Charles Pellat.[1]

ʿArīb ibn Saʿd gives information on the lunar mansions, the duration of day and night and twilight, as well as midday solar altitudes and shadows (expressed in words in terms of the gnomon length), for each 13, 14, 15 or 16 days. This information has recently been investigated by Julio Samsó.[2] The following table displays the values given for the last two quantities (the equinoxes are at III 16 and IX 18):

Table 5.1

Date	H	z ÷ n	Date	H	z ÷ n
I 2	$29^{2}/_{3}{}^{\circ}$	$1^{3}/_{4}$	VII 16	$73^{2}/_{5}$	$^{11}/_{36}$
I 16	$32^{1}/_{2}$	$1^{3}/_{5}$	VIII 1	$69^{2}/_{3}$	$^{3}/_{8}$
II 2	$36^{5}/_{6}$	$1^{1}/_{3}$	VIII 16	$65^{1}/_{4}$	$^{1}/_{2}$
II 15	$40^{1}/_{2}$	$1^{1}/_{6}$	IX 1	$55^{1}/_{4}$ (*sic*)	$^{7}/_{12}$
III 2	$46^{5}/_{6}$	$^{15}/_{16}$	IX 15	$54^{1}/_{2}$	$^{2}/_{3}$
III 18	$52^{1}/_{2}$	$^{3}/_{4}$	X 1	$47^{2}/_{3}$	$^{11}/_{12}$
IV 2	$58^{5}/_{6}$	$^{7}/_{12}$	X 8	*deest*	1
IV 17	$63^{4}/_{5}$	$^{1}/_{2}$	X 16	$41^{11}/_{12}$	$1^{1}/_{8}$
V 2	*deest*	*deest*	XI 1	$36^{1}/_{4}$	$1^{1}/_{3}$
V 14	$72^{4}/_{5}$	$^{1}/_{3}$	XI 15	$32^{2}/_{5}$	$1^{7}/_{12}$
VI 2	$75^{1}/_{3}$	$^{11}/_{40}$	XII 2	$29^{2}/_{3}$	$1^{3}/_{4}$
VI 16	$76^{1}/_{3}$	$^{1}/_{4}$	XII 16	$28^{2}/_{3}$	$1^{5}/_{6}$
VII 1	$75^{3}/_{4}$	$^{1}/_{4}$			

The midday solar altitudes are clearly computed, and underlying them are to be detected the parameters:

$$\phi = 37;30^{\circ} \quad \text{and} \quad \varepsilon = 23;50^{\circ} .$$

The first was no doubt intended to serve Cordova, although it is not attested in any other known medieval Islamic sources;[3] the accurate value is 37;53°. The second is an approximation for

---

1 On the context of all of this Andalusī material see Forcada, "*Anwāʾ* Books in al-Andalus". On the *Calendar of Cordova* see Pellat, *Le Calendrier de Cordoue*, pp. xiii-xiv, and, more recently, J. Martinez Garquez and J. Samsó, "Una nueva traducción latina del calendrio de Córdoba (siglo XIII)", in Vernet, ed., *Textos y estudios*, A, pp. 9-78, Samsó, "Sobre los materiales astronómicos en el 'Calendrio de Córdoba' y en su versión latina del siglo XIII", in Vernet, ed., *Textos y estudios*, B, pp. 125-138; and Forcada, *op. cit.* See also the article on the author by Charles Pellat in EI_2, and Sezgin, *GAS*, VII, pp. 355-356.

2 See Samsó, *op. cit.*, esp. pp. 126 ff. for details.

3 Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 95.

Ptolemy's 23;51,20°. Likewise computed are the ratios of the shadows to the gnomon. The tabulation of such quantities derived by computation borders on the mathematical tradition of Islamic astronomy.

5.2 al-Umawī

MS Escorial ár. 941 (38 fols., copied *ca*. 1300) is a unique but incomplete copy of a treatise on folk astronomy by al-Ḥasan ibn ʿAlī al-Umawī al-Qurṭubī, who was born in Cordova in 514 H [= 1120/21] and died in Seville in 602 H [= 1205/06].[4] The work is divided into two main parts, both incomplete, the first discussing *anwāʾ* for each of the *ʿajamī* (Christian) months, and the second giving information on a variety of topics including the times of prayer, the qibla, the winds and the seasons.

Our present concern is with the shadow lengths for the hours given at the end of the sections for each month. Unfortunately only three sets of values are preserved in the Escorial manuscript – see the following table – but these are adequate to reconstruct the entire set for the year.

Table 5.2

Month \ T	1/11	2/10	3/9	4/8	5/7	6	*ʿaṣr*
January	27	17	13	10	9	7	14
February	26	16	12	9	8	6	[13] [a]
March	25	15	11	8	7	5	12

[a] text: 14

The pattern is similar to, but not identical with, that of the Greek tables of type M – see **1.4** and also **7.2**. We may suppose, for example, that the midday shadows vary linearly between 8 (Dec.) and 2 (June). The increases for each hour are:

2 1 3 4 10

instead of the usual

1 2 3 4 10 ;

Neugebauer has noted similar absurdities in Latin tables.[5]

5.3 Ibn Sabʿīn (?)

In MS Cairo MJ 202,2 (fol. 33v, copied 1681), immediately following an astrological treatise *Kitāb al-Daraj* by the 13th-century Andulusian mystic Ibn Sabʿīn,[6] there is a rule for finding the time of day by shadows. Values for the shadow lengths are given in Eastern Arabic numerical notation, and the case-ending of the word *qadam*, "feet", indicates the the values given in the text are probably as originally intended. These are:

---

[4] On al-Umawī see Suter, *MAA*, no. 323.

[5] Neugebauer, *HAMA*, II, p. 745.

[6] On Ibn Sabʿīn see the article by A. Faure in EI_2, and also Brockelmann, *GAL*, I, p. 611, SI, p. 844, and SII, p. 1017, and *Cairo ENL Survey*, no. F15. His various astrological works deserve detailed investigation.

1	2	3	4	5	6	7	8	9	10	11	12
22	18	7	6	3	*	3	6	7	8	18	28

No base is stated. For the sixth hour the sun is said to be in "the pole of the sphere" (*quṭb al-falak*), perhaps here meaning the zenith (the scheme is not necessarily of Andalusī origin).[6] Notice the lack of symmetry and the fact that the compiler could not resist including a value for the twelfth hour. A similar set of values, albeit symmetrical, is presented by Ibn Tūmart – see **6.1**.

[6] Even before the advent of Islam the Arabs were aware of the celestial pole, which they called *quṭb* – see the article "al-Ḳuṭb" by Paul Kunitzsch in EI_2, repr. in *idem*, *Studies*, XVIII.

CHAPTER 6

MAGHRIBI SOURCES

6.1 Ibn Tūmart

Muḥammad ibn ʿAlī ibn Tūmart al-Andalusī was the author of a treatise entitled *Kitāb Kanz al-ʿulūm wa-'l-durr al-manẓūm fī ḥaqāʾiq ʿilm al-sharīʿa* and extant in several manuscripts.[1] The work deals with religion, law and the sciences, and is of considerable interest. It has been studied by Georges Vajda, who established that the author can not predate the beginning of the 13th century. I have used MS Cairo ḤJ 47,1 (fols. 1v-85r, copied *ca*. 1750), in which there is a section on simple timekeeping by shadows on fols. 80r-81v: see **Fig. 6.1**.

Although Ibn Tūmart was writing in al-Andalus or the Maghrib, the first shadow-scheme that he presents is for the Yemen and is introduced as such. It gives the midday shadows as the sun progresses through the 28 lunar mansions. For *al-Balda* (M_{21}) the shadow is $5^1/_2$ feet, decreasing $^1/_2$ foot for each mansion until *al-Dabarān* (M_4) when there is no shadow (*yanʿadim al-ẓill*). From *al-Haqʿa* (no. I) till *al-Jabha* (M_{10}) there is a very small midday shadow (*adnā ziyāda*), and then in *al-Zubra* (M_{11}, the text has incorrectly *al-Dabarān*) the shadow is $^1/_2$ foot, increasing $^1/_2$ foot each mansion until *al-Balda* (M_{21}). For each mansion the shadow at the *ʿaṣr* is $6^1/_2$ feet more. The maximum midday shadow length of $5^1/_2$ corresponds to $\phi = 15°$ for n = $6^1/_2$, but then the shadow should increase to 1 foot south in the mansions close to the summer solstice.

The above information is contained in the text of Ibn Tūmart's treatise and is summarized in a poem. Our author then presents a shadow-scheme in feet for the hours, which is as follows:

28 18 9 6 3 0 (*istiwāʾ*)

This scheme was also used by al-Amrānī / al-Imrānī (**3.5**). Ibn Tūmart concludes this section with a discussion of time-reckoning by night by means of the lunar mansions.

6.2 Abū Miqraʿ

Abū Miqraʿ was an authority on folk astronomy in the Maghrib in the 13th century.[2] His teachings are known to us through a poem called *al-Muqniʿ fī 'khtiṣār ʿilm Abī Miqraʿ* written by the 17th-century Maghribi astronomer Muḥammad ibn Saʿīd al-Sūsī al-Marghīthī, and they

---

[1] On Ibn Tūmart see the article in *EI*$_2$ by J. F. P. Hopkins, and Brockelmann, *GAL*, I, pp. 506, and SI, p. 697, and I, p. 274 and SI, p. 424 (are these references to the same individual?), as well as *Cairo ENL Survey*, no. F14. His treatise is investigated in Georges Vajda, "Une synthèse peu connue de la révélation et de la philosophie: le 'Kanz al-ʿUlūm' de Muḥammad ibn ʿAlī Ibn Tūmart al-Andalusī", in *Mélanges Louis Massignon*, Damas: Institut Français de Damas, 1957, vol. III, pp. 359-374.

[2] Colin & Renaud, "Abū Miqraʿ"; Renaud, "Additions et corrections à Suter", no. 531, and *Cairo ENL Survey*, no. F17.

Fig. 6.1: The section of the treatise of Ibn Tūmart dealing with time-keeping by shadows. [From MS Cairo ḤJ 47,1, fols. 80r-81v, courtesy of the Egyptian National Library.]

are outlined in greater detail in various commentaries, of which I have consulted one called *al-Mumtiʿ fī sharḥ al-Muqniʿ* by al-Sūsī.[3]

In MS Cairo ṬJ 201,2 (fols. 7r-12v, copied *ca*. 1600) of the *Mumtiʿ*, al-Sūsī quotes Abū Miqraʿ (fol. 11r) as having advocated the standard (Indian) formula for timekeeping (with n = $6^1/_2$), and the scheme:

9 7 5 3 2 1 1 2 4 ? 8 10

for the midday shadows for the months of the (Christian) calendar. The values are given in *abjad* notation, and the value for month X looks like a European 6, but this could be read as either a 5 or a 6. In MS Cairo DM 415 (55 fols., copied *ca*. 1700) of the *Mumtiʿ* the same values are given (fol. 41r) but with a *yāʾ* = 10 for month X; elsewhere (fol. 41v) the sequence 1 2 4 5 is mentioned for months VII to X, and it seems that a 5 was indeed intended for month X. al-Sūsī adds that the above values were specifically for latitude 32°, Marrakesh (the actual value is 31;49°), and that al-Jādarī had proposed the scheme:

10 8 5 3 2 1 1 2 4 6 8 10

for Fez. See further **6.4**.

6.3 Pseudo-Ibn al-Bannāʾ

Ibn al-Bannāʾ was a Morroccan astronomer and mathematician of some renown who worked in Marrakesh about 1300 and whose major work was a *zīj*, extant in several copies.[4] An almanac entitled *Risāla fī ʾl-Anwāʾ* is also attributed to him and has been published by Henri-Paul-Joseph Renaud.[5] Along with diverse information on agriculture, meteorology, and astronomical folklore given for each day of the year, this almanac displays the lengths of day and night for each 15 days of the solar year based on a latitude of about 38°, that is, Cordova, as well as the lengths of the gnomon shadow in feet (*aqdām*) at the *ẓuhr* and *ʿaṣr* for each month. Since Ibn al-Bannāʾ worked in Marrakesh ($\phi \approx 32°$) there are problems with the attribution of this work to him, but I shall not pursue these here.

Renaud used five manuscripts (S, P_1, P_2, A_1 and A_2[6]) for his translation of the almanac and noted the variants in the shadow lengths; there are clearly three main traditions – see **Table 6.3a**. Neugebauer, perhaps overeagerly, restored some of the values in the scheme represented by MS P_2 to produce the primitive scheme of Greek origin shown in **Table 6.3b**.[7]

---

3 Renaud, *op. cit*., no. 540.

4 On Ibn al-Bannāʾ see Suter, *MAA*, no. 399; Renaud, *op. cit*., no. 399; *idem*, "Ibn al-Bannāʾ"; *Cairo ENL Survey*, no. F23; and the article by J. Vernet in *DSB*.

5 Renaud, *Calendrier d'Ibn al-Bannāʾ*, esp. pp. 22-23. See now Forcada, "Les sources andalouses du Calendrier d'Ibn al-Bannāʾ".

6 These stand for: S – Salé (private owner); P_1 – Paris BNF ar. 4764; P_2 – Paris BNF ar. 6020; A_1 – Algiers 941; A_2 – Algiers 1468 (see Renaud, *op. cit*., p. 26).

7 Neugebauer, *HAMA*, II, pp. 743-744.

Table 6.3a

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
(i) MS S:												
ẓuhr	7	6	4	3	2	1	1	4	5	7	$9^1/_5$	11
ʿaṣr	14	13	11	10	9	8	8	10	10	11	15	17
(ii) MSS P_1, A_1 and A_2:												
ẓuhr	7	6	6	4	3	2	4	4[a]	5	7	$9^1/_5$	11
ʿaṣr	14	13	11	10	9	8	8	10[b]	10	11	15	15
(iii) MS P_2:												
ẓuhr	7	6	5	4	3	3	3	4	5	7	$9^1/_5$	11
ʿaṣr	14	13	11	10	9	8	8	10	10	11	15	15

[a] P_1 has $2^1/_3$ [b] P_1 has 8

Table 6.3b

Modifications to the scheme in MS P_2 made by Neugebauer are underlined.

ẓuhr	7	6	5	4	3	2	3	4	5	$\underline{6}$	$\underline{7}$	$\underline{8}$
ʿaṣr	14	13	11	10	9	8	$\underline{9}$	10	$1\underline{2}$	$1\underline{3}$	$1\underline{4}$	15
Δz	7	7	6	6	6	6	$\underline{6}$	6	$\underline{7}$	$\underline{7}$	$\underline{7}$	$\underline{7}$

The *ẓuhr* entries for months XI and XII, and perhaps also month X, are garbled. Virtually all of the *ʿaṣr* shadows are garbled since, at least according to the standard definition of the *ʿaṣr*, they should each be in excess of the corresponding midday entries by 7. (Even in Neugebauer's reconstruction the increases are either 6 or 7.) The *ẓuhr* shadows are clearly all midday shadows, with perhaps one exception (see below), rather than midday shadows augmented by one-quarter of the length of the gnomon, which was the standard definition of the time of the *ẓuhr* in al-Andalus and the Maghrib (see also **6.4**).[8]

The anomalous but well-attested value $9^1/_5$ for the *ẓuhr* in month XI appears to represent an attempt to add one-fifth of the gnomon length to the gnomon shadow. This corresponds to a time one seasonal hour after midday using the standard Indian formula (see **2.1** and **3.2**).[9] The value of the gnomon length varies between 6 and 7 feet in the table, but the value $9^1/_5$ could be derived from a midday shadow of 8 augmented by one-fifth of a gnomon length of six digits.

6.4 Abū ʿAbdallāh al-Muʾaddib

MS Oxford Pococke 249 (63 fols., copied *ca*. 1400) is a unique copy of a treatise on folk astronomy by a Maghribī astronomer named Abū ʿAbdallāh Muḥammad ibn ʿAbdallāh ibn ʿAmr al-Muʾaddib ("the teacher"), not otherwise known to the modern literature. On fols. 46v-47r there is a table displaying the shadow lengths in digits (n = 12) at each seasonal hour and at the *ẓuhr* and *ʿaṣr* for each date of the solar year when the shadow has an integral value – see **Fig. 6.4** and **Table 6.4** for extracts.

---

[8] On the motivation for the value $\Delta z = {}^1/_4$ n see **IV-4.4**.

[9] See further King, "Tunisian Sundial", p. 195, and now **IV-2**.

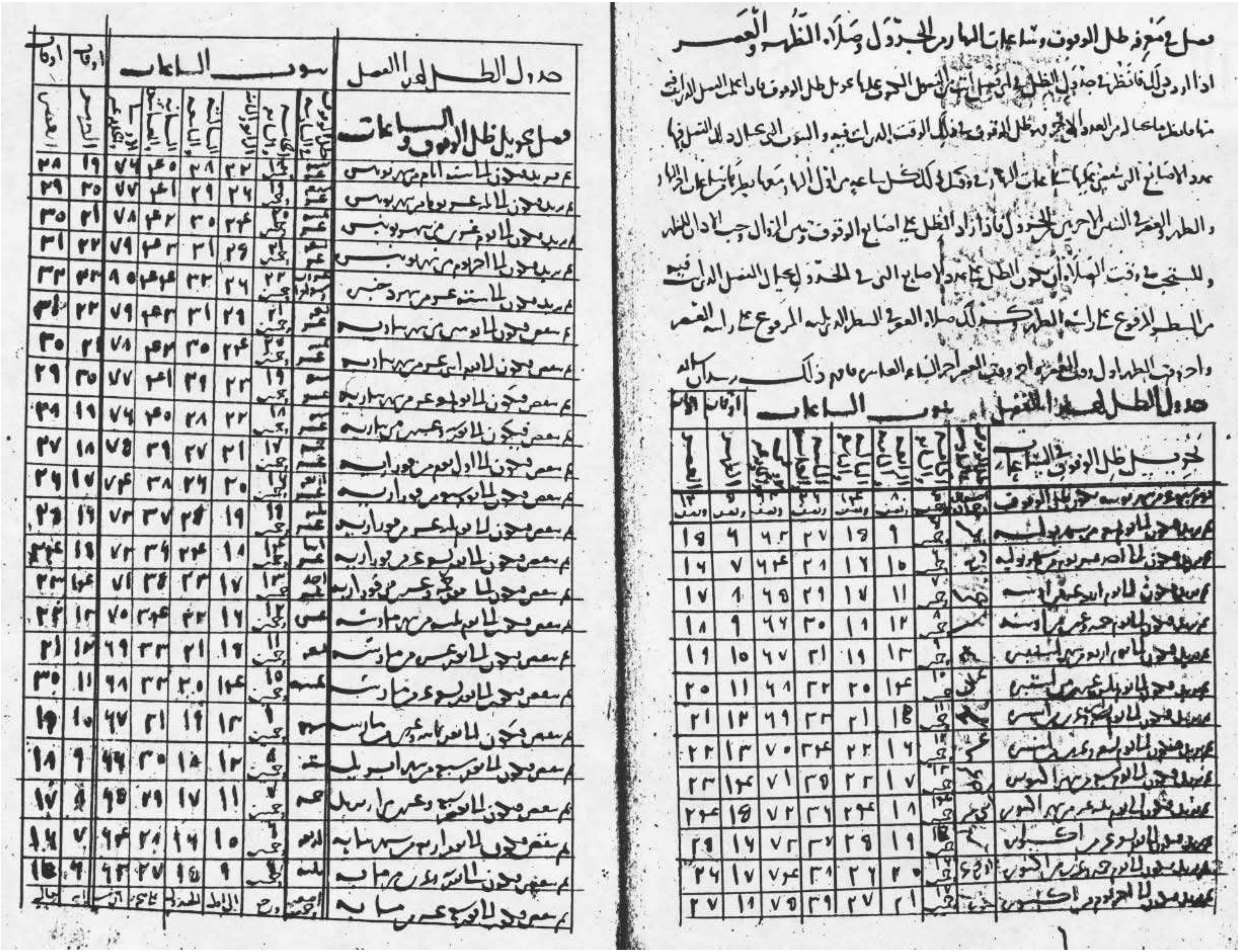


Fig. 6.4: A table for timekeeping by Abū ʿAbdallāh al-Muʾaddib. [From MS Oxford Pococke 249, fols. 46v-47r, courtesy of the Bodleian Library.]

Table 6.4

Date[a] | Hours	6	5	4	3	2	1	*ẓuhr*	*ʿaṣr*
VI 17	$2\,{}^{2}/_{5}$ [b]	5- [c]	$8\,{}^{1}/_{2}$	$14\,{}^{1}/_{2}$	$26\,{}^{1}/_{2}$	$62\,{}^{1}/_{2}$	$5\,{}^{1}/_{2}$	$14\,{}^{1}/_{2}$
VII 9	3	$5\,{}^{2}/_{5}$	9	15	27	63	6	15
VII 11 [d]	4	$6\,{}^{2}/_{5}$	10	16	28	64	7	16
VIII 14 - 25		.....						
IX 4 - 13 - 22 - 29		.....						
X 7 - 13 - 19 - 25 - 31		.....						
XI 6 - 13 - 20		.....						
XI 30	19	$21\,{}^{2}/_{5}$	25	31	43	79	22	31
XII 16	20;7 [e]	$22\,{}^{2}/_{5}$	26	32	44	80	23	32
I 2 19	$21\,{}^{2}/_{5}$	25	31	43	79	22	31	
I 12 - 19 - 26		.....						
II 1 - 7 - 13 - 19 - 25		.....						
III 3 - 10 - 19 - 28		.....						
IV 7 - 27 [f]		.....						
V 4		.....						
V 16(?)	3	$5\,{}^{2}/_{5}$	9	15	27	63	6	15
V 27(?)	$2\,{}^{2}/_{5}$	* [g]						

[a] the first date is followed by the phrase: *yakūn ẓill al-wuqūf* ... , meaning: the midday shadow will be ... . The remaining dates are preceded by the phrase: *thumma yazīd / yanquṣ fa-yakūn ilā* ... , meaning: then it increases / decreases until it becomes (amount) until (date). [b] *iṣbaʿān wa-khumsān*. [c] fraction illegible; one could read a *khumsayn* (= ${}^{2}/_{5}$) but there is more. [d] read 21? [e] *ʿishrūn wa-sabʿat ajzāʾ*. [f] read 17? [g] *wa-rajaʿ* (read: *wa-rjaʿ* ?) *ilā awwal al-jadwal ʿalā* ... (??) *in shāʾa llāh taʿālā*, meaning: go back to the beginning of the table ... ... (??) if God wills.

For the solstices non-integral values occur, namely:

$$2\,{}^{2}/_{5} \quad \text{and} \quad 20;7 \ .$$

These correspond to meridian altitudes:

$$78;42° \quad \text{and} \quad 30;49°,$$

whence one can derive:

$$\phi \approx 35;15° \quad \text{and} \quad \varepsilon \approx 23;55° \ .$$

It is, however, more realistic to conclude that these extremal values were computed for:

$$\phi \approx 35° \sim 36° \quad \text{and} \quad \varepsilon \approx 23;30° \ .$$

The values for the *ẓuhr* and the *ʿaṣr* are respectively 3 and 12 more than the midday values, corresponding to:

$$\Delta z = {}^{1}/_{4}n \quad \text{and} \quad n \ ,$$

the standard definitions used in al-Andalus and the Maghrib – see **6.3**. The shadow increases for the hours are according the following scheme:

T	5	4	3	2	1
Δz	$2\,{}^{2}/_{5}$	6	12	24	60
(=	${}^{1}/_{5}n$	${}^{1}/_{2}n$	n	2n	5n) ,

so that they are based on the standard Indian formula.

The dates on which the midday shadows are an integral number of digits were probably derived by observation.

6.5 al-Jādarī

ʿAbd al-Raḥmān ibn Abī Ghālib Muḥammad al-Jādarī was famous as the author of a short poem on folk astronomy compiled in 794 H [= 1391/92] and entitled *Rawḍat al-azhār*, which enjoyed several commentaries at the hands of later Maghribi astronomers.[10] In this poem there is no shadow-scheme, but one specifically for Fez is attributed to al-Jādarī by the 17th-century Maghribi astronomer al-Sūsī. This scheme is:

10 8 5 3 2 1 1 2 4 6 8 10

and is presented, for example, in MS Cairo DM 415, fol. 41v, of al-Sūsī's treatise entitled *al-Mumtiʿ*. See further **6.2**.

The only other known work of al-Jādarī is an almanac for Fez, extant in MS Istanbul Lalelı 3748,10. In this work values of the solar altitude at midday and the corresponding shadows to base $6^2/_3$ are given for each few days of the year, and I have not investigated these. However, the scheme

9 7 5 3 2 1 1 2 4 5 8 10

for the shadows at the *ẓuhr* beginning with January with corresponding values increased by 7 for the *ʿaṣr* is found in a note on fols. 131v, immediately following al-Jādarī's almanac.

6.6 al-Ṣafāqusī

ʿAlī al-Sharafī of Sfax compiled a navigational atlas *ca.* 1579.[11] I have examined two of the four available copies, namely, MSS Paris BNF ar. 2273 and Oxford Marsh 294 (both completed *ca*. 1600); I label these A and B, respectively.

Both copies contain a table displaying the shadow lengths at significant times of day – see **Table 6.6**. The table in A displays shadow lengths for the *ẓuhr* and the *ʿaṣr*, with difference six units: clearly the length of the gnomon is supposed to be 6. The scheme is not attested in any other known source. The table in B displays values for midday, the *ẓuhr* and the *ʿaṣr*. The difference between the first and second is 2 units. The difference between the first and the third is 7 units: clearly this is the length of the gnomon. The same scheme is found in other Maghribi sources (see **6.4** and **6.6**). A marginal note states that the shadow increase for the *ẓuhr* could be two units but that the value of three digits is precautionary (*iḥtiyāṭ*). These increases of 2 and 3 units probably represent approximations for $1^3/_4$ and $3^1/_2$ units, that is, one-quarter and one-half the length of the gnomon 7. In A, but not in B, the lengths of day and night for each month are also tabulated but they do not concern us here. The entries in both sources are displayed in the following table:

---

[10] On al-Jādarī see Suter, *MAA*, no. 424a; Renaud, "Additions et corrections à Suter", no. 424a; Brockelmann, *GAL*, SII, pp. 217-218; and *Cairo ENL Survey*, no. F26.

[11] On al-Ṣafāqusī and his work see Nallino, *Scritti*, V, pp. 533-548; and Brockelmann, *GAL*, SI, p. 710. A new study of these manuscripts is currently in preparation by Mónica Herrera Casais (La Laguna, Tenerife, and Frankfurt).

Table 6.6

	MS A				MS B		
Month	day	night	*ẓuhr*	*ʿaṣr*	midday	*ẓuhr*	*ʿaṣr*
III	$10\frac{1}{3}$	$13\frac{2}{3}$	7	13	5	8	12
IV	$12\frac{2}{3}$	$11\frac{1}{3}$	4	9	3	6	10
V	$13\frac{4}{5}$	$10\frac{1}{5}$	2	8	2	5	9
VI	$14\frac{2}{3}$	$9\frac{1}{3}$	1	7	1	4	8
VII	$14\frac{1}{2}$	$9\frac{1}{2}$	$1\frac{1}{3}$	$7\frac{1}{3}$	1	4	8
VIII	$13\frac{4}{5}$	$9\frac{1}{5}$	$2\frac{1}{3}$	$8\frac{1}{3}$	2	5	9
IX	$13\frac{2}{3}$	$11\frac{1}{3}$	4	10	4	7	11
X	$11\frac{1}{3}$	$12\frac{2}{3}$	5	11	5	8	12
XI	$10\frac{1}{3}$	$13\frac{2}{3}$	8	14	8	11	15
XII	$9\frac{2}{3}$	$14\frac{1}{3}$	10	16	10	13	17
I	$10\frac{1}{4}$	$13\frac{3}{4}$	9	15	9	11	16
II	$10\frac{1}{3}$	$13\frac{2}{3}$	7	13	7	10	14

6.7 Miscellaneous anonymi

(1) The standard approximate rule (with n = 7 and n = 12) is mentioned in an anonymous Maghribi treatise on the astrolabe extant in the unique MS Cairo DM 1169,6 (fols. 45v-47v, copied 1158 H [= 1745/46])[12] (see esp. fols. 47r-47v).
(2) Hoest (1781), Delphin (1891), Renaud (1948), and Roche (1965) have noted the use of the mnemonic

طزه جبا أبده حي

9 7 5 3 2 1 1 2 4 5 8 10

in the Maghrib.[13] This differs from the Egyptian version (see **9.7a**) only in that the shadow for month X is 5, not 6. Hoest records the text of an almanac in which these lengths occur. Also given in this source, but not the others, is the length of day and night for each month:

9;30	10+	12	13	14	15;40	14;30	13;20	12	10;30	9;20	8;55
14;30	13+	12	11	10	8;20	9;30	10;40	12	13;30	14;40	15; 5

Hoest reported that his informants had told him that they thought that the almanac had come to the Maghrib from Istanbul. The winter cold in the Maghrib was not so severe as implied by the almanac, and the shadows there were a little shorter.

---

[12] See **IV**, Arabic text 2.8.

[13] G. Hoest, *Nachrichten von Marokos und Fes im Lande selbst gesammelt, in den Jahren 1760 bis 1768*, Copenhagen: Christian Gottlob Prost, 1781, pp. 252-257; Delphin, "L'astronomie au Maroc", pp. 199-200; Renaud, *Calendrier d'Ibn al-Bannāʾ*, p. 23; and P. Roche, "L'irrigation et le statut juridique des eaux au Maroc", *Revue juridique et politique d'indépendance et coopération* 19 (1965), pp. 55-120 and 537-561, esp. p. 85, n. 68.

CHAPTER 7

ʿIRĀQĪ SOURCES

7.1 ʿAbd al-Qādir al-Kīlānī

ʿAbd al-Qādir ibn ʿAbdallāh al-Kīlānī was a celebrated *ṣūfī*, and founder of the Qādiriyya order.[1] He was born in 1078 in Gilan, studied in Baghdad, and died in 1167. His major work *al-Ghunya* contains some passages on reckoning time by shadow lengths, which I came across in MS Cairo MJ 180,17 (fols. 146v + 148r-149r, copied *ca*. 1750).[2]

al-Kīlānī first presents the midday shadow lengths for the Syrian months, beginning with Aylūl (= September), as follows:

IX	X	XI	XII	I	II	III	IV	V	VI	VII	VIII
5	6	7	8	7	6	5	4	3	2	3	4 .

He then presents "another opinion", which is that the midday "shadow of a person" is 3 feet on Ādhār (= III) 19, decreasing to ½ by Ḥazīrān (= VI) 19, then increasing at a rate of 1 foot each 36 days to 3 feet on Aylūl (= IX) 19 and then at 1 foot each 14 days to 7½ on Kānūn I (= XII) 19.

Yet another scheme is presented next. The shadows at midday and the *ʿaṣr* for the months beginning with Ḥazīrān are:

VI	VII	VIII	IX	X	XI	XII	I	II	III	IV	V
3	4	5	6	7	8	10½	9	7½	6	4½	3½
9½	10½	11½	12½	13½	14½	17	15	14½	12½	11	10

This scheme is clearly a combination of two linear schemes. The midday shadow increases from 3 in June to 8 in November at a rate of 1 foot each month and decreases from 10½ in December to 4½ in May at a rate of 1½ feet each month. No tables displaying this feature are recorded by Neugebauer. The values for the *ʿaṣr* are in general, but not always, 6½ units more than those for the *ẓuhr*. Since for $n = 6;30$ the minimum shadow 3 corresponds to a solar altitude of 65° and the maximum shadow 10½ corresponds to a solar altitude of 32°, even the limiting values of the shadow are inconsistent with reality for any latitude. The same scheme is recorded by Ibn Qudāma – see **8.1**.

7.2 al-Ḥiṣkafī

MS Paris BNF ar. 2540,2 (fols. 16r-27v, copied *ca*. 1450) contains a set of planetary tables compiled about the year 1400 by Nāṣir al-Dīn ibn ʿĪsā al-Ṭabīb al-Ḥiṣkafī (from Ḥiṣn Kayfā

[1] Article "ʿAbd al-Ḳādir al-Djīlānī" by W. Braune in EI_2, and Brockelmann, *GAL*, I, pp. 560-563, and SI, pp. 777-779.

[2] The passages extracted in this manuscript correspond to ʿAbd al-Qādir al-Kīlānī, *al-Ghunya*, 2 vols., Cairo, 1304 H [= 1886/87], II, pp. 89-90.

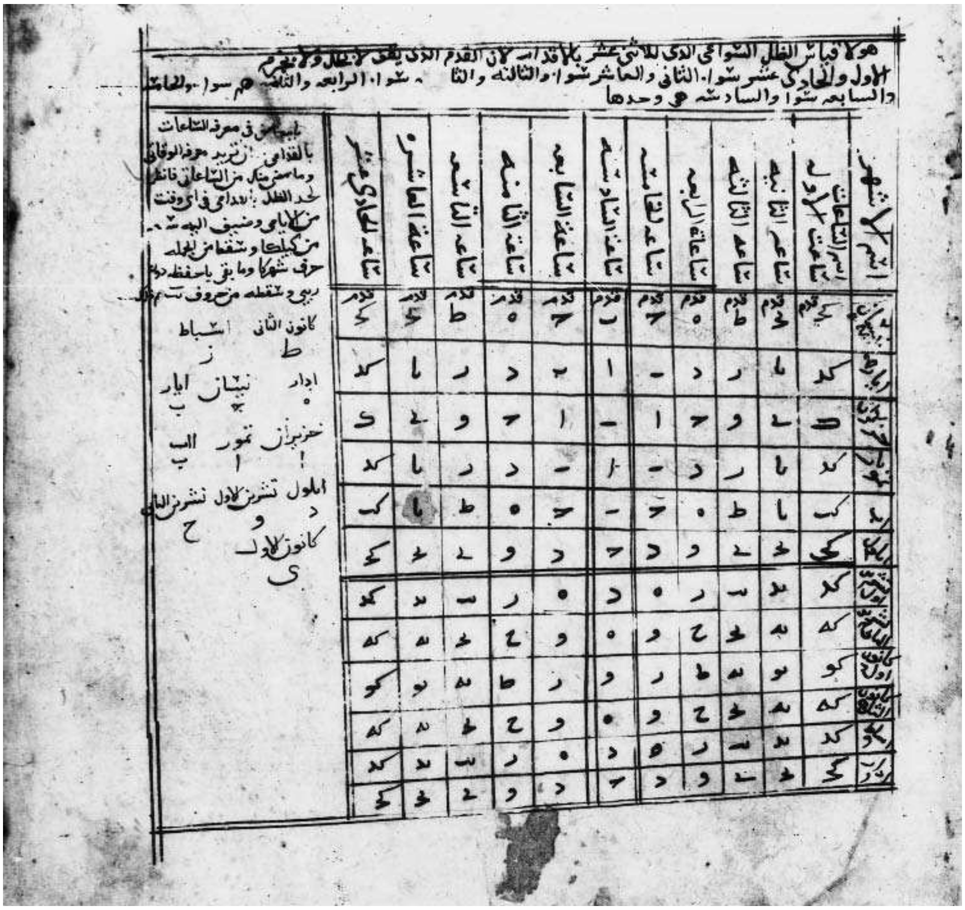


Fig. 7.2: A table for the seasonal hours based on a Hellenistic model and preserved in the planetary tables of al-Ḥiṣkafī. [From MS Paris BNF ar. 2540, fol. 27v, courtesy of the Bibliothèque Nationale de France.]

on the Tigris).[3] Another copy of this work is in MS Vatican ar. 1499 (copied 1576/77). I label these two sources A and B.

Both manuscripts contain (A: fol. 27v, B: fol. 13r) a table of the shadows at each seasonal hour from 1 to 11 for each month of the Syrian year starting with Nīsān (= April). See **Fig. 7.2** for the table as it appears in A. The values given in the table for hours 1 to 6 are displayed in **Table 7.2a**; those for 1 to 5 are confirmed by the identical entries for 11 to 7. The underlying scheme for midday is simply:

5 4 3 2 1 0 1 2 3 4 5 6 .

but the entries for the hours are in some cases, particularly for the first and third, corrupt. **Table 7.2b** shows a matrix of values based on the above midday scheme and increases at the hours of:

1 2 3 4 10 (individual) or 1 3 6 10 20 (cumulative).

[3] al-Ḥiṣkafī and the Paris copy of his tables are listed in Brockelmann, *GAL*, SI, p. 869.

The entries in **Table 7.2a** different from these are italicized. The entries in **Table 7.2b** are less by 2 than the corresponding ones in the standard classical tables of type M (see **1.4**) reproduced in **Table 7.2c**.[4]

Table 7.2a

Month \ T	1	2	3	4	5	6
IV	*23*	13	*9*	5	3	2
V	*24*	11	7	4	2	1
VI	*20* [a]	10	6	3	1	0 [b]
VII	*24*	11	7	4	2	1
VIII	22	11	*9*	5	3	2
IX	23	13	*10*	6	4	3
X	24	14	*12*	7	5	4
XI	25	15	*13*	8	6	5
XII	26	16	*15*	9	7	6
I	25	15	*13*	8	6	5
II	24	14	*12*	7	5	4
III	23	13	*10*	6	4	3

[a] thus in both A and B; [b] A: 2 (*sic*)

Table 7.2b

22	12	8	5	3	2
21	11	7	4	2	1
20	10	6	3	1	0
21	11	7	4	2	1
22	12	8	5	3	2
23	13	9	6	4	3
24	14	10	7	5	4
25	15	11	8	6	5
26	16	12	9	7	6
25	15	11	8	6	5
24	14	10	7	5	4
23	13	9	6	4	3

Table 7.2c

Month | T	1	2	3	4	5	6
I	24	14	10	7	5	4
II	23	13	9	6	4	3
III	22	12	8	5	3	2
IV	23	13	9	6	4	3
V	24	14	10	7	5	4
VI	25	15	11	8	6	5
VII	26	16	12	9	7	6
VIII	27	17	13	10	8	7
IX	28	18	14	11	9	8
X	27	17	13	10	8	7
XI	26	16	12	9	7	6
XII	25	15	11	8	6	5

Note that the midday June shadow of zero serves the tropic of Cancer, not the upper reaches of the Tigris. Since it is unlikely that the table was intended to display increases of the shadows

[4] Neugebauer, *HAMA*, II, p. 738.

over the summer midday minimum, we are left with the conclusion that it is simply the work of an incompetent.[5]

By the side of the main table in both copies is another in which the midday shadows for the Syrian months beginning with Kānūn II (= January) are given as:

9 7 5 3 2 1 1 2 4 6 8 10

This scheme is identical to the one attested in the Coptic and Arabic sources described in **9.7a**.

[5] Compare the Ethiopic tables (based on limits 0 and 9 for Z) in *idem*, *Ethiopic Astronomy and Computus*, pp. 212-213.

CHAPTER 8

SYRIAN SOURCES

8.1 Ibn Qudāma

MS Damascus Ẓāhiriyya 10732 (fols. 2r-4v, copied *ca*. 1600) contains a short treatise on the zodiacal signs, lunar mansions and simple timekeeping by Muwaffaq al-Dīn Abū Muḥammad ʿAbdallāh ibn Aḥmad ibn Muḥammad ibn Qudāma al-Maqdisī (1147-1223), a scholar of law and theology who worked mainly in Jerusalem and Damascus.[1]

The tract includes some prayer-tables (the underlying latitude is 30°, which corresponds more happily to Cairo),[2] and also a section on the shadows at the *ẓuhr* and the *ʿaṣr* for each month of the Syrian year beginning with Ḥazīrān (VI). The values given are the same as those in the third scheme of al-Kīlānī (see **7.1**) with midday values missing for months X and V and an *ʿaṣr* value $15^1/_2$ for month I.

In his major legal treatise, *al-Mughnī*, Ibn Qudāma presented a different scheme.[3] This is described as an approximation (*taqrīb*) and is attributed to Abu 'l-ʿAbbās al-*Ṣ-n-j-y* (?), an individual otherwise unknown to me. For the middle of each of the Syrian months, the shadow lengths in *qadam*s are as follows:

VI	VII/V	VIII/IV	IX/III	X/II	XI/I	XII
$1\ ^1/_3$	$1 + {}^1/_2 + {}^1/_3$	3	$4^1/_2$	$6^1/_2$	9	$10\ ^1/_6$

These are said to be valid for the provinces of Iraq and Syria and others with similar latitudes. The extremal values are roughly consistent with a latitude of *ca*. 34° for $n = 6^1/_2$.

8.2 Ibn al-Shāṭir

Ibn al-Shāṭir was the head of the astronomers at the Umayyad Mosque in Damascus in the mid-14th century, and he is well known to the history of science for his work in planetary astronomy, spherical astronomy and timekeeping.[4]

The introduction to a set of prayer-tables for latitude 34° by Ibn al-Shāṭir survives in the unique MS Leiden Or. 1111 (fols. 108r-113r, copied 1601), and the prayer-tables themselves survive in MS Cairo DM 1170,2 (fols. 11r-22v, copied *ca*. 1650). In the introduction to the

---

[1] On Ibn Qudāma see the article "Ibn Ḳudāma" by George Makdisi in EI_2. Only one scientific work is (incorrectly) attributed to him by Brockelmann (SI, p. 689, no. 20), namely, a work on comets which is actually by a later Egyptian writer also called ʿAbdallāh ibn Aḥmad al-Maqdisī (on whom see Brockelmann, *GAL*, II, p. 470, and SII, p. 486, and *Cairo ENL Survey*, no. D45).

[2] These are analyzed in **II-6.2**.

[3] Ibn Qudāma, *al-Mughnī*, 12 vols., Cairo, 1341-48 H [= 1922-30], I, p. 270.

[4] On Ibn al-Shāṭir (**II-9.3**) see my article in the *DSB* and the bibliography there cited (to which add Kennedy & Ghanem, eds., *Ibn al-Shāṭir*), and also King, "Astronomy of the Mamluks", pp. 538-539 and 547-548.

tables, which themselves are quite sophisticated and based on accurate formulae (**II-9.3**), Ibn al-Shāṭir mentions some approximate methods for timekeeping, including (fol. 111v) the standard approximate formula for n = 7.[5]

8.3 Anonymous

MS Princeton Yahuda 1168 (= Mach 4872), copied *ca*. 1800 (?)), contains a short text on the shadows at midday for the Syrian months, accompanied by a table: see **Fig. 8.3**. The text states that shadows in feet (*qadam*s, base 7) can be measured by one's feet lengthwise, and shadows in digits (*iṣba*ʿs, base 12) can be measured by one's feet widthwise (it is not completely clear what is intended here).

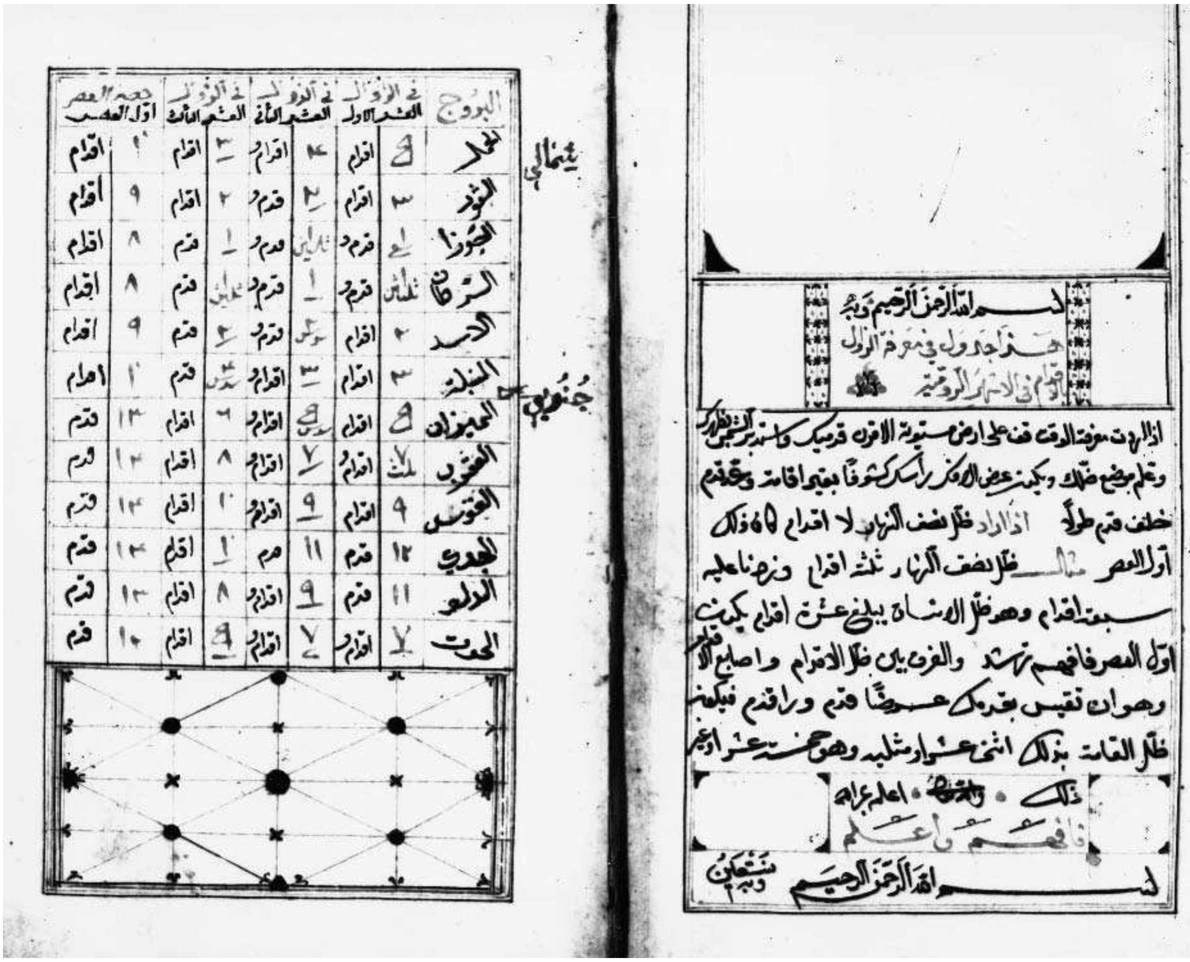


Fig. 8.3: The entire text of a short anonymous Syrian treatise on timekeeping by shadow-lengths. [From MS Princeton Yahuda 1168, courtesy of the Special Collections, Princeton University Library.]

[5] **IV**, Arabic text 2.9.

For each month in the Syrian calendar values are given in *qadam*s for the shadows at midday and the *ʿaṣr*. Those values of the midday shadow serve the first, middle and last ten days of each month.

Table 8.3

Months Days	1-10	10-20 Midday	20-30	*ʿaṣr*	[*ʿaṣr* - 7]
I	5	4	$3\frac{1}{2}$	10	3
II	3	$2\frac{1}{2}$	2	9	2
III	$1\frac{3}{4}$	$1\frac{2}{3}$	$1\frac{1}{2}$	8	1
IV	$1\frac{2}{3}$	$1\frac{1}{2}$	$1\frac{2}{3}$	8	1
V	2	$2\frac{1}{6}$	$2\frac{1}{2}$	9	2
VI	3	$3\frac{1}{2}$	$4\frac{1}{6}$	10	3
VII	5	$5\frac{1}{6}$	6	13	6
VIII	$7\frac{1}{3}$	$7\frac{1}{2}$	8	14	7
IX	9	$9\frac{1}{2}$	10	14	7
X	12	11	$10\frac{1}{2}$	14	7
XI	11	$9\frac{1}{2}$	8	13	6
XII	$7\frac{1}{2}$	$7\frac{1}{4}$	$5\frac{1}{2}$	12	5

These midday shadows range between $1\frac{1}{2}$ and 12, which correspond quite well to the latitude of Northern Syria ($\phi \approx 36°$). The values for the *ʿaṣr* are unrelated to these midday values. If we subtract 7 from each value we obtain a series of values for midday ranging from 1 to 7; these correspond to no terrestrial latitude.

CHAPTER 9

EGYPTIAN SOURCES

9.1 Ibn al-Ḥammāmī

A treatise on timekeeping entitled *Kitāb al-Fuṣūl al-muʿlama fī mawāqīt al-ṣalāt wa-'l-qibla* by an unidentified Egyptian astronomer named Ibn al-Ḥammāmī (**II-2.4**) is extant in the unique MS Istanbul Haci Mahmoud Ef. 5713,1 (fols. 1r-10v, copied *ca*. 1100). The author presents a series of shadow lengths for $\phi = 30°$ for each pair of zodiacal signs between the solstices as follows (fol. 3r):

♋	♌/♊	♍/♉	♎/♈	♏/♓	♐/♒	♑
1;21	2	4	7	10	14	16;16

The values 1;21 and 16;16 are accurately computed for $\phi = 30°$ and $n = 12$ (with $\varepsilon = 23;35°$). The others are, as Ibn al-Ḥammāmī says, rounded (*majbūr)*. Our author also prescribes the standard approximate formula with $n = 12$.

9.2 Najm al-Dīn al-Miṣrī

The early-14th-century Egyptian astronomer Najm al-Dīn al-Miṣrī compiled a treatise on aspects of timekeeping (**II-2.5**), which is extant in the unique copy MS Istanbul Hamidiye 1453 (fols. 219r-228v, copied 869 H [= 1464/65]). In this he gives a similar set of values for Cairo, with limiting values 1;20 and 16;15. He also presents the standard Indian formulae for timekeeping with bases $6^{2}/_{3}$ and 12.

Najm al-Dīn was not limited to such simple procedures: his major work was a universal table for timekeeping by day and night serving all latitudes and containing close to half a million entries (**I-2.6.1** and **9-3a**).

9.3 Anonymous (*Kitāb al-Durar wa-'l-yawāqīt*)

In a 13th-century Egyptian treatise on simple time-reckoning entitled *al-Durar wa-'l-yawāqīt* (**II-6.1**), extant in the unique copy MS Oxford Bodley 133,2 (fols. 94v-130r, copied 734 H [= 1333/34]), the author, who has not yet been properly identified, gives a set of shadow lengths for Egypt as follows:

1;21	2	4	7	10	13	16 .

See **9.1** above.

9.4 al-Dīrīnī

The early-13th-century Egyptian mystic al-Dīrīnī wrote a treatise on folk astronomy, partly in verse, which is extant in several manuscripts (**II-2.6**), including MS Cairo DM 651,5 (fols. 41r-58r, copied 828 H [= 1424/25]). al-Dīrīnī presents the following shadow-scheme in feet (*aqdām)* for the zodiacal signs:

♋	♌/♊	♍/♉	♎/♈	♏/♓	♐/♒	♑
$^2/_3$	1	2	$3^1/_2$	$5\ ^1/_4 + {}^1/_8$	$7\ ^1/_4$	$8\ ^1/_8$

These values correspond roughly to latitude 30° with base 6. al-Dīrīnī also presents the daily shadow increases for each sign in minutes (*daqāʾiq)* thus:

$1^1/_3$	4	6	$7^1/_2$	$7^1/_2$	$3^1/_2$

but these values correspond to base 12!

The same set of values together with daily rates of increase occurs on the inside cover of MS Cairo MM 168, which contains some miscellaneous tables from the 13th-century Egyptian *Muṣṭalaḥ Zīj* and some anonymous material on folk astronomy, copied *ca*. 1400-1500 probably in Egypt but possibly in the Yemen.[1]

9.5 Sirāj al-Dīn

MS Cairo ṬM 127 (59 fols., copied 1255 H [= 1839/40]) is a unique copy of a treatise entitled *Maṣābīḥ al-anwār wa-mafātīḥ al-asrār fī aʿmāl al-layl wa-'l-nahār* on folk astronomy and simple timekeeping. The work is attributed to an individual named simply Sirāj al-Dīn wa-'l-Dunyā.[2] It is indirectly related to a treatise on the same subject preserved in MS Princeton Yahuda 4657 (Mach 4983, fols. 40v-80r, copied *ca*. 1350), which – according to a spurious note at the beginning of the treatise – purports to be an abridgement of the *Maṣābīḥ al-anwār* by Sirāj.

The Cairo manuscript contains (fols. 50r-50v) a section on reckoning the time of day by shadows in which the schemes:

9	7	5	3	2	1	1	2	4	6	8	10

and

40	20	10	6	3	2

are advocated for the midday shadows for each month of the Syrian year and shadow increases for the seasonal hours, respectively. I suspect that this treatise is of Egyptian provenance, perhaps from the early 13th century; the same is the case for the treatise in the Princeton manuscript.

Of particular interest is a remark by the author of the treatise in the Cairo manuscript that the author of a book entitled *Kitāb Ḥall al-mushkilāt fī maʿrifat al-daqāʾiq wa-'l-daraj wa-'l-sāʿāt* – this title is new to the modern literature – devised a sundial called *al-basīṭa al-*

---

[1] *Cairo ENL Survey*, no. C12.

[2] The author – on whom see *Cairo ENL Survey*, no. C4 – is the celebrated Ḥanafī legal scholar Sirāj al-Dīn al-Sajāwandī (*fl. ca.* 1200 – see Brockelmann, *GAL*, I, pp. 470-471, and SI, pp. 650-651, and the article "al-Sadjāwandī" by Rudolf Sellheim in EI_2. See further **IV-3.3**.

sabʿīniyya because it bore seventy (*sabʿīn*) lines for regulating time of day according to these two rules. I do not understand the text of this passage, which merits further investigation. (See **10.3** on a sundial based on another simple shadow-scheme.) Sirāj al-Dīn points out that the author of this treatise, whom he calls *al-raʾīs al-fāḍil*,[3] used n = 7 and mentions that the longest shadow represented on the sundial was at one hour before sunset in the 4th Coptic month of Kayhak, namely, 10 + 60 = 85 (??). This suggests that the author was using a scheme for the hours other than the

40 20 10 6 3 2

scheme advocated by Sirāj al-Dīn. There is some confusion here, not least because $\Delta z = 60$ at $T = 1$ corresponds to a gnomon length of $n = 12$.

9.6 al-Ṣiqillī

MS Dublin CB 4538 (*ca*. 30 fols., copied *ca*. 1300) is a unique copy of a work on timekeeping by the lunar mansions. The title page is barely legible. I read *Anwāʾ ... ibn Hārūn* (? written *Hrwn*) *al-Ṣiqillī*, that is, "(Book on) the periods of the year arranged according to the lunar mansions (*nawʾ*) by ... ibn Hārūn al-Ṣiqillī". The name of this author is new to the modern literature. Although he or his family hailed from Sicily, his work was clearly compiled for use in Egypt. Perhaps he is identical with one Abu ʾl-Qāsim ibn ʿAbdallāh ibn ʿAbd al-Raḥmān ibn Ḥasan al-Qurashī al-Ṣiqillī, recently identified by Julio Samsó.[4] This man's date is uncertain, but he is clearly early enough to be a candidate. He is known to have made serious twilight measurements in Cairo and Alexandria and to have authored a book entitled *al-Intiqāʾ min al-yawāqīt fī ʿilm al-yawāqīt*, otherwise unknown to us but obviously dealing with more than timekeeping by night with the lunar mansions.

Each pair of pages is devoted to one lunar mansion: see **Fig. 9.6** for an example. Tables display the configuration of the lunar mansions with respect to the horizon and meridian at each of thirteen times during the night. Dates corresponding to each of the days of the *nawʾ* are given in the Coptic, Syrian and Western (Andalusī) calendars. Solar meridian altitudes are also given for each day. This information is accompanied by a diagram of the stars constituting the mansion and various additional items of information, such as the shadow lengths for the *nawʾ* in question. Values are given in digits (*iṣbaʿ*) and sixtieth divisions (*daqīqa*) thereof.

From the extremal values of the solar altitude, which are given to the nearest degree, namely, 83° and 36°, I conclude that these are truncated from values based on

$$\phi = 30;0° \text{ (Cairo)} \quad \text{and} \quad \varepsilon = 23;35° ,$$

although they might also be computed with $\varepsilon = 24°$. The values of the shadows are as follows:

[3] This is a common title given to the early-11th-century scholar Ibn Sīnā, on whom see the article by A. M. Goichon in EI_2. However, it is somewhat unlikely that Ibn Sīnā would have used the Coptic calendar.

[4] Samsó, "Astronomical Observations in the Maghrib", p. 174.

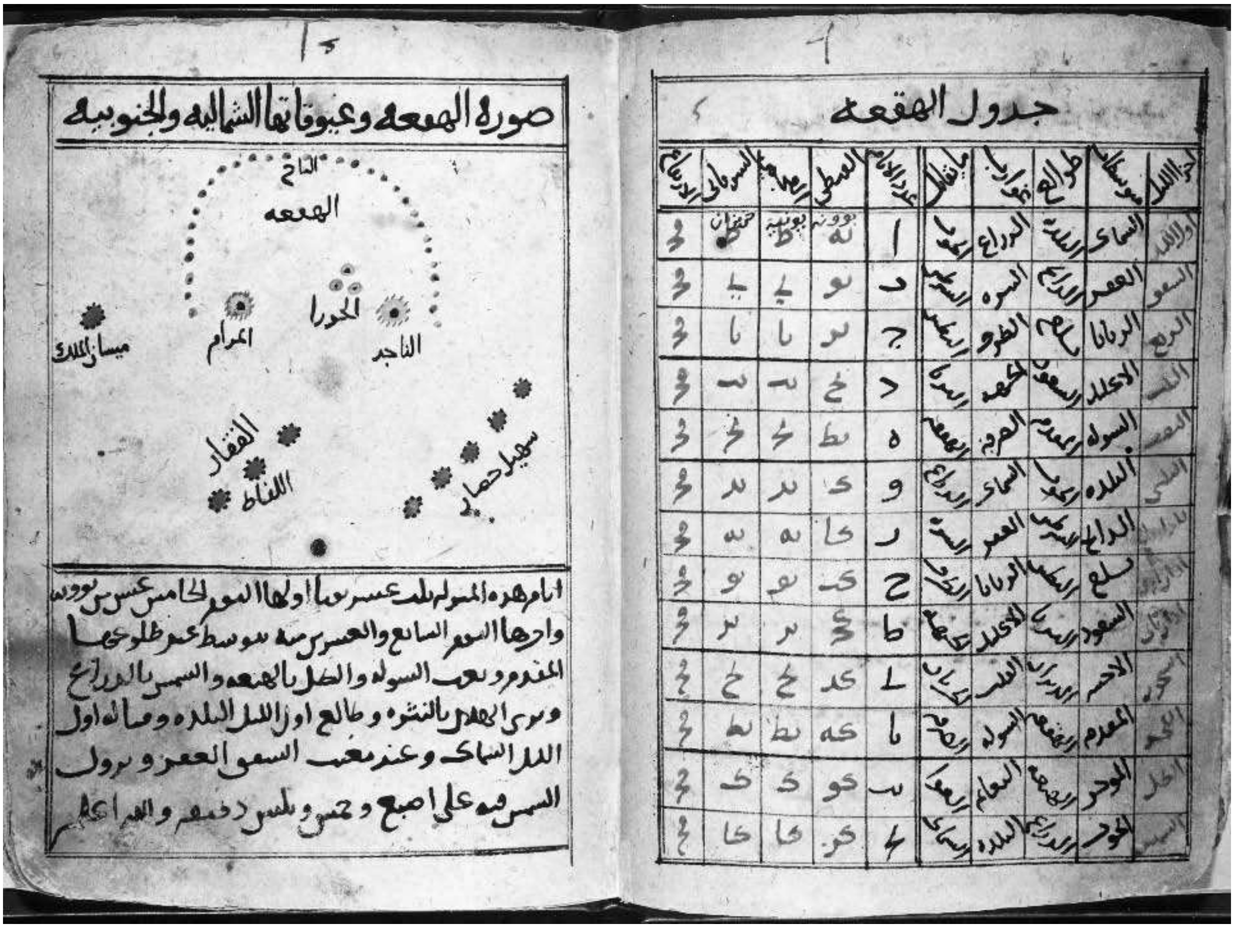


Fig. 9.6: An extract, one double page out of 28, from al-Ṣiqillī's tables for timekeeping by the lunar mansions with additional information about midday shadow lengths throughout the year. [From MS Dublin CB 4538, courtesy of the Chester Beatty Library.]

Table 9.6

M	Z	M	Z	M	Z	M	Z
1	3;?5	8	2;13	15	10;7	22	14;36
2	3;1	9	3;1	16	12;12	23	12;53
3	2;17	10	3;53	17	*deest*	24	11;4
4	1;15	11	5; 5	18	*deest*	25	9;20
5	1;35	12	6;20	19	16;50	26	7;43
6	1;33	13	7;45	20	16;49	27	6;16
7	1;49	14	9;23	21	16;1	28	5; 2

They were apparently computed (rather than observed), but extremal values 1;15 and 16;50 do not correspond properly to those for the above parameters, namely, 1;21 and 16;16. The corresponding values for $\varepsilon = 24°$ are 1;16 and 16;31. Thus al-Ṣiqillī's values are not fully explained.

9.7a Miscellaneous

Neugebauer has already drawn attention to a midday shadow-scheme for the months occurring in an Arabic text written for the use of the Copts and ascribed to the 12th patriarch of Alexandria, Demetrius (d. A.D. 230) but copied only in 1768.[5] The shadows for the twelve months beginning with Ṭūba (V) are:

9 7 5 3 2 1 1 2 4 6 8 10

As Neugebauer has pointed out, there are two arithmetical progressions underlying this primitive scheme. For latitude $\phi = 30°$ and $\varepsilon = 24°$ and base n = 7 we should expect maximum and minimum shadows of:

0;44 and 9;38 .

Neugebauer labelled the scheme as "badly distorted", but its simplistic nature did not detract from its popularity in medieval Egypt, as the following new sources show. See also **6.7** on a related scheme in various Maghribi sources and **7.2** on a Syrian version.

First I should mention that I have not found any such scheme in the one Coptic astronomical handbook from the medieval period which I have inspected, namely the work of the 13th-century Copt al-Asʿad ibn al-ʿAssāl,[6] extant in MS Cairo DM 910 (86 fols., copied *ca*. 1805). This work, which is of considerable interest and remains to be studied, contains solar and lunar tables and tables for calendar conversion and determining the feasts. However, it does not contain any material relating to timekeeping.

The same scheme for the Coptic months starting with Ṭūba that was noted by Neugebauer, now usually formulated as a mnemonic:

طزه جبا أبد وحي

is attested in the following sources:

(1) a poem on the midday shadows for the Coptic months attributed to the late-15th-/early-16th-century Egyptian polymath Jalāl al-Dīn al-Suyūṭī,[7] extant in MS Cairo MJ 39,3 (fol. 58r, copied 1069 H [= 1658/59]);
(2) a poem attributed to one Aḥmad al-Jālī in MS Cairo MM 171,6 (fols. 59v-60r, copied 1145 H [= 1732/33]);
(3) a treatise on timekeeping by the 17th-century Egyptian savant Shihāb al-Dīn al-Qalyūbī[8] in MS Cairo DM 203 (fol. 3r, copied 1094 [= 1683]);
(4) at the end of an anonymous almanac in MS Cairo MM 33,1 (fol. 5v, copied ca. 1600) – see **Fig. 9.7a**;
(5) amongst some anonymous notes in MS Cairo MM 181,1 (fol. 4r, copied *ca*. 1620); and

---

[5] G. Sobhy (Bey), "The Coptic Calendrical Computation and the System of Epacts known as "The Epact Computation" Ascribed to Abba Demetrius the XIIth Patriarch of Alexandria", *Bulletin de la Société d'Archéologie Copte* 8 (1942), pp. 169-199, and 9 (1943), pp. 237-252, p. 187 (French), p. 250 (Arabic), cited in Neugebauer, *HAMA*, II, p. 743.

[6] On Ibn al-ʿAssāl see the article by A. Atiya in EI_2, and *Cairo ENL Survey*, no. C10.

[7] On al-Suyūṭī see Suter, *MAA*, no. 449; Brockelmann, *GAL*, II, pp. 180-204, and SII, pp. 178-198; *Cairo ENL Survey*, no. D103; the article by E. Geoffroy in EI_2, and also n. **IV**-8:26.

[8] Brockelmann, *GAL*, II, pp. 478-479, and SII, pp. 492-493, and *Cairo ENL Survey*, no. D30.

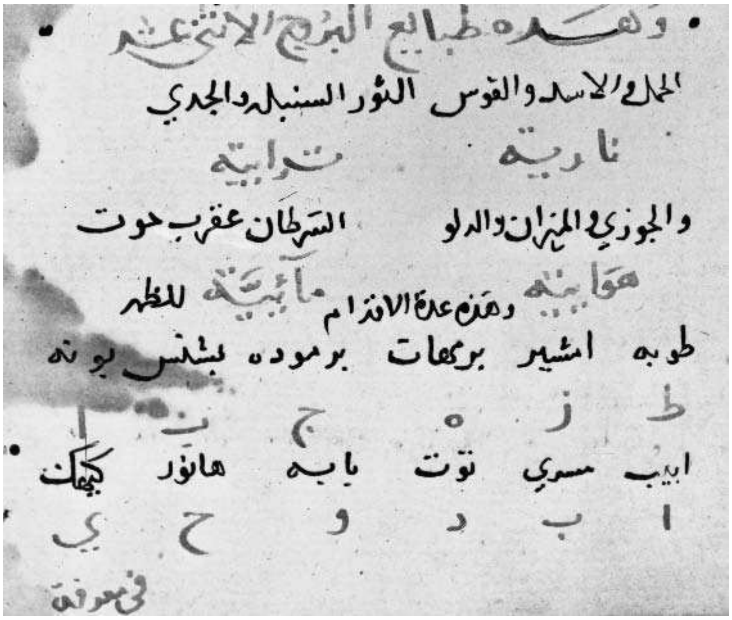


Fig. 9.7a: Midday shadow lengths for the Coptic months in a late Egyptian source. [From MS Cairo MM 33,1, courtesy of the Egyptian National Library.]

(6) a legal treatise by al-Bājūrī, who was *Shaykh al-Azhar* in Cairo *ca*. 1850. This source was noted by Eilhard Wiedemann, but the mnemonic as interpreted by him is garbled.[9]

A variation of this scheme is given in MS Cairo DJ 323,9 (fol. 39r, copied *ca*. 1900) in a poem attributed to one Abū Bakr al-Mīqātī, on whom I have no further information. The midday shadows for the Coptic months beginning with Tūt (I) are stated to be:

3 5 8 10 9 8 5 3 2 1 1 2 ,

these numbers being given in *abjad* notation thus:

جه حي طح هج با اب

In MSS Cairo DM 119,4 (fol. 9v, copied 1109 H [= 1697/98]) – see **Fig. 9.7b** – and 1112 (fol. 10v, copied 1310 H [= 1892/93]), both manuscripts of Egyptian provenance, there are notes (*fāʾida*) on the shadows increases for each hour to be added to the midday shadow, as follows:

T	1	2	3	4	5
Δz	30	12	6	3	1

For n = 6 the Indian formula yields the values:

30 12 6 3 $1^1/_5$,

so the rule is explained. The use of the base 6 is uncommon in post-Abbasid Islamic astronomy. The scheme

40 20 10 6 3 2

is presented in an anonymous Egyptian almanac containing mainly material relating to folk astronomy, preserved in MS Cairo DM 187 (recto of penultimate folio, copied *ca*. 1850).

9 Wiedemann, *Aufsätze*, II, p. 781. On al-Bājūrī see the article "al-Bād̲j̲ūrī" by Th. W. Juynboll in EI_2.

Fig. 9.7b: In these notes appended to a copy of a treatise on a special type of astrolabic quadrant by the late-15th-century Cairo astronomer Sibṭ al-Māridīnī, we find a reference to the sundial of the Caliph ʿUmar ibn ʿAbd al-ʿAzīz (**IV-7.1**), a poem on the prayer-times attributed to the Imām al-Shāfiʿī, elsewhere attributed to his descendant, the Fatimid astronomer Ibn Yūnus (**II-2.1**), and a simple shadow-scheme for timereckoning. [From MS Cairo DM 119,2-4, fols. 8v-9v, courtesy of the Egyptian National Library.]

Another mnemonic is attested in various Egyptian sources for the shadow lengths at the seasonal hours. This is:

مكي وجب

which represents the sequence

40	20	10	6	3	2

for the hours 1 to 6. In notes found in MSS Cairo MM 169,2 (fol. 39v, copied *ca*. 1750) and Cairo DM 830,2 (fol. 18r, copied 1257 H [= 1841/42]), the scheme is incorrectly represented as follows:

1	2	3	4	5	6	7	8	9	10	11	12
40	20	10	6	3	2	2	3	6	10	20	40

That this nonsense was actually intended is confirmed by the accompanying Arabic text in the second source; here the mnemonic:

مكي وجب بجو يكم

occurs. The sequence:

20	10	6	3

is reminiscent of the shadow increases:

20	10	6	3	1

for the seasonal hours in Greek shadow tables, in which the minimum midday shadow is also 2.[10] Notice also that, whereas in the Greek tables the hourly increases are:

1 2 3 4 10 (= 1 + 2 + 3 + 4) ,

in this scheme they are:

1 3 4 10 20 (= 1 + 2 + 3 + 4 + 10) .

9.7b Miscellaneous (time-reckoning by fingers)

The following procedure for reckoning time of day with the hand, using the thumb as gnomon, is outlined in MS Cairo DJ 323,7 (fol. 34r, copied *ca*. 1900). The text occurs amidst various astronomical treatise of Egyptian provenance – see **Fig. 9.7c**.[11]

> Translation: "A note on finding the hours with the shadow of the thumb (*ibhām*). You raise the thumb straight up with the shadow (falling) on the index finger (*sabbāba*). As long as the shadow has not reached the tip (*unmula*) of the index finger you are in the first hour. If the shadow falls on the tip of the index finger then you are in the second hour. If it has reached the first joint (*ʿuqda*) you are in the third hour, and if it has reached the second joint you are in the fourth hour. If it has reached the third mark (*ḥazza*) you are in the fifth hour, and if it has reached the fourth mark you are in the sixth hour. Then (use) the opposite (procedure) until the end of the day."

On an adult hand this corresponds approximately to a shadow-scheme

T	1	2	3	4	5	
z	5	4	3	2	1	(n = $2^1/_2$ inches)

I know of no other attestation of such a procedure in the Islamic sources. al-Bīrūnī, it is true,

[10] Neugebauer, *HAMA*, II, p. 738.
[11] *Cairo ENL Survey*, no. D228.

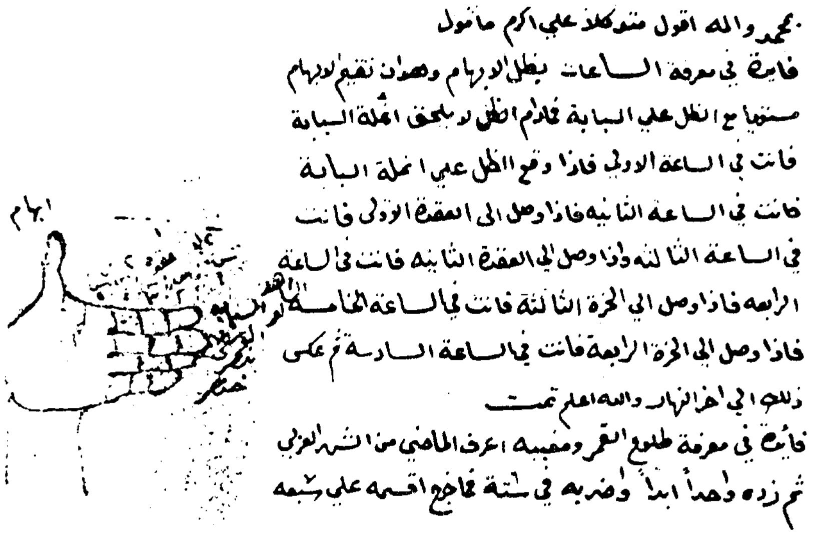


Fig. 9.7c: A scheme for reckoning the hours with the thumb and fingers. [From MS Cairo DJ 323,7, courtesy of the Egyptian National Library.]

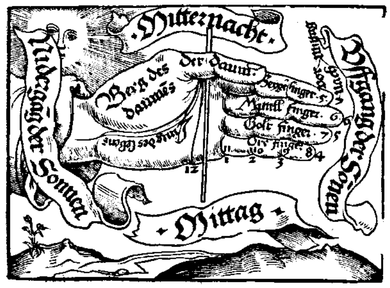


Fig. 9.7d: The scheme for timekeeping using the hand in a treatise by Jacob Koebel entitled *Eyn künstliche sonn Uhr inn eynes jeden menschen Lincken handt*, printed in Mainz in 1532. [From Gockerell, "Telling Time without a Clock", p. 133.]

mentions in the *Shadows* that he had observed the Indians using their middle finger extended vertically as a gnomon to cast shadows along their forearms held horizontally, measuring the shadows in finger lengths with the other hand.[12] But, as he says regretfully, he neglected to ask them the details. A method similar, but not identical, to that described in the Cairo manuscript is attested in a 16th-century German text (see **Fig. 9.7d**),[13] and no doubt such procedures were more widespread both in the Near East and in Europe than we could ever know from surviving textual evidence.

9.7c Miscellaneous (lost)

In the course of preparing this study I came across the following shadow-schemes for the monthly midday shadows in my notes on Ibn al-Bannāʾ's almanac, the first marked "Egypt". I have no recollection in which Cairo manuscript I originally found these schemes, which are as follows:

9	7	5	3	2	1	1	4	8	8	10	10
7	6	4	3	2	1	1	3	3	4	8	8

[12] al-Bīrūnī, *Shadows*, pp. 33-34, and Kennedy, *al-Bīrūnī's* Shadows, I, pp. 73-74, and II, p. 28.
[13] Gockerell, "Telling Time without a Clock", p. 133.

CHAPTER 10

MISCELLANEOUS

10.1 Sundry medieval European schemes

Neugebauer has described various wretched European shadow-schemes, concluding: "As usual the western tradition represents the lowest level in our material."[1] I include a brief summary:

(a) Two Latin tables:

midday shadows: 7 6 5 4 3 2 , hour differences: 1 3 7(!) 10 20 ;

(b) Three Spanish Mozarab (Latin) texts (11th century):

8 7 5 4 3 2 / 1 3 6 10 20 ;

(c) Palladius (4th-5th century):

9 7 5 4 3 2 / 1 3 6 10 20 ;

(d) Various missals (10th and 11th centuries):

11 9 7 5 3 1 with $\Delta z = 6$ for $T = 3^{sdh}$;

(e) Pseudo-Bede (date ?):

11 9 7 5 3 1 / 2 4 6 8 18 ;

(f) The German monk Wandalbert (9th century):

13 11 9 7 5 3 / 2 4 6 8 11 .

Did monks actually use these schemes to regulate their prayers at (the ends of) the third, sixth and ninth seasonal hours? For their sake, one could hope that they did not. In any case, not one of these schemes bears any trace of Islamic influence.

It would be amiss not to mention here the possibility of the transmission of some of the other material surveyed in this study to Europe. Only the scheme of the *Calendar of Cordova* is known to have been rendered into Latin, and the shadow tables compiled by Nicholas of Lynn in Oxford in the 14th century (strictly in the mathematical tradition) are a development of tables of this kind (**I-10.1**). The early European tables of solar altitudes or shadows at the hours specifically for marking cylindrical sundials also have their counterparts in yet earlier Islamic sources (**XI-4.4**). At least now that the ancient and medieval Islamic material is identified and classified we stand a better chance of understanding the medieval European material.

10.2 A late Ottoman navigational manual

MS Cairo DM 570 (182 fols., *ca.* 1875?) is a unique copy of a collection of texts, tables and diagrams relating to navigation. The work was apparently compiled *ca*. 1860, and it awaits

---

[1] Neugebauer, *HAMA*, II, pp. 745-746. For the table of Pseudo-Bede see also Ginzel, *Chronologie*, III, p. 88. On yet simpler forms of reckoning the hours in medieval Europe see Gockerell, "Telling Time without a Clock" (n. 9:13).

detailed study.[2] Of concern to the present investigation is a table (fol. 17v) of shadow lengths attributed to *al-shaykh* ʿAbdallāh ibn ʿUmar Bā Makhrama, a Yemeni scholar who flourished *ca*. 1500: see further **4.5**.

10.3 A sundial from Java

A horizontal sundial constructed in 1722 in the town of Gresik in Java (see **Fig. 10.3**) was published by the Batavian savant Tjondro Negoro in 1882.[3] The instrument measures *ca*. 60 × 60 cms., and its surface is covered with texts in Javanese, which were reproduced and rendered into Dutch by Negoro. It has since been discussed by various scholars of ethnography and ethnoastronomy interested mainly in the associated calendar, which is defined by means of midday shadows.[4]

The sundial, called a *bencet*, consists of a pair of perpendicular scales shaped like a "T". The shorter scale measures shadows at midday. It bears markings for each unit between 2 units north and 4 units south: the underlying gnomon length must be 7, as we see from the longer perpendicular scale. This measures shadows at the ʿ*aṣr* (Javanese: *asar*) based on the standard definition $\Delta z = n = 7$, and it is marked for each unit from 1 up to 11, although it is only the markings from 7 to 11 which are relevant to the function of the scale. In order to use the scale for the ʿ*aṣr*, the sundial should ideally be rotated away from the meridian so that the afternoon shadow can be compared with the markings on the scale, but we are not dealing with a precision instrument.

The year is divided in twelve periods called *mangsa*s, beginning with the summer solstice. These are defined as the intervals during which the shadow changes by 1 unit. The *mangsa*s vary in length between 23 and 43 days, an unusual feature for any calendar, and this has disturbed modern investigators. The inspiration for the calendar is clearly the fact that (for this latitude and gnomon length) the midsummer shadow is 4 units south and the midwinter one 2 units north, the total change in shadow length thus being 6 units. The midday shadows at the beginning of the *mangsa*s (m_i) are thus:

m_1	m_2/m_{12}	m_3/m_{11}	m_4/m_{10}	m_5/m_9	m_6/m_8	m_7
4 S	3 S	2 S	1 S	0	1 N	2 N

The limits 4 S and 2 N correspond reasonably well to the latitude of Gresik, namely *ca*. -7° (actually 4.12 S and 2.07 N for $n = 7$). The Dutch savant van der Stok, unaware of standard

[2] *Cairo ENL Survey*, no. Z41.

[3] See R. M. A. A. Tjondro Negoro, "De koperen zonnewijzer van Gresik", *Tijdschrift van het Bataviaasch Genootschap van Kunsten en Wetenschappen* (Batavia/Jakarta) 27 (1882), pp. 47-68; and also F. van den Bosch, "Der javanische Mangsa-Kalendar", *Bijdragen tot de Taal, Land en Volkenkunde* 126 (1980), pp. 248-282 and figs. 13-17, espeically pp. 251-253.

[4] See, for example, N. Daldjoeni, "Pranatamangsa, the Javanese Agricultural Calendar", *The Environmentalist* 4 (1984), supp. no. 7, pp. 15-17; *idem* and B. Hidayat, "Astronomical Aspects of "Pranotomongso" of the 19th-Century Central Java", in *New Delhi 1985 Colloquium Proceedings*, pp. 249-252; G. Ammarell, "Sky Calendars of the Indo-Malay Archipelago", *ibid*., pp. 241-248; as well as A. Maas, "Sternkunde und Sterndeuterei im malaiischen Archipel", *Tijdschrift voor Taal, Land en Volkenkunde* (Koninklijke Bataviaasch Genootschap van Kunsten en Wetenschappen, Batavia / Jakarta), 64 (1924).

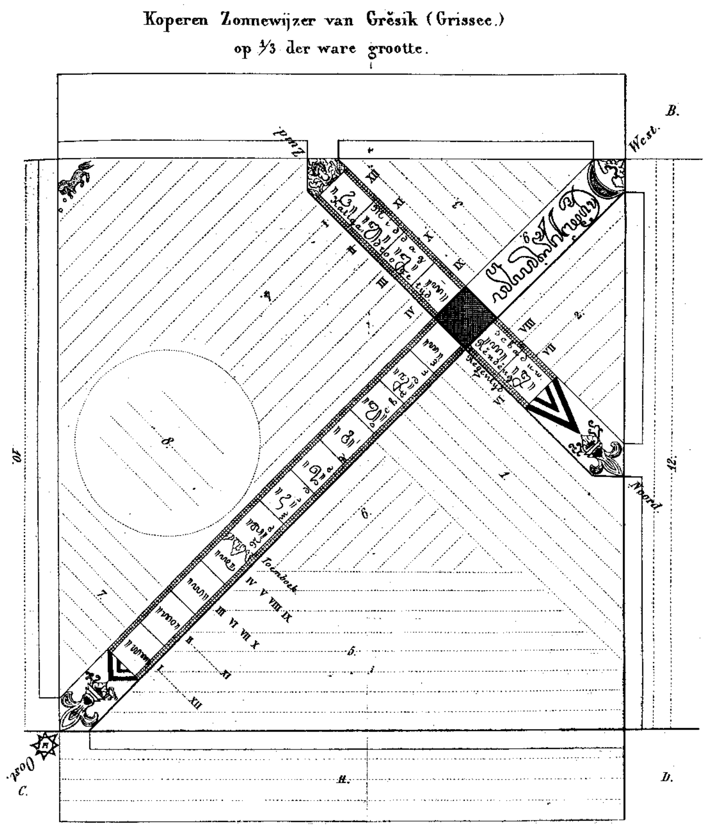


Fig. 10.3: The Gresik sundial. [From Negoro, "De koperen zonnewijzer van Gresik" (1882).]

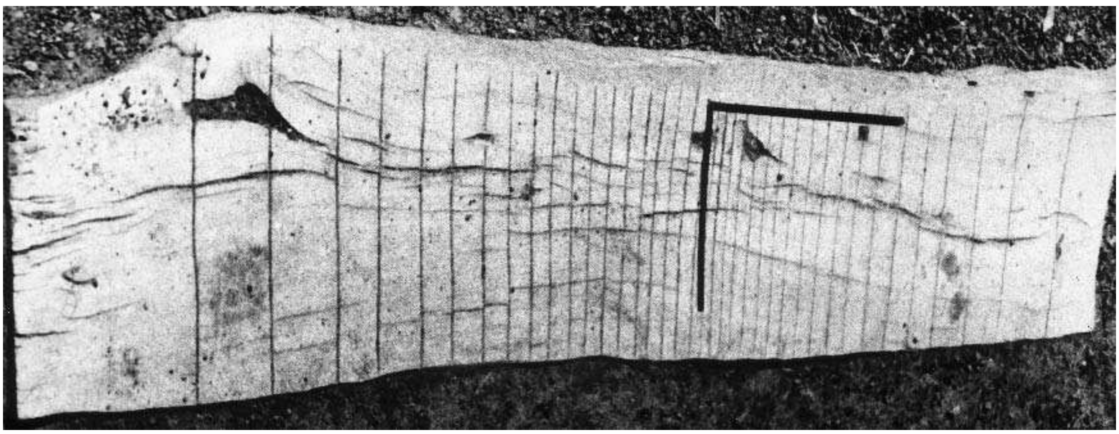


Fig. 10.4: A sundial for regulating water distribution from Muscat and Oman. I have no information on its precise location or the way in which it was used. The divisions are neither equal nor uniform, which may have made some *fallāḥīn* rave wildly if, as a result of this, they were deprived of their fair share of water. [From the archives of Alain Brieux, Paris.]

medieval Islamic practice, determined the most appropriate gnomon length as 7.11 units (but then the limiting shadows are 4.19 S and 2.1 N).[5]

The development of a calendar out of a shadow-scheme is not, as far as I know, attested in other societies. See, however, **6.4** for a first step in this direction.

10.4 20th-century Arab shadow-schemes for water regulation

From both Northern and Southern Arabia, as well as Egypt, we have accounts of the regulation of water for irrigation by means of folk astronomical techniques.[6] I do not doubt that similar practices were, and indeed in some cases still are, in vogue elsewhere in the rural areas of the Islamic world.

In the Oasis of al-ʿUlāʾ in North-Western Arabia, for example, the length of daylight is divided into 24 *asdās* (plural of *suds*, lit. sixth), and the passage of the *asdās* is regulated by observing the shadow of a 2-metre high wall pass over sets of stone markers for each *suds*.[7] There are three sets, two for the solstices and one for the equinoxes. The positions of the stones for the solstices, as recorded recently, approximate rather well to the hyperbolae of a horizontal sundial.

In the Buraimi Oasis of Northern Oman, the length of daylight is likewise divided into 24 *asdās*, but their passage is regulated by observing changes in the length of a man's shadow.[8] This is said to shorten by 40 *qadam*s during the first *suds* (that is, after the shadow becomes clearly identifiable), 20 in the second, 5 in the third, 3 in each of the fourth and fifth, 2 in the sixth and seventh, and 1 in each of the remaining *asdās* until midday. (Note that the clear shadow is thus 79, or around 80 *qadam*s long.)

In 1917 W. J. H. King published an account of a most curious procedure for regulating the distribution of water in the Dakhla Oasis to the west of the Nile.[9] The water supply for an entire day from sunrise to sunset is divided into 31 *qadam*s. Two men are involved in reckoning the passage of the *qadam*s. The meridian is marked and one man stands on it with his back to the sun and "bottom joint of his forefinger against the tip of his nose". The second man measures with his feet the distance from the shadow of the finger to the meridian. The *qadam*s are then regulated according to the information recorded in the following table.

---

[5] J. P. van der Stok, "Opmerkingen omtrent bovengemelden zonnewijzer", *Tijdschrift van het Bataviaasch Genootschap van Kunsten en Wetenschappen* (Batavia / Jakarta) 27 (1882), pp. 62-68, esp. p. 66.

[6] On irrigation in the Islamic world see the articles "Māʾ" in EI_2. A new study of links between irrigation and simple timekeeping is Thomas F. Glick and Simonne Teixeira, "Azaira, alhetma: Two medieval Arabisms Reflecting the Allocation of Irrigation Water", *Suhayl* 3 (2002-03), pp. 213-219. The duration of the *ẓuhr* in n. 3 on p. 214 is incorrectly cited.

[7] See A. A. Nasif, "Qanats at al-ʿUlāʾ", *Proceedings of the Seminar for Arabian Studies* 10 (1980), pp. 75-80, esp. pp. 76-77, and *idem*, *A Historical and Archaeological Survey of al-ʿUlāʾ with Special Reference to its Irrigation System*, 2 vols., unpublished Ph.D. thesis, Victoria University of Manchester, 1981, I, pp. 246-252, and II, pl. CCLIII.

[8] J. C. Wilkinson, *Water and Tribal Settlement in South-East Arabia*: *A Study of the* Aflāj *of Oman*, Oxford: Clarendon Press, 1977, pp. 106-110.

[9] See W. J. Harding King, "Irrigation in the Dakhla Oasis", *Geographical Journal* 50 (1917), pp. 358-364, for details.

Table 10.4

Morning (shadow west)	Perpendicular in *qadam*s	Afternoon (shadow east)
1	$\infty \leftarrow$ 26-26 $\rightarrow \infty$	31
2	26-22	30
3	22-18	29
4	18-14	28
5	14-12	27
6	12-10	26
7	10-9	25
8	9-8	24
9	8-7	23
10	7-6	22
11	6-5	21
12	5-4	20
13	4-3	19
14	3-2	18
15	2-1	17
16	1W-1E	16

As King noted, the 16th *qadam* provides a rather generous allowance, but he obtained his information from an old *fellāḥ* who "raved wildly on the beauties of the system in such a vile patois that even his village headman had some difficulty in following". Even in the 1910s the headman possessed a watch, and the old methods are most probably no longer used nowadays. King also described some methods of regulating water distribution by night using the moon. **Fig. 10.4** shows a sundial related to this kind of procedure from Muscat and Oman.

E. Stack, in his account of a journey through Persia in the late 19th century, presented a detailed account of contemporary irrigation practices,[10] and then – alas, without further information – added:

> "In some villages of Faraidan I found the water-distribution by day regulated by the length of a man's shadow, measured in feet; and at night by the stars. Elsewhere I believe a dial is used by day."

10.5 A late-20th-century almanac from Qaṭar

The almanac entitled *al-Taqwīm al-Qaṭarī* which is published annually in the Arabian Gulf state of Qaṭar (latitude *ca*. 25°) contains a set of prayer tables and agricultural information for each zodiacal sign and lunar mansion rising at dawn. I have examined a copy published in the 1980s. Occasional midday shadow lengths in *qadam*s are given and they were clearly lifted from some earlier work in a rather haphazard fashion. The following values are given for various signs (S) and mansions (M):

M_{10}: $7^1/_2$	M_2: $^1/_2$	S_{11}(♒): 6
M_{10}: 3	S_{12}(♓): 5	M_{12}: $5^1/_2$
M_{26}: $2^1/_2$	M_{14}: 6	M_{28}: 1

This information is, for all practical purposes, quite useless.

[10] E. Stack, *Six Months in Persia*, 2 vols., London, 1882, II, p. 269 (cited in EI_2, V, p. 870a).

10.6 A shadow-table from 18th-century Sind

For the sake of future researchers I include a reference to a shadow scheme for the solar months in Sindi verse that was in circulation in Sind in the 18th century.[11] It came to the attention of the jurist Makhdūm Muḥammad Hāshim of Tatta (d. 1760), who pointed out that the shadows had been calculated for some region other than Sind. He conducted his own observations in 1719-20 on a weekly / bi-weekly basis to produce to a table of the midday shadows in feet (*per*) for each two weeks of the solar year. Values are said to be valid for Lower Sind and are given to the nearest half a foot.

[11] See Baloch, "Measurement of Space and Time in Lower Sind", pp. 185-186, where the table is reproduced.

Index of Monthly Shadow-schemes

Scheme (beginning with longest shadows)												Section
13	11	9	7	5	3	3	5	7	9	11	13	10.1
13	10	8	6	4	3	2	3	4	6	8	10	1
11	9	7	5	3	1	1	3	5	7	9	11	10.1
11	7	6	6	4	3	2	4	4 / $2\tfrac{1}{3}$	5	7	$9\tfrac{1}{5}$	6.3
11	7	6	5	4	3	3	3	4	5	7	$9\tfrac{1}{5}$	6.3
11	7	6	4	3	2	1	1	4	5	7	$9\tfrac{1}{5}$	6.3
$10\tfrac{1}{2}$	9	$7\tfrac{1}{2}$	6	$4\tfrac{1}{2}$	$3\tfrac{1}{2}$	3	4	5	6	7	8	7.1, 8.1
$10\tfrac{1}{6}$	9	$6\tfrac{1}{2}$	$4\tfrac{1}{2}$	3	$1\tfrac{5}{6}$	$1\tfrac{1}{3}$	$1\tfrac{5}{6}$	3	$4\tfrac{1}{2}$	$6\tfrac{1}{2}$	9	8.1
10	10	9	7	5	3	2	1	1	4	8	8	9.7c
10	10	8	5	3	2	1	1	2	4	6	8	6.2, 6.4
10	9	8	5	3	2	1	1	2	3	5	8	9.7a
10	9	7	7	4	2	1	$1\tfrac{1}{3}$	$2\tfrac{1}{3}$	4	5	8	6.5
10	9	7	5	3	2	1	1	2	4	5	8	6.4, 6.5, 6.6
10	9	7	5	3	2	1	1	2	4	6	8	7.2, 9.5, 9.7a
10	9	7	5	3	2	1	1	2	4	?	8	6.2
10	9	7	5	3	2	1	1	2	3	5	8	3.5
9	7	5	4	3	2	2	3	4	5	7	9	10.1
8	8	7	6	4	3	2	1	1	3	3	4	9.7c
8	7	6	5	4	3	2	3	4	5	6	7	1, 5.2, 7.1
8	7	5	4	3	2	1	2	3	4	5	7	1
8	7	5	4	3	2	2	3	4	5	7	8	10.1
7	6	5	4	3	2	2	3	4	5	6	7	10.1
6	5	4	3	2	1	0	1	2	3	4	5	4.9, 7.2

Index of Hour Schemes

Type	Values						Section
T	6	5/7	4/8	3/9	2/10	1/11	
z	0	3	7	15	17	28	4.2
Δz	0	1+	$3\tfrac{1}{4}$	$6\tfrac{1}{2}$	$19\tfrac{1}{2}$	39	3.2
Δz / z (?)	0	3	6	7	18	22	5.3
″	3	6	7	8	18	28	“
z (?)	0	3	6	9	18	28	3.5, 6.1
z	0	3	6	9	18	38	4.9
″	0	3	5	9	15	35	“
Δz	0	1	3	6	10	20	1, 7.2, 10.1
Δz	0	1	3	6	12	30	9.7a
Δz	0	2	3	6	10	20	5.2
Δz / z (?)	0	3	5	7	13	26	4.9
″	0	3	5	7	13	24	“
z (?)	2	3	6	10	20	40	9.5, 9.7a
Δz	0	$\tfrac{1}{5}$n	$\tfrac{1}{2}$n	n	2n	5n	2.1, 4.1
Δz	0	$2\tfrac{2}{5}$	6	12	24	60	2.4, 6.4
z (?)	$\tfrac{2}{3}$	$3\tfrac{1}{4}$	$7\tfrac{1}{2}$	15	30	60	4.4
″	1	$3\tfrac{1}{4}$	$7\tfrac{1}{2}$	19	30	60	“
Δz	0	1	2	3	4	5	9.7b
Δz	?	?	?	?	?	60	9.5
Δz		1	3	7	10	20	10.1
″		2	4	6	8	18	“
″		2	4	6	8	11	“

See also the scheme for *muhurta*s in **2.4**.

Index of Solstitial and Equinoctial Shadow Lengths

	Scheme		Section
WS	EQ	SS	
$8^{7}/_{8}$	$3^{1}/_{2}$	$^{1}/_{2}$	2.2
$(1^{1}/_{48})$n	-	$^{1}/_{18}$n (S)	3.2
ca. 7	-	0;25 (S)	3.3
7	-	$^{1}/_{10}$	3.4
$5^{1}/_{5}$	-	$1 + ^{1}/_{4} + ^{1}/_{3} \cdot ^{1}/_{5}$ (S)	4.2
5	-	1 (S)	4.2
$5 - ^{1}/_{8}$	$1^{1}/_{3}$	$1 + ^{1}/_{12} +$ (S)	4.1
$5^{1}/_{4}$	$1^{1}/_{2}$	$1^{1}/_{10}$ (S)	4.3
$5^{1}/_{8}$	-	$1 - ^{1}/_{30}$ (S)	4.5
$5^{1}/_{2}$	-	0	4.6
$5 + ^{1}/_{4} + ^{1}/_{6}$	-	$2^{1}/_{6}$ (S)	4.6, 4,7
10	-	4 (S)	4.7
5	2 or $1^{4}/_{12}$ (!)	$^{4}/_{12}$ (N!)	4.10
$1^{5}/_{6}$	$^{3}/_{4}$	$^{1}/_{4}$	5.1
$5^{1}/_{2}$	-	$^{1}/_{2}$ (N!)	6.1
$7^{1}/_{2}$	3	$^{1}/_{2}$	7.1
$10^{1}/_{6}$	-	$1^{1}/_{3}$	8.1
12	-	$1^{1}/_{2}$	8.3
16;16	7	1;21	9.1
16;15	7	1;20	9.2
16	7	1;21	9.3
$8^{1}/_{8}$	$3^{1}/_{2}$	$^{2}/_{3}$	9.4
16;50	-	1;15	9.6
4 (S)	-	2 (N)	10.3
20;7	-	$2^{2}/_{5}$	6.4

See also the schemes for Aden and Taiz cited in **4.5**.

Manuscripts consulted

N.b.: An asterisk denotes that I have not been able to consult the manuscript or obtain a copy thereof. On the manuscripts of the almanac of Ibn al-Bannāʾ which I have not consulted, see n. 6:6.

Aleppo Awqāf 968	4.9
Baghdad Awqāf 2982/6276*	4.1
Berlin Staatsbibliothek Ahlwardt:	
5664	3.2
5731	4.2
Cairo Egyptian National Library:	
DM = Dār al-Kutub *mīqāt*	
119	9.7
130	3.3
187	9.7
203	9.7
415	6.2, 6.5
570	4.5, 10.2
651	9.4
830	9.7
899	4.7
910	9.7
948	2.2, 4.1, 4.8
1112	9.7
1169	6.7
1170	8.2
DJ = Dār al-Kutub *majāmīʿ*	
323	9.7
709	4.6
K = astronomy	
3769	4.9
ḤJ = Ḥalīm *majāmīʿ*	
47	6.1
MJ = Muṣṭafā Fāḍil *majāmīʿ*	
39	9.7
180	7.1
202	5.3
MM = Muṣṭafā Fāḍil *mīqāt*	
33	9.7
168	9.4
169	9.7
171	9.7
181	9.7
ṬJ = Ṭalʿat *majāmīʿ*	
201	6.2
ṬM = Ṭalʿat *mīqāt*	
127	9.5
Damascus Ẓāhiriyya	
5588	3.4, 3.5
10732	8.1
Dublin Chester Beatty 4538	9.6
Escorial ár. 941	5.2
Istanbul (Süleymaniye):	
Haci Mahmoud Ef. 5713	9.1
Hamidiye 1453	9.2
Lalelı 3748	6.5
Leiden Universiteitsbibliotheek 1111	8.2
Milan Ambrosiana Griffini 37	4.2
Oxford Bodleian Library:	
Bodley 133	9.3
Hunt. 233	4.3
Marsh 134	4.9
“ 294	6.6
Pococke 249	6.4
Paris BNF	
ar. 2273	6.6
ar. 2540	7.2
Princeton University Library:	
Garrett 1016	4.6
Yahuda 1168	8.3
“ 4657	9.5
Sanaa, private collection	4.4
Vatican	
ar. 962	4.7
ar. 1499	7.2